

$\tan \beta$ in the MSSM
Definitions, Gauge Invariance, Scheme Dependence
Applications

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based on arXiv:0710.1821, 0807.4668 and 0906.1665

The Higgs Potential, in the MSSM at tree-level

$$\begin{aligned}
 V &= m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_{12}^2 (H_1 \wedge H_2 + h.c.) \\
 &+ \frac{1}{8} (g^2 + g'^2) (|H_1|^2 - |H_2|^2)^2 + \frac{g^2}{2} |H_1^\dagger H_2|^2 \\
 &\text{with } H_1 \wedge H_2 = H_1^a H_2^b \epsilon_{ab} \quad (\epsilon_{12} = -\epsilon_{21} = 1, \epsilon_{ii} = 0).
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Opposite hypercharges , in principle distinguishable

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 \tan \beta = \frac{v_2}{v_1} \text{ ? } \implies \text{NOT INVARIANT?}$$

The tree-level Higgs potential

$$V = V_{const} + V_{linear} + V_{mass} + V_{cubic} + V_{quartic},$$

$$V_{linear} = T_{\phi_1^0} \phi_1^0 + T_{\phi_2^0} \phi_2^0,$$

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$$\det(N_{GP}) = 0 \quad \text{Tr}(N_{GP}) = v_1^2 + v_2^2$$

Tadpoles and invariants

- The requirement that v_1 and v_2 correspond to the true vacua is a requirement on the vanishing of the tadpoles. $T_{\phi_{1,2}^0} = 0$ can be seen as a trade off for m_1^2 and m_2^2 .
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$$c_{2\beta}^2, \text{ from } \underline{\text{book-keeping device}} \quad c_\beta = \frac{v_1}{v}, \quad s_\beta = \frac{v_2}{v}$$

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$c_{2\beta}^2$, from book-keeping device $c_\beta = \frac{v_1}{v}$, $s_\beta = \frac{v_2}{v}$

Usually one takes M_{A^0} , $M_{Z^0}(v^2)$, $t_\beta(c_{2\beta}^2)$ as input parameters, and derive M_{H^0} and M_{h^0}
but **what is $\tan \beta$?**

Basis and Rotations

The mass eigenstates in the Higgs sector are given, through rotation, by

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At the quantum level mixing between fields will be re-introduced, (like in the SM $Z - \gamma$ mixing,..) and one has to re-diagonalise again,

not exactly the same and equivalent as to how $\tan \beta$ will be renormalised, defined

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- scheme dependence
 - in particular this means that the corresponding counterterm (choice of input/definition) even if gauge invariant and leads to finite results has to be a good one: the (finite) corrections should not be excessively large because of a bad choice of input
 - (perturbation should be maintained or trusted).

$\tan \beta$ ubiquitous in the MSSM

Higgs Potential

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Higgs Potential

Higgs masses

Couplings of Higgses to fermions

Higgs Potential

Higgs masses

Couplings of Higgses to fermions

D terms fermion masses,...,
chargino and neutralino properties (mixing)

How to track gauge invariance

Practical, gauge parameter independence through a generalised gauge-fixing

slightly a bit more formal is Freitas-Stockinger hep-ph/0205281

Non-linear gauge implementation

$$\begin{aligned}\mathcal{L}_{GF} = & -\frac{1}{\xi_W} \left| \partial \cdot W^+ + \xi_W \frac{g}{2} v G^+ \right|^2 \\ & -\frac{1}{2\xi_Z} \left(\partial \cdot Z + \xi_Z \frac{g}{2c_W} v + G^0 \right)^2 - \frac{1}{2\xi_\gamma} (\partial \cdot A)^2\end{aligned}$$

This only affects the propagators. Usually calculations done with $\xi = 1$, otherwise large expressions, higher rank tensors, unphysical thresholds,...

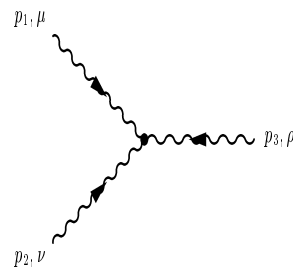
$$\frac{1}{k^2 - M_W^2} \left(g_{\mu\nu} - (1 - \xi_W) \frac{k_\mu k_\nu}{k^2 - \xi_W M_W^2} \right)$$

Non-linear gauge implementation

$$\mathcal{L}_{GF} = -\frac{1}{\xi_W} |(\partial_\mu - ie\tilde{\alpha}A_\mu - igc_W\tilde{\beta}Z_\mu)W^\mu|^2 + \xi_W \frac{g}{2} (v + \tilde{\delta}h + \tilde{\omega}H + i\tilde{\rho}A^0 + i\tilde{\kappa}G^0)G^+|^2$$

$$- \frac{1}{2\xi_Z} (\partial \cdot Z + \xi_Z \frac{g}{2c_W} (v + \tilde{\epsilon}h + \tilde{\gamma}H)G^0)^2 - \frac{1}{2\xi_\gamma} (\partial \cdot A)^2$$

- quite a handful of gauge parameters, but with $\xi_i = 1$, no “unphysical threshold”, no higher rank tensors, gauge parameter dependence in gauge/Goldstone/ghosts vertices.
- more important: no need for higher (than the minimal set) for higher rank tensors and tedious algebraic manipulations



$p_1 (\mu)$	$p_2 (\nu)$	$p_3 (\rho)$	
W^-	W^+	A	$e \left[g^{\mu\nu} (p_1 - p_2)^\rho + (1 + \tilde{\alpha}/\xi_W) (p_3^\nu g^{\mu\rho} - p_3^\mu g^{\nu\rho}) + (1 - \tilde{\alpha}/\xi_W) (p_2^\mu g^{\nu\rho} - p_1^\nu g^{\mu\rho}) \right]$
W^-	W^+	Z	$e \frac{c_W}{s_W} \left[g^{\mu\nu} (p_1 - p_2)^\rho + (1 + \tilde{\beta}/\xi_W) (p_3^\nu g^{\mu\rho} - p_3^\mu g^{\nu\rho}) + (1 - \tilde{\beta}/\xi_W) (p_2^\mu g^{\nu\rho} - p_1^\nu g^{\mu\rho}) \right]$

- we take the gauge fixing to be renormalised (not necessary to have *all* Green's functions finite.)

renormalisation: Parameters and counterterms

- From $X_L = (m_1, m_2, m_{12}, g, g', v_1, v_2)$ we take e, M_W, M_Z (as in SM) and $M_{A^0}, T_{\phi_1^0}, T_{\phi_2^0}$; On Shell scheme, GI with “ t_β ” to be defined.

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- shifts in parameters $X_L \rightarrow X_L + \delta X_L$ implies
 $e \rightarrow e + \delta e, T_{\phi_1^0} \rightarrow T_{\phi_1^0} + \delta T_{\phi_1^0}, \dots v_i \rightarrow v_i - \delta v_i (t_\beta \rightarrow t_\beta + \delta t_\beta)$

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- From $X_L = (m_1, m_2, m_{12}, g, g', v_1, v_2)$ we take e, M_W, M_Z (as in SM) and $M_{A^0}, T_{\phi_1^0}, T_{\phi_2^0}$; with “ t_β ” to be defined.
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- In any case field renormalisation (before or after rotation) still needed
this will imply

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix}_0 = \overbrace{U(\beta) Z_{\varphi^0} U(-\beta)}^{Z_P} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} Z_{G^0 G^0}^{1/2} & Z_{G^0 A^0}^{1/2} \\ Z_{A^0 G^0}^{1/2} & Z_{A^0 A^0}^{1/2} \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} .$$

Example of two-point functions

$$\begin{cases}
 \hat{\Sigma}_{G^0 G^0}(q^2) = \Sigma_{G^0 G^0}(q^2) + \delta M_{G^0}^2 - q^2 \delta Z_{G^0} \\
 \hat{\Sigma}_{G^0 A^0}(q^2) = \Sigma_{G^0 A^0}(q^2) + \delta M_{G^0 A^0}^2 - \frac{1}{2} q^2 \delta Z_{G^0 A^0} - \frac{1}{2} (q^2 - M_{A^0}^2) \delta Z_{A^0 G^0} \\
 \hat{\Sigma}_{A^0 A^0}(q^2) = \Sigma_{A^0 A^0}(q^2) + \delta M_{A^0}^2 - (q^2 - M_{A^0}^2) \delta Z_{A^0} \\
 \\
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 \hat{\Sigma}_{G^\pm H^\pm}(q^2) = \Sigma_{G^\pm H^\pm}(q^2) + \delta M_{G^\pm H^\pm}^2 - \frac{1}{2} q^2 \delta Z_{G^\pm H^\pm} - \frac{1}{2} (q^2 - M_{H^\pm}^2) \delta Z_{H^\pm G^\pm} \\
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 \hat{\Sigma}_{h^0 h^0}(q^2) = \Sigma_{h^0 h^0}(q^2) + \delta M_{h^0}^2 - (q^2 - M_{h^0}^2) \delta Z_{h^0}
 \end{cases}$$

Renormalisation Conditions, On-Shell in...Nut Shell

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What about $A^0 Z^0$ and $A^0 G^0$ transitions?

Dabelstein-Chankowski-Pokorski-Rosiek Scheme (DCPR)

$$\frac{\delta t_\beta}{t_\beta}^{\text{DCPR}} = -\frac{1}{M_{Z^0} s_{2\beta}} \text{Re} \Sigma_{A^0 Z^0}(M_{A^0}^2).$$

This is not gauge invariant! based on $\hat{\Sigma}_{A^0 Z^0}(M_{A^0}^2) = 0$ which is widely used (together with $\hat{\Sigma}_{A^0 G^0}(M_{A^0}^2) = 0$) but which is not true in all gauges.
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Moreover in our approach $\delta Z_{G^0 A^0}$ and $\delta \tan \beta$ come together

$$\hat{\Sigma}_{A^0 Z^0}(q^2) = \Sigma_{A^0 Z^0}(q^2) + \frac{M_{Z^0}}{2} \left(\delta Z_{G^0 A^0} + s_{2\beta} \frac{\delta t_\beta}{t_\beta} \right)$$

BRST transformation on the (“ghost”) operator

$$\langle 0 | \bar{c}^Z(x) A^0(y) | 0 \rangle = 0, \longrightarrow$$

$$\begin{aligned} q^2 \hat{\Sigma}_{A^0 Z^0}(q^2) + M_{Z^0} \hat{\Sigma}_{A^0 G^0}(q^2) &= (q^2 - M_{Z^0}^2) \frac{1}{(4\pi)^2} \frac{e^2 M_{Z^0}}{s_{2W}^2} s_{2\beta} \mathcal{F}_{GA}^{\tilde{\epsilon}, \tilde{\gamma}}(q^2) \\ &+ \frac{M_{Z^0}}{2} (q^2 - M_{A^0}^2) \left(\frac{1}{(4\pi)^2} \frac{2e^2}{s_{2W}^2} \mathcal{F}_{cc}^{\tilde{\epsilon}, \tilde{\gamma}}(q^2) + s_{2\beta} \frac{\delta t_\beta}{t_\beta} - \delta Z_{A^0 G^0} \right). \end{aligned}$$

$\mathcal{F}_{GA}^{\tilde{\epsilon}, \tilde{\gamma}}(q^2)$ and $\mathcal{F}_{cc}^{\tilde{\epsilon}, \tilde{\gamma}}(q^2)$ are functions which vanish in the linear gauge with $\tilde{\epsilon} = \tilde{\gamma} = 0$.

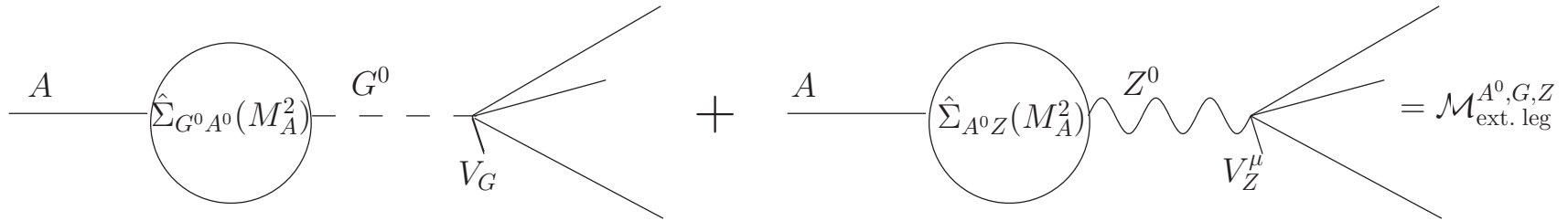
The constraint shows that even in the linear gauge $q^2 \hat{\Sigma}_{A^0 Z^0}(q^2) + M_{Z^0} \hat{\Sigma}_{A^0 G^0}(q^2)$ is zero only for $q^2 = M_{A^0}^2$ and not for *any* q^2 .

but in linear gauge can impose both $\hat{\Sigma}_{A^0 Z^0}(M_A^2) = \hat{\Sigma}_{A^0 G^0}(M_A^2) = 0$

no longer in a general gauge!

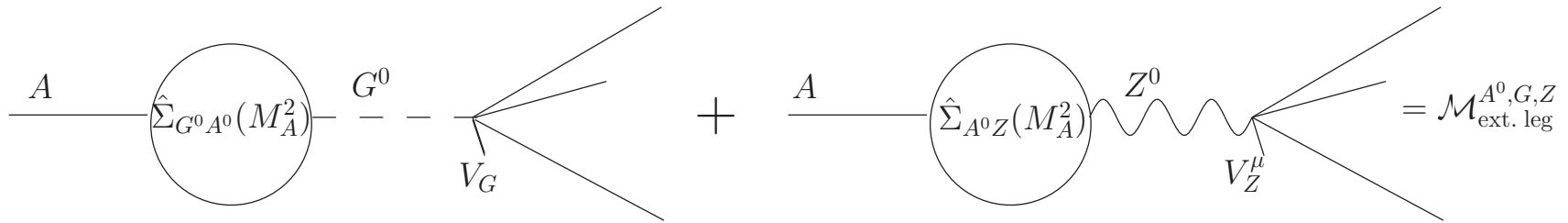
similar thing in the charged sector

we can still avoid one-loop corrections and counterterms in the external legs associated with an external pseudoscalar A^0 . Of concern to us are the transition $A^0 - Z^0$ and $A^0 - G^0$.



$$\begin{aligned}
 \mathcal{M}_{\text{ext. leg}}^{A^0, G, Z} &= \frac{\hat{\Sigma}_{A^0 G^0}(M_{A^0}^2)V_G + q.V_Z\hat{\Sigma}_{A^0 Z^0}(M_{A^0}^2)}{M_{A^0}^2 - M_{Z^0}^2} \\
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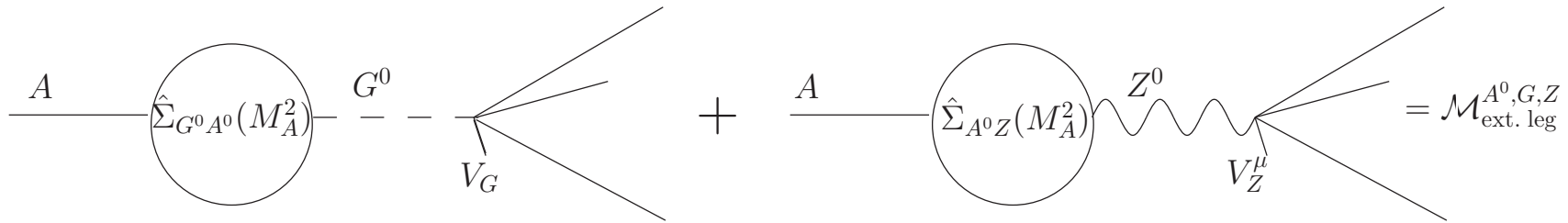


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impose

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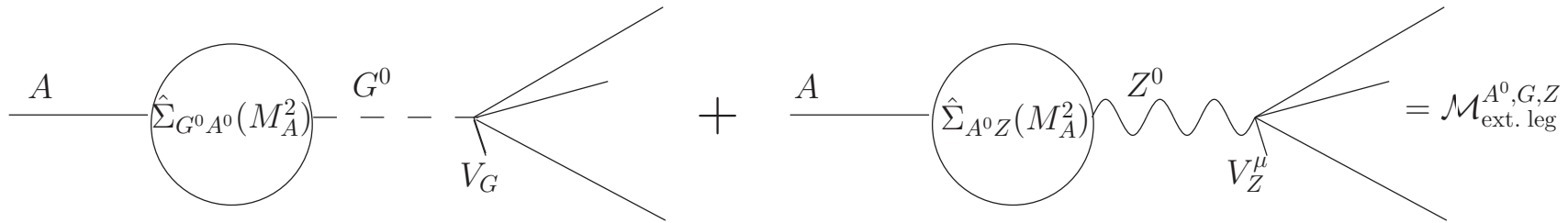


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To be consistent with the Ward identity

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$$\delta Z_{G^0 A^0} = -s_{2\beta} \frac{\delta t_\beta}{t_\beta} - 2 \frac{\Sigma_{A^0 Z^0}^{\text{tad}}(M_{A^0}^2)}{M_{Z^0}} + \frac{2}{(4\pi)^2} \frac{e^2}{s_{2W}^2} s_{2\beta} \mathcal{F}_{GA}^{\tilde{\epsilon}, \tilde{\gamma}}(M_{A^0}^2).$$

- $A_{\tau\tau}$ -scheme.

$$\mathcal{L}_{A_{\tau\tau}}^0 == i \frac{gm_\tau}{2M_{W^\pm}} \tan\beta \bar{\tau}\gamma_5\tau A^0$$

- t_β is extracted from the decay $A^0 \rightarrow \tau^+\tau^-$ to which the QED corrections have been subtracted, which in this neutral decay constitutes a gauge invariant subset. This leads to a gauge-independent counterterm and is physically unambiguous defined. Not exactly a definition from within the Higgs potential but nonetheless from Higgs physics/phenomenology.
- Criticism that it is not defined from 2–point functions is unfounded. Remember G_μ/M_W . Technically one has the tools
- sure it is flavour dependent But, one needs to measure this partial width with enough precision. !

Schemes for $\tan\beta$, 2

- *DCPR*-scheme .

$$\frac{\delta t_\beta}{t_\beta}{}^{DCPR} = -\frac{1}{M_Z s_{2\beta}} \text{Re} \Sigma_{A^0 Z^0}(M_{A^0}^2).$$

(in DCPR $H_i \rightarrow (1 + \frac{1}{2}\delta Z_{H_i})H_i$ $i = 1, 2$, then $v_i \rightarrow v_i \left(1 - \frac{\tilde{\delta}v_i}{v_i} + \frac{1}{2}\delta Z_{H_i}\right)$

impose $\frac{\tilde{\delta}v_1}{v_1} = \frac{\tilde{\delta}v_2}{v_2}$ such that in effect $\frac{\delta t_\beta}{t_\beta} = \frac{1}{2}(\delta Z_{H_2} - \delta Z_{H_1})$,

a physical quantity related to a wave function renormalisation constant is (almost) always dubious!)

Schemes for $\tan\beta$, 3

- MH -scheme.

$$\text{Re}\hat{\Sigma}_{H^0 H^0}(M_{H^0}^2) = 0$$

Here the heaviest CP-even Higgs mass M_{H^0} is taken as input. This definition is obviously gauge independent and process independent, but expect it to be **unstable**

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$$t_\beta = \sqrt{\frac{M_{A^0} M_{Z^0} + M_{H^0} \sqrt{M_{A^0}^2 + M_{Z^0}^2 - M_{H^0}^2}}{M_{A^0} M_{Z^0} - M_{H^0} \sqrt{M_{A^0}^2 + M_{Z^0}^2 - M_{H^0}^2}}}.$$

$$\frac{\delta t_\beta}{t_\beta} \simeq \frac{1}{M_{H^0}^2 / M_{A^0}^2 - 1} \left(-\frac{\delta M_{A^0}^2}{M_{A^0}^2} + \frac{\delta M_{H^0}^2}{M_{H^0}^2} \right).$$

→ 0 in the decoupling regime

Schemes for $\tan\beta$, 4

- \overline{DR} -scheme.

- In this scheme the counterterm for $\tan\beta$ is taken (from some **quantity** to be a pure divergence proportional to the ultraviolet (UV) factor, $C_{UV} = 1/\epsilon + \dots$, in dimensional reduction.
- In HHW prescription of Hollik, Heinemeyer and Weiglein (not GI in general) $\frac{\delta t_\beta}{t_\beta}^{\overline{DR}-\text{HHW}} = \frac{1}{2c_{2\alpha}} (Re\Sigma'_{h^0 h^0}(M_{h^0}^2) - Re\Sigma'_{H^0 H^0}(M_{H^0}^2))^\infty$.
- Pierce and Papadopoulos have defined δt_β by relating it to the *divergent* part of $M_{H^0}^2 - M_{h^0}^2$ (GI)

Examples, non gauge invariance

Parameter	Value	Parameter	Value	Constant	Value
s_W	0.48076	m_μ	0.1057	m_s	0.2
e	0.31345	m_τ	1.777	m_t	174.3
g_s	1.238	m_u	0.046	m_b	3
M_{Z^0}	91.1884	m_d	0.046	M_{A^0}	500
m_e	0.000511	m_c	1.42	t_β	3;50

$mhmax$	Value	$nomix$	Value	$large\ \mu$	Value
μ	-200	μ	-200	μ	1000
M_2	200	M_2	200	M_2	400
M_3	800	M_3	800	M_3	200
$M_{\tilde{F}_L}$	1000	$M_{\tilde{F}_L}$	1000	$M_{\tilde{F}_L}$	400
$M_{\tilde{f}_R}$	1000	$M_{\tilde{f}_R}$	1000	$M_{\tilde{f}_R}$	400
A_f	$2000+\mu/t_\beta$	A_f	μ/t_β	A_f	$-300+\mu/t_\beta$

Examples, finite and infinite part of $\tan \beta$

$$\delta t_\beta = \delta t_\beta^{\text{fin}} + \delta t_\beta^\infty C_{UV}$$

$$\text{nlg s} = 10 \rightarrow \tilde{\alpha} = 10, \tilde{\beta} = 10, \dots\dots\dots$$

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$$\delta t_\beta = \delta t_\beta^{\text{fin}} + \delta t_\beta^\infty C_{UV}$$

$$\text{nlg} = 10 \rightarrow \tilde{\alpha} = 10, \tilde{\beta} = 10, \dots\dots\dots$$

δt_β^∞	nlg = 0	nlg = 10
DCPR	-3.19×10^{-2}	-1.04×10^{-1}
OS_{M_H}	-3.19×10^{-2}	-3.19×10^{-2}
$\text{OS}_{A_{\tau\tau}}$	-3.19×10^{-2}	-3.19×10^{-2}
$\overline{\text{DR}}\text{-HHW}$	-3.19×10^{-2}	$+5.32 \times 10^{-2}$
$\overline{\text{DR}}\text{-PP}$	-3.19×10^{-2}	-3.19×10^{-2}

for the set *mhmax* at $t_\beta = 3$.

Examples, finite and infinite part of $\tan \beta$

$$\delta t_\beta = \delta t_\beta^{\text{fin}} + \delta t_\beta^\infty C_{UV}$$

$$\text{nlgs} = 10 \rightarrow \tilde{\alpha} = 10, \tilde{\beta} = 10, \dots\dots\dots$$

δt_β^∞	nlgs = 0	nlgs = 10	$\delta t_\beta^{\text{fin}}$	nlgs = 0	nlgs = 10
DCPR	-3.19×10^{-2}	-1.04×10^{-1}	DCPR	-0.10	-0.27
OS_{M_H}	-3.19×10^{-2}	-3.19×10^{-2}	OS_{M_H}	+0.92 (30%)	+0.92 (30%)
$\text{OS}_{A_{\tau\tau}}$	-3.19×10^{-2}	-3.19×10^{-2}	$\text{OS}_{A_{\tau\tau}}$	-0.10 (3%)	-0.10 (3%)
$\overline{\text{DR}}\text{-HHW}$	-3.19×10^{-2}	$+5.32 \times 10^{-2}$	$\overline{\text{DR}}\text{-HHW}$	0	0
$\overline{\text{DR}}\text{-PP}$	-3.19×10^{-2}	-3.19×10^{-2}	$\overline{\text{DR}}\text{-PP}$	0	0

for the set *mhmax* at $t_\beta = 3$.

scheme dependence in the usual linear gauge (finite part) with $\xi_{W,Z,\gamma} = 1$

$t_\beta = 3$	<i>mhmax</i>	<i>large μ</i>	<i>nomix</i>	$t_\beta = 50$	<i>mhmax</i>	<i>large μ</i>	<i>nomix</i>
DCPR	-0.10	-0.06	-0.08	DCPR	+3.42	+14.57	+0.48
OS_{M_H}	+0.92	-1.31	+0.64	OS_{M_H}	-385.53	-2010.84	-290.18
$OS_{A_{\tau\tau}}$	-0.10	-0.06	-0.08	$OS_{A_{\tau\tau}}$	+0.12	-4.72	+0.16
\overline{DR}	0	0	0	\overline{DR}	0	0	0

$$\frac{\delta t_\beta^{DCPR}}{t_\beta} \simeq -\frac{t_\beta}{s_{2\beta}} \frac{g^2}{c_W^2 M_Z^2} \frac{1}{4\pi^2} (3m_b^2 B_0(M_{A^0}^2, m_b^2, m_b^2) + m_\tau^2 B_0(M_{A^0}^2, m_\tau^2, m_\tau^2)) .$$

$$\propto t_\beta^2$$

Examples, Mass of M_h

$t_\beta = 3$	<i>mhmax</i>	<i>large μ</i>	<i>nomix</i>
$M_{h^0}^{TL} = 72.51$			
DCPR	134.28	97.57	112.26
OS_{M_H}	140.25	86.68	117.37
$\text{OS}_{A_{\tau\tau}}$	134.25	97.59	112.27
$\overline{\text{DR}} \ \bar{\mu} = M_{A^0}$	134.87	98.10	112.86
$\overline{\text{DR}} \ \bar{\mu} = M_t$	134.47	97.55	112.38
$t_\beta = 50$	<i>mhmax</i>	<i>large μ</i>	<i>nomix</i>
$M_{h^0}^{TL} = 91.11$			
DCPR	144.50	35.88	124.80
OS_{M_H}	143.76	13.21	124.16
$\text{OS}_{A_{\tau\tau}}$	144.50	35.73	124.80
$\overline{\text{DR}} \ \bar{\mu} = M_{A^0}$	144.50	35.77	124.80
$\overline{\text{DR}} \ \bar{\mu} = M_t$	144.50	35.77	124.80

$A^0 \rightarrow \tau^+ \tau^-$, the non QED one-loop corrections

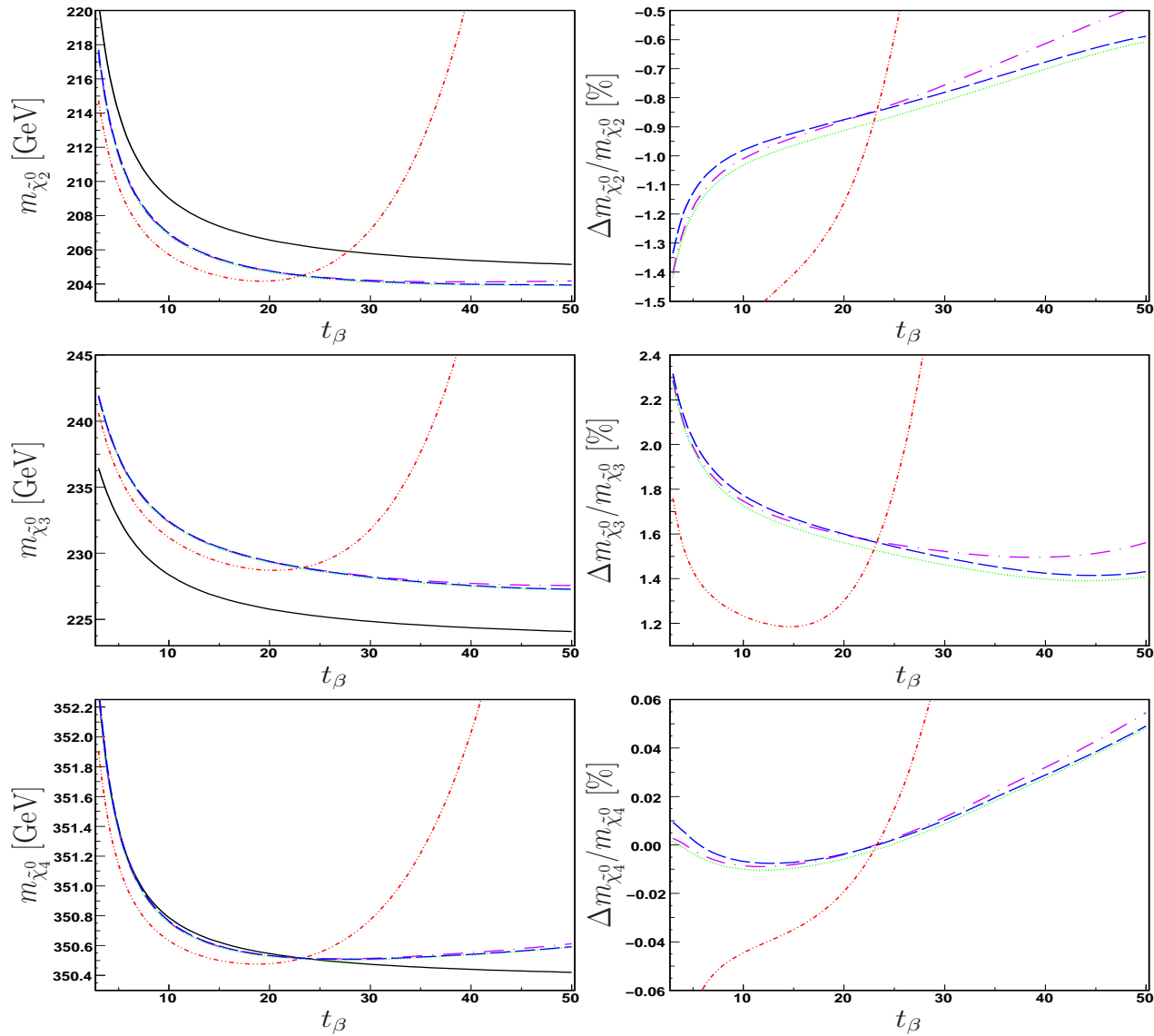
$t_\beta = 3$	<i>mhmax</i>	<i>large μ</i>	<i>nomix</i>
$\Gamma^{TL} = 9.40 \times 10^{-3}$			
DCPR	$+3.56 \times 10^{-5}$	-8.71×10^{-6}	-7.37×10^{-6}
OS_{M_H}	$+6.41 \times 10^{-3}$	-7.82×10^{-3}	$+4.56 \times 10^{-3}$
$OS_{A_{\tau\tau}}$	0	0	0
$\overline{DR} \, \bar{\mu} = M_{A^0}$	$+6.51 \times 10^{-4}$	$+3.94 \times 10^{-4}$	$+5.18 \times 10^{-4}$
$\overline{DR} \, \bar{\mu} = M_t$	$+2.30 \times 10^{-4}$	-2.66×10^{-5}	$+9.67 \times 10^{-5}$

$t_\beta = 50$	<i>mhmax</i>	<i>large μ</i>	<i>nomix</i>
$\Gamma^{TL} = 2.61 \times 10^0$			
DCPR	$+3.45 \times 10^{-1}$	$+2.01 \times 10^0$	$+3.35 \times 10^{-2}$
OS_{M_H}	-4.03×10^1	-2.09×10^2	-3.03×10^1
$OS_{A_{\tau\tau}}$	0	0	0
$\overline{DR} \, \bar{\mu} = M_{A^0}$	-1.21×10^{-2}	$+4.92 \times 10^{-1}$	-1.66×10^{-2}
$\overline{DR} \, \bar{\mu} = M_t$	-3.00×10^{-2}	$+4.75 \times 10^{-1}$	-3.44×10^{-2}

$H^0 \rightarrow Z^0 Z^0$ and $A^0 \rightarrow Z^0 h^0$ (suppressed at tree-level)

$t_\beta = 3$	$mhmax$	$large \mu$	$nomix$
$\Gamma^{TL} = 8.97 \times 10^{-3}$			
DCPR	$+1.59 \times 10^{-2}$	-6.32×10^{-3}	$+8.47 \times 10^{-3}$
OS_{M_H}	$+1.40 \times 10^{-2}$	-4.00×10^{-3}	$+7.12 \times 10^{-3}$
$OS_{A_{\tau\tau}}$	$+1.59 \times 10^{-2}$	-6.32×10^{-3}	$+8.47 \times 10^{-3}$
$\overline{DR} \bar{\mu} = M_{A^0}$	$+1.57 \times 10^{-2}$	-6.44×10^{-3}	$+8.32 \times 10^{-3}$
$\overline{DR} \bar{\mu} = M_t$	$+1.58 \times 10^{-2}$	-6.32×10^{-3}	$+8.44 \times 10^{-3}$
$t_\beta = 50$	$mhmax$	$large \mu$	$nomix$
$\Gamma^{TL} = 6.40 \times 10^{-5}$			
DCPR	$+2.18 \times 10^{-5}$	-5.14×10^{-4}	$+3.89 \times 10^{-5}$
OS_{M_H}	$+1.01 \times 10^{-2}$	$+4.66 \times 10^{-3}$	$+7.81 \times 10^{-4}$
$OS_{A_{\tau\tau}}$	$+3.02 \times 10^{-5}$	-4.65×10^{-4}	$+3.97 \times 10^{-5}$
$\overline{DR} \bar{\mu} = M_{A^0}$	$+3.05 \times 10^{-5}$	-4.77×10^{-4}	$+4.01 \times 10^{-5}$
$\overline{DR} \bar{\mu} = M_t$	$+3.09 \times 10^{-5}$	-4.76×10^{-4}	$+4.05 \times 10^{-5}$

Neutralino masses, ($M_A = 100\text{GeV}$)



Tree-level and at one-loop by using the $A_{\tau\tau}$ -scheme, the \overline{DR} scheme the $DCPR$ -scheme and the MH -scheme as a function of t_β .

Conclusions 1

- Many more examples, for other decays and cross sections (including relic density calc.), worked out and scheme dependence investigated

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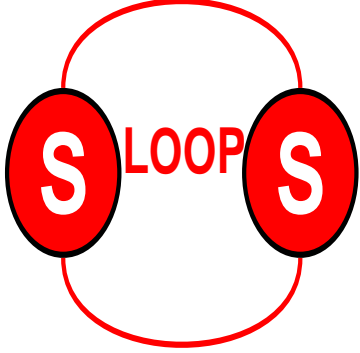
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- Scheme dependence of the MSSM needs to be further studied



N. Baro, FB, G. Chalons, S. Hao, Ninh Le Duc, A. Semenov, (D. Temes)

- Need for an automatic tool for susy calculations
- handles large numbers of diagrams both for tree-level
- and loop level
- able to compute loop diagrams at $v = 0$: dark matter, LSP, move at galactic velocities, $v = 10^{-3}$
- ability to check results: UV and IR finiteness but also gauge parameter independence for example
- ability to include different models easily and switch between different renormalisation schemes
- Used for SM one-loop multi-leg: new powerful loop libraries (with Ninh Le Duc)

Strategy: Exploiting and interfacing modules from different codes

Lagrangian of the model defined in LanHEP

- particle content
- interaction terms
- shifts in fields and parameters
- ghost terms constructed by BRST



Generic Model

-kinematical structures



Classes Model

-Feynman rules, including CT



Evaluation via FeynArts-FormCalc

LoopTools modified!!
tensor reduction inappropriate for small relative velocities
(Zero Gram determinants)



Renormalisation scheme

- definition of renorm. const. in the classes model
Non-Linear gauge-fixing constraints, gauge parameter dependence checks

```

vector
  A/A: (photon, gauge),
  Z/Z: ('Z boson', mass MZ = 91.1875, gauge),
  'W+'/'W-': ('W boson', mass MW = MZ*CW, gauge).
scalar  H/H: (Higgs, mass MH = 115).

transform A->A*(1+dZAA/2)+dZAZ*Z/2, Z->Z*(1+dZZZ/2)+dZZA*A/2,
  'W+'->'W+'*(1+dZW/2), 'W-'->'W-'*(1+dZW/2).
transform H->H*(1+dZH/2), 'Z.f'->'Z.f'*(1+dZZf/2),
  'W+.f'->'W+.f'*(1+dZWf/2), 'W-.f'->'W-.f'*(1+dZWf/2).

let pp = { -i*'W+.f', (vev(2*MW/EE*SW)+H+i*'Z.f')/Sqrt2 },
PP=anti(pp).

lterm -2*lambda*(pp*anti(pp)-v**2/2)**2
  where
  lambda=(EE*MH/MW/SW)**2/16, v=2*MW*SW/EE .

let Dpp^mu^a = (deriv^mu+i*g1/2*B0^mu)*pp^a +
  i*g/2*taupm^a^b^c*WW^mu^c*pp^b.
let DPP^mu^a = (deriv^mu-i*g1/2*B0^mu)*PP^a
  -i*g/2*taupm^a^b^c*{'W-'^mu,W3^mu,'W+'^mu}^c*PP^b.
lterm DPP*Dpp.

```

Gauge fixing and BRS transformation

```

let G_Z = deriv*Z+(MW/CW+EE/SW/CW/2*nle*H)*'Z.f'.

lterm -G_A**2/2 - G_Wp*G_Wm - G_Z**2/2.

lterm -'Z.C'*brst(G_Z).

```

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```

Output of Feynman Rules with Counterterms !!

```

M$CouplingMatrices = {

  (*-----      H   H   -----*)
  C[ S[3], S[3] ] == - I *
  {
    { 0 , dZH },
    { 0 , MH^2 dZH + dMHsq }
  },
  (*-----      W+.f  W-.f  -----*)
  C[ S[2], -S[2] ] == - I *
  {
    { 0 , dZWf },
    { 0 , 0 }
  },
  (*-----      A   Z   -----*)
  C[ V[1], V[2] ] == 1/2 I / CW^2 MW^2 *
  {
    { 0 , 0 },
    { 0 , dZZA },
    { 0 , 0 }
  },

  (*-----      H   H   H   -----*)
  C[ S[3], S[3], S[3] ] == -3/4 I EE / MW / SW *
  {
    { 2 MH^2 , 3 MH^2 dZH -2 MH^2 / SW dSW - MH^2 / MW^2 dMWsq
  },
  (*-----      H   W+.f  W-.f  -----*)
  C[ S[3], S[2], -S[2] ] == -1/4 I EE / MW / SW *
  {
    { 2 MH^2 , MH^2 dZH + 2 MH^2 dZWf -2 MH^2 / SW dSW - MH^2
  },

  (*-----      W-.C   A.c   W+   -----*)
  C[ -U[3], U[1], V[3] ] == - I EE *
  {
    { 1 },
    { - nla }
  },
}

```

```

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transform A->A*(1+dZAA/2)+dZAZ*Z/2, Z->Z*(1+dZZZ/2)+dZZA*A/2,
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let G_Z = deriv*Z+(MW/CW+EE/SW/CW/2*nle*H)*'Z.f'.

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```

```

RenConst[ dMHsq ] := ReTilde[SelfEnergy[prt["H"] -> prt["H"], MH]]
RenConst[ dZH ] := -ReTilde[DSelfEnergy[prt["H"] -> prt["H"], MH]]
RenConst[ dZZf ] := -ReTilde[DSelfEnergy[prt["Z.f"] -> prt["Z.f"],
MZ]] RenConst[ dZWf ] := -ReTilde[DSelfEnergy[prt["W+.f"] ->
prt["W+.f"], MW]]

```

```

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  C[ S[3], S[3] ] == - I *
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  },
  (*----- W+.f W-.f -----*)
  C[ S[2], -S[2] ] == - I *
  {
    { 0 , dZWf },
    { 0 , 0 }
  },
  (*----- A Z -----*)
  C[ V[1], V[2] ] == 1/2 I / CW^2 MW^2 *
  {
    { 0 , 0 },
    { 0 , dZZA },
    { 0 , 0 }
  },

  (*----- H H H -----*)
  C[ S[3], S[3], S[3] ] == -3/4 I EE / MW / SW *
  {
    { 2 MH^2 , 3 MH^2 dZH -2 MH^2 / SW dSW - MH^2 / MW^2 dMWsq }
  },
  (*----- H W+.f W-.f -----*)
  C[ S[3], S[2], -S[2] ] == -1/4 I EE / MW / SW *
  {
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  },

  (*----- W-.C A.c W+ -----*)
  C[ -U[3], U[1], V[3] ] == - I EE *
  {
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    { - nla }
  },
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```


TREE LEVEL CALCULATIONS

Comparison with public codes: Grace and CompHEP

Cross-section [pb]	SloopS	CompHEP	Grace
$h^0 h^0 \rightarrow h^0 h^0$	3.932×10^{-2}	3.932×10^{-2}	3.929×10^{-2}
$W^+ W^- \rightarrow \tilde{t}_1 \bar{\tilde{t}}_1$	7.082×10^{-1}	7.082×10^{-1}	7.083×10^{-1}
$e^+ e^- \rightarrow \tilde{\tau}_1 \bar{\tilde{\tau}}_2$	2.854×10^{-3}	2.854×10^{-3}	2.854×10^{-3}
$H^+ H^- \rightarrow W^+ W^-$	6.643×10^{-1}	6.643×10^{-1}	6.644×10^{-1}
Decay [GeV] # 200 processes checked		
$A^0 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$	1.137×10^0	1.137×10^0	1.137×10^0
$\tilde{\chi}_1^+ \rightarrow t \bar{b}_1$	5.428×10^0	5.428×10^0	5.428×10^0
$H^0 \rightarrow \tilde{\tau}_1 \bar{\tilde{\tau}}_1$	7.579×10^{-3}	7.579×10^{-3}	7.579×10^{-3}
$H^+ \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^0$	1.113×10^{-1}	1.113×10^{-1}	1.113×10^{-1}

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- Default: on-shell, GI, renormalisation in **ALL** sectors

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- Issues with definition of $\tan \beta$, many defs not gauge invariant!
- Same for mixing angle in the sfermion sector.
- Good scale dependence of ren. csts.