The evolution of vacuum states and phase transitions in 2HDM during cooling of Universe.

I.F.Ginzburg, I.P.Ivanov, K.A.Kanishev

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Motivation

Minimal SM

- One Higgs doublet – single vacuum, conserving charge and CP.
- Early universe – high temperatures T.
- Vacuum expectations are given by minimisation of Gibbs potential
  $$\Phi = V(\phi) + aT^2\phi^2$$
- Cooling $\langle\phi\rangle = 0 \to \langle\phi\rangle \neq 0$ – single EWSB phase transition.

Two Higgs doublet model (2HDM)

- Two Higgs doublets, up to 5 observable Higgses, $H^\pm$, h_1, h_2, h_3
- Vacua with different properties can exist.
- It is possible to have a sequence of phase transitions (between different vacua) during cooling of the Universe. [Ginzburg,Kanishev,2007]

2HDM Potential

- Two scalar doublets: $\phi_1 = \begin{pmatrix} \phi_{11} \\ \phi_{12} \end{pmatrix} \qquad \phi_2 = \begin{pmatrix} \phi_{21} \\ \phi_{22} \end{pmatrix}$
- Scalar combinations: $x_1 = \phi_1^\dagger \phi_1, \quad x_2 = \phi_2^\dagger \phi_2, \quad x_3 = \phi_1^\dagger \phi_2\,.$

Most general form of potential

$$V = \tfrac{1}{2}\lambda_1 x_1^2 + \tfrac{1}{2}\lambda_2 x_2^2 + \lambda_3 x_1 x_3 + \lambda_4 x_3^\dagger x_3 + \left[\tfrac{1}{2}\lambda_5 x_3^2 + (\lambda_6 x_1 + \lambda_7 x_2)\, x_3 + h.c.\right]$$
$$-\tfrac{1}{2}\left[m_{11}^2 x_1 + m_{22}^2 x_2 + \left(m_{12}^2 x_3 + h.c.\right)\right]$$

- Most general case: λ_{1-4}, m_{11}^2, m_{22}^2 - real, λ_{5-7}, m_{12}^2 - complex. Positivity constraints: $V > 0$ at large values of ϕ_i
- Explicitly CP conserving potential – all λ_i and m_{ij}^2 are real.
- Soft Z_2 violation: $\lambda_6 = \lambda_7 = 0$ (Z_2 is violated only by quadratic term)

Useful parametrization

$$\lambda_2/\lambda_1 = k^4, \quad m_{11}^2 = m^2(1-\delta), \quad m_{22}^2 = k^2 m^2 (1+\delta)$$

At $\delta = 0$: k-symmetry of potential – $\phi_1 \leftrightarrow k\phi_2$

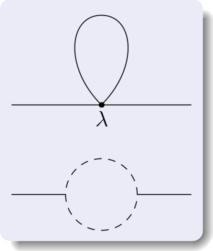
Abbreviations

$$\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5, \quad \tilde{\lambda}_{345} = \lambda_3 + \lambda_4 - \lambda_5.$$

$$\Lambda_{345\pm} = \sqrt{\lambda_1\lambda_2} \pm \lambda_{345}, \quad \tilde{\Lambda}_{345\pm} = \sqrt{\lambda_1\lambda_2} \pm \tilde{\lambda}_{345}, \quad \Lambda_{3\pm} = \sqrt{\lambda_1\lambda_2} \pm \lambda_3.$$

Temperature dependence

- Gibbs potential: $V_G = \frac{Tr\left\{V\,e^{-H/T}\right\}}{Tr\left\{e^{-H/T}\right\}} = V + \Delta V$
- ΔV is calculated with Matsubara diagram technique



$$m_{11}^2(T) = m_{11}^2(0) - 2c_1 m^2 w, \quad m_{22}^2(T) = m_{22}^2(0) - 2k^2 c_2 m^2 w,$$
$$m_{12}^2(T) = m_{12}^2(0); \quad c_i = c_i^s + c_i^g + c_i^f, \quad w = \frac{T^2}{12m^2}$$
$$c_1^s = \frac{3\lambda_1 + 2\lambda_3 + \lambda_4}{2}, \quad c_2^s = \frac{3\lambda_2 + 2\lambda_3 + \lambda_4}{2k^2}, \quad c_1^g = c_2^g = (3g^2 + g'^2)B\,.$$

(μ, δ) plane

Rewriting $m^2_{ij}(T)$ for m^2, μ and δ parameters:

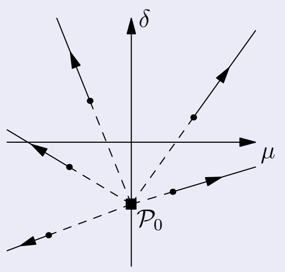
$$m^2(T) = m^2\left[1 - (c_1 + c_2)w\right]$$

$$\delta(T) = \frac{m^2}{m^2(T)}\left[\delta - (c_2 - c_1)w\right], \quad \mu(T) = \mu\frac{m^2}{m^2(T)}$$

- On the (μ, δ) plane its a straight ray:

$$\delta(T) = \frac{\mu(T)}{\mu}(\delta - P) + P, \quad P = \frac{c_2 - c_1}{c_2 + c_1}$$

- All rays converge at the point $\mathcal{P}_0 = (0, P)$ in the $T \to -\infty$ limit
- Two sheets:
  - 1st sheet $\mu(T)/\mu > 0$
  - 2nd sheet $\mu(T)/\mu < 0$



Extremum, minimum, vacuum

Usual definitions

Extremum : point with $\left.\frac{\partial V}{\partial \phi_i}\right|_{\langle\phi_j\rangle} = \left.\frac{\partial V}{\partial \phi_i^\dagger}\right|_{\langle\phi_j\rangle} = 0$ (extremum conditions)

Minimum : local minimum $\left.\frac{\partial^2 V}{\partial \phi_i \partial \phi_j^\dagger}\right|_{\langle\phi_j\rangle} \geq 0 \quad \Leftrightarrow \quad$ masses $M^2 > 0$

Vacuum = global minimum: local minimum with lowest E.

Classification of extrema in 2HDM

$$\langle\phi_1\rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \qquad \langle\phi_2\rangle = \begin{pmatrix} u \\ v_2 e^{-i\xi} \end{pmatrix}, \quad u, v_1, v_2 > 0$$

$$v^2 = v_1^2 + v_2^2 + u^2, \quad \tan\beta = v_2/v_1$$

Electroweak symmetric (EWs): $u = v_1 = v_2 = 0$

Electroweak symmetry is not broken.
Gauge bosons and fermions are massless.

Neutral extremum: $u = 0$

Electric charge is conserved, 5 physical Higgs particles (h_1, h_2, h_3, and $H^\pm$)

Spontaneous CP violating (sCPv) $\xi \neq 0$ – two degenerate extrema

CP conserving (CPc) $\xi = 0$ – up to 4 such extrema [A.Barroso, R.Santos]

Charge-breaking extremum: $u \neq 0$

Electric charge is not conserved, photon is massive

Evolution of parameters result in change of vacuum states during cooling of universe

1. What are possible sequences of phase states are allowed in 2HDM?
2. Which parameters correspond to a particular sequence of phase transition?
3. How can physical parameters vary during cooling of the Universe?
4. What are the consequences for cosmology?

EWSB phase transition

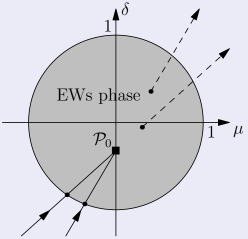
- EW point $\langle \phi_1 \rangle = \langle \phi_2 \rangle = 0$ is the vacuum,
  if $m_{11}^2 < 0, m_{22}^2 < 0 \text{and} m_{11}^2 m_{22}^2 \geq |m_{12}^2|$.
- In (δ, μ) terms: $\delta^2 + \mu^2 < 0$ on the 2nd sheet $\mu(T)/\mu < 0$.

EWSB phase transitions occur at the crossing of circle with the ray.

At $c_1 > 0$ and $c_2 > 0$:
$\mathcal{P}_0$ inside the circle,
single EWSB phase transition

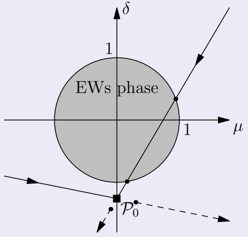
At $c_1 c_2 < 0$:
$\mathcal{P}_0$ outside the circle,
none or two phase transitions

$c_1 > 0$ and $c_2 > 0$



One EWSB transition

$c_1 c_2 < 0$



None or two EWSB transitions

Sectors in λ_i space

The space of all λ_i that satisfy positivity constraints, is subdivided into 4 non-overlapping sectors that allow for certain type of phases and phase transitions.

$$\begin{array}{rccc}
\text{Sector I:} & \Lambda_{345-} > 0 & \lambda_5 < 0 & \lambda_4 + \lambda_5 < 0 \\
\text{Sector II:} & \Lambda_{345-} < 0 & \Lambda_{3-} < 0 & \tilde{\Lambda}_{345-} < 0 \\
\text{Sector III:} & \lambda_5 > \lambda_4 & \lambda_5 > 0 & \tilde{\Lambda}_{345-} > 0 \\
\text{Sector IV:} & \lambda_5 < \lambda_4 & \Lambda_{3-} > 0 & \lambda_4 + \lambda_5 > 0
\end{array}$$

CPc extremum, general

- Can exist in any sector in λ_i space.
- Extremum equations rewritten for v^2 and $\tau = k\tan\beta$:

$$v^2 = m^2(k^2+\tau^2)\frac{1-\delta+\mu\tau}{\lambda_{345}\tau^2+\sqrt{\lambda_1\lambda_2}}\,,$$

$$\sqrt{\lambda_1\lambda_2}\mu\tau^4 + (\Lambda_{345-} - \Lambda_{345+}\delta)\tau^3 - (\Lambda_{345-} + \Lambda_{345+}\delta)\tau - \sqrt{\lambda_1\lambda_2}\mu = 0\,.$$

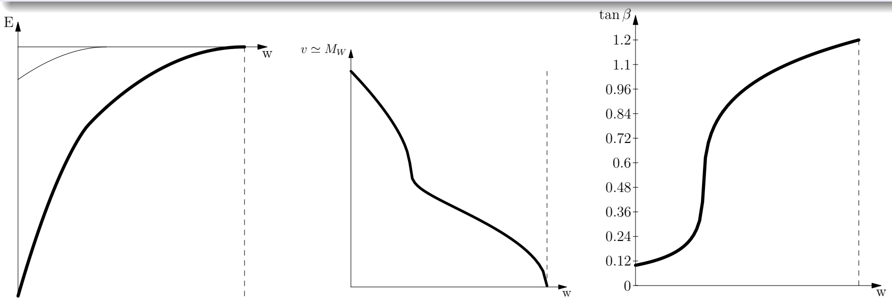
$\Rightarrow$ Up to 4 CPc extrema can exist

- Energy of CPc extremum:

$$\mathcal{E}_{CPc} = -\frac{m^4k^2}{8}\cdot\frac{(1-\delta+\mu\tau)[1-\delta+2\mu\tau+\tau^2(1+\delta)]}{\lambda_{345}\tau^2+\sqrt{\lambda_1\lambda_2}}\,.$$

Sector I: CPc extrema, only EWSB phase transition

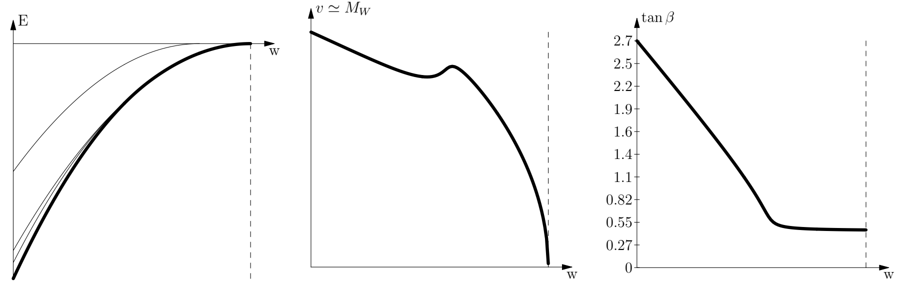
Sector I: $\Lambda_{345-} > 0,\ \lambda_5 < 0,\ \lambda_4 + \lambda_5 < 0$



- Similar situation can occur in any other sector in λ_i space

Sector I: CPc extrema, only EWSB phase transition

Nonmonotonic evolution of $v \simeq M_W$ is possible:



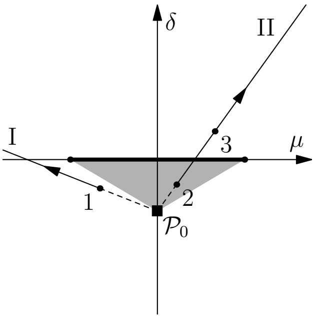
Sector II: First order phase transition is possible

Sector II: $\Lambda_{345-} < 0,\ \Lambda_{3-} < 0,\ \tilde{\Lambda}_{345-} < 0$

- At cetrain temperature $\delta = 0$ $\Rightarrow$ potential is k-symmetric $\Rightarrow$ two degenerate extrema.
- At other temperatures potential is not k-symmetric $\Rightarrow$ extrema are non-degenerate.
- Possibility to have first order phase transition between such extrema.

The (μ, δ) plane. Sector II.

- First order transition segment — very thick line.
- The possible evolution of physical states — rays, directed to the side of growth of temperature.
- Dots — variants of present values of parameters.
- The shaded area cover all present values of (μ, δ) in which in the past first order phase transition occurs.

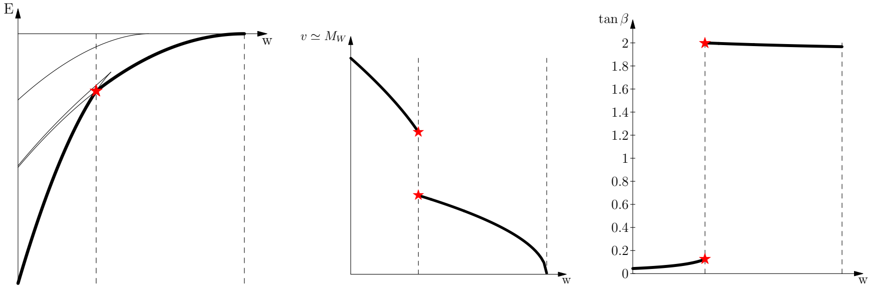


Possible sequences of phase transitions

For present point 2 — EWs → CPc1 → CPc2

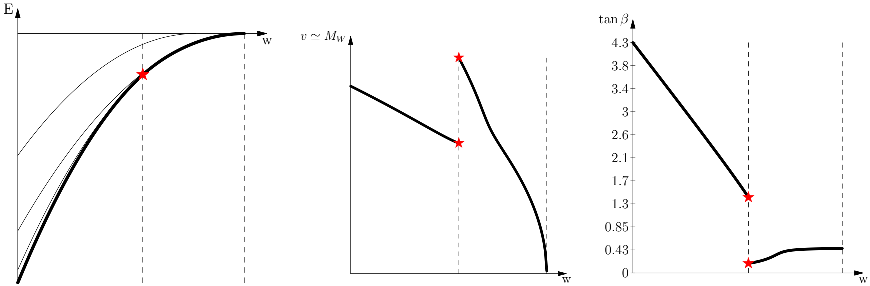
For present points 1 or 3 — EWs → CPc.

Sector II. 1-st order transition



- Discontinuity in v^2 and $\tan\beta$ evolution.
- On the left plot energies of ALL extremum states are presented.

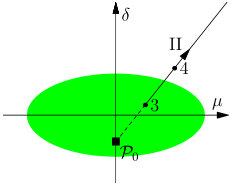
Sector II. Another set of parameters



- Before phase transition v^2 is bigger than after phase transition.

Sector III: $\lambda_5 > \lambda_4,\ \lambda_5 > 0,\ \tilde{\Lambda}_{345-} > 0$

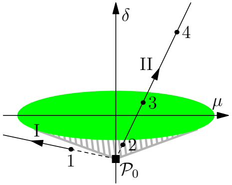
- $\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{-i\xi} \end{pmatrix}$
- Potential is CP symmetric $\Rightarrow$ sCPv extremum is doubly degenerated (in the sign of ξ).
- If sCPv is minimum $\Rightarrow$ it is the vacuum.



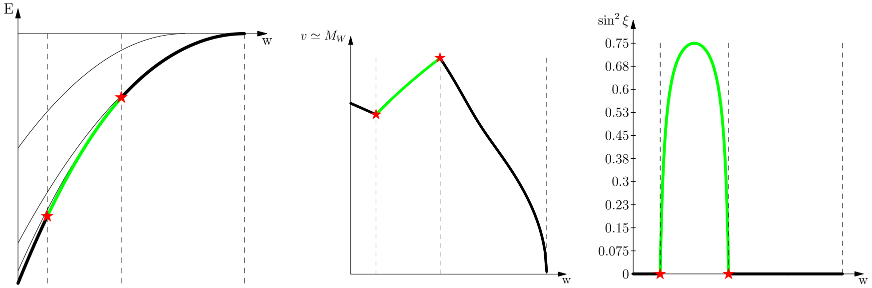
(μ, δ) plot

- Green area – sCPv vacuum.
- Hatched area – modern points, with transition through sCPv phase .
- Possible sequences of phase transitions:

  Point 2 – EWs $\xrightarrow{II}$ CPc1 $\xrightarrow{II}$ sCPv $\xrightarrow{II}$ CPc2
  Point 3 – EWs $\xrightarrow{II}$ CPc1 $\xrightarrow{II}$ sCPv
  Points 1,4 – EWs $\xrightarrow{II}$ CPc1



Sector III: transition through sCPv vacuum

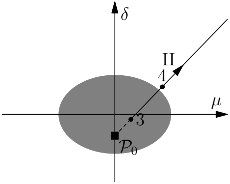


- Order parameter $\sin^2 \xi$
- Small energy gap between sCPv vacuum and CPc extermum.
- Discontinuity in first derivative in transition points.

Sector IV: Charge-breaking vacuum

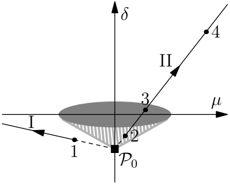
Sector IV: $\lambda_5 < \lambda_4,\ \Lambda_{3-} > 0,\ \lambda_4 + \lambda_5 > 0$

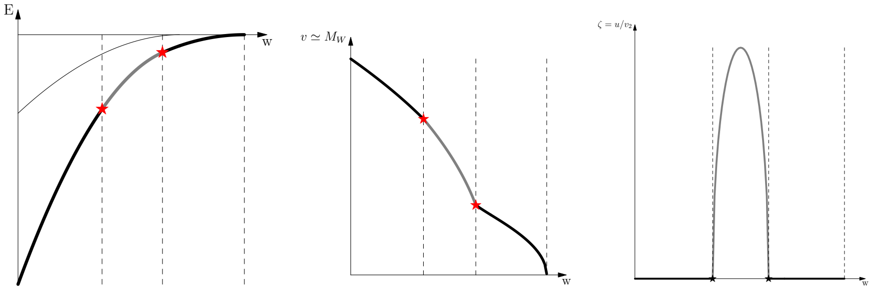
- Electric charge is not conserved
- 4 (not 5) massive Higgs bosons
- 4 (not 3) massive gauge bosons
- If charge breaking extremum is minimum $\Rightarrow$ it is the vacuum.
- Cannot be a modern state



(μ, δ) plot

- Gray area – charge-breaking vacuum.
- Possible sequences of phase transitions:
  Point 2 – EWs $\xrightarrow{II}$ CPc1 $\xrightarrow{II}$ charged $\xrightarrow{II}$ CPc2
  Points 1,4 – EWs $\xrightarrow{II}$ CPc1





- Order parameter: $\zeta = \dfrac{u}{\sqrt{v_1^2 + v_2^2}}$

Possible sequences of phase transitions

- At $T \simeq 10 M_W$ – EWSB phase transition
- Modern state – either sCPv or CPc
- List of all possible sequences:
  - EW $\xrightarrow{II}$ CPc – Any sector
  - EW $\xrightarrow{II}$ CPc $\xrightarrow{I}$ CPc – Sector II
  - EW $\xrightarrow{II}$ CPc $\xrightarrow{II}$ sCPv – Sector III
  - EW $\xrightarrow{II}$ CPc $\xrightarrow{II}$ sCPv $\xrightarrow{II}$ CPc – Sector III
  - EW $\xrightarrow{II}$ CPc $\xrightarrow{II}$ Charged $\xrightarrow{II}$ CPc – Sector IV

Particle mass evolution

- During evolution $v = v(T)$, $v_1 = v_1(T)$, $v_2 = v_2(T)$.
- Assuming Model II for Yukawa.

Masses of particles evolve

$$M_W(T) = M_W \frac{v(T)}{v}, \quad m_d(T) = m_d \frac{v_1(T)}{v_1}, \quad m_u(T) = m_u \frac{v_2(T)}{v_2}$$

Ratio of masses of quarks of the same charge remains unchanged

$$m_t(T) : m_c(T) : m_u(T) = m_t : m_c : m_u$$

Within one generation:

$$\frac{m_u(T)}{M_W(T)} = \frac{m_u}{M_W} \frac{cos\beta(T)}{cos\beta}, \quad \frac{m_d(T)}{M_W(T)} = \frac{m_d}{M_W} \frac{sin\beta(T)}{sin\beta}$$

$$\frac{m_u(T)}{m_d(T)} = \frac{m_u}{m_d} \frac{tan\beta(T)}{tan\beta}$$

Rearrangement of particle mass spectrum

Having in mind Model II:

- Most of considered examples show strong variation of $\tan\beta$. (Stepwise for sector II.)

$\Rightarrow$ In the past quark mass spectrum can be rearranged:

  - ⋆ Either $m_u > m_d \Rightarrow M_n < M_p$
  - ⋆ or $m_c < m_s$
  - ⋆ or even $m_t < m_b$,...

- Note that ratio $m_t : m_c : m_u$ is fixed.

- Very unusual initial state for bariogenesis and galaxies formation.
- Looks like one more phase transition in fermion sector.

First relations to cosmology ?

1. In the standard approach the temperature of phase transition is given by electroweak scale, it is very high. In our model the same is valid for the first EWSB transition. However, the temperature of last phase transition can be low enough (the point, representing modern state can be close to phase separation line).
2. If the charged phase was the intermediate phase, then electroneutrality was strongly violated $\Rightarrow$ strong relative motions after transition to modern neutral phase – ? relation to bariogenesis?. This picture can influence for Microwave Background radiation and structure of proto-galaxies.
3. New phase transitions $\Rightarrow$ new critical fluctuations.
4. Small gaps between vacuum and nearest CPc extrema in sCPv and charged phases $\Rightarrow$ small energy barrier $\Rightarrow$ big fluctuations, bubbles.