The evolution of the Universe to the present Inert phase

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based on collaboration with Ilya Ginzburg, Konstantin Kanishev (Novosibirsk) and Maria Krawczyk (Warsaw)

Outline

- 2HDM potential and vacuum structure
- Thermal corrections to the potential
- Possible sequences of phase transitions leading to Inert Model

2HDM

T.D Lee, Weinberg, …

Potential with Z_2 symmetry:

$$\boxed{\begin{array}{c} V = -\frac{1}{2}(m_{11}^2 x_1 + m_{22}^2 x_2) + \\ +\frac{1}{2}(\lambda_1 x_1^2 + \lambda_2 x_2^2) + \lambda_3 x_1 x_2 + \lambda_4 x_3 x_3^\dagger \\ + \frac{1}{2}\lambda_5 (x_3^2 + x_3^{\dagger 2}) \end{array}}$$

$$\boxed{\begin{aligned} x_1 &= \varphi_1^\dagger \varphi_1 , \\ x_2 &= \varphi_2^\dagger \varphi_2 , \\ x_3 &= \varphi_1^\dagger \varphi_2 , \\ x_{3*} \equiv x_3^\dagger &= \varphi_2^\dagger \varphi_1 \, . \end{aligned}}$$

Useful parametrization:

$$\boxed{\lambda_2/\lambda_1 = k^4 \, , \quad m_{11}^2 = m^2(1-\delta) \, , \quad m_{22}^2 = k^2 m^2 (1+\delta) \, ,}$$

$$\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5 , \quad \lambda_{345} = \lambda_3 + \lambda_4 - \lambda_5 ,$$

$$\Lambda_{345\pm} = \sqrt{\lambda_1 \lambda_2} \pm \lambda_{345} , \quad \tilde{\Lambda}_{345\pm} = \sqrt{\lambda_1 \lambda_2} \pm \tilde{\lambda}_{345} , \quad \Lambda_{3\pm} = \sqrt{\lambda_1 \lambda_2} \pm \lambda_3 .$$

Positivity constrains:

$$\boxed{\lambda_1 > 0 \, , \quad \lambda_2 > 0 , \;\; \Lambda_{3+} > 0 , \;\; \Lambda_{345+} > 0 , \;\; \tilde{\Lambda}_{345+} > 0 .}$$

Interaction with fermions: Model I

2HDM

Extremum conditions:

$$\partial V/\partial \varphi_i|_{\varphi_i=\langle\varphi_i\rangle} = 0\,,\ \partial V/\partial \varphi_i^\dagger|_{\varphi_i=\langle\varphi_i\rangle} = 0$$

EW symmetric extremum:

$$\boxed{\langle\varphi_1\rangle = \langle\varphi_2\rangle \;=\; 0}$$

- local maximum if $m_{11,22}^2 > 0$
- local minimum if $m_{11,22}^2 < 0$

General type of SSB VEV:

$$\boxed{\langle\varphi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle\varphi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u \\ v_2 \end{pmatrix}}$$

Positivity constrains guarantee that the extremum with the lowest energy is the **global minimum**.

Global minimum = vacuum.

Charge breaking and Normal

Barroso et al.

Charge breaking extremum:

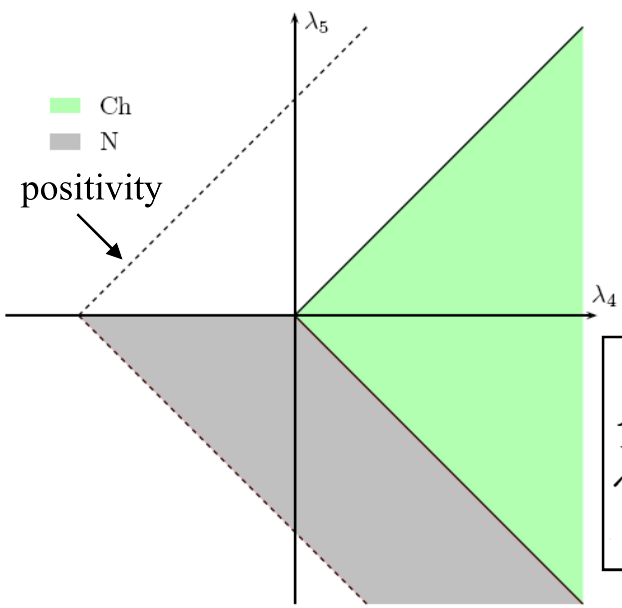
$$\langle \varphi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \varphi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} u \\ 0 \end{pmatrix}$$

$U(1)_{EM}$ broken
Electric charge not conserved
Massive photon

Normal (=CP conserving) extremum:

$$\langle \varphi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \varphi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

$U(1)_{EM}$ (charge conserved)
Standard case (as in MSSM)

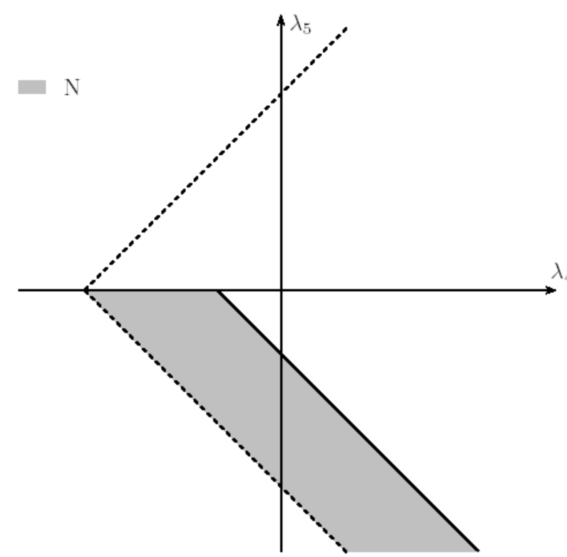


$\Lambda_{3-} > 0$

N and Ch can be vacuum in separate regions of (λ_4, λ_5) plane with additional conditions realized:

Normal:
$\Lambda_{345-} > 0,$
$\lambda_4 + \lambda_5 < 0$
$\lambda_5 < 0$

Charge breaking:
$\Lambda_{3-} > 0,$
$\lambda_4 + \lambda_5 > 0$
$\lambda_4 - \lambda_5 < 0$



$\Lambda_{3-} < 0$

Inert and B

Deshpande, Ma (1978)

Barbieri, Rychkov (2006)

Inert

$$\langle\varphi_1\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}, \langle\varphi_2\rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$U(1)_{EM}$ (charge conserved)
Model I – fermion masses from φ_1
Not the limit of N for $v_2 \to 0$

- Z_2 symmetry ($\varphi_1 \to \varphi_1$, $\varphi_2 \to - \varphi_2$) both in L and in vacuum.
- φ_1 as in SM (SM-like Higgs boson h), φ_2 - 4 scalars (no Higgs bosons!)
- Z_2 symmetry conserved: only φ_2 has odd Z_2 –parity, so the lightest scalar is a candidate for dark matter (φ_2 dark doublet with dark scalars).
- Viable model for today.

Phase B

$$\langle\varphi_1\rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \langle\varphi_2\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$U(1)_{EM}$ (charge conserved)
Model I – fermion masses from φ_1
Not the limit of N for $v_1 \to 0$

- φ_1 and φ_2 change roles comparing to the Inert Model.
- **Model I – fermion masses only from loops.**

Inert Model

$$X = \sqrt{\lambda_1 \lambda_2} + \lambda_3 > 0$$

(positivity)

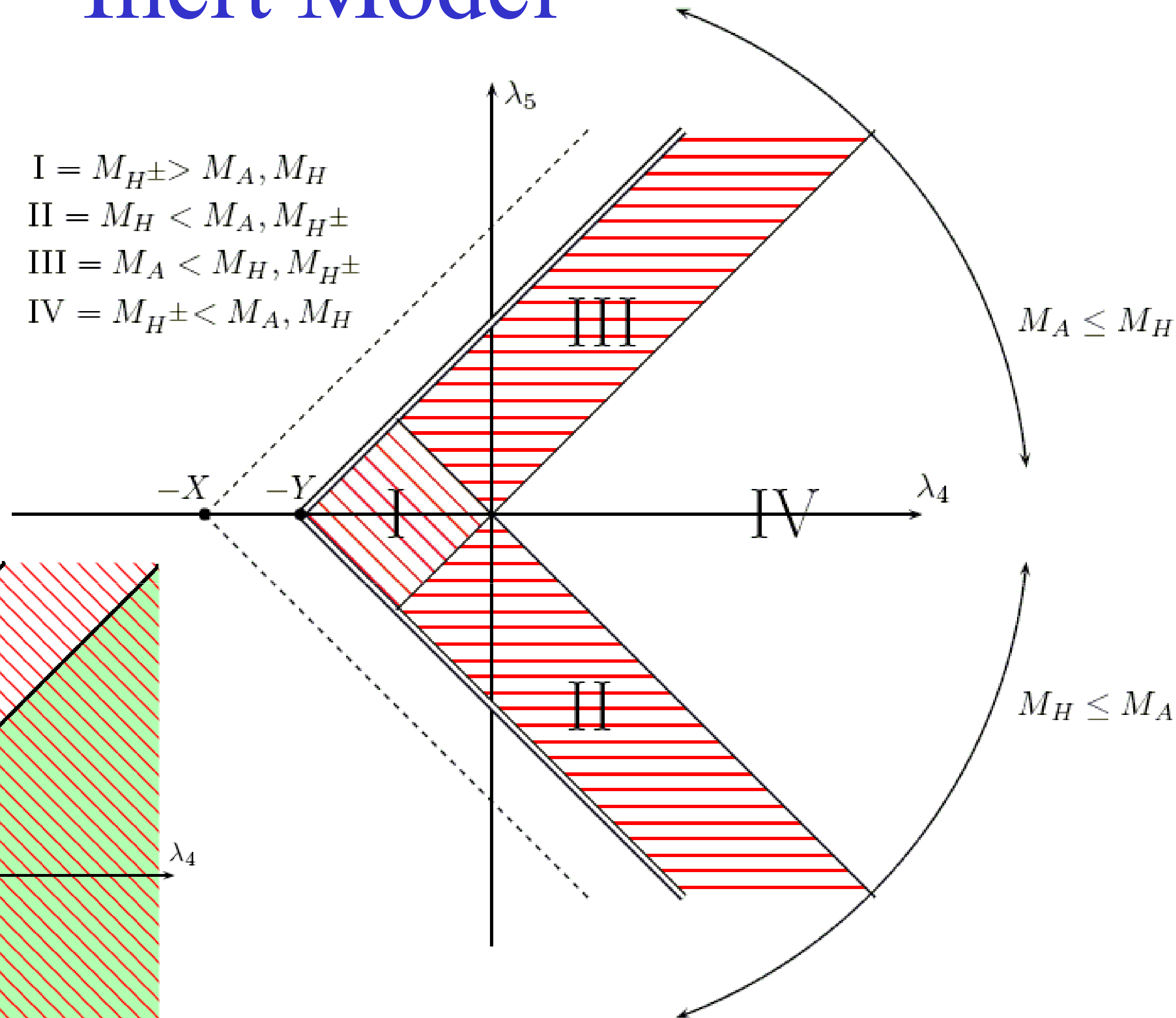
$$Y = \frac{2M^2_{H^\pm}}{v^2} > 0$$

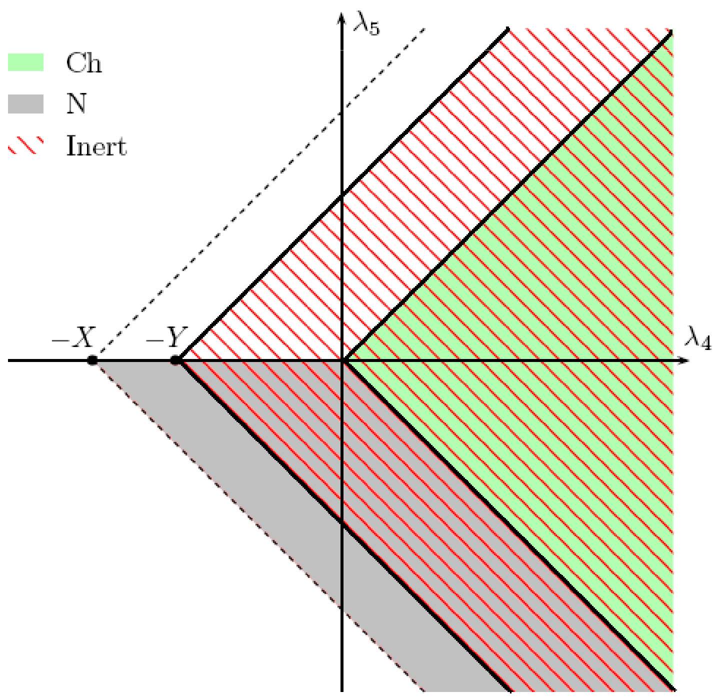
$\mathrm{I} = M_{H^\pm} > M_A, M_H$

$\mathrm{II} = M_H < M_A, M_{H^\pm}$

$\mathrm{III} = M_A < M_H, M_{H^\pm}$

$\mathrm{IV} = M_{H^\pm} < M_A, M_H$





Interaction with fermions and gauge bosons

- For the N Phase (α – mixing angle, χ – relative coupling)
  - Interaction with fermions:

$$\chi_t^{H^0} : \frac{\sin\alpha}{\sin\beta} \;;\; \chi_b^{H^0} : \frac{\sin\alpha}{\sin\beta} \;;\; \chi_t^{h^0} : \frac{\cos\alpha}{\sin\beta} \;;\; \chi_b^{h^0} : \frac{\cos\alpha}{\sin\beta}$$

$$\chi_t^{A^0} : -i\gamma_5 \cot\beta \;;\; \chi_b^{A^0} : i\gamma_5 \cot\beta$$

  - Interaction with gauge bosons

$$\chi_h^V = \cos\alpha, \qquad \chi_H^V = \sin\alpha, \qquad \chi_A^V = 0.$$

- For the Inert Model:
  - Dark scalars from φ_2 do not interact with fermions
  - They have no triple interactions with gauge bosons $H_i V_1 V_2$, but there are vertices $H_i H_j V$ and $H_i H_j V_1 V_2$
  - h from φ_1 interacts with fermions and gauge bosons like SM Higgs boson

Evolution of the Universe

Ginzburg, Ivanov, Kanishev

We assume that today Inert Model is realized.
However, in the past some other extrema could have been lower than Inert due to the thermal corrections to the potential.

Matsubara method (temperature $T \gg m^2$) – only quadratic (mass) parameters change with T:

$$m_{11}^2(T) = m_{11}^2(0) - 2c_1 m^2 w\,, \quad m_{22}^2(T) = m_{22}^2(0) - 2k^2 c_2 m^2 w\,;$$

$$c_1 = \frac{3\lambda_1 + 2\lambda_3 + \lambda_4}{2}\,, \qquad c_2 = \frac{3\lambda_2 + 2\lambda_3 + \lambda_4}{2k^2}\,; \quad \boxed{w = \frac{T^2}{12m^2}\,,}$$

$$m^2(T) = m^2\left(1 - (c_2 + c_1)w\right)\,,$$

$$\delta(T) = \frac{m^2}{m^2(T)}\left(\delta - \frac{c_2 - c_1}{c_2 + c_1}\right) + \frac{c_2 - c_1}{c_2 + c_1}.$$

c_1 and c_2 positive to restore EW symmetry in the past

What other vacua could have been realized in the past?

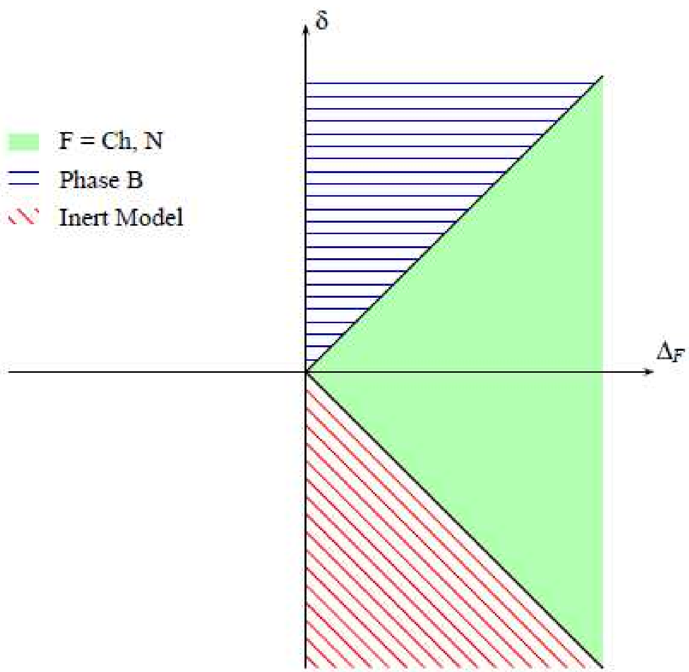
Evolution of the Universe

The extremum with the lowest energy is the vacuum:

$$\boxed{\mathcal{E}_I - \mathcal{E}_N} = \frac{m^4}{8}\,\frac{(\Lambda_{345-} + \delta\Lambda_{345+})^2}{\lambda_1 \Lambda_{345-}\Lambda_{345+}}, \quad \boxed{\mathcal{E}_I - \mathcal{E}_{Ch}} = \frac{m^4}{8}\,\frac{(\Lambda_{3-} + \delta\Lambda_{3+})^2}{\lambda_1 \Lambda_{3-}\Lambda_{3+}}$$

$$\boxed{\mathcal{E}_N - \mathcal{E}_{Ch}} = \frac{k^2 m^2}{4}\lambda_{45}\left[\frac{1}{\Lambda_{3+}\Lambda_{345+}} - \frac{\delta^2}{\Lambda_{3-}\Lambda_{345-}}\right], \quad \boxed{\mathcal{E}_I - \mathcal{E}_B} = \frac{m^4\delta}{2\lambda_1}\,.$$

Delta evolves with time (with temperature): this lead to different phases



$$\mathcal{S}_N : \ \Lambda_{345-} > 0, \quad \lambda_4 + \lambda_5 < 0$$
with
$$\Delta_N = \Lambda_{345-}/\Lambda_{345+}.$$

$$\mathcal{S}_{Ch} : \Lambda_{3-} > 0, \quad \lambda_4 + \lambda_5 > 0$$
with
$$\Delta_{Ch} = \Lambda_{3-}/\Lambda_{3+}.$$

Evolution of the Universe

Possible phase transitions from the EW symmetric state that lead to the Inert Model:

- EW $\xrightarrow{II}$ Phase B $\xrightarrow{I}$ Inert phase
- EW $\xrightarrow{II}$ Phase B $\xrightarrow{II}$ Phase N $\xrightarrow{II}$ Inert phase
- EW $\xrightarrow{II}$ Phase B $\xrightarrow{II}$ Charged phase $\xrightarrow{II}$ Inert phase
- EW $\xrightarrow{II}$ Inert phase

Extremum conditions constrain the parameters of the potential:

- from the moment that Phase B becomes a possible extremum (not necessarily the vacuum) $m^2_{22}>0$. This limits the value of λ_3.
- from the moment that Inert Phase becomes a possible extremum (not necessarily the vacuum) $m^2_{11}>0$

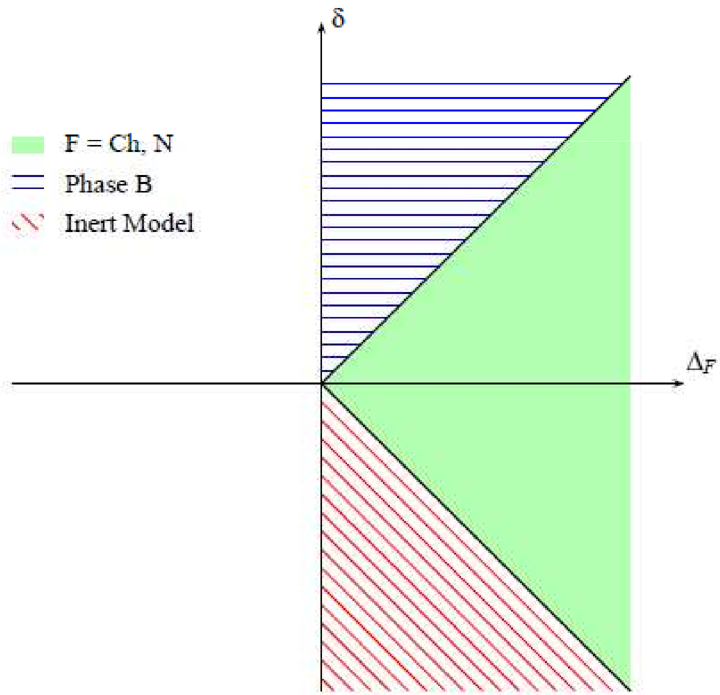
If Phase B or Inert Phase becomes an extremum at some point in time, then it is an extremum to the end of the evolution – it is an extremum now.

EW – B

The EW to B transition takes place when m^2_{22} becomes positive at:

$$w_{EWSB,B} = \frac{1+\delta}{c_2}\,.$$

The further evolution depends on the parameters:



- through the N phase:

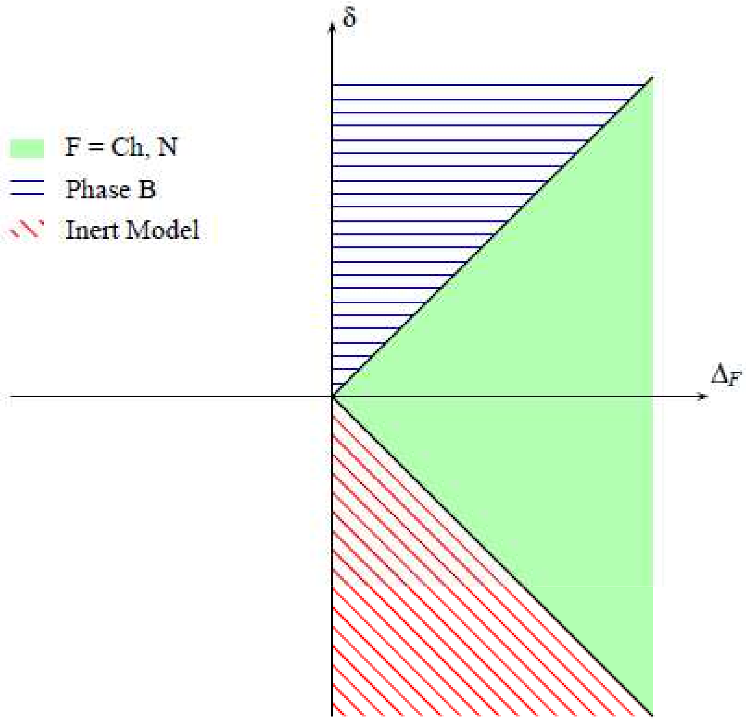
$$\mathcal{S}_N:\ \Lambda_{345-} > 0, \quad \lambda_4+\lambda_5 < 0$$
with
$$\Delta_N = \Lambda_{345-}/\Lambda_{345+}.$$

- through the Ch phase:

$$\mathcal{S}_{Ch}:\Lambda_{3-} > 0, \quad \lambda_4+\lambda_5 > 0$$
with
$$\Delta_{Ch} = \Lambda_{3-}/\Lambda_{3+}.$$

- straight to the Inert phase in the complementary region to S_N and S_{Ch}

EW – B – N – I



For this sequence to happen we need to be in S_N region of parameters:

$$\mathcal{S}_N: \ \Lambda_{345-} > 0, \quad \lambda_4 + \lambda_5 < 0$$
$$\text{with}$$
$$\Delta_N = \Lambda_{345-}/\Lambda_{345+}.$$

The transitions happen at:

$$\delta(T_{N\pm}) = \pm\Delta_N.$$

This transition is possible only if at the present time:

$$0 > \delta > \frac{c_2 - c_1}{c_2 + c_1}$$

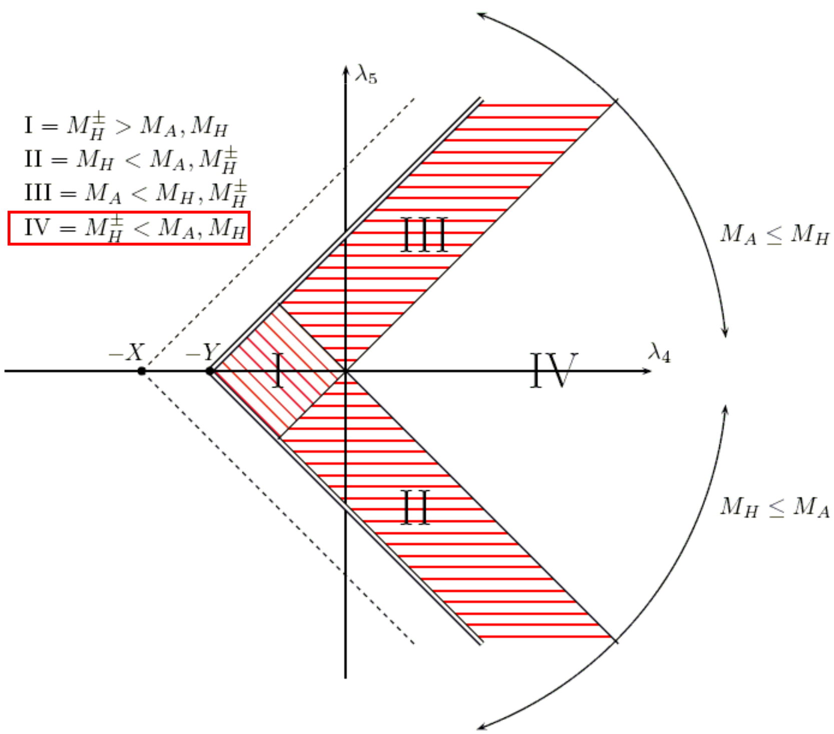
Because both B and I are extrema today we know that: $m^2_{22}>0$ and $m^2_{11}>0$.

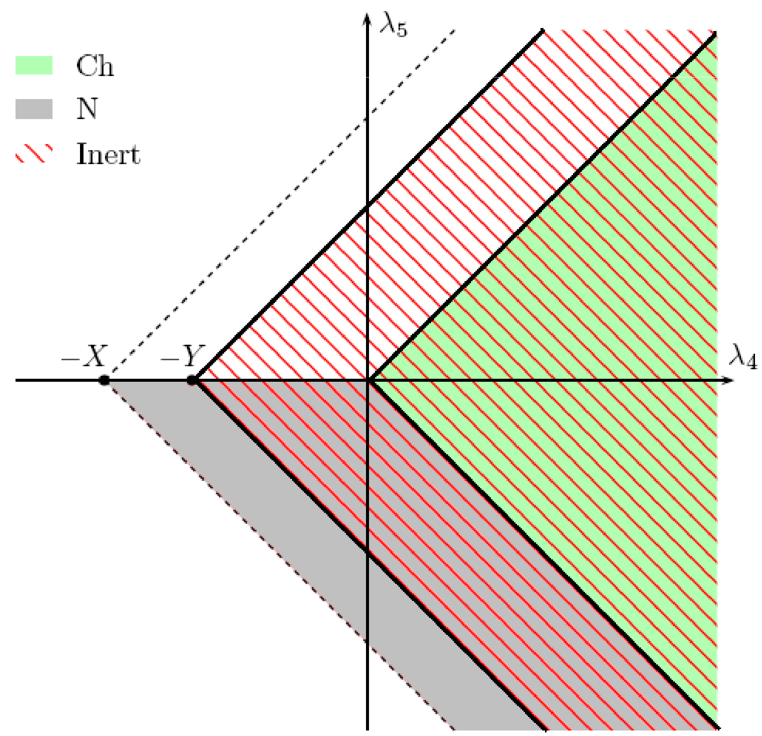
EW – B – Ch – I

If we are in S_{Ch} region from Phase B we go into Charged Phase and then into Inert Phase at temperature:

$$\delta(T_{Ch\pm}) = \pm\Delta_{Ch}$$

This phase transition can be realized only if today we have charged Dark Matter particle.





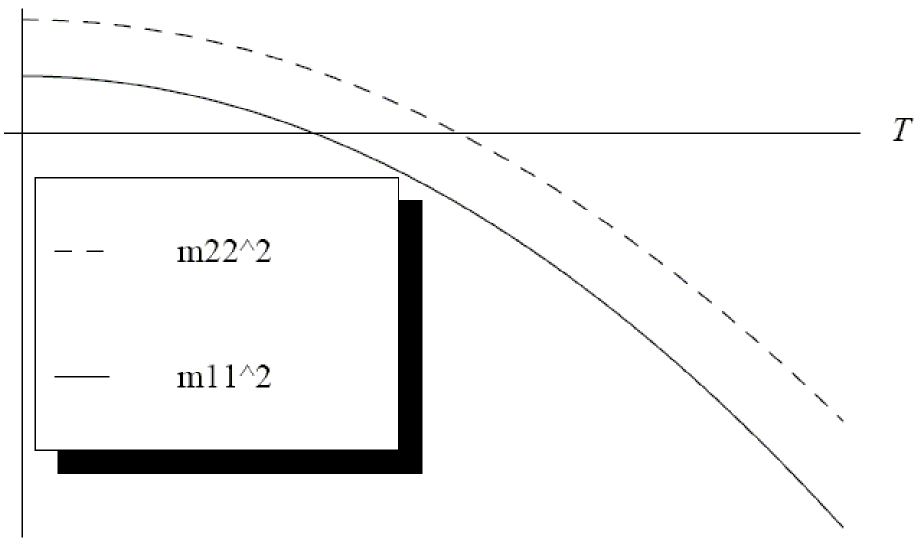
EW – B – I

The direct Phase B to Inert transition will be realized if we are in the region of parameters complementary to the previous to cases (S_N and S_{Ch}).

The transition happens at:

$$w_{BI} = -\frac{\delta}{c_1 - c_2}, \qquad \delta(T) = 0$$

$$Q_{B\to I} = T\frac{\partial \mathcal{E}_B}{\partial T} - T\frac{\partial \mathcal{E}_I}{\partial T}\Big|_{\delta(T)\to 0} = \frac{m^2k^2}{2}\cdot\frac{(c_1-c_2)\delta}{(c_1-c_2)\delta + c_1 + c_2}$$



First we have EW to B transition then B to I transition: $m^2_{22}>0$, $m^2_{11}>0$

EW – B – I: an example

Let's consider an example when:

$$\lambda_1 = 0.217, \quad \lambda_2 = 1.1, \quad \lambda_3 = 1.116, \quad \lambda_4 = -0.48, \quad \lambda_5 = -0.046$$

$$M_h = 115\, GeV, \quad M_H = 60\, GeV, \quad M_A = 80\, GeV, \quad M_{H^\pm} = 140\, GeV$$

$$k^2 = \frac{9}{4}, \quad \delta = -0.016$$

Delta is negative, so we are in the Inert Phase.

The following conditions are satisfied:

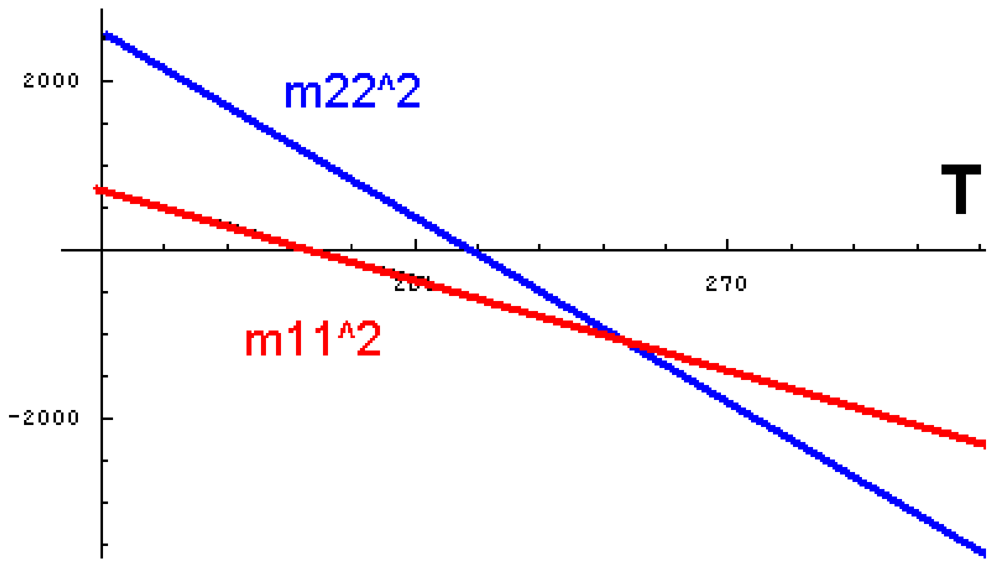
$$0 > \delta > \frac{c_2 - c_1}{c_2 + c_1} \qquad \Lambda_{3-} < 0, \quad \Lambda_{345-} < 0$$

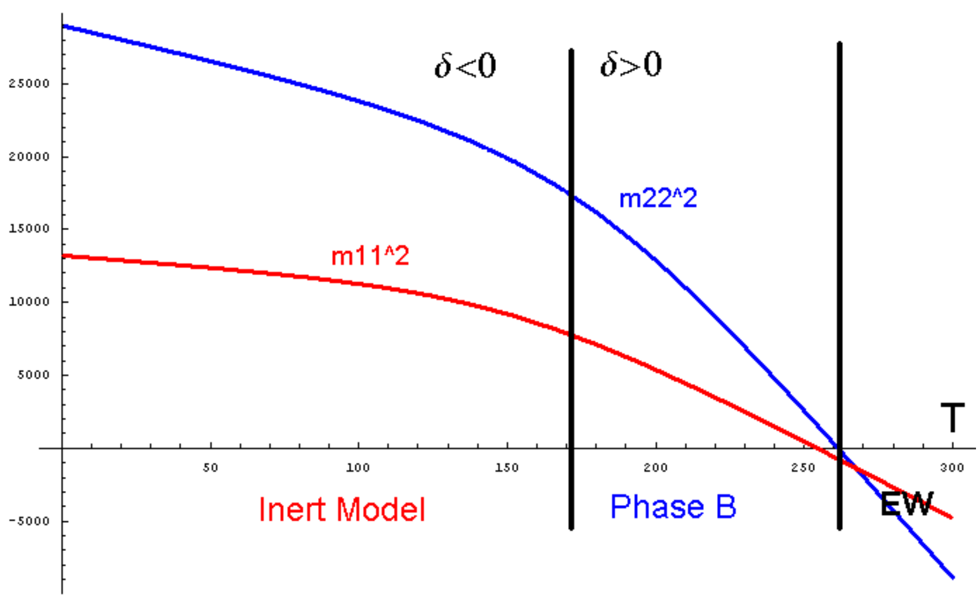
so we went through Phase B

but going through N Phase or Charged Phase was not possible.

The only possibility is B to I transition.

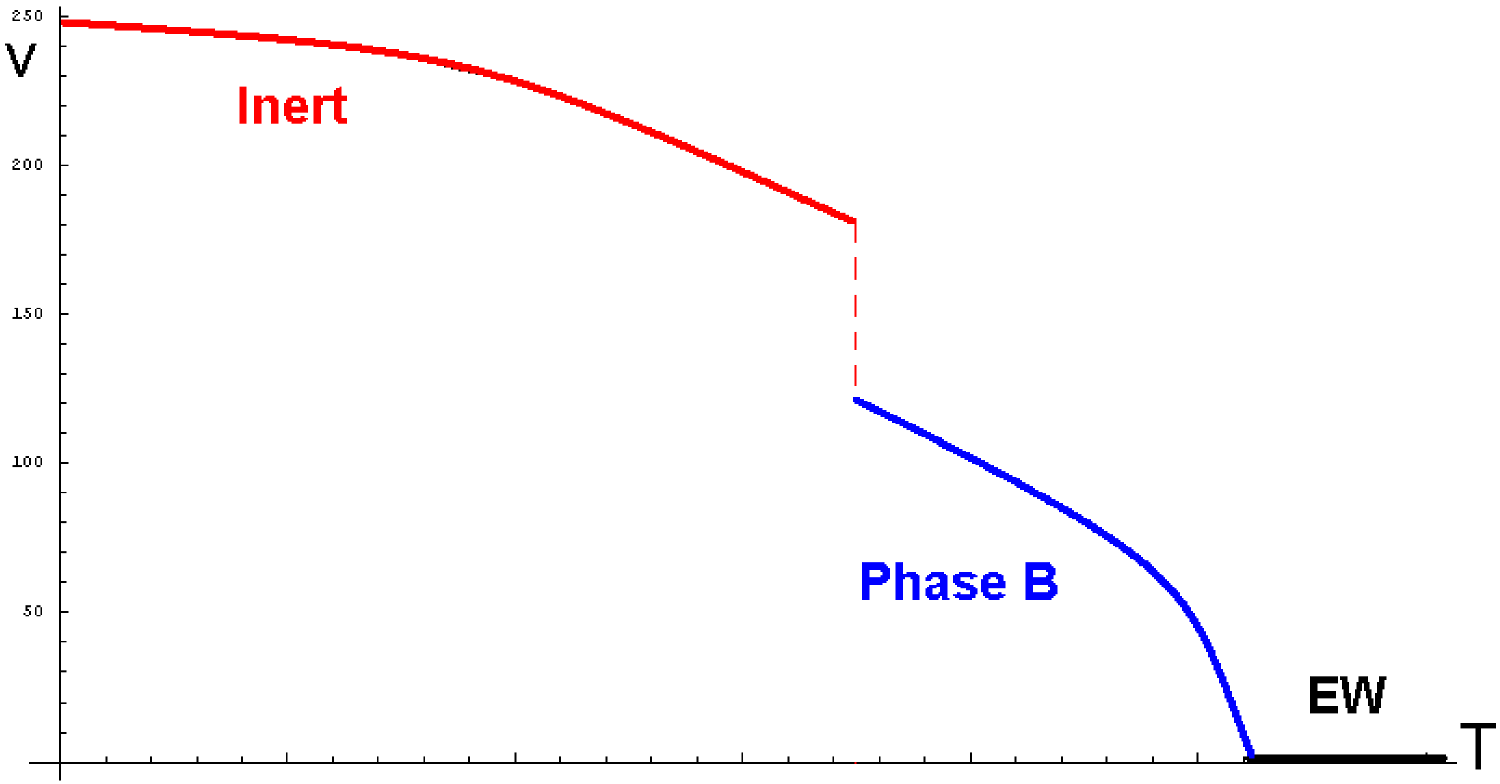
EW – B – I: an example





- EW symmetric phase ($m^2{}_{11}$, $m^2{}_{22}$ negative)
- $m^2{}_{22} > 0$, $m^2{}_{11} < 0$: Phase B becomes a vacuum ($\delta > 0$), Inert is not an extremum
- $m^2{}_{11} > 0$: Phase B still a vacuum ($\delta > 0$), Inert becomes an extremum
- $\delta < 0$: Inert becomes a vacuum, B is an extremum

EW – B – I: an example



1st order phase transition – discontinuity in v

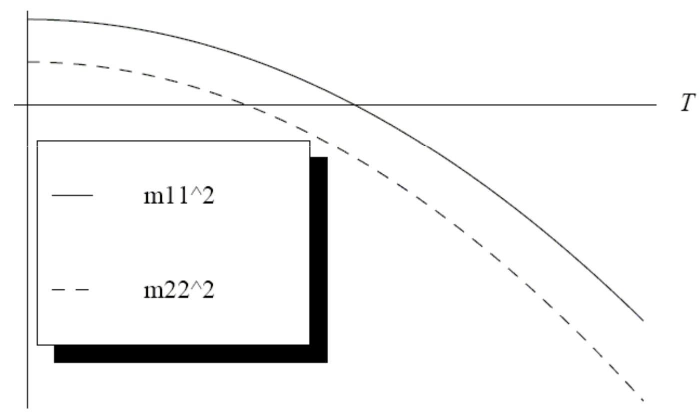
EW – I

Inert vacuum exists during an entire history of the Universe after SSB if δ is permanently negative which is satisfied if:

$$\delta - \frac{c_2 - c_1}{c_2 + c_1} < 0$$

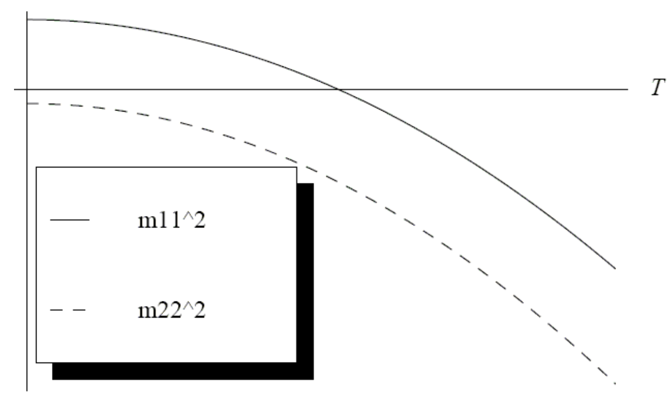
In this case δ(T) decreases with the growth of temperature T. Only one phase transition took place at the temperature when m^2_{11} becomes positive at:

$$w_{EWSB,I} = \frac{1-\delta}{c_1}$$



Phase B can be an extremum (and so $m^2_{22}>0$) but not a vacuum if:

$$-1 < \delta < \frac{c_2 - c_1}{c_2 + c_1} < 0$$



Phase B can never be an extremum (and so $m^2_{22}<0$) if:

$$\delta < -1$$

Conclusions

- The 2HDM has a rich vacuum structure – different types of extrema can be realized during the history of the Universe
- We use Matsubara method (T^2 corrections to mass parameters)
- Possible phase transitions:
  - EW $\xrightarrow{II}$ Phase B $\xrightarrow{I}$ Inert phase
  - EW $\xrightarrow{II}$ Phase B $\xrightarrow{II}$ Phase N $\xrightarrow{II}$ Inert phase
  - EW $\xrightarrow{II}$ Phase B $\xrightarrow{II}$ Charged phase $\xrightarrow{II}$ Inert phase
  - EW $\xrightarrow{II}$ Inert phase
- **EW – B – Ch – I** transition requires the charged Dark Matter – definitely not the standard DM candidate
- **B – I** is 1st order phase transition – discontinuity in physical parameters
- It is possible to have no DM for high temperatures (going through Phase B)
- Only in **EW – I** sequence DM existed during the entire time after SSB

Backup