A Two-Higgs-Doublet Model with maximal CP symmetry, the MCPM. Yukawa couplings and predictions for the LHC

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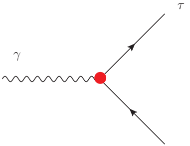
Introduction

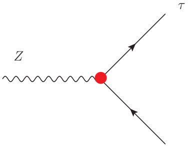
- How did we come to look at models with extended Higgs sector?
- From 1988 onwards we have been studying anomalous $\not{CP}$ couplings in various reactions. (W. Bernreuther, J.P. Ma, M. Diehl, O.N. et al.)
- Example: W. Bernreuther, O.N., P. Overmann, PRD 48, 78 (1993)

$$e^+ + e^- \to \gamma, Z \to \tau^+ + \tau^-$$

- $\not{CP}$ couplings: electric and weak dipole moments, $d_\tau^{\gamma,Z}$ chirality changing, dimension $d = 5$

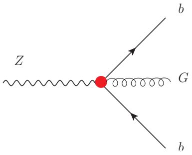
$$\mathcal{L}'_{\text{eff}} = -\frac{i}{2}\bar{\tau}\sigma^{\mu\nu}\gamma_5\tau\left[d_\tau^\gamma(\partial_\mu A_\nu - \partial_\nu A_\mu) + d_\tau^Z(\partial_\mu Z_\nu - \partial_\nu Z_\mu)\right]$$





- Chirality conserving $d = 6$ $\not{CP}$ couplings, e.g. for $Z \to b\bar{b}G$:

$$\mathcal{L}'_{\text{eff}} = \bar{b}\frac{\lambda_a}{2}\gamma^\nu(h_{Vb} + h_{Ab}\gamma_5)bZ^\mu G^a_{\mu\nu}$$



- Generation of such couplings is possible in multi-Higgs models at one loop level.

W. Bernreuther et al. PLB 387, 155 (1996)

Flavour dynamics with general scalar fields.

W. Bernreuther, O.N. EPJC 9, 319 (1999)

$\Rightarrow$ Minimal requirement for such couplings:

- 3 Higgs doublets + 1 charged Higgs singlet.
- If such couplings are found e.g. at a Giga Z facility we have an indication of a very rich Higgs sector.

- This motivated us from 1996 onwards to look in detail into multi-Higgs models, in particular, into the simplest extension of the SM Higgs sector:

  Two-Higgs-Doublet Models

- Collaborators:

  F. Nagel ($\leq$ 2004)
  M. Maniatis
  A. von Manteuffel

Two-Higgs-Doublet Model

- Higgs fields, hypercharge $y = 1/2$

$$\varphi_1(x) = \begin{pmatrix} \varphi_1^+(x) \\ \varphi_1^0(x) \end{pmatrix}, \quad \varphi_2(x) = \begin{pmatrix} \varphi_2^+(x) \\ \varphi_2^0(x) \end{pmatrix}$$

- Lagrangian:

$$\mathcal{L}_\varphi = \sum_{i=1,2} (D_\mu \varphi_i)^\dagger (D^\mu \varphi_i) - V(\varphi_1, \varphi_2)$$

- Potential

$V(\varphi_1, \varphi_2)$ is required to be **gauge invariant** $\Rightarrow$
it must be built from $\varphi_i^\dagger \varphi_j$,
and **renormalisable** $\Rightarrow$ at most quartic in φ_i

- **Gauge invariant functions**: We write

F. Nagel, thesis, available at SPIRES (2004)
Maniatis et al, EPJC 48, 805 (2006)

Nishi, PRD 74, 036003 (2006)

$$\phi(x) = \begin{pmatrix} \varphi_1^+(x) & \varphi_1^0(x) \\ \varphi_2^+(x) & \varphi_2^0(x) \end{pmatrix}$$

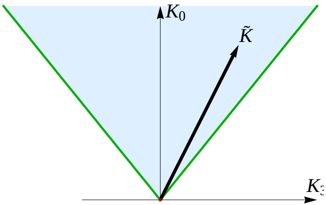
and define gauge invariant quantities:

$$\begin{aligned} \underline{K}(x) &= \phi(x)\phi^\dagger(x) = \begin{pmatrix} \varphi_1^\dagger\varphi_1 & \varphi_2^\dagger\varphi_1 \\ \varphi_1^\dagger\varphi_2 & \varphi_2^\dagger\varphi_2 \end{pmatrix} \\ &= \frac{1}{2}(K_0(x)\mathbb{1}_2 + \mathbf{K}(x)\sigma). \\ \underline{K}(x) &\geq 0 \Rightarrow K_0(x) \geq 0\,,\; K_0^2(x) - \mathbf{K}^2(x) \geq 0 \end{aligned}$$

- The **gauge orbits** of the Higgs fields in the THDM are parametrised by Minkowski-type four-vectors

$$\tilde{K}(x) = \begin{pmatrix} K_0(x) \\ \mathbf{K}(x) \end{pmatrix}$$

lying inside or on the forward light cone in K-space.



- Basis transformations of the Higgs fields

$$\varphi_i'(x) = U_{ij}\,\varphi_j(x)\ ,\ U = (U_{ij}) \in U(2)$$

correspond to **rotations** in K-space.

$$K_0'(x) = K_0(x) \qquad U^\dagger \sigma^a U = R_{ab}(U)\sigma^b$$
$$\mathbf{K}'(x) = R(U)\mathbf{K}(x) \qquad R(U)R^{\mathrm{T}}(U) = \mathbb{1}_3$$
$$\det R(U) = 1$$

Extension to boost transformations: I. Ivanov, PR D75, 035001 (2007)

- The most general potential V can now be written as

$$V = \xi_0 K_0 + \xi \mathbf{K} + \eta_{00} K_0^2 + 2K_0 \eta \mathbf{K} + \mathbf{K}^{\mathrm{T}} E \mathbf{K}.$$

- 14 Parameters, all real: $\xi_0,\ \eta_{00}$,

$$\xi = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix}, \quad \eta = \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}, \quad E = E^{\mathrm{T}} \ (3 \times 3 \text{ Matrix})$$

- By a basis transformation we can diagonalise E:

$$E = \text{diag}\,(\mu_1\ ,\ \mu_2\ ,\ \mu_3).$$

- We have given the precise criteria for V to be stable in a very concise form.
- The global minimum of V determines the vacuum expectation values of the Higgs fields (vevs). We have developed a general algebraic method for determining these vevs. One result is as follows. Let us write

$$\phi_{\mathsf{v}} = \langle \phi(x) \rangle = \begin{pmatrix} \langle \varphi_1^+ \rangle & \langle \varphi_1^0 \rangle \\ \langle \varphi_2^+ \rangle & \langle \varphi_2^0 \rangle \end{pmatrix}\ ,\ \underline{K}_{\mathsf{v}} = \phi_{\mathsf{v}} \phi_{\mathsf{v}}^\dagger = \frac{1}{2}(K_{0\mathsf{v}} \mathbb{1}_2 + \mathbf{K}_{\mathsf{v}} \sigma)$$

- We have the correct EWSB

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \longrightarrow SU(3)_c \otimes U(1)_{em}$$

if and only if

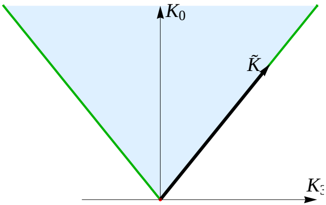
$$\tilde{K}_{\mathrm{v}} = \begin{pmatrix} K_{0\mathrm{v}} \\ \mathbf{K}_{\mathrm{v}} \end{pmatrix} \qquad \text{with } K_{0\mathrm{v}} = |\mathbf{K}_{\mathrm{v}}| > 0$$

- That is, we must have $\tilde{K}_{\mathrm{v}} \neq 0$ and $\tilde{K}_{\mathrm{v}}$ must be **on** the forward light cone.

In a suitable basis:

$$\tilde{K}_{\mathrm{v}} = \frac{\mathrm{v}_0^2}{2} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$\mathrm{v}_0 \approx 246$ GeV (standard vev)



CP violation in the Higgs sector

Gunion, Haber, PRD 72, 095002 (2005)
Nishi, PRD 74, 036003 (2006)
Maniatis et al. EPJC 57, 719 (2008)

Ferreira, Haber, Silva, PRD 79 (2009)

- Standard CP transformation:

$$CP_s : \varphi_i(x) \longrightarrow \varphi_i^*(x') \ , \ x = \begin{pmatrix} x^0 \\ \mathbf{x} \end{pmatrix} \ , \ x' = \begin{pmatrix} x^0 \\ -\mathbf{x} \end{pmatrix}$$

- Generalised CP transformations:

Ecker et al. NPB 191, 465 (1981)
Bernabeu et al. PL 169B, 243 (1986)

Ecker et al. J.Phys. A 20, L807 (1987)

$$\begin{aligned} CP_g \quad : \qquad & \varphi_i(x) \rightarrow U_{ij}\varphi_j^*(x') \ , \ U = (U_{ij}) \in U(2) \\ & K_0(x) \rightarrow K_0(x'), \\ & \mathbf{K}(x) \rightarrow \bar{R}\mathbf{K}(x') \end{aligned}$$

$\bar{R}$ is an **improper rotation**: $\bar{R}\bar{R}^{\mathrm{T}} = \mathbb{1}_3 \ , \ \det \bar{R} = -1$

- We require that $CP_g \circ CP_g$ gives the identity transformation for the gauge invariant functions $\tilde{K}(x)$.

$$\Rightarrow \bar{R}\bar{R} = \mathbb{1}_3 \Rightarrow \bar{R} = \bar{R}^{\mathrm{T}} \Rightarrow$$

- $\bar{R}$ can be diagonalised by a basis change of the Higgs fields.

$$\bar{R}' = R(U)\bar{R}R^{\mathrm{T}}(U) = \text{diagonal}$$
$$\bar{R}'\bar{R}' = \mathbb{1}_3\ ,\ \det \bar{R}' = -1$$

$$(i) \quad \bar{R}' = -\mathbb{1}_3 = \text{diag}(-1,-1,-1)$$

$$(ii) \quad \bar{R}' = \left\{ \begin{array}{ll} \text{diag}\ (-1,1,1) & =: R_1 \\ \text{diag}\ (1,-1,1) & =: R_2 \\ \text{diag}\ (1,1,-1) & =: R_3 \end{array} \right\} \text{ These are equivalent by a basis change}$$

$$\begin{aligned} \mathcal{L}_\varphi &= \sum_{i=1,2} (D_\mu \varphi_i)^\dagger (D^\mu \varphi_i) - V(\varphi_1, \varphi_2), \\ V &= \xi_0 K_0 + \xi \mathbf{K} + \eta_{00} K_0^2 + 2 K_0 \eta \mathbf{K} + \mathbf{K}^\mathrm{T} E \mathbf{K}. \end{aligned}$$

- We have CP_g invariance of $\mathcal{L}_\varphi$ if and only if

$$\bar{R}\xi = \xi \ , \ \bar{R}\eta = \eta \ , \ \bar{R}E\bar{R}^\mathrm{T} = E$$

- A CP_g invariance of $\mathcal{L}_\varphi$ is spontaneously broken if and only if

$$\bar{R}\mathbf{K}_\mathrm{v} \neq \mathbf{K}_\mathrm{v}$$

Theories with type (i) CP_g invariance

$$CP_g^{(i)} \qquad \varphi_i(x) \to \epsilon_{ij}\varphi_j^*(x')\ ,\ \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\mathbf{K}(x) \to -\mathbf{K}(x')\ ,\ \bar{R} = -\mathbb{1}_3$$

(For the fields φ_i this transformation was also considered in Davidson et al., PRD 72, 035004 (2005))

- Conditions for the parameters of the potential:

$$\begin{aligned} -\xi &= \xi\ ,\ -\eta = \eta \Rightarrow \xi = 0\ ,\ \eta = 0 \\ V &= \xi_0 K_0 + \eta_{00} K_0^2 + \mathbf{K}^{\mathrm{T}} E \mathbf{K}, \\ E &= \text{diag}\ (\mu_1, \mu_2, \mu_3)\ ,\ \mu_1 \geq \mu_2 \geq \mu_3 \end{aligned}$$

- Stability and correct EWSB if and only if

$$\eta_{00} > 0\ , \quad \mu_a + \eta_{00} > 0 \quad (a = 1, 2, 3), \quad \xi_0 < 0, \quad \mu_3 < 0$$

- These models have automatically **three additional type (ii)** CP_g symmetries corresponding to the reflections on the coordinate planes in K space.
  $CP^{(ii)}_{g,a}$, reflection R_a , $a = 1, 2, 3$
- Example:

$$R_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad R_3\xi = \xi \text{ for } \xi = 0, \text{ etc.}$$

- EWSB always breaks $CP^{(i)}_g$ and $CP^{(ii)}_{g,3}$ spontaneously.
  Remember:

$$\mathbf{K}_{\mathrm{v}} = \frac{\mathrm{v}_0^2}{2} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \Rightarrow -\mathbf{K}_{\mathrm{v}} \neq \mathbf{K}_{\mathrm{v}} \, , \; R_3\mathbf{K}_{\mathrm{v}} \neq \mathbf{K}_{\mathrm{v}}$$

- vev: $\mathrm{v}_0^2 = \frac{-\xi_0}{\eta_{00}+\mu_3}$
- Spectrum of physical Higgs particles:

neutral:

$$\begin{aligned} \rho' : \quad & m_{\rho'}^2 &= & \; 2(-\xi_0) \\ h' : \quad & m_{h'}^2 &= & \; 2\mathrm{v}_0^2(\mu_1 - \mu_3) \\ h'' : \quad & m_{h''}^2 &= & \; 2\mathrm{v}_0^2(\mu_2 - \mu_3) \end{aligned}$$

charged:

$$H^\pm : \quad m_{H^\pm}^2 = 2\mathrm{v}_0^2(-\mu_3)$$

The maximally-CP-symmetric model, MCPM

M. Maniatis et al. EPJC 57, 739 (2008), JHEP 05: 028 (2009)

- We require as a principle that the complete Lagrangian

$$\mathcal{L} = \mathcal{L}_{FB} + \mathcal{L}_{Yuk} + \mathcal{L}_{\varphi}$$

should respect all four generalised CP symmetries,

$$CP_g^{(i)} \ , \ CP_{g,a}^{(ii)} \quad a = 1, 2, 3.$$

- This leads to $\mathcal{L}_{\varphi}$ as just discussed, gives no restriction for $\mathcal{L}_{FB}$ but restricts $\mathcal{L}_{Yuk}$ severely.
- First we have to discuss the CP_g transformations of the fermions. For ease of presentation we give only formulae for leptons.

One fermion family

$$CP_g : \begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix} \longrightarrow e^{i\xi_1}\gamma^0 S(C) \begin{pmatrix} \bar{\nu}^{\mathrm{T}}_{eL}(x') \\ \bar{e}^{\mathrm{T}}_L(x') \end{pmatrix}$$
$$e_R(x) \longrightarrow e^{i\xi_2}\gamma^0 S(C) \bar{e}^{\mathrm{T}}_R(x')$$

$$S(C) = i\gamma^2\gamma^0$$
$$\mathcal{L}_{Yuk} = -\bar{e}_R(x) c_{l,i} \varphi_i^\dagger(x) \begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix} + h.c.$$

- Invariance under $CP_g^{(i)}$ requires:

$$\left.\begin{aligned} c^*_{l,1} &= -e^{i(\xi_1-\xi_2)} c_{l,2} \\ c_{l,2} &= e^{-i(\xi_1-\xi_2)} c^*_{l,1} \end{aligned}\right\} \Rightarrow c_{l,1} = c_{l,2} = 0$$

- **$\mathcal{L}_{Yuk} \neq 0$ requires family replication.**

Two fermion families

- We consider the second and third families

$$l_2 \equiv \mu \; , \; \nu_2 \equiv \nu_\mu \; , \; l_3 \equiv \tau \; , \; \nu_3 \equiv \nu_\tau$$

- Most general ansatz:

$$\begin{aligned} \mathcal{L}_{Yuk} &= -\bar{l}_{\alpha R}(x) C^{(j)}_{\alpha\beta} \varphi^\dagger_j(x) \begin{pmatrix} \nu_{\beta L}(x) \\ l_{\beta L}(x) \end{pmatrix} + h.c. \\ C^{(j)} &= \left(C^{(j)}_{\alpha\beta} \right) \quad \text{arbitrary complex } 2 \times 2 \text{ matrices} \\ j &\in \{1,2\} \; , \; \alpha, \beta \in \{2,3\} \end{aligned}$$

- Ansatz for CP_g transformations of fermions:

$$\begin{aligned} CP_g: \quad \begin{pmatrix} \nu_{\alpha L}(x) \\ l_{\alpha L}(x) \end{pmatrix} \quad &\longrightarrow U_{L\alpha\beta}\gamma^0 S(C) \begin{pmatrix} \bar{\nu}^{\mathrm{T}}_{\beta L}(x') \\ \bar{l}^{\mathrm{T}}_{\beta L}(x') \end{pmatrix}, \\ l_{\alpha R}(x) \quad &\longrightarrow U_{R\alpha\beta}\gamma^0 S(C) \bar{l}^{\mathrm{T}}_{\beta R}(x'), \\ U_L = (U_{L\alpha\beta}) \in U(2) \quad , \quad & U_R = (U_{R\alpha\beta}) \in U(2). \end{aligned}$$

- We require $CP_g \circ CP_g$ to give back the original fields up to phases,

$$\Rightarrow \quad -U_L U_L^* = \pm \mathbb{1}_2 \, , \; -U_R U_R^* = \pm \mathbb{1}_2$$

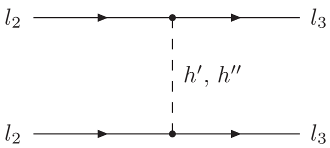
- U_L and U_R are in general different of each other and depend on the CP_g considered.

It is now straightforward but lengthy to work out the forms of $\mathcal{L}_{Yuk}$ which allow implementation of **all four CP_g symmetries**, $CP_g^{(i)}$, $CP_{g,a}^{(ii)}; a = 1, 2, 3$. We find the following results:

- Unequal and non zero masses for μ and τ lead necessarily to large lepton-flavour changing neutral currents, FCNCs.

$$\left.\begin{array}{l} m_\mu \neq 0 \\ m_\tau \neq 0 \\ m_\mu \neq m_\tau \end{array}\right\} \Longrightarrow$$

$$\mu^- + \mu^- \to \tau^- + \tau^-$$



- We require absence of FCNCs on phenomenological grounds. We have then only the possibilities:
- Equal masses: $m_\mu = m_\tau \neq 0$
- A **large mass hierarchy:** $m_\mu = 0\ ,\ m_\tau \neq 0$
  This leads to a highly symmetric $\mathcal{L}_{Yuk}$

$$\mathcal{L}_{Yuk} = -\frac{m_\tau \sqrt{2}}{\mathrm{v}_0} \left\{ \bar{\tau}_R(x)\varphi_1^\dagger(x) \begin{pmatrix} \nu_{\tau L}(x) \\ \tau_L(x) \end{pmatrix} - \bar{\mu}_R(x)\varphi_2^\dagger(x) \begin{pmatrix} \nu_{\mu L}(x) \\ \mu_L(x) \end{pmatrix} + h.c. \right\}$$

- After EWSB:

$$
\begin{aligned}
\mathcal{L}_{Yuk} = & - m_\tau \left(1 + \frac{\rho'(x)}{\mathrm{v}_0}\right) \bar{\tau}(x)\tau(x) \\
& + \frac{m_\tau}{\mathrm{v}_0} h'(x)\bar{\mu}(x)\mu(x) + i\frac{m_\tau}{\mathrm{v}_0} h''(x)\bar{\mu}(x)\gamma_5\mu(x) \\
& + \left\{ \frac{m_\tau}{\mathrm{v}_0\sqrt{2}} H^+(x)\bar{\nu}_\mu(x)(1+\gamma_5)\mu(x) + h.c. \right\}
\end{aligned}
$$

- Couplings of physical Higgs particles:

ρ'	couples exclusively to	τ
$h', h'', H^\pm$	couple exclusively to	μ, ν_μ

- The inclusion of quarks is straightforward.

Properties and predictions of the MCPM

- Two-Higgs-Doublet model
- First fermion family (ν_e, e, u, d) is uncoupled to Higgs fields.
- Second and third family are coupled to the Higgs fields with four $\mathrm{CP_g}$ invariances:
  $\mathrm{CP}_g^{(i)}$, $\mathrm{CP}_{ga}^{(ii)}$, $a = 1, 2, 3$.

- The symmetries leading to the highly symmetric $\mathcal{L}_{Yuk}$ are

$$
\begin{aligned}
CP_g : \qquad & \varphi_i(x) \longrightarrow W_{ij}\varphi_j^*(x')\ ,\ W \in U(2) \\
& \psi_{\alpha L}(x) \longrightarrow U_{L\alpha\beta}\gamma^0 S(C)\bar{\psi}^{\mathrm{T}}_{\beta L}(x') \\
& \psi_{\alpha R}(x) \longrightarrow U_{R\alpha\beta}\gamma^0 S(C)\bar{\psi}^{\mathrm{T}}_{\beta R}(x') \qquad \alpha, \beta \in \{2,3\}
\end{aligned}
$$

CP_g	W	U_R	U_L
$CP_g^{(i)}$	ϵ	ϵ	σ^1
$CP_{g,1}^{(ii)}$	σ^3	$-\sigma^3$	$\mathbb{1}_2$
$CP_{g,2}^{(ii)}$	$\mathbb{1}_2$	$\mathbb{1}_2$	$\mathbb{1}_2$
$CP_{g,3}^{(ii)}$	σ^1	$-\sigma^1$	σ^1

- The third family (τ, t, b) is massive, the second (μ, c, s) and first (e, u, d) families are massless
(at tree level in the symmetry limit).
- Experiment:

$$\frac{m_e}{m_\tau} = 2.9 \times 10^{-4}\,, \quad \left.\frac{m_u}{m_t}\right|_{\mathrm{v}_0} = 9.9 \times 10^{-6}\,, \quad \left.\frac{m_d}{m_b}\right|_{\mathrm{v}_0} = 1.0 \times 10^{-3}$$

$$\frac{m_\mu}{m_\tau} = 5.9 \times 10^{-2}\,, \quad \left.\frac{m_c}{m_t}\right|_{\mathrm{v}_0} = 3.6 \times 10^{-3}\,, \quad \left.\frac{m_s}{m_b}\right|_{\mathrm{v}_0} = 1.8 \times 10^{-2}$$

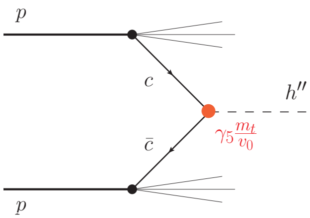
- The CKM matrix $V = \mathbb{1}_3$
(at tree level in the symmetry limit)
- Experiment:

$$\begin{pmatrix} |V_{11}| & |V_{12}| & |V_{13}| \\ |V_{21}| & |V_{22}| & |V_{33}| \\ |V_{31}| & |V_{32}| & |V_{33}| \end{pmatrix} = \begin{pmatrix} 0.974 & 0.227 & 0.004 \\ 0.227 & 0.973 & 0.042 \\ 0.008 & 0.042 & 0.999 \end{pmatrix}$$

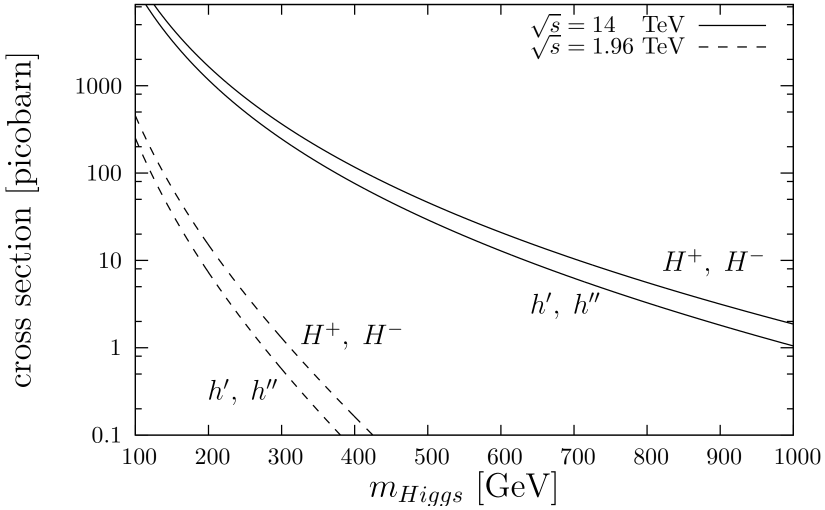
Predictions to be checked at LHC:

- Physical Higgs particles: $\rho', h', h'', H^{\pm}$.
- ρ' couples exclusively to (τ, t, b) family.
  ρ' behaves $\approx$ SM Higgs.
- $h', h'', H^{\pm}$ couple exclusively to (μ, c, s) family with strengths given by the third generation fermion masses.
  h': scalar couplings to fermions,
  h'': pseudoscalar couplings,
  $m_{h'} > m_{h''}$
- The (e, u, d) family is (at tree level in the symmetry limit) uncoupled to Higgs particles.

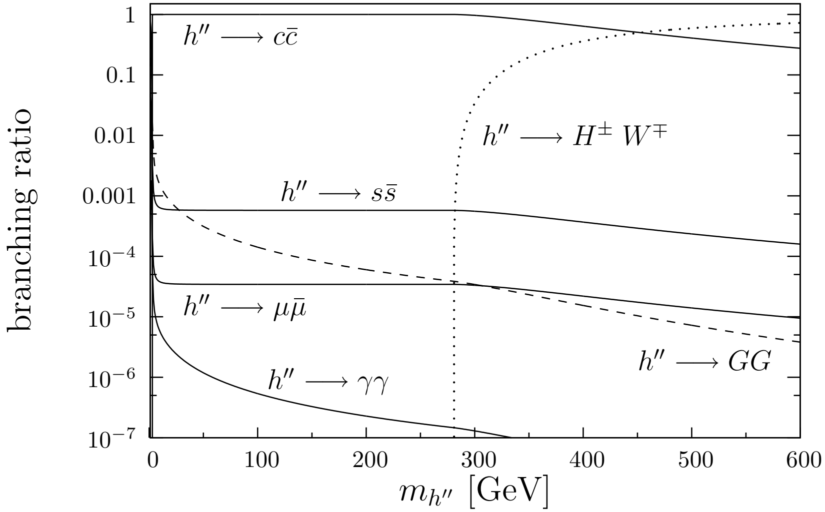
- We predict large cross sections for $h', h'', H^{\pm}$ production via a Drell–Yan type mechanism at the LHC.
  Example:



- h'' decay mostly to $c\bar{c}$ jets.
- $Br(h'' \to \mu\bar{\mu}) = 3 \cdot 10^{-5}$.



- $\int \mathcal{L}dt = 30$ fb^{-1}, $m_{h''} = 250$ GeV
  $\Rightarrow$ $30,000,000$ h'' produced, 900 $\mu^+\mu^-$ pairs.



- Assumption: $m_{H^{\pm}} = 200$ GeV

Conclusions

- **Optimist's view:** The theory has some features which look like a first approximation to what is observed in Nature.
- **Pessimist's view:** We got only a caricature of Nature.
- **Realist's view:** The theory is at least falsifiable.