Physical parameters and basis transformations in the Two-Higgs-Doublet model

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Topics

The scalar sector of SM

- The less known sector of SM (less constrained)
- One Higgs doublet
- The “Higgs” is yet to be discovered

- The Higgs mechanism is responsible to
  - give masses to all massive particles: fermions, gauge bosons;
  - hide the $SU(2)_L \otimes U(1)_Y$ symmetry.
- SM could be an effective low energy theory of a more fundamental one
- MSSM, GUTs, extra dimensions, . . . , etc. predict more fundamental scalars.
- Phenomenological necessity:
  - new sources of CP violation (bariogenesis, leptogenesis)
  - new nearly stable, weakly interacting particle(s) (darkmatter)

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  ... maybe in the LHC
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Extending the scalar sector

- With more than one doublet it is possible to break spontaneously
  - CP symmetry (SCPV, Lee, 1973)
  - EM symmetry (charge breaking vacuum)
- Technical difficulties:
  - more than one local minima with different symmetries
  - reparametrization freedom (horizontal space)
- 2HDMs (MSSM):
  - SCPV without NFC and with real CKM (FCNC)
  - neutral vacuum deeper when exists (Ferreira, Santos, Barroso)
  - Minkowski structure (Ivanov)
- 3HDMs: SCPV with NFC but with real CKM (Weinberg, Branco)
- What about more? Manageable? (huge parameter space)

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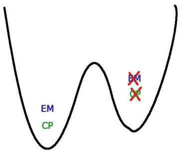

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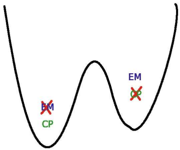


Horizontal space (HS)

- Identical N Higgs doublets:
  - same gauge quantum numbers,
  - masses different in general (after EWSB).
- “Rotations” in horizontal space (HS) are physically irrelevant: reparametrization invariance $[SU(N)_H]$
- Unless hidden quantum numbers exist.
  e.g. family (flavor) symmetries: continuous or discrete
- The theory is not invariant by horizontal transf.
- In the 3 family quarks and leptons $\rightarrow$ the flavor problem: family replication $\rightarrow$ mixing $\rightarrow$ CP violation
- Physical quantities are reparametrization invariant
- CP violation effect is rephasing invariant in SM (Jarslkog inv.).
- Less physical parameters than initially supposed
- Before v.s. after EWSB? Physical parameters?
- How do we parametrize the general cases?

The general Two-Higgs-doublet model (2HDM)

- Simplest extension $\rightarrow$ complete study (still difficult)
- MSSM $\rightarrow$ 2HDM
- 2 Higgs doublets $\Phi_a = (\phi_{a1}, \phi_{a2})^{\mathrm{T}}$, $a = 1, 2$.

General potential

$$\begin{aligned} V(\Phi) &= \mu_{11}\Phi_1^\dagger\Phi_1 + \mu_{22}\Phi_2^\dagger\Phi_2 + (\mu_{12}\Phi_1^\dagger\Phi_2 + h.c.) \\ &\quad + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_1^\dagger\Phi_1)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4|\Phi_1^\dagger\Phi_2|^2 \\ &\quad + \left\{\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 + [\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)](\Phi_1^\dagger\Phi_2) + h.c.\right\} \end{aligned}$$

- $\{\mu_{11}, \mu_{22}, \lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ real
- $\{\mu_{12}, \lambda_5, \lambda_6, \lambda_7\}$ complex

$$\Bigg\} \underbrace{2 + 2\times 1}_{\mu_{ab}} + \underbrace{4 + 2\times 3}_{\lambda_a} = 14$$

Basis transformations in the 2HDM

Basis transf.:

$$\Phi_a \to \Phi'_a = U_{ab}\Phi_b$$

- $U \in SU(2)_H$ $[U(2)_H]$
- $V(\Phi';\mu,\lambda) = V(\Phi;\mu',\lambda')$ (reparametrization)
- $\mu_{ab} \to \mu'_{ab}$, $\lambda_i \to \lambda'_i$ $\sim$ tensors of **2**

- Not all potentials $V(\Phi;\mu,\lambda)$ are distinct
- Choose a basis: 14-$D(SU(2))$=11 essential parameters
- Gauge freedom: use gauge invariant variables

$$\Phi^\dagger_a\Phi_b \sim \bar{\mathbf{2}} \otimes \mathbf{2} \sim \text{adjoint} \oplus \mathbf{1}$$

- Reparametrization: use variables irreducible by $SU(2)_H$

$$r^\mu(\Phi) \equiv \tfrac{1}{2}\sigma^\mu_{ab}\Phi^\dagger_a\Phi_b$$

$\sigma^\mu \equiv (\mathbb{1}_2, \sigma^i)$, $\quad i = 1,2,3$

- $\mu = 0 \sim$ singlet
- $\mu = i \sim$ adjoint ($\mathbb{R}^3$ vector)

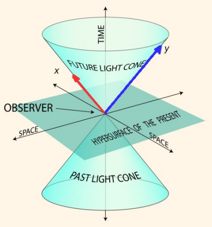
Minkowski structure: variable space

- r^μ satisfy

$$r_\mu r^\mu = \phi^*_{a1}\phi_{a1}\phi^*_{b2}\phi_{b2} - |\phi^*_{a1}\phi_{a2}|^2 \geq 0$$

Thus $r^\mu(\Phi)$ live inside and on the future lightcone in a Minkowski spacetime $\mathbb{R}^{1,3}$:

$$LC^\uparrow \equiv \{x^\mu \in \mathbb{R}^{1,3} \,|\, x^\mu x_\mu \geq 0, x_0 > 0\}$$



$LC^\uparrow$

- $r^\mu(\Phi)$ spans $LC^\uparrow$

- It is natural to extend

$$\begin{array}{ccccc} \Phi_a : & SU(2)_H & \rightarrow & SL(2,c) \\ & \wr & & \wr \\ r^\mu : & \text{rotations} & \rightarrow & \text{Lorentz} \end{array}$$

Analysis of (Ivanov's talk)

- bounded below condition
- number and type of minima
- remaining symmetries of minima

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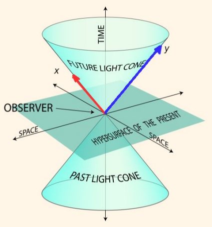

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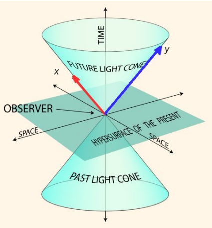

$LC^\uparrow$

- $r^\mu(\Phi)$ spans $LC^\uparrow$ ($[\Phi] \sim 4(N-1)$ and $[r^\mu] \sim N^2$)

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2HDM: parameter space and special bases

General potential

$$V(\Phi) = V(r^\mu) = \underbrace{M_\mu r^\mu}_{V_2} + \underbrace{\tfrac{1}{2}\Lambda_{\mu\nu} r^\mu r^\nu}_{V_4}$$

scalars M_0, Λ_{00}

vectors $\{M_i\}, \{\Lambda_{0i}\}$

tensor $\tilde\Lambda = \{\Lambda_{ij}\}$

$$\{M_\mu\} = (\mu_{11}+\mu_{22}, 2\mathrm{Re}\,\mu_{12}, -2\mathrm{Im}\,\mu_{12}, \mu_{11}-\mu_{22})$$

$$\{\Lambda_{\mu\nu}\} = \left(\begin{array}{c|ccc} \bar\lambda+\lambda_3 & \mathrm{Re}(\lambda_6+\lambda_7) & -\mathrm{Im}(\lambda_6+\lambda_7) & \Delta\lambda/2 \\ \hline \mathrm{Re}(\lambda_6+\lambda_7) & \lambda_4+\mathrm{Re}\lambda_5 & -\mathrm{Im}\lambda_5 & \mathrm{Re}(\lambda_6-\lambda_7) \\ -\mathrm{Im}(\lambda_6+\lambda_7) & -\mathrm{Im}\lambda_5 & \lambda_4-\mathrm{Re}\lambda_5 & -\mathrm{Im}(\lambda_6-\lambda_7) \\ \Delta\lambda/2 & \mathrm{Re}(\lambda_6-\lambda_7) & -\mathrm{Im}(\lambda_6-\lambda_7) & \bar\lambda-\lambda_3 \end{array}\right)$$

$\bar\lambda = (\lambda_1+\lambda_2)/2,\ \Delta\lambda = \lambda_1-\lambda_2$

- Special basis **1**: $M_\mu = (M_0, 0, 0, M_3)$
- Special basis **2**: diagonal $\tilde\Lambda$

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- Special basis **1**: $M_\mu = (M_0, 0, 0, M_3)$ $\sim$ 14-2=12 param.
- Special basis **2**: diagonal $\tilde{\Lambda}$ $\sim$ 14-3=11 param.

2HDM: Conditions for CP invariance for the potential

- Canonical basis (real): $\tilde{\Lambda}' = O\tilde{\Lambda}O^{\mathsf{T}} = \mathrm{diag}(\tilde{\lambda}_i)$, $O \in SO(3)$

$$V(r', M', \Lambda') = M_0' r^{0\prime} + \Lambda_{00}' (r^{0\prime})^2 + M_i' r^{i\prime} + 2\Lambda_{0i}' r^{0\prime} r^{i\prime} + \tilde{\lambda}_i (r^{i\prime})^2$$

- Canonical CP transf.:
  - $\Phi_a' \xrightarrow{\text{CPC}} \Phi_a'^{*}$
  - $r^{i\prime} \xrightarrow{\text{CPC}} (I_2)^i_j r^{j\prime}$

  canonical CP reflection!
- $V(r; M, \Lambda)$ is CP inv. if, and only if,

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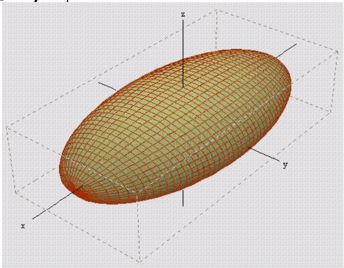

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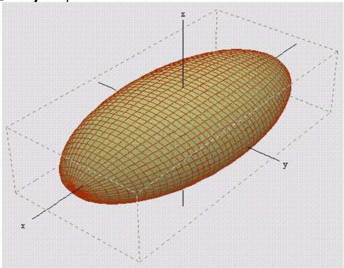

or ...

... in terms of $SU(2)_H$ invariants

Pseudoscalar inv.:

$$I(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3) = \varepsilon_{ijk} v_{1i} v_{2j} v_{3k} = (\mathbf{v}_1 \times \mathbf{v}_2) \cdot \mathbf{v}_3$$

- $I_M = I(\mathbf{M}, \tilde{\Lambda}\mathbf{M}, \tilde{\Lambda}^2\mathbf{M})$
- $I_{\Lambda_0} = I(\mathbf{\Lambda}_0, \tilde{\Lambda}\mathbf{\Lambda}_0, \tilde{\Lambda}^2\mathbf{\Lambda}_0)$
- $I_1 = I(\mathbf{M}, \mathbf{\Lambda}_0, \tilde{\Lambda}\mathbf{M})$
- $I_2 = I(\mathbf{M}, \mathbf{\Lambda}_0, \tilde{\Lambda}\mathbf{\Lambda}_0)$

(A) If $\mathbf{M} \times \mathbf{\Lambda}_0 \neq 0$,
then $I_1 = 0$ and $I_2 = 0 \Leftrightarrow V(r)$ is CP inv.
CP reflection direction: $\mathbf{M} \times \mathbf{\Lambda}_0$ (eigenvector of $\tilde{\Lambda}$)

(B) If $\mathbf{M} \parallel \mathbf{\Lambda}_0$,
then $I_M = 0$ (or $I_{\Lambda_0} = 0$) $\Leftrightarrow V(r)$ is CP inv.
CP reflection direction: $\mathbf{M} \times \tilde{\Lambda}\mathbf{M}$ ($\neq 0$)
eigenvector of $\tilde{\Lambda} \perp \mathbf{M}$ ($\tilde{\Lambda}\mathbf{M} \parallel \mathbf{M}$)

(C) All pseudoscalar invariants of superior order are null if conditions (A) or (B) are satisfied

- For a CP invariant potential to have SCPV, it is necessary and sufficient to have the breaking direction (vacuum) out of the CP principal plane.

EW symmetry breaking: **another special basis**

- Nontrivial minimum $\langle\Phi\rangle \neq 0 \leftrightarrow \langle r^\mu\rangle \neq 0$
- Parametrization

$$\langle\Phi_1\rangle = \begin{pmatrix}\langle u_1\rangle\\ \langle w_1\rangle\end{pmatrix} \qquad \langle\Phi_2\rangle = \begin{pmatrix}\langle u_2\rangle\\ \langle w_2\rangle\end{pmatrix}$$

- Charge breaking (CB): $\langle u\rangle \neq \alpha\langle w\rangle$ — $\langle r^\mu\rangle$ inside $LC^\uparrow$
- Neutral (N): $\langle u\rangle = (0,0)$, $\langle w\rangle \neq (0,0)$ — $\langle r^\mu\rangle$ on $LC^\uparrow$
- Higgs basis: $\langle w\rangle = (0, \frac{v}{\sqrt{2}})$

Extremum equation:

$$\frac{\partial}{\partial\phi^*_{ai}} V(\Phi) = \mathbb{M}_{ab}\phi_{bi} = 0 \qquad \begin{cases}\langle\mathbb{M}u\rangle = 0\\ \langle\mathbb{M}w\rangle = 0\end{cases}$$

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The potential after EWSB

If there is a nontrivial neutral vacuum, the potential after EWSB is

$$V(\Phi + \langle\Phi\rangle) = V_0 + V_2 + V_3 + V_4$$

$$\begin{aligned} V_2 &= \Phi^\dagger \langle\mathbb{M}\rangle \Phi + \tfrac{1}{2}\Lambda_{\mu\nu} s^\mu s^\nu \\ V_3 &= \Lambda_{\mu\nu} s^\mu r^\nu \\ V_4 &= \tfrac{1}{2}\Lambda_{\mu\nu} r^\mu r^\nu \end{aligned}$$

where

$$\begin{aligned} \Phi &\to \Phi + \langle\Phi\rangle \\ r^\mu &\to r^\mu + \langle r^\mu\rangle + s^\mu \end{aligned}$$

$$\begin{aligned} s^\mu &= \tfrac{1}{2}\langle w\rangle^\dagger \sigma^\mu w + \ h.c. \\ &\sim \qquad w_i \end{aligned}$$

- Condition for local minimum: mass matrix $\sim \langle\mathbb{M}\rangle + \Lambda$ is positive semidefinite
- The masses of the fields u_1, u_2 are defined by $\langle\mathbb{M}\rangle$ (charged)
- For μ, ν such that $s^\mu = 0$, $\Lambda_{\mu\nu}$ only contributes to the interactions but not to the mass matrix

The potential after EWSB

If there is a nontrivial neutral vacuum, the potential after EWSB is

$$V(\Phi + \langle\Phi\rangle) = V_0 + V_2 + V_3 + V_4$$

$$\begin{aligned} V_2 &= \Phi^\dagger \langle\mathbb{M}\rangle \Phi + \tfrac{1}{2}\Lambda_{\mu\nu} s^\mu s^\nu \\ V_3 &= \Lambda_{\mu\nu} s^\mu r^\nu \\ V_4 &= \tfrac{1}{2}\Lambda_{\mu\nu} r^\mu r^\nu \end{aligned}$$

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Physical charged Higgs basis (PCH) or Higgs basis

Physical fields of Φ_1

$$\Phi_1 \to \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(t_1 - i\, t_2) \end{pmatrix}$$

Physical fields of Φ_2

$$\Phi_2 \to \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} + \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(-t_3 + i\, G^0) \end{pmatrix}$$

- Physical fields: 1 charged $H^\pm$, 3 neutral t_1, t_2, t_3
- Goldstone bosons: 1 charged $G^\pm$, 1 neutral G^0
- $s^\mu = \frac{v}{2}(-t_3, \vec{t}) \to$ no trilinear interactions with Goldstones!
- Mass term: $V_2 = m^2 H^+ H^- + \frac{1}{2}(\mathcal{M}_N)_{ij} t_i t_j$

$$\mathcal{M}_N = m^2 \mathrm{diag}(1,1,0) + \frac{v^2}{4}\tilde{\Lambda} + \frac{v^2}{4}\begin{pmatrix} 0 & 0 & -\Lambda_{01} \\ 0 & 0 & -\Lambda_{02} \\ -\Lambda_{01} & -\Lambda_{02} & \Lambda_{00} - 2\Lambda_{03} \end{pmatrix}$$

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Basis transformations after EWSB

- $SU(2)_H$: $\boxed{\Phi_a \to \Phi'_a = U_{ab}\Phi_b} \longleftrightarrow \boxed{u' = U\,u, \quad w' = U\,w}$
- Covariant definition for H^+, t_i, G^+, G^0?

$$w = \frac{1}{\sqrt{2}}\begin{pmatrix} t_1 - it_2 \\ -t_3 + iG^0 \end{pmatrix} \not\longrightarrow \quad w' = \frac{1}{\sqrt{2}}\begin{pmatrix} t'_1 - it'_2 \\ -t'_3 + iG^0 \end{pmatrix}$$

Yes: $u_i \sim SU(2)_H$

- $u_1, u_2 \sim \alpha h^+ + \beta G^+$
- $\langle u \rangle = 0$, $\langle w \rangle$ covariant
- $\langle \mathbb{M} \rangle \sim$ covariant

Yes: $t_i \sim SO(3)_H$

- $t_i \equiv \frac{2}{v} s_i \sim$ vector
- $w = \frac{1}{\sqrt{2}}(iG^0 \mathbb{1}_2 + t_i\sigma_i)\langle \hat{w} \rangle$
- $(\mathcal{M}_N)_{ij} \sim$ tensor

- Physical covariant parameters:

$$\{M_0, \Lambda_{00}, \mathbf{\Lambda_0}, \mathbf{M}, \tilde{\Lambda}\} \to \{m^2, \Lambda_{00}, \mathbf{\Lambda_0}, \langle \mathbf{r} \rangle, \mathcal{M}_N\}$$

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Physical Basis (P-basis) and reparametrization

- Physical Charged Higgs basis (PCH): $\langle w \rangle = (0, \frac{v}{\sqrt{2}})$
- Physical Neutral Higgs basis (PNH): $\mathcal{M}_N = \text{diag}(m_1^2, m_2^2, m_3^2)$
- True Physical basis: physical fields with definite masses!
- General horizontal (basis) transformations

HT1: BEFORE EWSB: preserve G_{EW} $\quad u \to Uu, \quad w \to Uw$

$\downarrow \qquad \downarrow$

HT2: AFTER EWSB: preserve $U(1)_{\text{EM}}$ $\quad u \to U_u u, \quad w \to U_w w$

- HT2 $\sim SU(2) \times SO(3) \neq SU(2) \times SO(4)$
- HT2: G^+ mix with H^+ but G^0 does not mix with t_i
- P-basis can be achieved only with HT2:

$$\text{general basis} \xrightarrow{\textbf{HT1}} \text{PCH} \xrightarrow{\textbf{HT2}} \text{P-basis}$$

- Reparametrization is only induced by HT1: $V(u_a, t_i, G^0; \mu, \lambda) \sim$

$$V(U_{ab}u_b, R_{ij}(U)t_j, G^0; \mu, \lambda) = V(u_i, t_i, G^0; \mu', \lambda')$$

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Physical Consequences and Some Observations

- The depth of the potential is bounded by $-\frac{v^2}{8}m_3^2 \leq V_0 \leq -\frac{v^2}{8}m_1^2$
- The mass of the charged Higgs can not be arbitrarily larger than the neutral Higgs masses:

$$m^2 - m_3^2 < \frac{v^2}{4}\Lambda_{00}$$

- Observability of $\{m^2, \Lambda_{00}, \mathbf{\Lambda_0}, \langle\mathbf{r}\rangle, \mathcal{M}_N\}$ is not direct: some parameters are extractable only from trilinear and quartic interactions (basis inv.)
- The use of $\{m^2, \Lambda_{00}, \mathbf{\Lambda_0}, \langle\mathbf{r}\rangle, \mathcal{M}_N\}$ guarantee $\langle\mathbf{r}\rangle$ is a local minimum but not the global minimum (two are possible)
- Remaining reparametrization freedom ($U(1)_{\rm EM}$): rotation in t_1, t_2

$$\Phi_1 \to e^{i\theta}\Phi_1, \quad \Phi_2 \to \Phi_2$$

- $G^0 \overset{\rm CP}{\to} -G^0$ is a pseudoscalar irrespective of V (Haber& O'Neil, PRD06')

$$\begin{aligned} y_i = \tfrac{1}{2}w^\dagger\sigma_i w \quad &\sim \quad (\text{even})\langle\hat{\mathbf{r}}\rangle + t_{\parallel}\, t_i + G^0(\mathbf{t}\times\langle\hat{\mathbf{r}}\rangle)_i \\ |D_\mu(\Phi_a + \langle\Phi_a\rangle)|^2 \quad &\to \quad v\, Z^\mu \partial_\mu G^0 \end{aligned}$$

CP direction orthogonal to $\langle\mathbf{r}\rangle$

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Conclusions

- Horizontal transformations can be transposed to the fields after EWSB: $(H^+, G^+) \sim SU(2)_H$, $t_i \sim SO(3)_H$ (HT1, Haber& O'Neil)
- Similar quantities under HT1 before and after EWSB: $\tilde{\Lambda} \sim \mathcal{M}_N$
- To achieve the P-basis HT2 $\sim SU(2) \times SO(3) \supset HT1$ is necessary
- HT1 induces reparametrization
- HT2 transfers some parameters to mixing parameters
- HT2 is larger than HT1
- Basis transformations may not induce reparametrization
- Basis transf. (or horizontal transf.) are specified when the preserved gauge structure is specified
- For NHDMs:
  - $t_i \sim$ vectors of $\mathrm{adj}SU(N)_H$ [dim=$N^2 - 1$]
  - $\mathcal{M}_N$ needs $SO(N^2 - 1)$ for diagonalization $\Rightarrow$ PNH basis can not be reached by reparametrization
  - general HT2 after EWSB: $SU(N) \times SO(N^2 - 1)$

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Covariant form of V

$$
\begin{aligned}
V_2|_{\text{SSB}} &= m^2 H^+ H^- + \tfrac{1}{2}(\mathcal{M}_N)_{ij} t_i t_j \\
V_3|_{\text{SSB}} &= \Lambda_{\mu\nu} s_\mu r_\nu \\
V_4|_{\text{SSB}} &= \tfrac{1}{2}\Lambda_{\mu\nu} r_\mu r_\nu \\
|\langle \mathbf{r} \rangle| &= v^2/4
\end{aligned}
$$

$$
\begin{aligned}
s_\mu &= \frac{v}{2}(\langle \hat{\mathbf{r}} \rangle \cdot \mathbf{t}, \mathbf{t}) \\
r_\mu &= x_\mu + y_\mu \\
x_\mu &= \tfrac{1}{2} u^\dagger \sigma_\mu u \\
y_\mu &= \tfrac{1}{2} w^\dagger \sigma_\mu w
\end{aligned}
$$

$$
\mathcal{M}_N = m^2 \left[\mathbb{1}_3 - \frac{\langle \mathbf{r} \rangle \langle \mathbf{r} \rangle^\mathsf{T}}{|\langle \mathbf{r} \rangle|^2} \right] + |\langle \mathbf{r} \rangle| \left[\tilde{\Lambda} + \Lambda_{00} \frac{\langle \mathbf{r} \rangle \langle \mathbf{r} \rangle^\mathsf{T}}{|\langle \mathbf{r} \rangle|^2} \right] + \langle \mathbf{r} \rangle \boldsymbol{\Lambda}_\mathbf{0}{}^\mathsf{T} + \boldsymbol{\Lambda}_\mathbf{0} \langle \mathbf{r} \rangle^\mathsf{T}
$$

$$
Y = \tfrac{1}{2}[m^2 - \langle r_0 \rangle \Lambda_{00} - \boldsymbol{\Lambda}_\mathbf{0} \cdot \langle \mathbf{r} \rangle]\big(\mathbb{1}_2 - \sigma_i \langle \hat{r}_i \rangle\big) - \tfrac{1}{2}\sigma_i (\mathcal{M}_N)_{ij} \langle \hat{r}_j \rangle
$$

$$
\begin{aligned}
\frac{v}{2} V_3\Big|_{\text{SSB}} = & -\left[u^\dagger Y u + w^\dagger Y w\right] t_\parallel + m^2 [x_0\, t_\parallel - \mathbf{x} \cdot \mathbf{t}] \\
& + \langle r_0 \rangle \boldsymbol{\Lambda}_{\mathbf{0}\perp} \cdot \mathbf{t}_\perp \left[|u_\perp|^2 + \tfrac{1}{2}\mathbf{t}_\perp^2 \right] + (\mathbf{x} + \mathbf{y})^\mathsf{T} \mathcal{M}_N \, \mathbf{t} \,,
\end{aligned}
$$

$$
V_4\Big|_{\text{SSB}} = \tfrac{1}{2}\Lambda_{00}(r_0^2 - \mathbf{r}^2) + (r_0 - \langle \hat{\mathbf{r}} \rangle \cdot \mathbf{r}) \boldsymbol{\Lambda}_\mathbf{0} \cdot \mathbf{r} + \frac{\mathbf{r}^\mathsf{T} \mathcal{M}_N \mathbf{r}}{2\langle r_0 \rangle} + \tfrac{1}{2}\Big(\Lambda_{00} - \frac{m^2}{\langle r_0 \rangle}\Big)\mathbf{r}_\perp^2
$$

where

$$
\begin{aligned}
u_\perp &\equiv u - \langle\hat{w}\rangle\langle\hat{w}\rangle^\dagger u\,, &(1)\\
t_\| &\equiv \mathbf{t}\cdot\langle\hat{\mathbf{r}}\rangle\,, &(2)\\
\mathbf{t}_\perp &\equiv \mathbf{t} - \langle\hat{\mathbf{r}}\rangle t_\|\,, &(3)\\
|u_\perp|^2 &= x_0 - \mathbf{x}\cdot\langle\hat{\mathbf{r}}\rangle\,, &(4)\\
\mathbf{t}_\perp^2 &= 2(y_0 - \mathbf{y}\cdot\langle\hat{\mathbf{r}}\rangle)\,, &(5)
\end{aligned}
$$

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Parametrization of $V(r^\mu)$

A large class of bounded below potentials can be parametrized by

1. Take $\Lambda_{\mu\nu} = \mathrm{diag}(\Lambda_0, \Lambda_i)$ obeying the positivity constraint and a general M_μ.
2. "Boost" Λ by a particular transf. $SO(1,d)/\mathrm{adj}SU(N)_H$.

- Parametrization of Λ is easier than Z.
- For $N = 2$ the procedure is sufficient to parametrize all potentials with positive V_4 and the boost is indeed the Lorentz boost (4D)
- Number of parameters for $N = 2$:

$$4(\Lambda) + 4(M) + 3(\text{boost}) = 11$$

- Number of parameters for general N:

$$N^2(\Lambda)+N^2(M)+\left[\tfrac{1}{2}N^2(N^2-1)-(N^2-1)\right](\text{boost}) = \tfrac{1}{2}N^2(N^2+1)+1$$

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Minimal parametrization of the vacuum

General vacuum:

$$\langle\Phi\rangle = \begin{pmatrix} \begin{pmatrix} \langle u_1\rangle \\ \langle w_1\rangle \end{pmatrix} \\ \begin{pmatrix} \langle u_2\rangle \\ \langle w_2\rangle \end{pmatrix} \\ \vdots \\ \begin{pmatrix} \langle u_N\rangle \\ \langle w_N\rangle \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \langle u\rangle + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \langle w\rangle$$

1. Gauge $SU(2)_L$: $\boxed{\langle u\rangle^\dagger\langle w\rangle = 0}$
2. Horizontal U_N: $\langle u\rangle \to U_N\langle u\rangle$, $\langle w\rangle \to U_N\langle w\rangle$
3. Horizontal U_{N-1}: $\langle u\rangle \to U_{N-1}\langle u\rangle$, $\langle w\rangle \to U_{N-1}\langle w\rangle = \langle w\rangle$

- $\sqrt{2}\sqrt{|\langle u\rangle|^2 + |\langle w\rangle|^2} = v = 246\mathrm{GeV}$

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$$\langle w\rangle \to |\langle w\rangle| \begin{pmatrix}0\\ \vdots \\ 0\\ 1\end{pmatrix}, \quad \langle u\rangle \to \begin{pmatrix}\langle u_1\rangle \\ \vdots \\ \langle u_{N-1}\rangle \\ 0\end{pmatrix}$$

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$$\langle\Phi_N\rangle = |\langle w\rangle| \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \langle\Phi_{N-1}\rangle = |\langle u\rangle| \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ldots$$

- $\sqrt{2}\sqrt{|\langle u\rangle|^2 + |\langle w\rangle|^2} = v = 246\mathrm{GeV}$