5D spontaneously broken 2HD Models

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- **Review of the 5D2HD models**
- **Equivalence Theorem, Unitarity bounds**
- **Scalar vacuum configurations**
- **Conclusions**

De Curtis, DD, Pelaez, Phys. Lett., 2003
De Curtis, DD, Pelaez, Phys. Rev. D, 2003
Coradeschi, De Curtis, DD, Pelaez, JHEP 2008

Related work

Equivalence theorem and Unitarity in 5D

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- R. S. Chivukula and H. J. He, Phys. Lett. B 532 (2002) 121.
- R. S. Chivukula, D. A. Dicus, H. J. He and S. Nandi, Phys. Lett. B 562 (2003) 109
- A. Muck, L. Nilse, A. Pilaftsis and R. Ruckl, Phys. Rev. D 71 (2005) 066004
- Y. Abe, N. Haba, K. Hayakawa, Y. Matsumoto, M. Matsunaga and K. Miyachi, Prog. Theor. Phys. 113 (2005) 199
- T. Ohl and C. Schwinn, Phys. Rev. D 70 (2004) 045019
- A. Falkowski, S. Pokorski and J. P. Roberts, JHEP 0712 (2007) 063
- R. S. Chivukula, H. J. He, M. Kurachi, E. H. Simmons and M. Tanabashi, arXiv:0808.1682 [hep-ph]
- N. Haba, Y. Sakamura and T. Yamashita, arXiv:0908.1042 [hep-ph].

Scalar vacuum solutions

- V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B 125 (1983) 136.
- B. Grzadkowski and M. Toharia, Nucl. Phys. B 686 (2004) 165
- C.P. Burgess, C. de Rham and L. van Nierop, ArXiv:0802.4221 [hep-ph]

The framework

5D 2HDM: a $SU(2) \times U(1)$ bulk gauge theory, with two scalars and fermions living in the bulk or on the brane.

(Antoniadis, Benakli; Masip, Pomarol; Nath and Yamaguchi; Rizzo, Wells, De Curtis, Casalbuoni, D., Gatto; Delgado, Pomarol, Quiros; Strumia, Muck, Pilaftsis, Rückl)

- The fifth is a compactified orbifold S^1/Z_2 (a circle with the identification $y \to -y$, $y \in [0, \pi R]$), and flat metric
- Large extra dimension: the scale of the compact dimension $R \sim$ TeV^{-1}
- KK excitations mix with W and Z, modify their couplings and masses
- KK excitations induce new four fermion operators
- From high precision electroweak data $M \equiv R^{-1} \sim 2 - 6$ TeV depending on the model, fermion localization...
- Very interesting possibility: can be tested in future colliders

5D 2HD models

The 5D lagrangian is given by (Delgado, Pomarol, Quiros)

$$\begin{aligned}\mathcal{L}_{5D} &= -\frac{1}{4g_5^2}F_{MN}^2 - \frac{1}{4g_5'^2}B_{MN}^2 + \\ &+ \sum_i (1-\varepsilon^{\Phi_i})|D_M\Phi_i|^2 + \sum_\psi (1-\varepsilon^\psi) i\bar{\psi}\Gamma^M D_M\psi \\ &+ \Big[\sum_i \varepsilon^{\Phi_i}|D_\mu\Phi_i|^2 + \sum_\psi \varepsilon^\psi i\bar{\psi}\sigma^\mu D_\mu\psi\Big]\delta(y)\end{aligned}$$

where ε defined as $\varepsilon^F = 1$ (0) for the F-field living in the boundary (bulk); $D_M = \partial_M + iV_M$, $M = (\mu, 5)$.

The fields living in the bulk even or odd under the $\mathbb{Z}_2$-parity, i.e. $\phi_\pm(y) = \pm\phi_\pm(-y)$.

Fourier-expansion as

$$\begin{aligned}\phi_+(x,y) &= \sum_{n=0}^{\infty} \cos\frac{ny}{R}\phi_+^{(n)}(x)\,,\\ \phi_-(x,y) &= \sum_{n=1}^{\infty} \sin\frac{ny}{R}\phi_-^{(n)}(x)\end{aligned}$$

where $\phi_\pm^{(n)}$ are the KK excitations of the 5D fields. Gauge and Higgs bosons living in the 5D bulk will be assumed to be even under the $\mathbb{Z}_2$.

Fermions in 5D have two chiralities, ψ_L and ψ_R: choose the even assignment for the ψ_L (ψ_R) components of fermions ψ, which are doublets (singlets) under SU(2)$_L$. As a consequence only the ψ_L of SU(2)$_L$ doublets and ψ_R of SU(2)$_L$ singlets have zero modes.

After integrating over the fifth dimension, in the charged sector:

$$
\begin{aligned}
\mathcal{L}^{ch} &= \sum_{a=1}^{2}\Big[\frac{1}{2}m_W^2\left\{W_a^2+2\sqrt{2}s_\alpha^2\sum_{n=1}^{\infty}W_a\cdot W_a^{(n)}\right\}+\frac{1}{2}M^2\sum_{n=1}^{\infty}n^2\,(W_a^{(n)})^2\\
&- g\,W_a\cdot J_a - g\,\sqrt{2}J_a^{KK}\cdot\sum_{n=1}^{\infty}W_a^{(n)}\Big]
\end{aligned}
$$

$$
J_{a\mu}=\sum_\psi\bar\psi_L\gamma_\mu\frac{\sigma_a}{2}\psi_L\ ,\ \ J_{a\mu}^{KK}=\sum_\psi\varepsilon^{\psi_L}\bar\psi_L\gamma_\mu\frac{\sigma_a}{2}\psi_L\ ,
$$

with $s_\alpha^2=\varepsilon^{\Phi_2}\,s_\beta^2+\varepsilon^{\Phi_1}\,c_\beta^2$, $\tan\beta=\langle\Phi_2\rangle/\langle\Phi_1\rangle$. In the neutral sector

$$
\begin{aligned}
\mathcal{L}^{neutral} &= \frac{1}{2}m_Z^2\left\{Z^2+2\sqrt{2}s_\alpha^2\sum_{n=1}^{\infty}Z\cdot Z^{(n)}\right\}+\frac{1}{2}M^2\sum_{n=1}^{\infty}n^2\,\left[(Z^{(n)})^2+A^{(n)}\cdot A^{(n)}\right]\\
&- \frac{e}{s_\theta c_\theta}\left[Z\cdot J_Z+\sqrt{2}\sum_{n=1}^{\infty}Z^{(n)}\cdot J_Z^{KK}\right]\\
&- e\left[A\cdot J_{em}+\sqrt{2}\sum_{n=1}^{\infty}A^{(n)}\cdot J_{em}^{KK}\right]
\end{aligned}
$$

$$
J_{\mu\,Z}=\sum_\psi\bar\psi\gamma_\mu(g_V^\psi+\gamma_5 g_A^\psi)\psi\ ,\ \ J_{\mu\,Z}^{KK}=\sum_\psi\bar\psi\gamma_\mu(g_V^{\psi,KK}+\gamma_5 g_A^{\psi,KK})\psi
$$

$$
J_{\mu\,em}^{KK}=\sum_\psi\bar\psi\gamma_\mu(g_{em,V}^{\psi,KK}+\gamma_5 g_{em,A}^{\psi,KK})\psi
$$

For momenta $p^2 \sim M_W^2 \ll M_c^2$ we can integrate out the KK modes $W_a^{(n)}$, $Z^{(n)}$, $A^{(n)}$ using their equations of motion and neglecting their kinetic terms:

$$\mathcal{L}^{ch}_{a,eff} = \frac{1}{2} M_W^2 W_a^2 - g W_a \cdot \left[J_a - s_\alpha^2\, c_\theta^2\, X\, J_a^{KK} \right] - \frac{g^2}{2\, m_Z^2} X\, J_a^{KK} \cdot J_a^{KK}$$

where

$$\begin{aligned} \mathcal{L}^{neutral}_{eff} &= \frac{1}{2} M_Z^2\, Z^2 - \frac{e}{s_\theta\, c_\theta} Z \cdot \left[J_Z - s_\alpha^2 X\, J_Z^{KK} \right] - eA \cdot J_{em} \\ &- \frac{1}{2\, M_Z^2} \frac{e^2}{s_\theta^2\, c_\theta^2} X J_Z^{KK} \cdot J_Z^{KK} - \frac{e^2}{2\, M_Z^2} X\, J_{em}^{KK} \cdot J_{em}^{KK} \end{aligned}$$

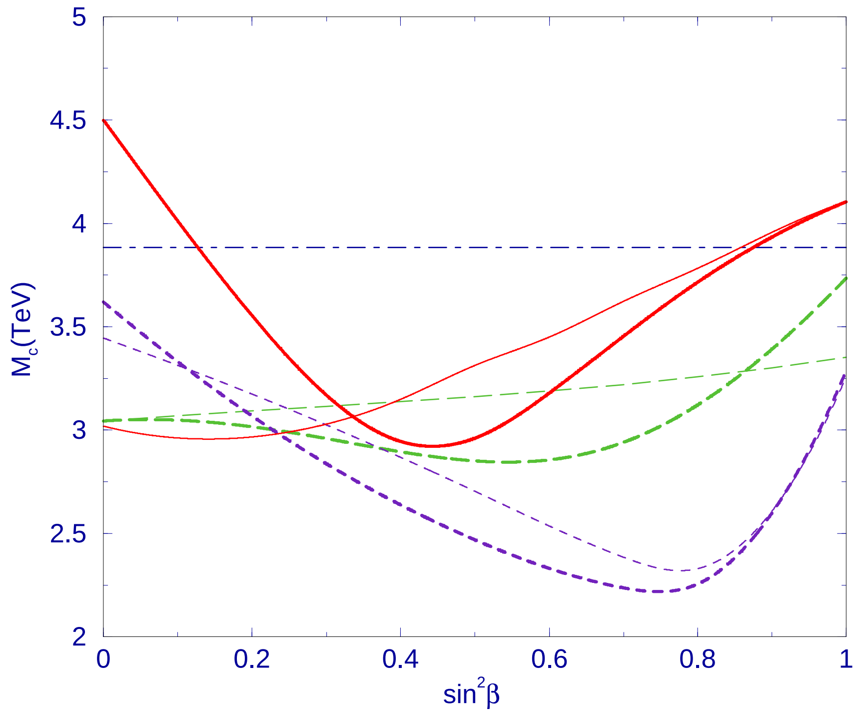
where

$$M_W^2 = \left[1 - s_\alpha^4 c_\theta^2 X \right]\, m_W^2 \quad M_Z^2 = \left[1 - s_\alpha^4 X \right] m_Z^2$$

$$X = \frac{\pi^2}{3} \frac{m_Z^2}{M^2}$$

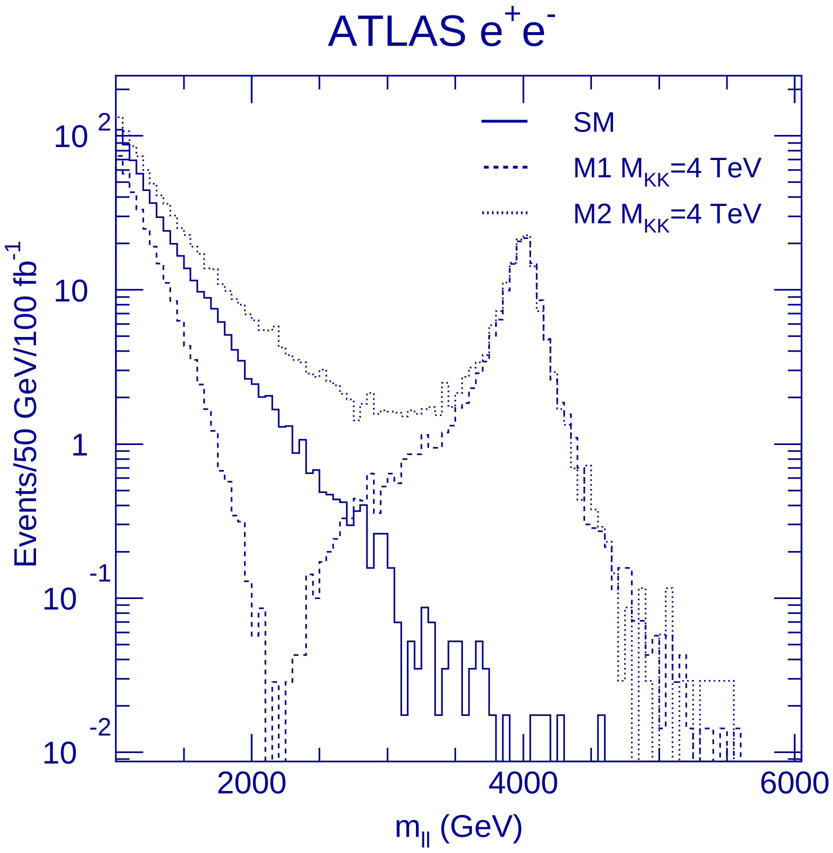
Electroweak bounds on $M = R^{-1}$

(Delgado, Pomarol, Quiros)



95% CL lower bounds on M corresponding to the case $\varepsilon^{\Phi_1} = 1$, $\varepsilon^{\Phi_2} = 0$, $\varepsilon^{q_L} = \varepsilon^{\ell_L} = \varepsilon^{u_R} = 1$ (solid), $\varepsilon^{\Phi_1} = 1$, $\varepsilon^{\Phi_2} = 0$, $\varepsilon^{q_L} = \varepsilon^{u_R} = \varepsilon^{e_R} = 1$ (short-dashed), $\varepsilon^{\Phi_1} = 0$, $\varepsilon^{\Phi_2} = 1$, $\varepsilon^{q_L} = \varepsilon^{\ell_L} = \varepsilon^{d_R} = \varepsilon^{e_R} = 1$ (long-dashed) and $\varepsilon^{\Phi_1} = \varepsilon^{\Phi_2} = 0$, $\varepsilon^{q_L} = \varepsilon^{\ell_L} = \varepsilon^{u_R} = \varepsilon^{d_R} = \varepsilon^{e_R} = 1$ (dash-dotted). Thin (thick) lines correspond to SM (MSSM) 5D extensions.

Atlas Simulation (Azuelos, Polesello)



Invariant mass distribution of e^+e^- pairs for the Standard Model (full line) and for models M1 (dashed line) and M2 (dotted line). The mass of the lowest lying KK excitation is 4 TeV. The histograms are normalized to 100 fb^{-1}.
Peaks visible up 5.8 TeV with 100 fb^{-1}.

Equivalence Theorem for 5D 2HDM

Equivalence theorem in the SM

(Lee, Quigg, Thacker; Cornwall, Levine, Tiktopoulos; Chanowitz, Gaillard)

At high energy

$$T(V_L, V_L, ...) = T(G^V, G^V, ...) + O(M_V/E)$$

with $V_L = W_L, Z_L$ and $G^V = G^W, G^Z$ their associated Goldstones.

Very useful

Amplitudes with complicated longitudinal fields replaced by amplitudes with simple scalar fields.

Sketch of the proof

Trick to prove the ET: identify the G, or the gauge fixing

$$L_{GF} = \frac{1}{2\xi} F(x)^2 = \frac{1}{2\xi} (\partial_\mu V^\mu(x) - \xi M G(x))^2$$

and the identity

$$< A, out|T[F(x_1)....F(x_n)]|B, in >_{con} = 0$$

Passing to S matrix elements, taking into account the relation between the inverse propagators of Goldstones and vectors and the property

$$\epsilon_\mu^L \sim \frac{p_\mu}{M} \qquad \Rightarrow V_L \sim G$$

$$T(V_L, V_L, ...) = T(G^V, G^V, ...) + O(M_V/E)$$

A 5D two Higgs model

Consider a 1 Bulk Higgs, 1 Brane Higgs model

$$\int_{-\pi R}^{\pi R} dy \int dx \Big\{ -\frac{1}{4} B_{MN} B^{MN} - \frac{1}{4} F^a_{MN} F^{aMN} + \mathcal{L}_{GF}(x,y)$$
$$+ \ (D_M \Phi_1)^\dagger (D^M \Phi_1) + \delta(y) (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2) - V(\Phi_1, \Phi_2) \Big\}$$

$D_M = \partial_M - i g_5 A^a_M \tau^a/2 - i g_5' B_M/2$. **For simplicity we will require a discrete symmetry** $\Phi_2 \rightarrow -\Phi_2$**,**

$$\begin{aligned} V(\Phi_1, \Phi_2) = & \ \mu_1^2 \, (\Phi_1^\dagger \Phi_1) + \lambda_1 \, (\Phi_1^\dagger \Phi_1)^2 \\ & + \ \delta(y) \Big[\frac{1}{2} \mu_2^2 \, (\Phi_2^\dagger \Phi_2) + \frac{1}{2} \lambda_2 \, (\Phi_2^\dagger \Phi_2)^2 \\ & + \frac{1}{2} \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \frac{1}{2} \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \lambda_5 \, (\Phi_1^\dagger \Phi_2)^2 + \text{ h.c.} \Big] \end{aligned}$$

Scalar vacuum state

The vacuum state manifold corresponds to configurations which are both energy minima and solutions of

$$
\begin{aligned}
(-\partial_y^2 + \Box)\Phi_1 &= \mu_1^2 \Phi_1 + 2\lambda_1 (\Phi_1^\dagger \Phi_1)\Phi_1 \\
&+ \delta(y) \left[\lambda_3 \Phi_1 (\Phi_2^\dagger \Phi_2) + \lambda_4 \Phi_2 (\Phi_2^\dagger \Phi_1) + 2\lambda_5 (\Phi_1^\dagger \Phi_2)\Phi_2\right], \\
\Box \Phi_2 &= \mu_2^2 \Phi_2 + 2\lambda_2 (\Phi_2^\dagger \Phi_2)\Phi_2 + \Big[\lambda_3 (\Phi_1^\dagger \Phi_1)\Phi_2 \\
&\qquad + \lambda_4 (\Phi_2^\dagger \Phi_1)\Phi_1 + 2\lambda_5 (\Phi_1^\dagger \Phi_2)\Phi_1\Big]\Big|_{y=0}
\end{aligned}
$$

Customarily assumed (Masip, Pomarol, Casalbuoni et al , Delgado et al , Rizzo et , Muck et al)**:**

$$
\Phi_1 = (0, v_1/\sqrt{4\pi R}) \quad \Phi_2 = (0, v_2/\sqrt{2})
$$

Vacuum state

However, if we substitute such constant solutions into the equations of motion,

$$
\begin{aligned}
0 &= v_1 \left(\mu_1^2 + 2\lambda_1 \frac{v_1^2}{4\pi R} \right) , \\
0 &= v_2 \left(\mu_2^2 + \lambda_2 v_2^2 + \frac{v_1^2}{4\pi R} (\lambda_3 + \lambda_4 + 2\lambda_5) \right) , \\
0 &= v_1 \, v_2^2 \, (\lambda_3 + \lambda_4 + 2\lambda_5)
\end{aligned}
$$

This implies

$$\lambda_3 + \lambda_4 + 2\lambda_5 = 0$$

A fine tuning, with no theoretical justification, is thus required to get a constant vacuum configuration.
Higgs fields are expanded in the standard form

$$\Phi_1(x,y) = \begin{pmatrix} \frac{i}{\sqrt{2}}(\omega^1 - i\omega^2) \\ \frac{1}{\sqrt{2}}(\frac{v_1}{\sqrt{2\pi R}} + h_1 - i\omega^3) \end{pmatrix}, \quad \Phi_2(x) = \begin{pmatrix} \frac{i}{\sqrt{2}}(\pi^1 - i\pi^2) \\ \frac{1}{\sqrt{2}}(v_2 + h_2 - i\pi^3) \end{pmatrix}$$

Gauge fixing after KK decomposition

New features: different and non diagonal mass matrices for V and GB's. This implies a mixing of GB$^{(n)}$ and $V_5^{(n)}$.

$$\mathcal{L}_{GF}(x) = -\frac{1}{\xi}\sum_{n=0}^{\infty}\left\{\frac{1}{2}\left[\partial_\mu A^\mu_{(n)} - \xi\frac{n}{R}\, A^5_{(n)}\right]^2 + \left|\partial_\mu \hat{W}^{+\mu}_{(n)} - \xi m_{W(n)}\hat{G}^{+}_{(n)}\right|^2 + \frac{1}{2}\left[\partial_\mu \hat{Z}^\mu_{(n)} - \xi m_{Z(n)}\hat{G}^{Z}_{(n)}\right]^2\right\}$$

GF in terms of mass eigenstate gauge bosons, $\hat{\mathrm{V}}^\mu = P_V^T\,\mathrm{V}^\mu$, and would be GB, $\hat{\mathrm{G}}^{\mathrm{V}} = Q_V^T\,\mathrm{G}^{\mathrm{V}}$. P_V^T, Q_V^T non diagonal when $\tan\beta = v_2/v_1 \neq 0$.

$$G^{\pm}_{(0)} = -\omega^{\pm}_{(0)}, \quad G^{\pm}_{(n)} = c^W_n\, W^{\pm}_{5\,(n)} + s^W_n\,\omega^{\pm}_{(n)},\ n \geq 1,$$
$$G^{Z}_{(0)} = -\omega^{3}_{(0)}, \quad G^{Z}_{(n)} = c^Z_n\, Z_{5\,(n)} + s^Z_n\,\omega^{3}_{(n)},\ n \geq 1$$

with $s^V_n = -m_V c_\beta/\sqrt{n^2/R^2 + m_V^2 c_\beta^2}$.

Equivalence theorem

The ET proof proceeds as usual simply by substituting $V_L \longrightarrow \hat{V}_L$ and the would-be GB by $\hat{G}^V$:

$$T(\hat{V}^{\mu}_{L\,(m)}, \hat{V}^{\mu}_{L\,(n)}, \ldots) \simeq T(\hat{G}^{V}_{(m)}, \hat{G}^{V}_{(n)}, \ldots) + O(M_k/E)$$

In general, for the calculations of amplitudes we would also need the orthogonal Higgs combinations

$$a^{\pm}_{(n)} = -s^{W}_{n}\, W^{\pm}_{5\,(n)} + c^{W}_{n}\, \omega^{\pm}_{(n)}, \qquad a^{Z}_{(n)} = -s^{Z}_{n}\, Z_{5\,(n)} + c^{Z}_{n}\, \omega^{3}_{(n)}$$

with masses

$$m^2_{a_{(n)}} = m^2_V + \frac{n^2}{R^2}$$

Vertices can be read from the lagrangian in terms of $\omega_{(n)}$ once reexpressed in terms of $a_{(n)}$ and $\hat{G}_{(n)}$.

Simplest case: one Higgs in the bulk

Tree level unitarity bounds from scattering of longitudinal bosons $W^+_{L(0)}W^-_{L(0)}$, $Z_{L(0)}Z_{L(0)}$ and Higgs. We use ET.

$$t^{J=0}_{G^+_{(0)}G^-_{(0)}\to G^+_{(0)}G^-_{(0)}} = \frac{-G_F m^2_{h(0)}}{8\pi\sqrt{2}}\left[2+\frac{m^2_{h(0)}}{s-m^2_{h(0)}}-\frac{m^2_{h(0)}}{s}\log\left(1+\frac{s}{m^2_{h(0)}}\right)\right]$$

$$t^{J=0}_{G^+_{(0)}G^-_{(0)}\to G^Z_{(0)}G^Z_{(0)}} = \frac{-G_F\, m^2_{h(0)}}{16\pi}\left[1+\frac{m^2_{h(0)}}{s-m^2_{h(0)}}\right]$$

$$t^{J=0}_{G^+_{(0)}G^-_{(0)}\to h_{(0)}h_{(0)}} = \frac{-G_F\, m^2_{h(0)}}{16\pi}\left[1+\frac{3m^2_{h(0)}}{s-m^2_{h(0)}}+\frac{4m^2_{h(0)}}{s\,\sigma_{h(0)}}\log\left(\frac{s-2m^2_{h(0)}-s\,\sigma_{h(0)}}{2m^2_{h(0)}}\right)\right]$$

$$t^{J=0}_{G^+_{(0)}G^-_{(0)}\to a^+_{(n)}a^-_{(n)}} = \frac{-G_F m^2_{h(0)}}{8\pi\sqrt{2}}\left[2+\frac{m^2_{h(0)}}{s-m^2_{h(0)}}-\frac{m^2_{h(0)}}{s\sigma_{a^\pm(n)}}\log\left(\frac{2m^2_{a^\pm(n)}-2m^2_{h(n)}-s(1+\sigma_{a^\pm(n)})}{2m^2_{a^\pm(n)}-2m^2_{h(n)}-s(1-\sigma_{a^\pm(n)})}\right)\right]$$

$$t^{J=0}_{G^+_{(0)}G^-_{(0)}\to a^Z_{(n)}a^Z_{(n)}} = \frac{-G_F\, m^2_{h(0)}}{16\pi}\left[1+\frac{m^2_{h(0)}}{s-m^2_{h(0)}}\right]$$

$$t^{J=0}_{G^+_{(0)}G^-_{(0)}\to h_{(n)}h_{(n)}} = \frac{-G_F\, m^2_{h(0)}}{16\pi}\left[1+\frac{3m^2_{h(0)}}{s-m^2_{h(0)}}-\frac{2m^2_{h(0)}}{s\,\sigma_{h(n)}}\log\left(\frac{2m^2_{h(n)}-2m^2_{a^\pm(n)}-s(1+\sigma_{h(n)})}{2m^2_{h(n)}-2m^2_{a^\pm(n)}-s(1-\sigma_{h(n)})}\right)\right]$$

where $\sigma_\Phi = \sqrt{1-4m^2_\Phi/s}$.

Unitarity bounds with coupled channels

The S matrix unitarity relation $SS^\dagger = 1$, when

$$S_{\beta\alpha} = \delta_{\beta\alpha} + i(2\pi)^4\delta^4(p_\alpha - p_\beta)T_{\beta\alpha}$$

translates as

$$T_{\alpha\beta} - T^\dagger_{\beta\alpha} = i\sum_\gamma T_{\alpha\gamma}T^\dagger_{\beta\gamma}(2\pi)^4\delta^4(p_\alpha - p_\beta)$$

where $\alpha, \beta, \ldots$ denote the different states. Define the $J - th$ partial wave

$$t^J_{\alpha\beta}(s) = \frac{1}{32\pi}\int_{-1}^{1} d(\cos\theta)T_{\alpha\beta}(s,t,u)P_J(\cos\theta),$$

If there is only one two-body accessible state, α, each partial wave $t^J_{\alpha\alpha}$ satisfies

$$\mathrm{Im}\, t^J_{\alpha\alpha} = \sigma_\alpha \,|\, t^J_{\alpha\alpha}|^2 \quad ,$$

where $\sigma_\alpha = 2q_\alpha/\sqrt{s}$ and q_α is the C.M. momentum of the state α.

By writing $t^J_{\alpha\alpha} = |t^J_{\alpha\alpha}| \exp(i\delta^J_{\alpha\alpha})$, this implies the following bound

$$\sigma_\alpha \, |t^J_{\alpha\alpha}| \leq 1 \qquad \stackrel{s\to\infty}{\Longrightarrow} \qquad |t^J_{\alpha\alpha}| \leq 1.$$

If β is different two-particle state, the unitarity relation for the partial waves can be written as

$$\left.\begin{aligned} \mathrm{Im}\, t^J_{\alpha\alpha} &= \sigma_\alpha \, |\, t^J_{\alpha\alpha}|^2 + \sigma_\beta \, |\, t^J_{\alpha\beta}|^2 \\ \mathrm{Im}\, t^J_{\alpha\beta} &= \sigma_\alpha \, t^J_{\alpha\alpha} \, (t^J_{\alpha\beta})^* + \sigma_\beta \, t^J_{\alpha\beta} \, (t^J_{\beta\beta})^* \\ \mathrm{Im}\, t^J_{\beta\beta} &= \sigma_\alpha \, |\, t^J_{\alpha\beta}|^2 + \sigma_\beta \, |\, t^J_{\beta\beta}|^2 \end{aligned}\right\} \longrightarrow \quad \mathrm{Im}\, T^J = T^J \, \Sigma \, T^{J\,*}.$$

Example: the SM

For finite number of states the strongest bound from the largest T^J eigenvalue. In the SM neutral channel, only three states are relevant, namely, $\alpha = W_L^+ W_L^-$, $\beta = Z_L Z_L/\sqrt{2}$, $\gamma = HH/\sqrt{2}$:

$$T^{J=0}_{s\to\infty} = \frac{G_F M_H^2}{4\pi\sqrt{2}} \begin{pmatrix} 1 & 1/\sqrt{8} & 1/\sqrt{8} \\ 1/\sqrt{8} & 3/4 & 1/4 \\ 1/\sqrt{8} & 1/4 & 3/4 \end{pmatrix} \longrightarrow \frac{G_F M_H^2}{4\pi\sqrt{2}}(3/2, 1/2, 1/2)$$

The largest one, 3/2, provides the stringent unitarity bound:

$$M_H^2 \leq 8\pi\sqrt{2}/(3G_F) \simeq 2.7\pi\sqrt{2}/G_F \sim (1\ \mathrm{TeV})^2$$

However, the calculation of the determinant can be extremely complicated.

An alternative method: from

$$\text{Im}\, t^J_{\alpha\alpha} = \sigma_\alpha \,|\, t^J_{\alpha\alpha}|^2 + \sum_{\beta\neq\alpha} \sigma_\beta \,|\, t^J_{\alpha\beta}|^2,$$

by recalling that $t^J_{\alpha\alpha} = |t^J_{\alpha\alpha}| \exp(i\delta^J_{\alpha\alpha})$ it is straightforward to arrive to the following bound:

$$\text{Unit}_{\alpha\to\alpha} \equiv \sigma_\alpha |t^J_{\alpha\alpha}| + \frac{1}{|t^J_{\alpha\alpha}|} \sum_{\beta\neq\alpha} \sigma_\beta \,|\, t^J_{\alpha\beta}|^2 \leq 1.$$

Example: the SM, T matrix in the $s \to \infty$ limit, if we choose $\alpha = W^+_L W^-_L$, then the bound:

$$M^2_H \leq 16\pi\sqrt{2}/(5G_F) \simeq 3.2\pi\sqrt{2}/G_F \sim (1.1\ \text{TeV})^2$$

much closer to the determinant bound than the naive bound (1.2 TeV).

Simplest case: one Higgs in the bulk

$$t^{J=0}_{G^+_{(0)}G^-_{(0)}\to G^+_{(0)}G^-_{(0)}} = \frac{-G_F m^2_{h(0)}}{8\pi\sqrt{2}}\left[2+\frac{m^2_{h(0)}}{s-m^2_{h(0)}}-\frac{m^2_{h(0)}}{s}\log\ \ 1+\frac{s}{m^2_{h(0)}}\ \ \right]$$

$$t^{J=0}_{G^+_{(0)}G^-_{(0)}\to G^Z_{(0)}G^Z_{(0)}} = \frac{-G_F\, m^2_{h(0)}}{16\pi}\left[1+\frac{m^2_{h(0)}}{s-m^2_{h(0)}}\right]$$

$$t^{J=0}_{G^+_{(0)}G^-_{(0)}\to h_{(0)}h_{(0)}} = \frac{-G_F\, m^2_{h(0)}}{16\pi}\left[1+\frac{3m^2_{h(0)}}{s-m^2_{h(0)}}+\frac{4m^2_{h(0)}}{s\,\sigma_{h(0)}}\log\ \ \frac{s-2m^2_{h(0)}-s\,\sigma_{h(0)}}{2m^2_{h(0)}}\ \ \right]$$

$$t^{J=0}_{G^+_{(0)}G^-_{(0)}\to a^+_{(n)}a^-_{(n)}} = \frac{-G_F m^2_{h(0)}}{8\pi\sqrt{2}}\left[2+\frac{m^2_{h(0)}}{s-m^2_{h(0)}}-\frac{m^2_{h(0)}}{s\sigma_{a^\pm(n)}}\log\ \ \frac{2m^2_{a^\pm(n)}-2m^2_{h(n)}-s(1+\sigma_{a^\pm(n)}}{2m^2_{a^\pm(n)}-2m^2_{h(n)}-s(1-\sigma_{a^\pm(n)}}\right.$$

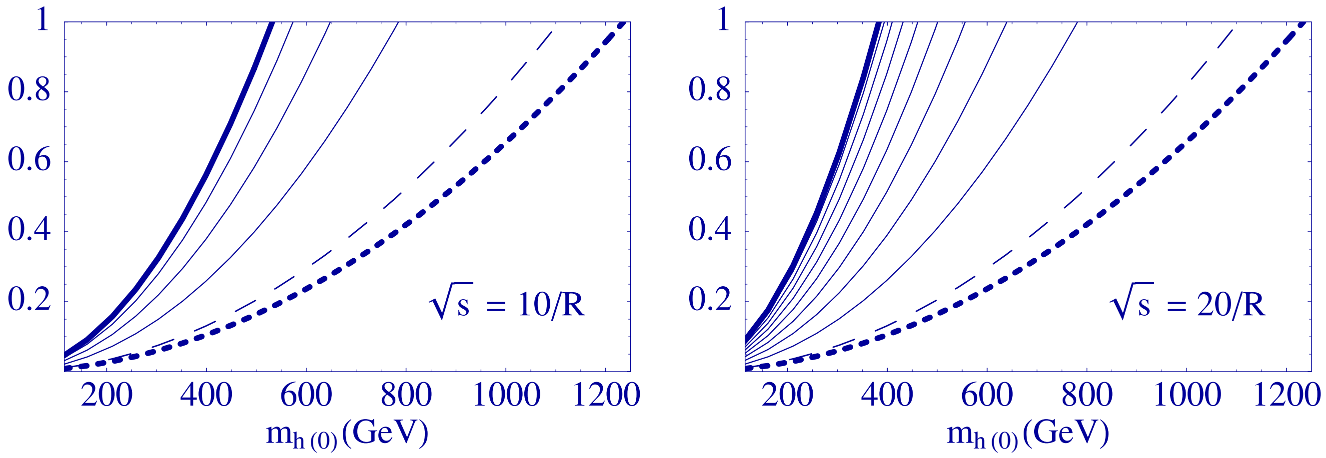
$$t^{J=0}_{G^+_{(0)}G^-_{(0)}\to a^Z_{(n)}a^Z_{(n)}} = \frac{-G_F\, m^2_{h(0)}}{16\pi}\left[1+\frac{m^2_{h(0)}}{s-m^2_{h(0)}}\right]$$

$$t^{J=0}_{G^+_{(0)}G^-_{(0)}\to h_{(n)}h_{(n)}} = \frac{-G_F\, m^2_{h(0)}}{16\pi}\left[1+\frac{3m^2_{h(0)}}{s-m^2_{h(0)}}-\frac{2m^2_{h(0)}}{s\,\sigma_{h(n)}}\log\ \ \frac{2m^2_{h(n)}-2m^2_{a^\pm(n)}-s(1+\sigma_{h(n)})}{2m^2_{h(n)}-2m^2_{a^\pm(n)}-s(1-\sigma_{h(n)})}\right.$$

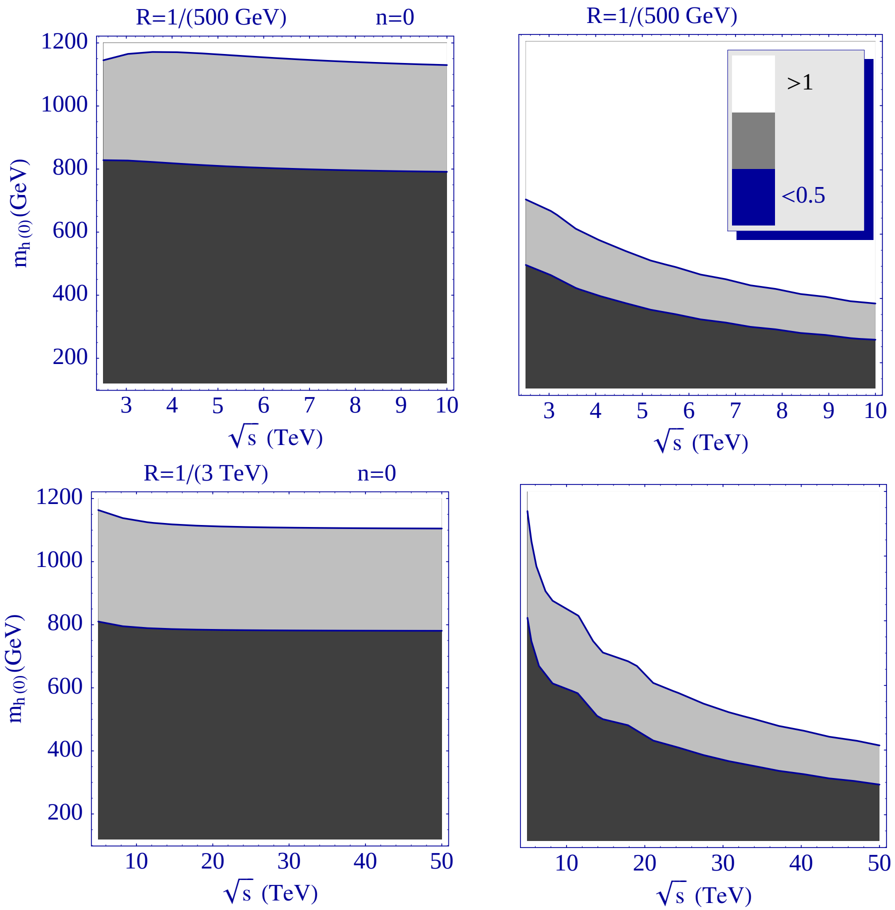
where $\sigma_\Phi = \sqrt{1-4m^2_\Phi/s}$.

The tree level unitarity bound on $m_{h(0)}$

The thick dotted line: SM ($n = 0$) results only from the $W^+_{L\,(0)} W^-_{L\,(0)}$ elastic scattering, whereas the dashed line includes also the $h_{(0)} h_{(0)}$, $Z_{L\,(0)} Z_{L\,(0)}$ coupled states. The continuous lines correspond to considering the first, second, etc... KK excitations of the previous states. Thick continuous line: the complete calculation including all the kinematically allowed states, which, for $\sqrt{s} = 10/R$ and $20/R$, are 4 and 9 KK levels, respectively.



The white (grey) areas: the regions where the tree level calculation violates the unitarity bound: Unit$_{W^+_{L\,(0)}W^-_{L\,(0)}\to W^+_{L\,(0)}W^-_{L\,(0)}} > 1(0.5)$ suggesting a strongly interacting regime. The bounds obtained using only the SM fields (left), and those including the KK excitations (right).



Vacuum solutions

Non constant vacuum state

The bulk scalar needs to interpolate between different vacuum choices. Generalize our request for the VEV:

$$\langle \Phi_1(x,y) \rangle = \begin{pmatrix} 0 \\ \varphi_1(y) \end{pmatrix}, \quad \langle \Phi_2(x) \rangle = \begin{pmatrix} 0 \\ \varphi_2 \end{pmatrix}$$

For ease of illustration, $\lambda_4 = \lambda_5 = 0$ leading to

$$\partial_y^2 \varphi_1(y) - \varphi_1(y) \left[\mu_1^2 + 2\lambda_1 \varphi_1(y)^2 + \delta(y)\, \lambda_3 \varphi_2^2\right] = 0$$
$$\varphi_2 \left[\mu_2^2 + 2\lambda_2\, \varphi_2^2 + \lambda_3\, \varphi_1(y)^2\right] |_{y=0} = 0$$

The above solutions have an associated energy density per unit volume:

$$\begin{aligned} \mathcal{H} &= \int_{-\pi R}^{\pi R} dy [(\partial_y \varphi_1(y))^2 + \mu_1^2 \varphi_1(y)^2 + \lambda_1 \varphi_1(y)^4 \\ &+ \delta(y) \left(\mu_2^2 \varphi_2^2 + \lambda_2 \varphi_2^4 + \lambda_3 \varphi_1(y)^2 \varphi_2^2\right)] \end{aligned}$$

Energy of trivial solutions $\varphi_{1,2} = 0$ is zero.

Non constant vacuum state

As usually done, we solve

$$\partial_y^2 \varphi_1(y) - \varphi_1(y)\left[\mu_1^2 + 2\lambda_1 \varphi_1(y)^2\right] = 0$$

in the bulk regions $y < 0$ and $y > 0$ separately, and then connecting both pieces using the following boundary conditions

- continuity in $y = 0$:

$$\varphi_1(0^-) = \varphi_1(0^+) \equiv \varphi_1(0);$$

- discontinuity of the first derivative in $y = 0$ with a gap $\lambda_3 \varphi_2^2 \varphi_1(0)$:

$$\varphi_1'(0^+) - \varphi_1'(0^-) = \lambda_3 \varphi_2^2 \varphi_1(0),$$

where we should have

$$\varphi_2^2 = -\frac{\mu_2^2}{2\lambda_2} - \frac{\varphi_1(0)^2 \lambda_3}{2\lambda_2}, \quad \text{with} \quad \varphi_2^2 > 0.$$

Non constant solutions in terms of Jacobi elliptic functions

For $\alpha = \frac{8e_0\lambda_1}{\mu_1^4} < 0$:

$$\varphi_1^A(y) = \pm\frac{|\mu_1|}{\sqrt{2\lambda_1}}\sqrt{1+\beta^2}\ \mathrm{nc}\left(|\mu_1|\beta(y-y_0), \frac{1}{2}(1-\frac{1}{\beta^2})\right),$$

For $0 < \alpha < 1$:

$$\varphi_1^{B1}(y) = \pm\frac{|\mu_1|}{\sqrt{2\lambda_1}}\sqrt{1-\beta^2}\ \mathrm{sn}\left(\frac{|\mu_1|}{\sqrt{2}}\sqrt{1+\beta^2}(y-y_0), \frac{1-\beta^2}{1+\beta^2}\right),$$

with $\beta^2 = \sqrt{1-\alpha}$.
For $\alpha > 1$ a more complicated combination of Jacobi functions.

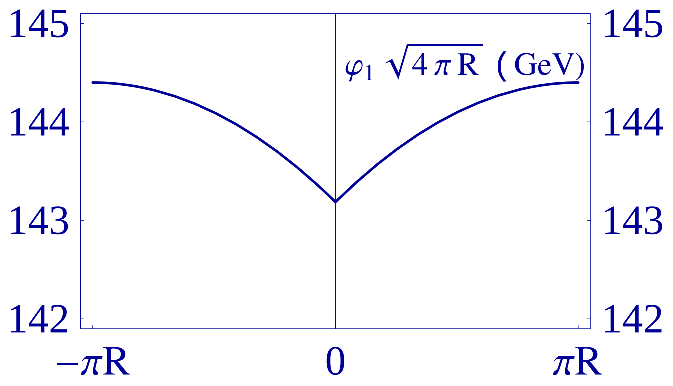
e_0, y_0 to be determined by BC's.

Imposing BC's

- **The presence of brane terms implies non trivial BC's**
- **Different choices possible**
- **In general BC's not analytically solvable**
- **We have built explicit examples to illustrate different vacuum behaviour**

Example 1

- With the choice $\pi R = (1\ \mathrm{TeV})^{-1}$, $|\mu_1| = 165\ \mathrm{GeV}$, $\lambda_1 = 0.5 \times 2\pi \mathrm{R}$, $\lambda_2 = 1, \lambda_3 = 0.85 \times 2\pi \mathrm{R}$, $|\mu_2| = 220\ \ \mathrm{GeV}$ we found a vacuum solution of the type B1 with $\beta \simeq 0.79, y_0 \simeq \mathrm{GeV}^{-1}$.
- The energy density $\sim -(179\ \mathrm{GeV})^4$, which is less than the $(0\ \mathrm{GeV})^4$ associated with the trivial static solution, $\Rightarrow$ spontaneous symmetry breakdown.



Vacuum configuration almost constant. Only a negligeable distortion in KK modes.

Example 2

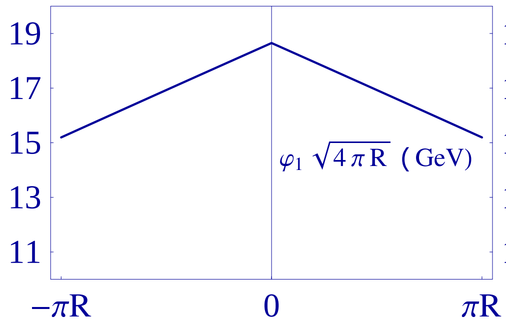
Let us allow for a discontinuity of the first derivative in $y = \pi R$: the simplest possibility a brane mass term

$$\delta(y - \pi R)(-2\kappa \Phi_1^\dagger \Phi_1)$$

The BC on the second brane

$$\partial_y \Phi_1|_{\pi R^-} = k\Phi_1|_{\pi R^-}$$

Parameters: $\pi R = (1\text{TeV})^{-1}$, $|\mu_1| = 60$ GeV, $\lambda_1 = 0.5 \times 2\pi R$, $\lambda_2 = 2$, $\lambda_3 = 10 \times 2\pi R$, $|\mu_2| \simeq 349\,\text{GeV}$ $\quad \text{k} = 229\,\text{GeV}$ **with** $\beta \simeq 0.1$ **and** $y_0 \simeq 0.15\,\text{GeV}^{-1}$. **The energy density in this case is** $\simeq -(245\,\text{GeV})^4$, **again indicating a spontaneous symmetry breaking.**



The constant approximation would not be appropriate, since the difference between $\varphi_1(0)$ and $\varphi_1(\pi R)$ is more than 20%.

Effective 4D vacuum state via holography

The holographic procedure requires the explicitation of the y-dependence of the Φ_1 field; solve the equations of motion, with suitable boundary conditions. Then this y dependence is integrated out to get an effective 4-dimensional action.

$$\Phi_1(x,y) \to \begin{pmatrix} 0 \\ \frac{h_1(x,y)}{\sqrt{2}} \end{pmatrix}, \ \Phi_2(x) \to \begin{pmatrix} 0 \\ \frac{h_2(x)}{\sqrt{2}} \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}_S &= \frac{1}{2}\partial_M h_1 \partial^M h_1 - \frac{1}{2}\mu_1^2 h_1^2 - \frac{\lambda_1}{4}h_1^4 + \delta(y)\left(\frac{1}{2}\partial_\mu h_2 \partial^\mu h_2 \right. \\ &- \left. \frac{1}{2}\mu_2^2 h_2^2 \frac{\lambda_2}{4} h_2^4 - \frac{\lambda_3}{4} h_1^2 h_2^2\right) - \delta(y-\pi R) k h_1^2 \end{aligned}$$

$$\partial_M \partial^M h_1 - |\mu_1|^2 h_1 + \lambda_1 h_1^3 + \text{Brane terms} = 0$$

First approximation, neglecting the λ_1-proportional interaction term while solving the e.o.m.;

$$\partial_y^2 h_1(p,y) = -(|\mu_1|^2 + p^2) h_1(p,y),$$

where $p^2 = p_\mu p^\mu$. The general solution

$$h_1 = A(p)\cos(\omega(y - y_0)),\ \omega = \sqrt{|\mu_1|^2 + p^2}.$$

Impose appropriate BC's to fix the integration constants A and y_0. On the $y = 0$ brane h_1 is equal to a purely 4-dimensional "source field", $\tilde{h}_1$:

$$h_1(y = 0, p) = \tilde{h}_1(p),$$

while on the $y = \pi R$ brane the BC used both for the vacuum configuration:

$$\partial_y h_1(y = \pi R, p) = \frac{1}{2}\frac{\delta V_R}{\delta h_1}\bigg|_{\pi R} = \frac{k}{2} h_1(y = \pi R, p).$$

Holography with no brane terms at $y = \pi R$

$$h_1(y,p) = \tilde{h}_1(p)\frac{\cos\omega(y-\pi R)}{\cos\omega\pi R}.$$

Bulk Lagrangian

$$\int_{-\pi R}^{\pi R} dy\,\mathcal{L}_S = \omega\tan(\omega\pi R)\,\tilde{h}_1^2$$

$$-\frac{\lambda_1}{2\cos^4(\omega\pi R)}\left(\frac{3}{8}\pi R + \frac{1}{4\omega}\sin(2\omega\pi R) + \frac{1}{32\omega}\sin(4\omega\pi R)\right)\tilde{h}_1^4$$

$$+\frac{1}{2}\partial_\mu h_2\partial^\mu h_2 + \frac{|\mu_2|^2}{2}h_2^2 - \frac{\lambda_2}{4}h_2^4 - \frac{\lambda_3}{4}\tilde{h}_1^2 h_2^2.$$

Expand for $p^2 << (\pi R)^{-1}$, $\tilde{h}_1 \to \frac{\tilde{h}_1}{\sqrt{2\pi R}}$

$$\mathcal{L}_S^{eff} = \frac{1}{2}\partial_\mu\tilde{h}_1\partial^\mu\tilde{h}_1 + \frac{1}{2}|\mu_1|^2\tilde{h}_1^2 - \frac{\lambda_1}{8\pi R}\tilde{h}_1^4$$

$$+\frac{1}{2}\partial_\mu h_2\partial^\mu h_2 + \frac{|\mu_2|^2}{2}h_2^2 - \frac{\lambda_2}{4}h_2^4 - \frac{\lambda_3}{8\pi R}\tilde{h}_1^2 h_2^2.$$

The potential energy in this Lagrangian has nontrivial minima at

$$\langle \tilde{h}_1^2 \rangle = \frac{16(\pi R)^2 \lambda_2 |\mu_1|^2 - 4\pi R \lambda_3 |\mu_2|^2}{8\pi R \lambda_1 \lambda_2 - \lambda_3^2}$$

$$\langle \tilde{h}_2^2 \rangle = \frac{4\pi R(2\lambda_1 |\mu_2|^2 - \lambda_3 |\mu_1|^2)}{8\pi R \lambda_1 \lambda_2 - \lambda_3^2}.$$

A comparison with the exact calculation using the numerical values of example 2 shows almost perfect agreement.

Holography with brane terms at $y = \pi R$

$$h_1(y,p) = \tilde{h}_1(p)\frac{\omega\cos(\omega(y-\pi R)) + k\sin(\omega(y-\pi R))}{\omega\cos(\omega\pi R) - k\sin(\omega\pi R)}.$$

Same procedure and good agreement.

Conclusions

- 5D 2HDM with compactification scale $M = R^{-1} \sim$ TeV. Present bounds, 2-6 TeV, can be tested at LHC.
- Extension of ET to spontaneously broken 5D 2HDM
- Derivation of stronger unitarity bound on m_H when many intermediate states are available
- Check how the explicit breaking of translational invariance on the extra dimension induced by delta-like interactions between scalar bulk and brane fields modifies the naively expected pattern of spontaneous symmetry breakdown
- Constant non trivial solutions for the scalar field in the bulk cannot be found. Vacuum configuration for the scalar bulk field depends on the extra coordinate y.
- We have illustrated the shape of two examples: in the first one, the y dependence is weak, so that a constant configuration may still be a good approximation; in the second example, the constant solution would only be a poor approximation to the actual vacuum configuration.

Further developments

- How the Kaluza-Klein spectrum of both the scalar and gauge fields is modified in a model with brane-bulk interaction
- How these effects modify the scattering of longitudinal gauge bosons among themselves and with Higgs bosons.
- In addition, one can test how a y-dependent vacuum expectation value of the Higgs field would modify the generated fermion masses.