MINIMAL FLAVOUR VIOLATION IN 2HDM

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Contents

1 THE FLAVOUR STRUCTURE OF THE SM

The flavour structure of the SM is fully contained in the Yukawa sector

$$L_Y = -\overline{Q}_L \, \Gamma \Phi d_R - \overline{Q}_L \Delta \tilde{\Phi} u_R - \overline{L}_L \, \Pi \Phi l_R + h.c.$$

and controlled by the so called Yukawa coupling matrices: Γ, Δ and Π.

In the absence of Yukawa coupling the SM has a large global $U(3)^5$ symmetry

$$G_{global}(\Gamma = \Delta = \Pi = 0) = SU(3)^3_q \otimes SU(3)^2_l \otimes U(1)^5$$

$$\begin{aligned} SU(3)^3_q &= SU(3)_{Q_L} \otimes SU(3)_{u_R} \otimes SU(3)_{d_R} \\ SU(3)^3_l &= SU(3)_{L_L} \otimes SU(3)_{l_R} \end{aligned}$$

The gauge and pure Higgs Lagrangians are invariant under the non abelian flavour symmetry $SU(3)_q^3 \otimes SU(3)_l^2$:

$$\begin{array}{ccccc} SU(3)_{Q_L} & ; & SU(3)_{u_R} & ; & SU(3)_{d_R} \\ Q_L \to W_L Q_L & ; & u_R \to W_R^u u_R & ; & d_R \to W_R^d d_R \end{array}$$

$$\begin{array}{ccc} SU(3)_{L_L} & ; & SU(3)_{l_R} \\ L_L \to W_L^L L_L & ; & l_R \to W_R^l l_R \end{array}$$

The Yukawa couplings break this flavour symmetries. So the entire SM is not invariant under these -sometimes called- weak basis (WB) transformations.

Nevertheless, after performing a WB transformation we have a new theory equivalent to the previous one but with new Yukawa couplings (let us concentrate in the quark sector)

$$\boxed{\begin{array}{c}\Gamma \rightarrow W_L^{\dagger} \Gamma W_R^d \\ \Delta \rightarrow W_L^{\dagger} \Delta W_R^u\end{array}}$$

Clearly any physical observable should be independent of the weak basis we chose to formulate the theory. So two theories whose Yukawa are related by the previous transformation should be identical.

The diagonalization of the mass matrices $M_d = \upsilon\Gamma/\sqrt{2}$ and $M_u = \upsilon\Delta/\sqrt{2}$ are done with

$$\begin{array}{l}U_L^{d\dagger} M_d U_R^d = D_d = diag(m_d, m_s, m_b) = \frac{\upsilon}{\sqrt{2}} diag(y_d, y_s, y_b) \\ U_L^{u\dagger} M_u U_R^u = D_u = diag(m_u, m_c, m_t) = \frac{\upsilon}{\sqrt{2}} diag(y_u, y_c, y_t)\end{array}$$

Note that this is not a WBT. Without loose of generality, sometime is useful to choose a special WBT

$$W_L = U_L^d \quad ; \quad W_R^u = U_R^u \quad ; \quad W_R^d = U_R^d$$

such that

$$\Gamma = Y^d = diag(y_d, y_s, y_b) \quad ; \quad \Delta = V^\dagger Y^u = V^\dagger diag(y_u, y_c, y_t)$$

Y^d and Y^u are the diagonal Yukawa coupling and all the changes of flavour are encoded in the CKM matrix V.

2 BASIC FLAVOUR STRUCTURES

Flavour related Physical Observables (PO) should not depend on the WB we chose, so they must depend on quantities made up with Γ and Δ that are invariant under

$$\boxed{\begin{array}{c}\Gamma \rightarrow W_L^\dagger \Gamma W_R^d \\ \Delta \rightarrow W_L^\dagger \Delta W_R^u\end{array}}$$

In general that means that PO should be of the form

$$\boxed{Tr\left[(H_u)^\alpha (H_d)^\beta (H_u)^\gamma (H_d)^\delta \ldots.\right]}$$

where

$$H_d = M_d M_d^\dagger = \frac{v^2}{2}\Gamma\Gamma^\dagger$$
$$H_u = M_u M_u^\dagger = \frac{v^2}{2}\Delta\Delta^\dagger$$

and they transform as

$$H_d \rightarrow W_L^\dagger H_d W_L$$
$$H_u \rightarrow W_L^\dagger H_u W_L$$

Some examples are

$$Tr\,[H_u] = m_t^2 + m_c^2 + m_u^2$$
$$Tr\,[H_u H_d] = \sum_{i,j} \left|V_{ij}\right|^2 m_{u_i}^2 m_{d_j}^2$$

A well-known example of invariant that signals CP violation in the SM is:

$$\begin{aligned} J &= -\frac{i}{6} Tr\left([H_u, H_d]^3\right) = \\ \mathrm{Im}\, Tr\left[H_u H_d (H_u)^2 (H_d)^2\right] &= \left(m_t^2 - m_c^2\right)\left(m_t^2 - m_u^2\right)\left(m_c^2 - m_u^2\right) \\ &\times \left(m_b^2 - m_s^2\right)\left(m_b^2 - m_d^2\right)\left(m_s^2 - m_d^2\right) \\ &\times \mathrm{Im}\,[V_{22} V_{33} V_{23}^* V_{32}^*] \end{aligned}$$

Note that there is no other CP violating invariant before we arrive at order 12 in Yukawa Couplings

This invariant encodes the necessary and sufficient conditions to have CP violation in the SM, but it is not trivially related to the standard CP violating observables measured at Babar, Belle, LHCb, etc.. In fact J only appears as a physical observable when we sum up over the quarks. This is the case of the Baryon Asymmetry and the CKM contribution to the electron DM.

Is there any simple way to write PO in terms of WB invariants?

Is there any simple way to write $\left|V_{ij}\right|$ in terms of WB invariants?

3 PROJECTOR OPERATORS

The key point is that there are other matrices in addition to H_d and H_u that transform under a WB transformation as

$$\mathcal{O} \to W_L^\dagger \mathcal{O} W_L$$

From

$$M_d = U_L^d D_d U_R^{d\dagger}$$

$$\boxed{H_d = M_d M_d^\dagger = U_L^d D_d^2 U_L^{d\dagger} = U_L^d \sum_{i=1}^{3} m_{d_i}^2 P_i U_L^{d\dagger} = \sum_{i=1}^{3} m_{d_i}^2 \mathcal{P}_i^{d_L}}$$

$$\mathcal{P}_i^{d_L} = U_L^d P_i U_L^{d\dagger} \quad ; \quad (P_i)_{jk} = \delta_{ij}\delta_{ik}$$

are the projector operators over the lefthanded down quarks in an arbitrary weak basis.

$$\mathcal{P}_i^{d_L}\mathcal{P}_j^{d_L} = \delta_{ij}\mathcal{P}_i^{d_L}$$
$$\sum_{i=1}^{3}\mathcal{P}_i^{d_L} = I$$

and obviously they transform as H_d :

$$\mathcal{P}_i^{d_L} \rightarrow W_L^\dagger \mathcal{P}_i^{d_L} W_L$$

In general we can define up, down, left and right projection operators:

$$\begin{array}{rcl} H_d = M_d M_d^\dagger = \sum_{i=1}^3 m_{d_i}^2 \mathcal{P}_i^{d_L} & ; & \boxed{\mathcal{P}_i^{d_L} = U_L^d P_i U_L^{d\dagger}} \\ H_u = M_u M_u^\dagger = \sum_{i=1}^3 m_{u_i}^2 \mathcal{P}_i^{u_L} & ; & \boxed{\mathcal{P}_i^{u_L} = U_L^u P_i U_L^{u\dagger}} \\ M_d^\dagger M_d = \sum_{i=1}^3 m_{d_i}^2 \mathcal{P}_i^{d_R} & ; & \mathcal{P}_i^{d_R} = U_R^d P_i U_R^{d\dagger} \\ M_u^\dagger M_u = \sum_{i=1}^3 m_{d_i}^2 \mathcal{P}_i^{u_R} & ; & \mathcal{P}_i^{u_R} = U_R^u P_i U_R^{u\dagger} \end{array}$$

That transform under a WB transformation as

$$\boxed{\mathcal{P}_i^{d_L} \to W_L^\dagger \mathcal{P}_i^{d_L} W_L} \quad ; \quad \boxed{\mathcal{P}_i^{u_L} \to W_L^\dagger \mathcal{P}_i^{u_L} W_L}$$

$$\mathcal{P}_i^{d_R} \to W_R^{d\dagger} \mathcal{P}_i^{d_R} W_R^d \quad ; \quad \mathcal{P}_i^{u_R} \to W_R^{u\dagger} \mathcal{P}_i^{u_R} W_R^u$$

It is important to emphasize that in the SM we do not need to use right handed projectors. The WB transformation properties of $\mathcal{P}_i^{d_R}$, for example, enforces it to be be sandwich between M_d ... $M_d^\dagger$:

$$M_d \mathcal{P}_i^{d_R} M_d^\dagger = \left(U_L^d D_d U_R^{d\dagger} \right) \left(U_R^d P_i U_R^{d\dagger} \right) \left(U_R^d D_d U_L^{d\dagger} \right) = m_{d_i}^2 \mathcal{P}_i^{d_L}$$

4 INVARIANTS FROM PROJECTOR OPERATORS

Now PO should have the form:

$$Tr\left[\mathcal{P}_i^{u_L}\mathcal{P}_j^{d_L}\right]$$
$$Tr\left[\mathcal{P}_i^{u_L}\mathcal{P}_j^{d_L}\mathcal{P}_k^{u_L}\mathcal{P}_l^{d_L}\right]$$

$$\begin{aligned} Tr\left[\mathcal{P}_i^{u_L}\mathcal{P}_j^{d_L}\right] &= Tr\left[U_L^u P_i U_L^{u\dagger} U_L^d P_j U_L^{d\dagger}\right] = Tr\left[U_L^{d\dagger} U_L^u P_i U_L^{u\dagger} U_L^d P_j\right] = \\ &= Tr\left[V^\dagger P_i V P_j\right] = \left(V^\dagger\right)_{ji} (V)_{ij} = \left|V_{ij}\right|^2 \end{aligned}$$

We have answered the previous question

$$\boxed{\left|V_{ij}\right|^2 = Tr\left[\mathcal{P}_i^{u_L}\mathcal{P}_j^{d_L}\right] \propto \Gamma\left(d_{L_j} \to u_{L_j} "W"\right)}$$

Other interesting result is

$$\begin{aligned}\mathsf{Im}\left[V_{11}V_{22}V_{12}^{*}V_{21}^{*}\right] &= \mathsf{Im}\, Tr\left[\mathcal{P}_1^{u_L}\mathcal{P}_1^{d_L}\mathcal{P}_2^{u_L}\mathcal{P}_2^{d_L}\right] \propto \\ &\propto \Gamma\left(D_s^{+} \to K^{0}\pi^{+}\right) - \Gamma\left(D_s^{-} \to \overline{K}^{0}\pi^{-}\right)\end{aligned}$$

Note that because we tag the quarks u,c,d,s we know they are different from t and b and therefore we have direct CP violation with the interference of two tree level decay amplitudes: we do not have any mass suppression factor (of course we need non zero FSI phase differences).

These equalities are in some sense trivial. The first one is the modulus square of the amplitude

$$\begin{aligned}\Gamma\left(d_{L_j} \to u_{L_j}\text{"}W\text{"}\right) &\propto \left\langle u_{L_i}\right| I \left|d_{L_j}\right\rangle \left\langle d_{L_j}\right| I \left|u_{L_i}\right\rangle = Tr\left[\left|u_{L_i}\right\rangle\left\langle u_{L_i}\right|\left|d_{L_j}\right\rangle\left\langle d_{L_j}\right|\right] \\ &= Tr\left[\mathcal{P}_i^{u_L}\mathcal{P}_j^{d_L}\right] = \left|V_{ij}\right|^2\end{aligned}$$

And the second one is the interference of two tree level amplitudes: decay $c \to ud\bar{d}$ and an annihilation one $(c\bar{s} \to u\bar{s})^*$. Clearly this interference involve the left-handed projectors $c\bar{c}, u\bar{u}, d\bar{d}, s\bar{s}$.

5 MINIMAL FLAVOUR VIOLATION

The legacy of B factories can be summarized by:

Very likely, flavour violation and CP violation in flavour changing processes are dominated by the CKM mechanism.

But even more, if to the SM Lagrangian we add for $K^0 - \overline{K}^0$, $B^0 - \overline{B}^0$, $B_s^0 - \overline{B}_s^0$ mixing the New Physics (NP) Lagrangian

$$\mathcal{L}_{NP} = \sum_{i \neq j} \frac{C_{ij}^2}{\Lambda^2} \left(\overline{Q}_{L_i} \gamma^\mu Q_{L_j} \right)^2$$

The condition

$$|A_{NP}| < |A_{SM}|$$

Implies

$$\Lambda > \begin{cases} \left(10^4 \text{ TeV}\right) (C_{sd}) \\ \left(10^2 \text{ TeV}\right) \left(C_{bq}\right) \end{cases}$$

If one insist, for example, that the scale of NP is at the TeV, we need to suppress very strongly c_{ij}.they cannot be order 1, the NP flavour structure is highly non generic.

A popular way to implement this non generic structure of the NP operators is the so called Minimal Flavour Violation (MFV) hypothesis. It consists of two ingredients:

1. A flavour symmetry: $SU(3)_{Q_L} \otimes SU(3)_{u_R} \otimes SU(3)_{d_R}$

2. A set of symmetry breaking flavour terms: The Yukawa Couplings Γ and Δ are the unique sources of flavour symmetry breaking in the NP model we are considering.

In our previous example we have

$$C = \left(aI + b\Gamma\Gamma^{\dagger} + c\Delta\Delta^{\dagger} + d\Gamma\Gamma^{\dagger}\Gamma\Gamma^{\dagger} + e\Delta\Delta^{\dagger}\Delta\Delta^{\dagger} + \cdots\cdots\cdots\cdots\right)$$

$$\Gamma = \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} \quad ; \quad \Delta = V^\dagger \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}$$

Taking into account the differences of the eigenvalues of the Yukawa couplings, neglecting all of them but the top, we get that the unique relevant structure is:

$$\begin{aligned} \left(\Delta\Delta^\dagger\right)^n_{i\neq j} &\propto \left(V^\dagger y_t^2 P_3 V\right)^n_{i\neq j} = y_t^{2n} V_{3i}^* V_{3j} \\ C_{i\neq j} &\propto y_t^{2n} V_{3i}^* V_{3j} \end{aligned}$$

So in theories of NP with MFV one gets:

$$\mathcal{A}\left(d_j \to d_i\right)_{MFV} = \left(V_{ti}^* V_{tj}\right) \mathcal{A}_{SM}^{(\Delta F=1))} \left[1 + a_1 \frac{16\pi^2 M_W^2}{\Lambda^2}\right]$$

$$\mathcal{A}\left(M_{ij} \to \overline{M}_{ij}\right)_{MFV} = \left(V_{ti}^* V_{tj}\right)^2 \mathcal{A}\left(M_{ij} \to \overline{M}_{ij}\right)_{SM}^{(\Delta F=2))} \left[1 + a_2 \frac{16\pi^2 M_W^2}{\Lambda^2}\right]$$

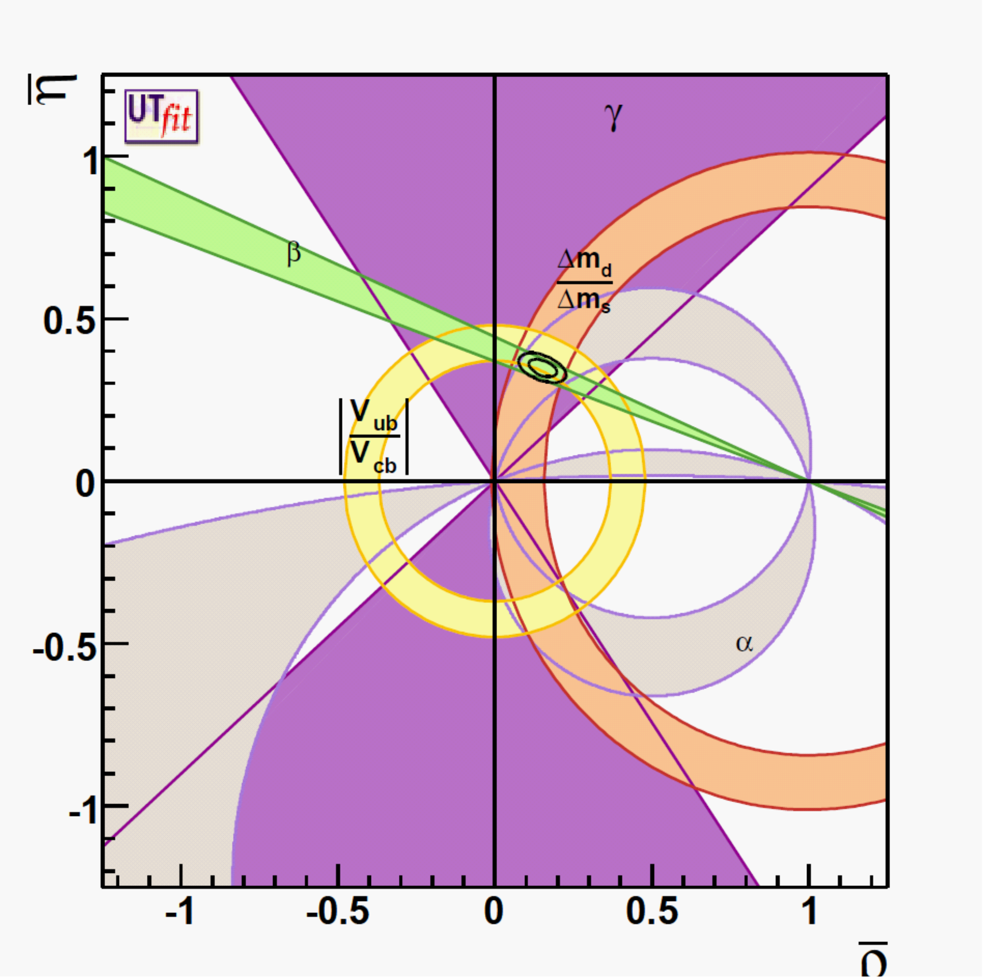
that essentially implies the same relative correction for $b \longrightarrow s$, $b \longrightarrow d$ and $s \longrightarrow d$ transitions. Some important predictions are

$$\frac{\Gamma\left(B_d \to l^+ l^-\right)}{\Gamma\left(B_s \to l^+ l^-\right)} \approx \frac{f_{B_d}^2 m_{B_d}}{f_{B_s}^2 m_{B_s}} \frac{|V_{td}|^2}{|V_{ts}|^2}$$

$$\frac{\Delta M_{B_d}}{\Delta M_{B_s}} = \frac{f_{B_d}^2 m_{B_d} B_{B_d}}{f_{B_s}^2 m_{B_s} B_{B_s}} \frac{|V_{td}|^2}{|V_{ts}|^2}$$

$$\frac{B_r\left(B \to X_d \nu \overline{\nu}\right)}{B_r\left(B \to X_s \nu \overline{\nu}\right)} = \frac{|V_{td}|^2}{|V_{ts}|^2}$$

That allows, for example, the determination of the Universal unitarity triangle



UTfit
γ
β
α
$\frac{\Delta m_d}{\Delta m_s}$
$\left|\frac{V_{ub}}{V_{cb}}\right|$
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1
$\bar{\eta}$
$\bar{\rho}$

6 MFV IN 2HDM I

In the two Higgs doublet model, the flavour structure is much more involved ($\tilde{\Phi}_j = i\sigma_2 \Phi_j^*$)

$$L_Y = -\overline{Q}_L \left(\Gamma_1 \Phi_1 + \Gamma_2 \Phi_2\right) d_R - \overline{Q}_L \left(\Delta_1 \tilde{\Phi}_1 + \Delta_2 \tilde{\Phi}_2\right) u_R + h.c.$$

In the Higgs basis: $\langle H_1 \rangle^T = \begin{pmatrix} 0 & \frac{v}{2} \end{pmatrix}, \langle H_2 \rangle^T = \begin{pmatrix} 0 & 0 \end{pmatrix}$

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = \begin{pmatrix} e^{i\theta_1} \frac{v_1}{v} & e^{i\theta_1} \frac{v_2}{v} \\ e^{i\theta_2} \frac{v_2}{v} & -e^{i\theta_2} \frac{v_1}{v} \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}$$

$$L_Y = -\overline{Q}_L \frac{\sqrt{2}}{v} \left[M_d H_1 + N_d H_2\right] d_R - \overline{Q}_L \frac{\sqrt{2}}{v} \left[M_u \widetilde{H}_1 + N_u \widetilde{H}_2\right] u_R + h.c.$$

$$M_d = \frac{1}{\sqrt{2}}\left(\Gamma_1 \upsilon_1 e^{i\theta_1} + \Gamma_2 \upsilon_2 e^{i\theta_2}\right)$$
$$N_d = \frac{1}{\sqrt{2}}\left(\Gamma_1 \upsilon_2 e^{i\theta_1} - \Gamma_2 \upsilon_1 e^{i\theta_2}\right)$$
$$M_u = \frac{1}{\sqrt{2}}\left(\Delta_1 \upsilon_1 e^{-i\theta_1} + \Delta_2 \upsilon_2 e^{-i\theta_2}\right)$$
$$N_u = \frac{1}{\sqrt{2}}\left(\Delta_1 \upsilon_2 e^{-i\theta_1} - \Delta_2 \upsilon_1 e^{-i\theta_2}\right)$$

In this framework the MFV hypothesis would be that all the flavour symmetries are broken by only two independent structures transforming as

$$\boxed{\begin{array}{c} O^d \to W_L^\dagger O^d W_R^d \\ O^u \to W_L^\dagger O^u W_R^u \end{array}}$$

One could choose $\left(\frac{\sqrt{2}}{\upsilon}\right) M_d$ and $\left(\frac{\sqrt{2}}{\upsilon}\right) M_u$. Traditionally it is used: Γ_1 and Δ_2. And the MFV expansion is now

$$\Gamma_2 = \left[\epsilon_0 I + \epsilon_1 \Gamma_1 \Gamma_1^\dagger + \epsilon_2 \Delta_2 \Delta_2^\dagger + \epsilon_3 \Delta_2 \Delta_2^\dagger \Gamma_1 \Gamma_1^\dagger + \epsilon_4 \Gamma_1 \Gamma_1^\dagger \Delta_2 \Delta_2^\dagger + \cdot\right] \Gamma_1$$
$$\Delta_1 = \left[\epsilon'_0 I + \epsilon'_1 \Gamma_1 \Gamma_1^\dagger + \epsilon'_2 \Delta_2 \Delta_2^\dagger + \epsilon'_3 \Delta_2 \Delta_2^\dagger \Gamma_1 \Gamma_1^\dagger + \epsilon'_4 \Gamma_1 \Gamma_1^\dagger \Delta_2 \Delta_2^\dagger + \cdot\right] \Delta_2$$

The presence of two vacuum expectation values invalidates the argument previously used to neglect the down Yukawa couplings. Therefore here one has to keep

$$\Gamma_1\Gamma_1^\dagger \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b^2 \end{pmatrix} \;\; ; \;\; \Delta_2\Delta_2^\dagger \sim V^\dagger \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t^2 \end{pmatrix} V$$

7 MFV IN 2HDM II

We can trye again the MFV in the 2HDM but this time we will use M_d and M_u as the basic objects that brakes the flavour symmetries

$$L_Y = -\overline{Q}_L \frac{\sqrt{2}}{v} \left[M_d H_1 + N_d H_2 \right] d_R - \overline{Q}_L \, \frac{\sqrt{2}}{v} \left[M_u \widetilde{H}_1 + N_u \widetilde{H}_2 \right] u_R + h.c.$$

$$M_d = \frac{1}{\sqrt{2}}\left(\Gamma_1 \upsilon_1 e^{i\theta_1} + \Gamma_2 \upsilon_2 e^{i\theta_2}\right) \quad ; \quad M_u = \frac{1}{\sqrt{2}}\left(\Delta_1 \upsilon_1 e^{-i\theta_1} + \Delta_2 \upsilon_2 e^{-i\theta_2}\right)$$
$$N_d = \frac{1}{\sqrt{2}}\left(\Gamma_1 \upsilon_2 e^{i\theta_1} - \Gamma_2 \upsilon_1 e^{i\theta_2}\right) \quad ; \quad N_u = \frac{1}{\sqrt{2}}\left(\Delta_1 \upsilon_2 e^{-i\theta_1} - \Delta_2 \upsilon_1 e^{-i\theta_2}\right)$$

So we need the MFV expansion of N_d and N_u. This time one would write ($H_d = M_d M_d^\dagger$, $H_u = M_u M_u^\dagger$)

$$\begin{aligned} N_d &= [\epsilon_0 I + \epsilon_1 H_d + \epsilon_2 H_u + \epsilon_3 H_u H_d + \epsilon_4 H_d H_u + \cdot]\, M_d \\ N_u &= \left[\epsilon'_0 I + \epsilon'_1 H_d + \epsilon'_2 H_u + \epsilon'_3 H_u H_d + \epsilon'_4 H_d H_u + \cdot\right] M_u \end{aligned}$$

But let us use our projection operators:

$$\begin{aligned} N_d &= \left[a_0 I + a_{1j}\mathcal{P}_j^{d_L} + a_{2j}\mathcal{P}_j^{u_L} + a_{3ij}\mathcal{P}_i^{u_L}\mathcal{P}_j^{d_L} + a_{4ij}\mathcal{P}_i^{d_L}\mathcal{P}_j^{u_L} + \cdot\right] M_d \left[b_k \mathcal{P}_k^{d_R}\right] \\ N_u &= \left[a'_0 I + a'_{1j}\mathcal{P}_j^{d_L} + a'_{2j}\mathcal{P}_j^{u_L} + a'_{3ij}\mathcal{P}_i^{u_L}\mathcal{P}_j^{d_L} + a'_{4ij}\mathcal{P}_i^{d_L}\mathcal{P}_j^{u_L} + \cdot\right] M_u \left[b'_k \mathcal{P}_k^{u_R}\right] \end{aligned}$$

But

$$M_d\left[b_k \mathcal{P}_k^{d_R}\right] = \left(U_L^d D_d U_R^{d\dagger}\right) b_k U_R^d P_k U_R^{d\dagger} = b_k U_L^d P_k U_L^{d\dagger} U_L^d D_d U_R^{d\dagger} = \left[b_k \mathcal{P}_k^{d_L}\right] M_d$$
$$M_u\left[b'_k \mathcal{P}_k^{u_R}\right] = \left[b'_k \mathcal{P}_k^{u_L}\right] M_u$$

So the most general MFV expansion in 2HDM is

$$\boxed{\begin{array}{l} N_d = \left(a_0 I + a_{1j} P_j^{d_L} + a_{2j} P_j^{u_L} + a_{3ij} P_i^{u_L} P_j^{d_L} + a_{4ij} P_i^{d_L} P_j^{u_L} + \cdot\right) M_d \\ N_u = \left(a'_0 I + a'_{1j} P_j^{d_L} + a'_{2j} P_j^{u_L} + a'_{3ij} P_i^{u_L} P_j^{d_L} + a'_{4ij} P_i^{d_L} P_j^{u_L} + \cdot\right) M_u \end{array}}$$

Remarkably enough it can be shown that renormalizable models known long time ago and enforced by flavour symmetries (Branco, Grimus, Lavoura) realize the most simple MFV expansion with controlled FCYC.

For example one BGL model is enforced by

$$Q_{L_3} \to e^{i\alpha} Q_{L_3} \quad ; \quad u_{R_3} \to e^{i2\alpha} u_{R_3} \quad ; \quad \Phi_2 \to e^{i\alpha} \Phi_2$$

It correspond to the model defined by the MFV expansion

$$\begin{array}{l} N_d = \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_3^{u_L}\right] M_d \\ N_u = \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_3^{u_L}\right] M_u \end{array}$$

This is the Up3 model and obviously there are other two Up1,2 models. These models have FCYC in the down sector controlled by quark masses and CKM matrix elements:

$$\widehat{N}_d = U_L^{d\dagger} N_d U_R^d = \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) V^\dagger P_3 V\right] D_d$$
$$\widehat{N}_u = U_L^{u\dagger} N_u U_R^u = \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) P_3\right] D_u$$

$$\left(\widehat{N}_d\right)_{ij} = \left[\frac{v_2}{v_1}\delta_{ij} - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) V_{3i}^* V_{3j}\right] m_{dj}$$
$$\left(\widehat{N}_u\right)_{ij} = \left[\frac{v_2}{v_1}\delta_{ij} - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \delta_{ij}\delta_{i3}\right] m_{u_j}$$

In a similar way there are three Down models with FCYC in the up sector defined by

$$Q_{L_3} \to e^{i\alpha} Q_{L_3} \quad ; \quad d_{R_3} \to e^{i2\alpha} d_{R_3} \quad ; \quad \Phi_2 \to e^{i\alpha} \Phi_2$$

$$N_d = \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_3^{d_L}\right] M_d$$
$$N_u = \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_3^{d_L}\right] M_u$$

Because two exact relations of the 2HDM are

$$\begin{aligned} N_d &= \frac{v_2}{v_1} M_d - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) \frac{v_2}{\sqrt{2}} e^{i\theta_2} \Gamma_2 \\ N_u &= \frac{v_2}{v_1} M_d - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) \frac{v_2}{\sqrt{2}} e^{-i\theta_2} \Delta_2 \end{aligned}$$

These BGL models are fully defined by the relations (Up3)

$$\begin{aligned} \frac{v_2}{\sqrt{2}} e^{i\theta_2} \Gamma_2 &= \mathcal{P}_3^{u_L} M_d \\ \frac{v_2}{\sqrt{2}} e^{-i\theta_2} \Delta_2 &= \mathcal{P}_3^{u_L} M_u \end{aligned}$$

$$N_d = \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) \mathcal{P}_3^{u_L} \right] M_d \iff \mathcal{P}_3^{u_L} \Gamma_2 = \Gamma_2 \text{ and } \mathcal{P}_3^{u_L} \Gamma_1 = 0$$

$$N_u = \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) \mathcal{P}_3^{u_L} \right] M_u \iff \mathcal{P}_3^{u_L} \Delta_2 = \Delta_2 \text{ and } \mathcal{P}_3^{u_L} \Delta_1 = 0$$

In some basis this implies

$$\Gamma_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix} \quad ; \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}$$

$$\Delta_1 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad ; \quad \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}$$

The simplicity of the relations that defines BGL model suggest immediately to propose several new models that we do not know if they can be derived from symmetries. But the simplicity of its definition makes it easy to study its stability under RGE. For example

$$N_d = \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_3^{u_L}\right] M_d \iff \mathcal{P}_3^{u_L}\Gamma_2 = \Gamma_2 \text{ and } \mathcal{P}_3^{u_L}\Gamma_1 = 0$$
$$N_u = \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_3^{d_L}\right] M_u \iff \mathcal{P}_3^{d_L}\Delta_2 = \Delta_2 \text{ and } \mathcal{P}_3^{d_L}\Delta_1 = 0$$

$$N_d = \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_3^{u_L}\right] M_d \iff \mathcal{P}_3^{u_L}\Gamma_2 = \Gamma_2 \text{ and } \mathcal{P}_3^{u_L}\Gamma_1 = 0$$
$$N_u = \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_2^{u_L}\right] M_u \iff \mathcal{P}_2^{u_L}\Delta_2 = \Delta_2 \text{ and } \mathcal{P}_2^{u_L}\Delta_1 = 0$$

These models are not stable under RGE.There other models stable under RGE than in one sense or another are trivially equivalent to BGL.

8 CONCLUSIONS

We have shown the usefulness of projection operators to write physical observables in term of Weak Basis transformation invariants.

We have made a brief review of MFV violation and have shown that instead of powers of $\Gamma\Gamma^\dagger$ and $\Delta\Delta^\dagger$ one can use left-handed projection operators $\mathcal{P}_i^{u_L}$ and $\mathcal{P}_j^{d_L}$.

In the 2HDM the most general MFV expansion is

$$\boxed{\begin{array}{l} N_d = \left(a_0 I + a_{1j} P_j^{d_L} + a_{2j} P_j^{u_L} + a_{3ij} P_i^{u_L} P_j^{d_L} + a_{4ij} P_i^{d_L} P_j^{u_L} + \cdot\right) M_d \\ N_u = \left(a'_0 I + a'_{1j} P_j^{d_L} + a'_{2j} P_j^{u_L} + a'_{3ij} P_i^{u_L} P_j^{d_L} + a'_{4ij} P_i^{d_L} P_j^{u_L} + \cdot\right) M_u \end{array}}$$

In these sense we have find that the first models that implement the MFV ansantz in a fully renormalizable theory: The Branco Grimus Lavoura models with the flavour symmetries

$$Q_{L_j} \to e^{i\alpha} Q_{L_j} \quad ; \quad u_{R_j} \to e^{i2\alpha} u_{R_j} \quad ; \quad \Phi_2 \to e^{i\alpha} \Phi_2$$

$$Q_{L_j} \to e^{i\alpha} Q_{L_j} \quad ; \quad d_{R_j} \to e^{i2\alpha} d_{R_j} \quad ; \quad \Phi_2 \to e^{i\alpha} \Phi_2$$

meet this MFV expansion in a very simple way

$$a_0 = a'_0 = \frac{v_2}{v_1} \quad ; \quad a_{2j} = a'_{2j} = -\left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \quad ; \quad a_{2i\neq j} = a'_{2i\neq j} = \cdot\cdot = 0$$

$$a_0 = a'_0 = \frac{v_2}{v_1} \quad ; \quad a_{1j} = a'_{1j} = -\left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \quad ; \quad a_{1i\neq j} = a'_{1i\neq j} = \cdot\cdot = 0$$