The Higgs Triplet Model and the LHC results

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Based on PRD 84 (2011) & JHEP 1204 (2012).

Outline

- Higgs history....
- Motivations
- The Higgs Triplet Model (HTM)
- Theoretical constraints:
- Discussions
- HTM phenomenology and LHC
  - $h \to \gamma\gamma$
  - $h \to ZZ, WW, \tau\tau$ and $b\bar{b}$
  - $h \to \gamma Z$
- Summary and Conclusions

Brief Suvey of Higgs history...

- 1964 an important year: Birthday of Higgs boson.
  - July, 24th: Brout and Englert' paper accepted by **PLB**
  - July, 31st: Higgs sent his paper to **PLB**, but.....**rejected!**
  - August, 31th: received by **PRL** and finally....**accepted!**
  - October, 12th: Guralnik, Hagen and Kibble sent their paper to **PRL**
  - july, 4th: ATLAS and CMS collaborations reported observation of a Higgs boson at around 125 GeV.
- 2012, 28 - 31 August: Multi Higgs Models
  Last 3 days: Talks about 2HDM, 4HDM,....IDM.
  It is time to see a different SM extension....
  the Higgs Triplet Model.

Motivations...

There are several possible extensions to the SM:

- fermionic singlets, called right-handed neutrinos; this is Type I seesaw; [Minkowski'77, Mohapatra, Senjanović'79,Yanagida'79, Glashow'79, Gell-Mann, Ramond, Slansky'79]
- a scalar SU(2)$_L$ triplet ($Y = 2$): Type II seesaw; [Magg, Wetterich'80,Lazarides, Shafi, Wetterich '81, Mohapatra, Senjanović'81]
- a fermionic SU(2)$_L$ triplet ($Y = 0$): Type III seesaw [Foot, Lew, He, Joshi'89]

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- Hybrid seesaw: a combination of I and III [Ma'98,B. Bajc and G. Senjanović'06, Fileviez Pérez'07]
- Left-Right symmetric models both Type I and II seesaw are present [Mohapatra, Pati'75, Senjanović, Mohapatra'75]

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- Alternatively, neutrino masses can be generated by radiative corrections [Zee'80, Babu'88]

Motivations...

a scalar SU(2)$_L$ triplet ($Y = 2$);

$$\mathcal{L}_{\text{Yukawa}} \supset -Y_\nu L^T C \otimes i\sigma^2 \Delta L + \text{h.c.} \Rightarrow m_\nu = Y_\nu v_\Delta$$

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$$V(\Delta, H) = M_\Delta^2 Tr(\Delta^\dagger \Delta) + \mu (H^T i\tau_2 \Delta^\dagger H) + \dots ,$$

$$EWSB \Rightarrow v_\Delta = \mu v_d^2 / M_\Delta^2$$

$$\boxed{m_\nu = Y_\nu \mu v_d^2 / M_\Delta^2}$$

- μ term breaks lepton number by 2 units ($l_\Delta = -2$).

Motivations...

$$M_\Delta^2 Tr(\Delta^\dagger \Delta) + \mu (H^T i\tau_2 \Delta^\dagger H) \ ,$$

$$\boxed{m_\nu = Y_\nu \mu v_d^2 / M_\Delta^2}$$

- If $m_\nu \approx 1$ eV with $Y_\nu \approx 1$, then $M_\Delta \approx \mu \approx 10^{14-15}$ GeV
  Not testable at the LHC
- If $m_\nu \approx 1$ eV and $M_\Delta \approx 1$ TeV , $Y_\nu \mu \approx 10^{-8}$ GeV
- Small μ can be viewed as soft breaking term of lepton number.

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- Real Triplet Higgs: Cold dark matter candidate (CDM) if $vev = 0$
  Cirelli/Fornengo/Strumia, NPB'753 (2006)
  Cirelli/Strumia/Tamburini, NPB'787 (2007)

The Higgs Triplet Model (HTM)

HTM is an extension of SM with an extra scalar $SU(2)_L$ Higgs triplet Δ with $Y_\Delta = 2$ ($H \sim (1,2,1)$ and $\Delta \sim (1,3,2)$ under $SU(3)_c \times SU(2)_L \times U(1)_Y$)

$$\Delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix} \qquad \text{and} \qquad H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

The most general renormalizable Lagrangian:

$$\mathcal{L} = (D_\mu H)^\dagger (D^\mu H) + Tr(D_\mu \Delta)^\dagger (D^\mu \Delta) - V(H,\Delta) + \mathcal{L}_{\text{Yukawa}}$$

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$$V = M_\Delta^2 Tr\Delta^\dagger\Delta - m_H^2 H^\dagger H + \frac{\lambda}{4}(H^\dagger H)^2 + \lambda_1 (H^\dagger H) Tr(\Delta^\dagger\Delta)$$
$$+\lambda_2 (Tr\Delta^\dagger\Delta)^2 + \lambda_3 Tr(\Delta^\dagger\Delta)^2 + \lambda_4 H^\dagger \Delta\Delta^\dagger H + \mu H^T i\tau_2 \Delta^\dagger H + h$$

- Inclusion of the μ term eliminates the Majoron.

The Higgs Boson Spectrum

After the EWSB, when

$$\Delta = \begin{pmatrix} 0 & 0 \\ v_t/\sqrt{2} & 0 \end{pmatrix} \qquad \text{and} \qquad H = \begin{pmatrix} 0 \\ v_d/\sqrt{2} \end{pmatrix}$$

Minimization of the potential yields:

$$\begin{aligned} M_\Delta^2 &= \frac{2\mu v_d^2 - \sqrt{2}(\lambda_1+\lambda_4)v_d^2 v_t - 2\sqrt{2}(\lambda_2+\lambda_3)v_t^3}{2\sqrt{2}v_t} \\ m_H^2 &= \frac{\lambda v_d^2}{4} - \sqrt{2}\mu v_t + \frac{(\lambda_1+\lambda_4)}{2}v_t^2 \end{aligned}$$

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$$m_H^2 = \frac{\lambda v_d^2}{4} - \sqrt{2}\mu v_t + \frac{(\lambda_1+\lambda_4)}{2}v_t^2$$

- Thanks to : $M_W^2 = g^2(v_d^2 + 2v_t^2)/4$
  Independents parameters= 7 (10-3): 5 λ_i's, μ, 2 vev.

- The HTM has 7 Higgs bosons: h, H (CP-even), A (CP-odd), $H^\pm$, $H^{\pm\pm}$.

Doublet-Triplet Mixing

- $H^{\pm\pm}$ is purely triplet: $\Delta^{++} = \delta^{++}$
- $H^{\pm}$ and A are dominated by the triplet δ fields,

  Mixing $\approx v_t/v$, small ($v_t/v \leq 0.03$)
- h and H are mixtures of doublet ϕ and triplet δ fields,

$$\tan 2\alpha = \frac{2\mathcal{M}_{12}^2}{\mathcal{M}_{11}^2 - \mathcal{M}_{22}^2}$$

maximal mixing is possible for $\mathcal{M}_{11}^2 = \mathcal{M}_{22}^2$ (when h and H are close to degenerate)

[Akeroyd/Chiang, PRD'10]
[Dey/Kundu/Mukhopadhyaya, J. Phys'09]

Constraint on Higgs triplet vev: v_t

The parameter $\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_w}$

In HTM, Z and W masses are,

$$M_Z^2 = \frac{(g^2+g'^2)(v_d^2+4v_t^2)}{4} = \frac{g^2(v_d^2+4v_t^2)}{4\cos^2\theta_W}$$

$$M_W^2 = \frac{g^2(v_d^2+2v_t^2)}{4}$$

hence,

$$\rho = \frac{M_W^2}{c_W^2 M_Z^2} = \frac{v_d^2+2v_t^2}{v_d^2+4v_t^2} \simeq 1 - 2\frac{v_t^2}{v_d^2} \approx 1+\delta\rho$$

At the 2 σ level, $\rho_0 = 1.0004^{+0.0029}_{-0.0011}$ (or $\rho_0 = 1.0008^{+0.0017}_{-0.0010}$),

Compatible with $\delta\rho < 0$, and provides an upper bound on v_t:

$v_t \leq$ 4.6 GeV

onstraint on μ: Absence of tachyonic mod

Positivity of Higgses masses requires that:

$$m_A^2 = \frac{\mu(v_d^2 + 4v_t^2)}{\sqrt{2}v_t} \Rightarrow \mu > 0$$

$$m_{H^{\pm\pm}}^2 = \frac{\sqrt{2}\mu v_d^2 - \lambda_4 v_d^2 v_t - 2\lambda_3 v_t^3}{2v_t} \Rightarrow \mu > \frac{\lambda_4 v_t}{\sqrt{2}} + \sqrt{2}\frac{\lambda_3 v_t^3}{v_d^2}$$

$$m_{H^{\pm}}^2 = \frac{(v_d^2 + 2v_t^2)[2\sqrt{2}\mu - \lambda_4 v_t]}{4v_t} \Rightarrow \mu > \frac{\lambda_4 v_t}{2\sqrt{2}}$$

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In CP-even sector, tachyonless condition is more involved:

$$-8\mu^2 v_t + \sqrt{2}\mu(\lambda v_d^2 + 8\lambda_{14} v_t^2) + 4(\lambda\lambda_{23} - \lambda_{14})v_t^3 > 0$$

$$(\lambda_{14}^2 - \lambda\lambda_{23})\frac{2\sqrt{2}}{\lambda}\frac{v_t^3}{v_d^2} + \mathcal{O}(v_t^4) < \mu < \frac{\lambda}{4\sqrt{2}}\frac{v_d^2}{v_t} + \sqrt{2}\lambda_{14} v_t + \mathcal{O}(v_t^2).$$

with $\lambda_{ij} = \lambda_i + \lambda_j$

Boundedness From Below (BFB)

Vacuum stability ($V > V_{min}$) requires that the potential should be BFB. At large field values, V dominated by quartic terms

$$V^{(4)}(H,\Delta) = \frac{\lambda}{4}(H^\dagger H)^2 + \lambda_1(H^\dagger H)Tr(\Delta^\dagger\Delta) + \lambda_2(Tr\Delta^\dagger\Delta)^2$$
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- If we pick up neutral directions: (what is usually done)

$$V_0^{(4)} = \frac{\lambda}{4}|\phi^0|^4 + (\lambda_2+\lambda_3)|\delta^0|^4 + (\lambda_1+\lambda_4)|\phi^0|^2|\delta^0|^2 =$$

$$[\frac{\sqrt{\lambda}}{2}|\phi^0|^2 - \sqrt{\lambda_2+\lambda_3}|\delta^0|^2]^2 + (\lambda_{14} + \sqrt{\lambda(\lambda_{23})})|\phi^0|^2|\delta^0|^2$$

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- $\lambda > 0 \;\&\; \lambda_2 + \lambda_3 > 0 \;\&\; \lambda_1 + \lambda_4 + \sqrt{\lambda(\lambda_2+\lambda_3)} > 0$

BFB

What about the other 10 directions:
$(\phi^0\ ,\ \delta^{++})$, $(\phi^0\ ,\ \delta^{+})$, $(\phi^0\ ,\ \phi^{+})$, $(\delta^{+}\ ,\ \phi^{+})$ ...

- $(\phi^0\ ,\ \delta^0)$ neutral direction:

  $$\lambda > 0\ \&\ \lambda_2 + \lambda_3 > 0\ \&\ \lambda_1 + \lambda_4 + \sqrt{\lambda(\lambda_2 + \lambda_3)} > 0$$

- $(\phi^0\ ,\ \delta^{++})$ direction:

  $$\lambda > 0\ \&\ \lambda_2 + \lambda_3 > 0\ \&\ \lambda_1 + \sqrt{\lambda(\lambda_2 + \lambda_3)} > 0$$

- $(\delta^{+}\ ,\ \phi^{+})$ direction:

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- (ϕ^0 , δ^{++}) direction:
  $\lambda > 0 \;\&\; \lambda_2 + \lambda_3 > 0 \;\&\; \lambda_1 + \sqrt{\lambda(\lambda_2 + \lambda_3)} > 0$

- (δ^{+} , ϕ^{+}) direction:
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- It is obvious that these sets are neither equivalent nor contained in the neutral direction. Neutral direction BFB constraint is neither necessary nor sufficient

BFB: General proof

A more convenient parameterization,

$$r \equiv \sqrt{H^\dagger H + Tr\Delta^\dagger\Delta} > 0$$

$$H^\dagger H \equiv r^2 \cos^2\gamma$$

$$Tr(\Delta^\dagger\Delta) \equiv r^2 \sin^2\gamma \quad ; \quad -\frac{\pi}{2} < \gamma < +\frac{\pi}{2}$$

$$Tr(\Delta^\dagger\Delta)^2/(Tr\Delta^\dagger\Delta)^2 \equiv \zeta \in [\frac{1}{2}, 1]$$

$$(H^\dagger\Delta\Delta^\dagger H)/(H^\dagger H Tr\Delta^\dagger\Delta) \equiv \xi \in [0, 1]$$

$$V_0^{(4)} = \frac{r^4\cos^4\gamma}{4}(\lambda + 4(\lambda_1 + \xi\lambda_4)\tan^2\gamma + 4(\lambda_2 + \zeta\lambda_3)\tan^4\gamma)$$

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$$= a + b\chi^2 + c\chi^4 = (\sqrt{a} - \sqrt{c}\chi^2)^2 + (b + 2\sqrt{ac})\chi^2 \Rightarrow a > 0 \,\&\, c > 0 \,\&\, b + 2\sqrt{ac} > 0$$

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$$\lambda > 0 \;\&\; \lambda_2 + \zeta\lambda_3 > 0 \;\&\; \lambda_1 + \xi\lambda_4 + \sqrt{\lambda(\lambda_2 + \zeta\lambda_3)} > 0$$

Kaffas, Khater, Ogreid and Osland, NPB'45, 2007

BFB: General proof

$$\lambda > 0 \,\&\, \lambda_2 + \zeta\lambda_3 > 0 \;\&\, \lambda_1 + \xi\lambda_4 + \sqrt{\lambda(\lambda_2 + \zeta\lambda_3)} > 0$$

$$\forall\zeta \in [\frac{1}{2}, 1], \forall\xi \in [0, 1]$$

Since dependence on ξ and ζ is monotonic, then

$$\lambda > 0 \;\&\; \lambda_2 + \lambda_3 > 0 \;\&\; \lambda_2 + \frac{\lambda_3}{2} > 0$$

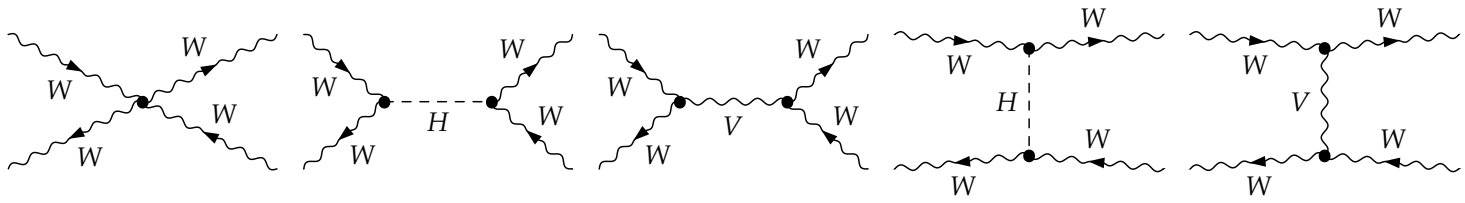
$$\&\; \lambda_1 + \sqrt{\lambda(\lambda_2 + \lambda_3)} > 0 \;\&\; \lambda_1 + \sqrt{\lambda(\lambda_2 + \frac{\lambda_3}{2})} > 0$$

$$\&\; \lambda_1 + \lambda_4 + \sqrt{\lambda(\lambda_2 + \lambda_3)} > 0 \;\&\; \lambda_1 + \lambda_4 + \sqrt{\lambda(\lambda_2 + \frac{\lambda_3}{2})} > 0$$

Most general all directions necessary and sufficient BFB conditions (at tree level).

Perturbative Unitarity

In the SM: $W_L W_L$, $Z_L Z_L$, HH, $W_L H$, $Z_L H$



the scattering amplitude $\mathcal{M}$ can be written as:

$$\mathcal{M}(s,t,u) = 16\pi \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) a_l(s)$$

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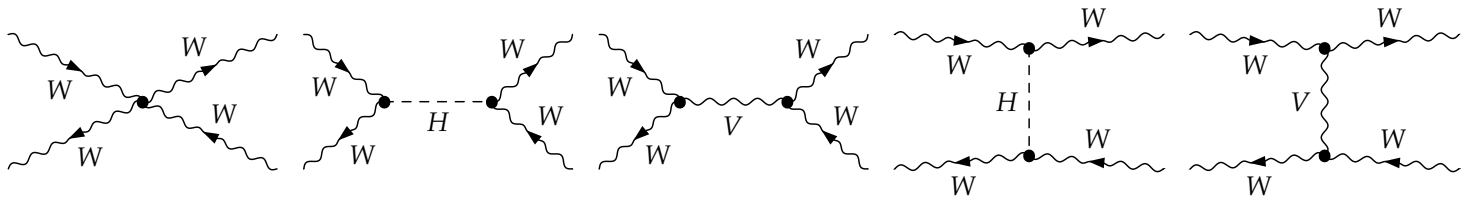

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If we limit ourselves to the ($l = 0$) s–wave amplitude $a_0(s)$

$$a_0 = \frac{M_{\phi^0}^2}{v^2} \begin{pmatrix} 1 & \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & 0 & 0 \\ \frac{\sqrt{2}}{4} & \frac{3}{4} & \frac{1}{4} & 0 & 0 \\ \frac{\sqrt{2}}{4} & \frac{1}{4} & \frac{3}{4} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad , \quad \begin{array}{r} |a_0| \leq 1 \\ \\ M_{\phi^0} \leq 710\text{GeV} \end{array}$$

Perturbative Unitarity

In the HTM: the scattering amplitude is 35×35 matrix which can be cast to 7 sub-matrices:

- $S_1(6 \times 6)$, $S_2(7 \times 7)$, $S_3(2 \times 2)$, (0-charge channels): $\delta^0\delta^0$, $\phi^+\phi^-$, $\delta^{++}\delta^{--}$
- $S_{(4)}(10 \times 10)$, (1-charge channels) : $\delta^0\phi^+$
- $S_{(5)}(7 \times 7)$, (2-charge channels) : $\phi^+\phi^+$
- $S_{(6)}(2 \times 2)$, (3-charge channels): $\delta^{++}\phi^+$
- $S_{(7)}(1 \times 1)$, (4-charge channels): $\delta^{++}\delta^{++}$

Perturbative Unitarity

As an illustrative example, consider the submatrix corresponding to scattering in the basis: $(\phi^+\phi^-, \delta^+\delta^-, \delta^{++}\delta^{--}, Im(\phi^0)Im(\phi^0), Im(\delta^0)Im(\delta^0), Re(\phi^0)Re(\phi^0), Re(\delta^0)Re(\delta^0)$

$$\mathcal{M}_2 = \begin{pmatrix}
\lambda & \frac{\tilde{\lambda}_{14}}{2} & \lambda^+_{14} & \frac{\lambda}{2\sqrt{2}} & \frac{\lambda_1}{\sqrt{2}} & \frac{\lambda}{2\sqrt{2}} & \frac{\lambda_1}{\sqrt{2}} \\
\frac{\tilde{\lambda}_{14}}{2} & 2\tilde{\lambda}_{23} & 2\lambda^+_{23} & \frac{\tilde{\lambda}_{14}}{2\sqrt{2}} & \sqrt{2}\lambda^+_{23} & \frac{\tilde{\lambda}_{14}}{2\sqrt{2}} & \sqrt{2}\lambda^+_{23} \\
\lambda^+_{14} & 2\lambda^+_{23} & 4\lambda^+_{23} & \frac{\lambda_1}{\sqrt{2}} & \sqrt{2}\lambda_2 & \frac{\lambda_1}{\sqrt{2}} & \sqrt{2}\lambda_2 \\
\frac{\lambda}{2\sqrt{2}} & \frac{\tilde{\lambda}_{14}}{2\sqrt{2}} & \frac{\lambda_1}{\sqrt{2}} & \frac{3}{4}\lambda & \frac{\lambda^+_{14}}{2} & \frac{\lambda}{4} & \frac{\lambda^+_{14}}{2} \\
\frac{\lambda_1}{\sqrt{2}} & \sqrt{2}\lambda^+_{23} & \sqrt{2}\lambda_2 & \frac{\lambda^+_{14}}{2} & 3\lambda^+_{23} & \frac{\lambda^+_{14}}{2} & \lambda^+_{23} \\
\frac{\lambda}{2\sqrt{2}} & \frac{\tilde{\lambda}_{14}}{2\sqrt{2}} & \frac{\lambda_1}{\sqrt{2}} & \frac{\lambda}{4} & \frac{\lambda^+_{14}}{2} & \frac{3\lambda}{4} & \frac{\lambda^+_{14}}{2} \\
\frac{\lambda_1}{\sqrt{2}} & \sqrt{2}\lambda^+_{23} & \sqrt{2}\lambda_2 & \frac{\lambda^+_{14}}{2} & \lambda^+_{23} & \frac{\lambda^+_{14}}{2} & 3\lambda^+_{23}
\end{pmatrix}$$

$$f_1 = \frac{\lambda}{2} \quad , \quad f_2 = 2\lambda_2 \quad , \quad f_3 = 2(\lambda_2 + \lambda_3)$$

$$a_\pm = \frac{1}{4}[\lambda + 4\lambda_2 + 8\lambda_3 \pm \sqrt{(\lambda - 4\lambda_2 - 8\lambda_3)^2 + 16\lambda_4^2}\,]$$

Unitarity

Unitarity:

$$|\lambda_1+\lambda_4|\leq\kappa\pi \quad ; \quad |\lambda_1|\leq\kappa\pi \quad ; \quad |2\lambda_1+3\lambda_4|\leq 2\kappa\pi$$

$$|\lambda|\leq 2\kappa\pi \quad ; \quad |\lambda_2|\leq\frac{\kappa}{2}\pi \quad ; \quad |\lambda_2+\lambda_3|\leq\frac{\kappa}{2}\pi$$

$$|2\lambda_1-\lambda_4|\leq 2\kappa\pi \quad ; \quad |2\lambda_2-\lambda_3|\leq\kappa\pi$$

$$|\lambda+4\lambda_2+8\lambda_3\pm\sqrt{(\lambda-4\lambda_2-8\lambda_3)^2+16\lambda_4^2}\,|\leq 4\kappa\pi$$

$$|3\lambda+16\lambda_2+12\lambda_3\pm\sqrt{(3\lambda-16\lambda_2-12\lambda_3)^2+24(2\lambda_1+\lambda_4)^2}\,|$$
$$\leq 4\kappa\pi$$

$\kappa=16$ *or* 8, *depending on :* $|a_0|<1$ *or* $|\Re e a_0|<\frac{1}{2}$

BFB & Unitarity

Combined set of unitarity and BFB constraints written in a compact and analytical simple form:

$$0 \leq \lambda \leq \frac{2}{3}\kappa\pi$$

$$\lambda_2 + \lambda_3 \geq 0 \;\; \& \;\; \lambda_2 + \frac{\lambda_3}{2} \geq 0$$

$$\lambda_2 + 2\lambda_3 \leq \frac{\kappa}{2}\pi$$

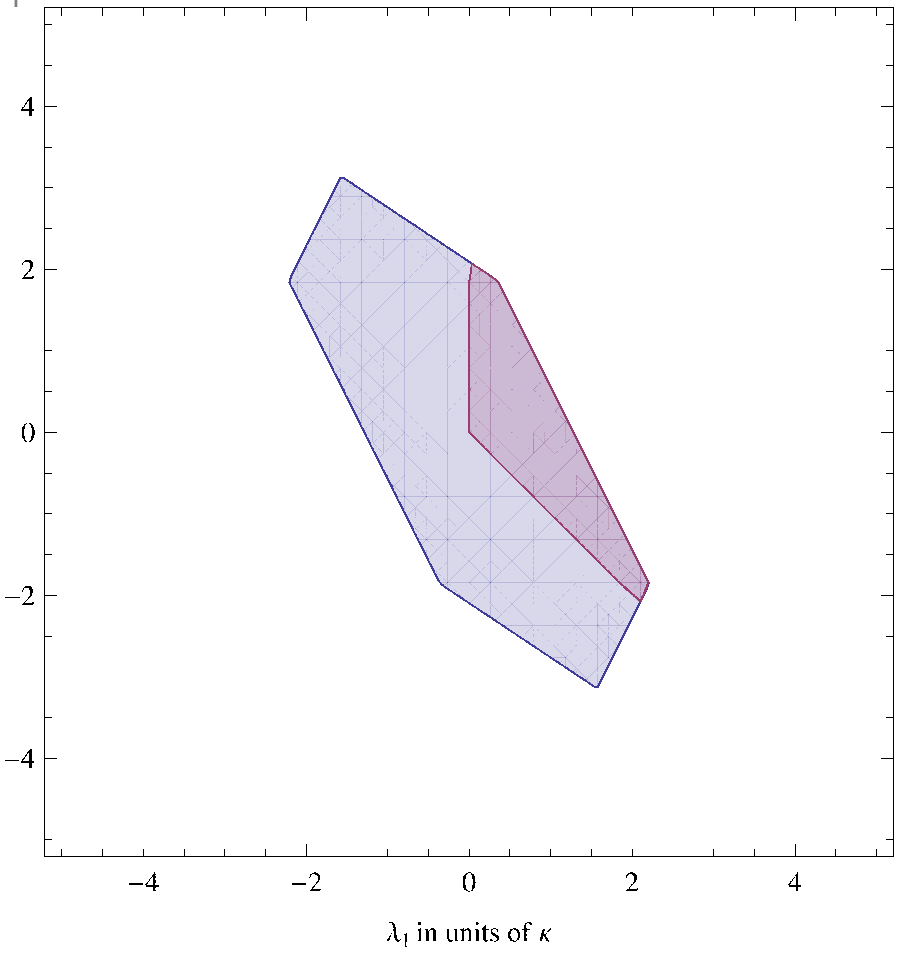
$$4\lambda_2 + 3\lambda_3 \leq \frac{\kappa}{2}\pi$$

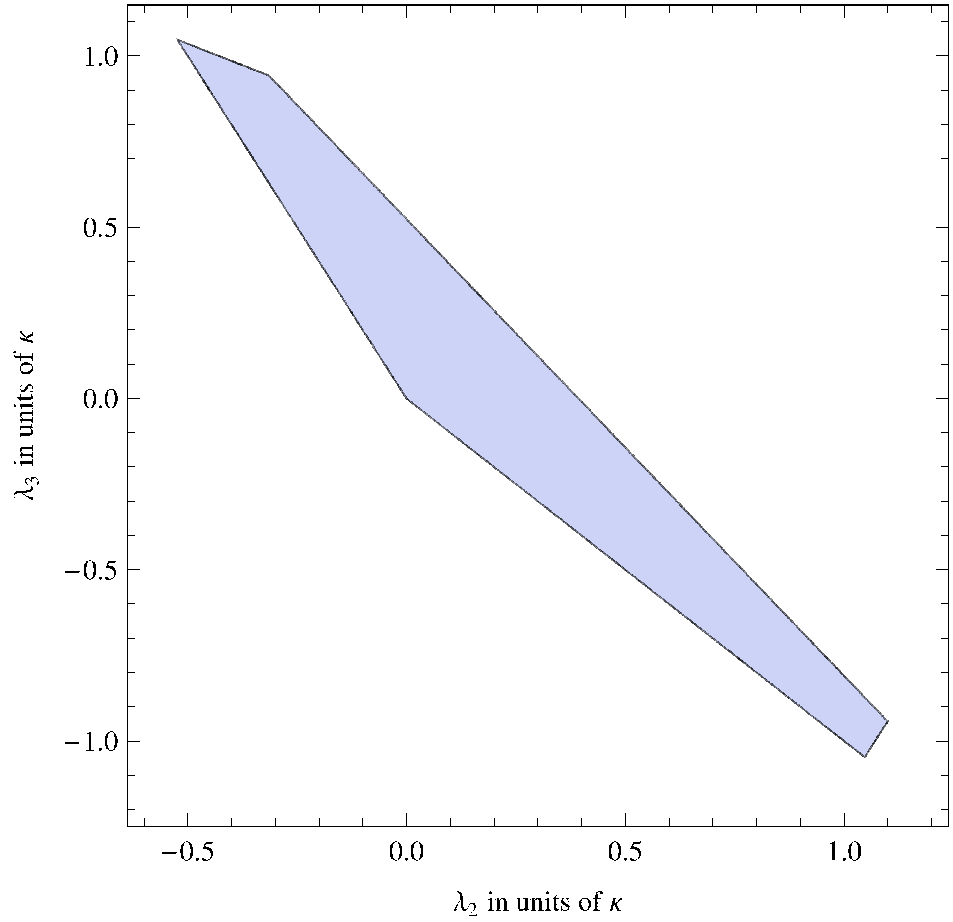
$$2\lambda_2 - \lambda_3 \leq \kappa\pi$$

$$|\lambda_4| \leq \min \sqrt{(\lambda \pm 2\kappa\pi)(\lambda_2 + 2\lambda_3 \pm \frac{\kappa}{2}\pi)}$$

$$|2\lambda_1 + \lambda_4| \leq \sqrt{2(\lambda - \frac{2}{3}\kappa\pi)(4\lambda_2 + 3\lambda_3 - \frac{\kappa}{2}\pi)}$$

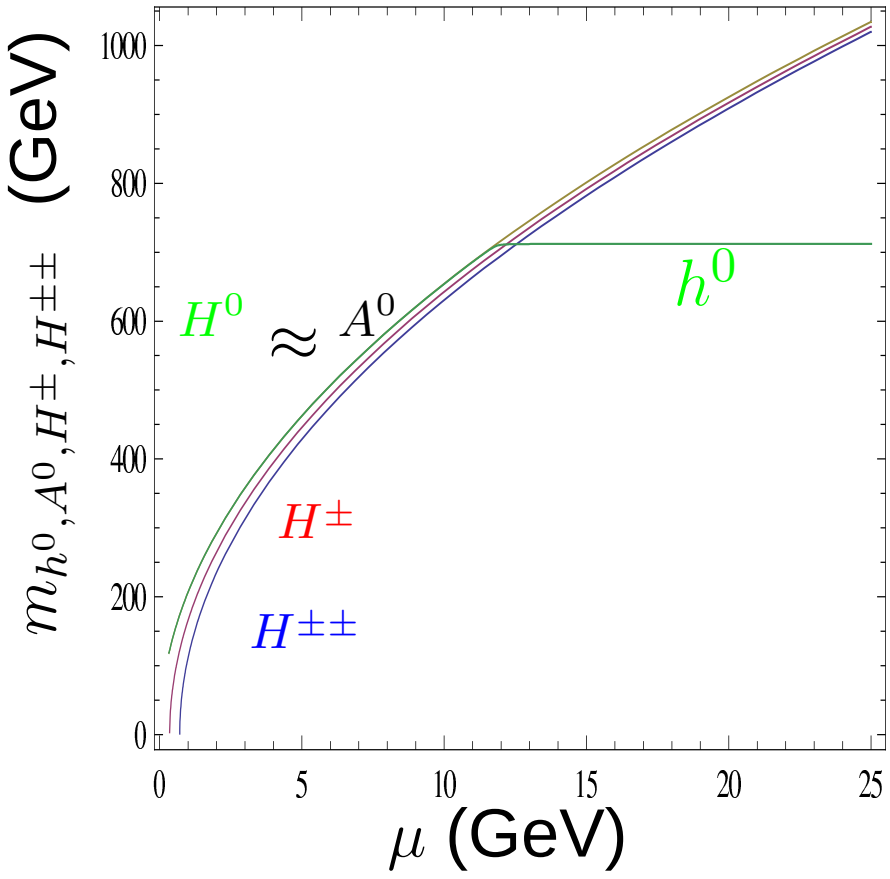
Discussions

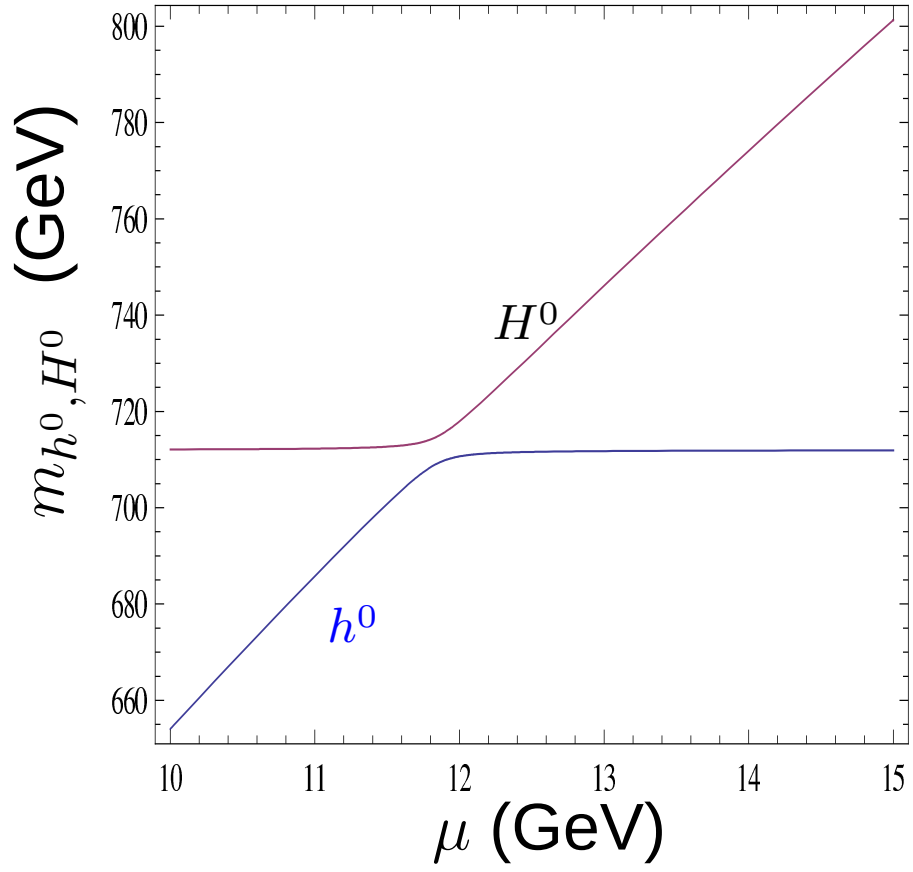




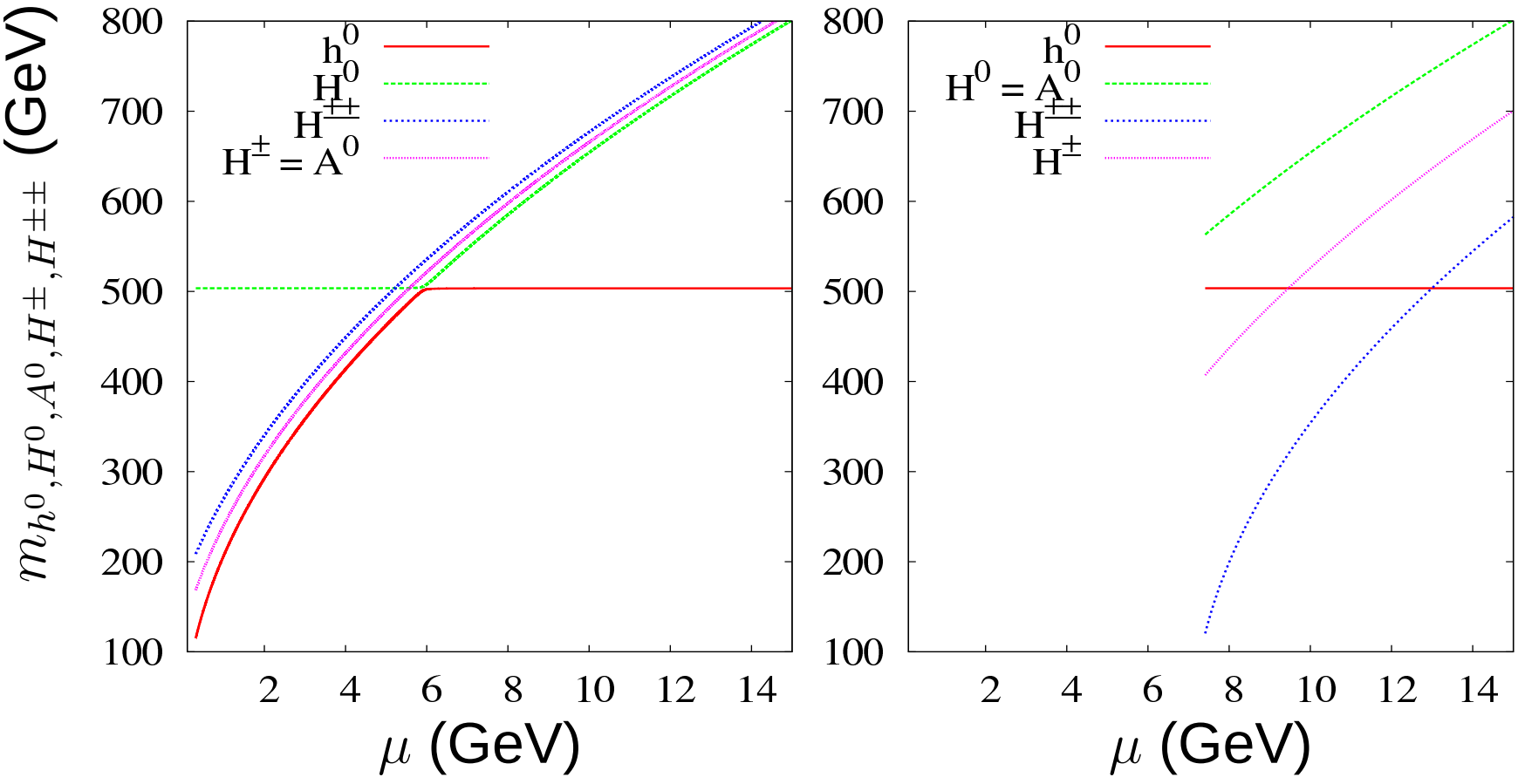
Left: $\lambda_2 = \lambda_3 = 0$, Right: $\lambda_1 = \lambda_4 = 0$

Results



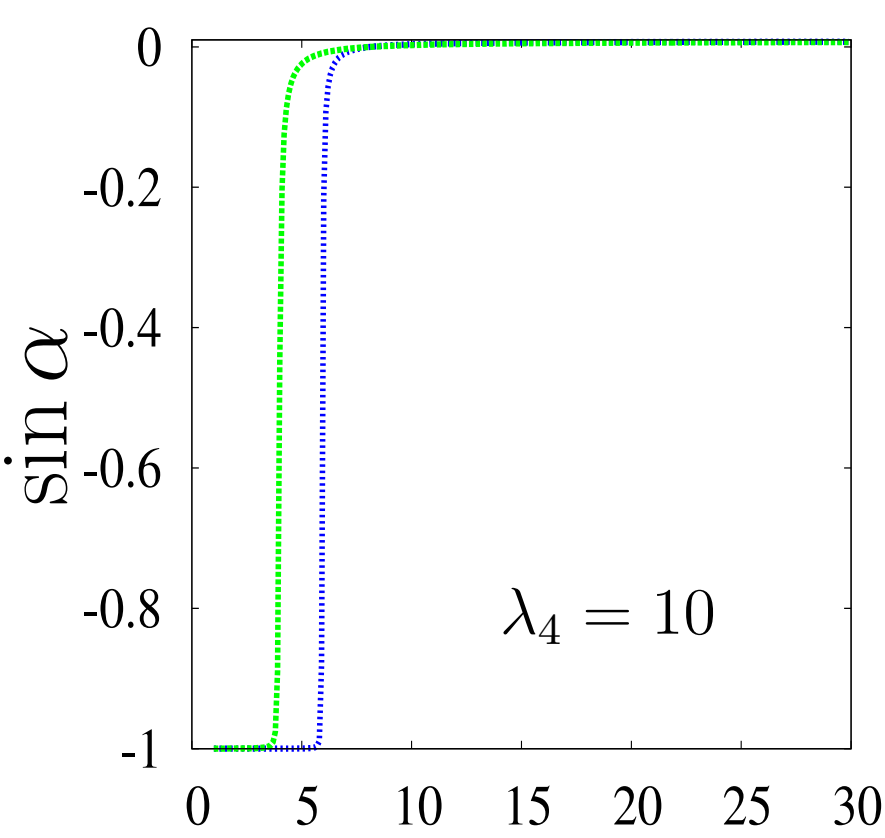


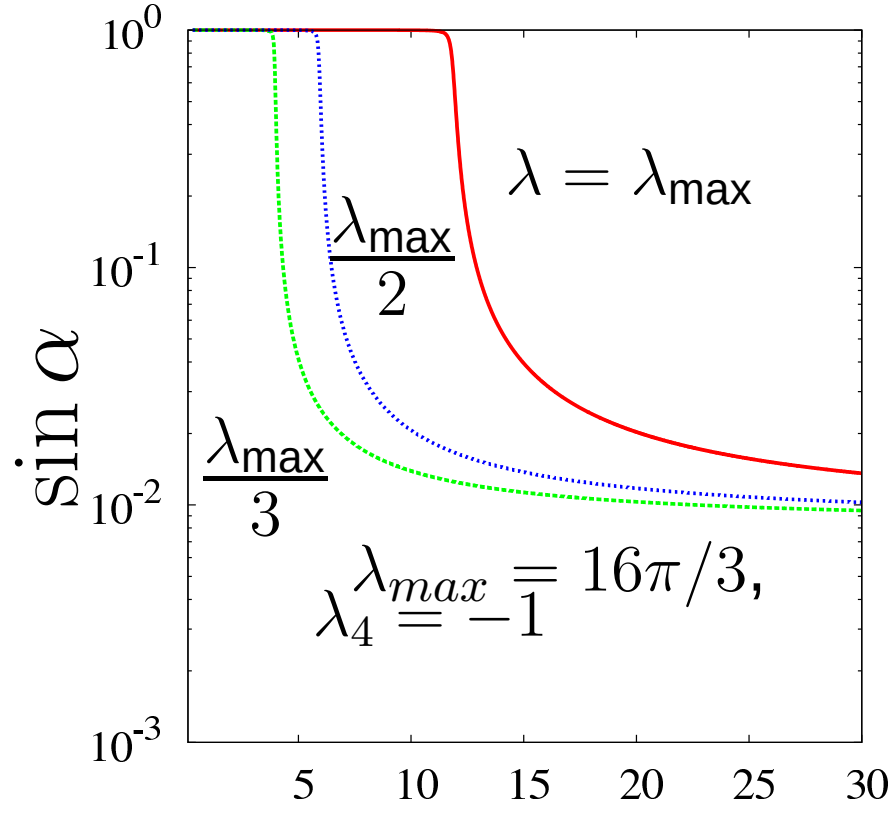
$$\lambda = \frac{16\pi}{3}, \lambda_2 = 0.1, \lambda_3 = 0.2, \lambda_1 = -0.5, \lambda_4 = 1, v_t = 1 \text{ GeV}$$



Higgs boson masses as a function of μ with $v_t = 1$ GeV, $\lambda = 8\pi/3$, $\lambda_1 = 0.5$, $\lambda_2 = \lambda_3 = 0.1$, $\lambda_4 = -1$ (left) and $\lambda_4 = 10$ (right)

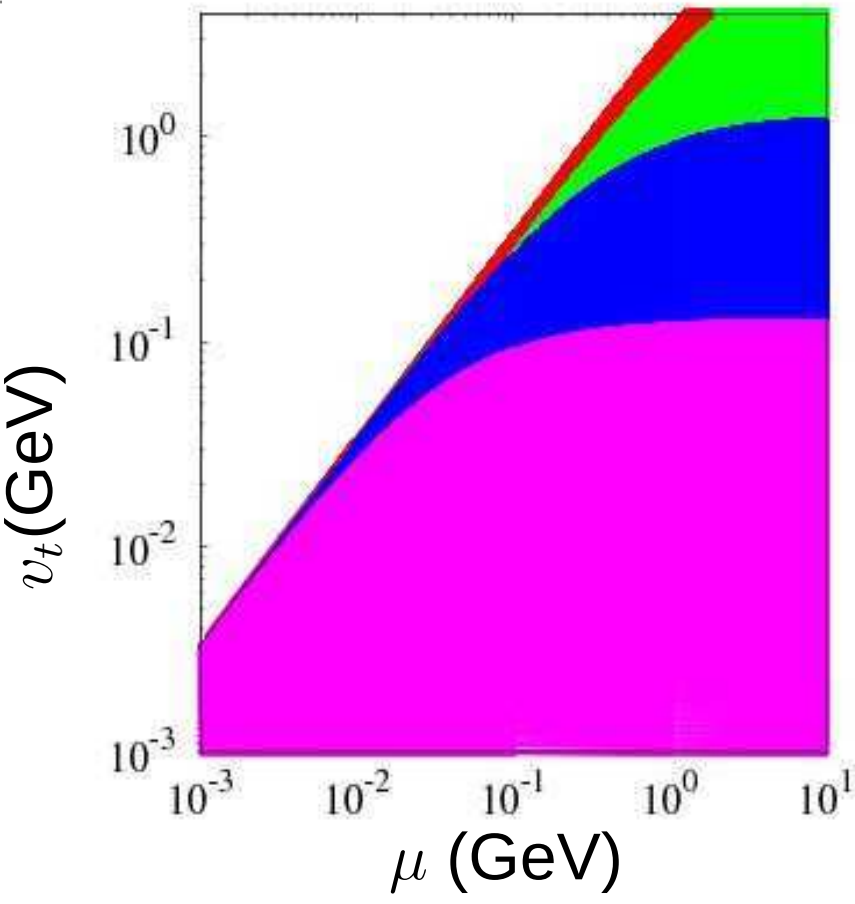
$$h^0 = \sin\alpha \Re e(\phi^0) + \cos\alpha \Re e(\delta^0)$$

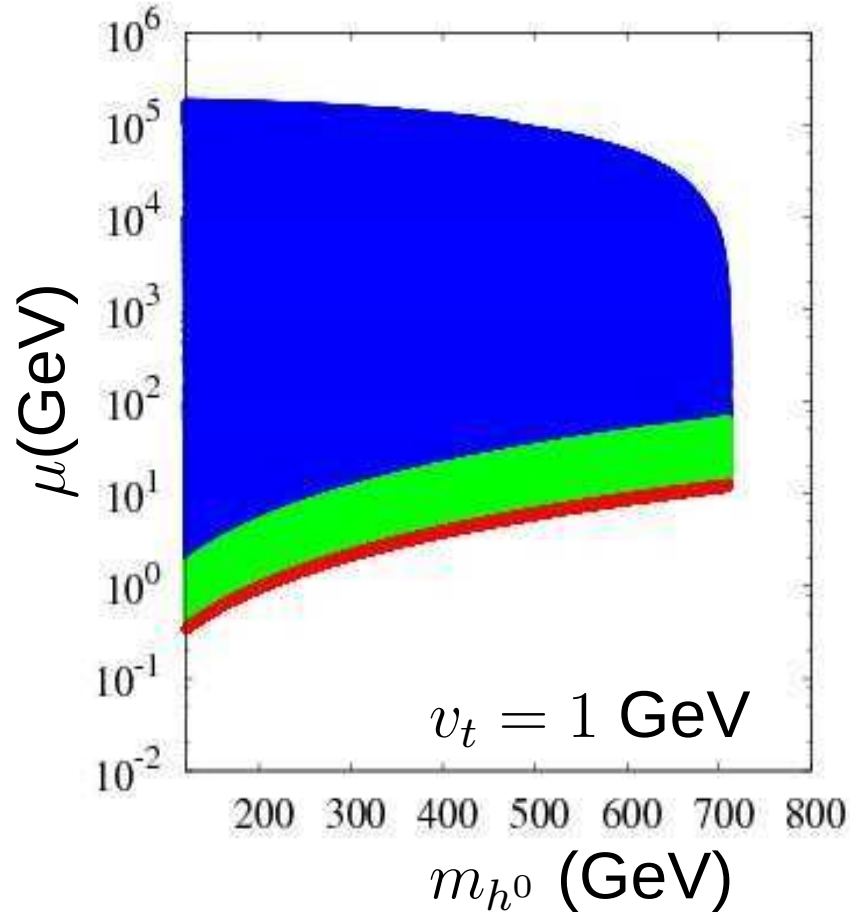




μ (GeV)

$\lambda_2 = \lambda_3 = 0.1,\ \lambda_1 = 0.5, v_t = 1$ GeV





(a) Correlation between μ and v_t with $\lambda = 0.48$, (b) Correlation between μ and m_{h^0}, with: $0.44 \leq \lambda \leq 16\pi/3$ $10^{-1} \leq s_\alpha \leq 1$, $10^{-2} \leq s_\alpha \leq 10^{-1}$, $10^{-3} \leq s_\alpha \leq 10^{-2}$

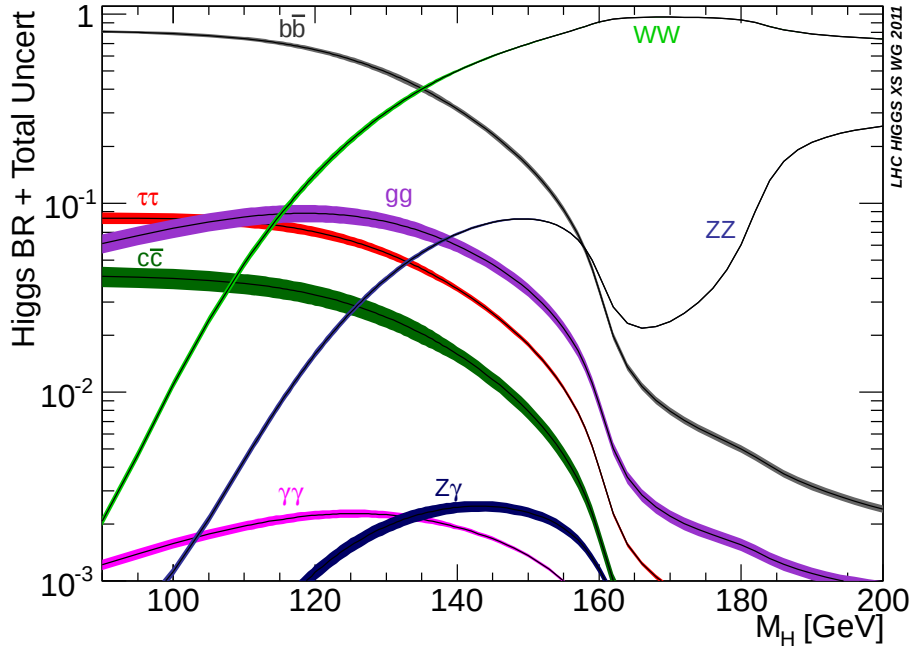
$\lambda_1 = -\lambda_4 = 1$, $\lambda_2 = \lambda_3 = 0$

Higgs phenomenology and LHC

- ATLAS AND CMS diphoton search results: July 4th, 2012.
  Observation of new particle with Higgs like properties
- CDF and D0: Support evidence for a Higgs boson in decay to a pair of bottom quarks.
- Best fit m_h in the range $125 - 126 GeV$
  ATLAS-CONF-2012-093 and CMS-HIG-2012-028
- Golden era for Higgs phenomenology after discovery.
- Top priority of colliders: Support Higgs boson discovery.
- Rich low mass region: 5 decay modes can be exploited
  $\tau\tau$, $b\bar{b}$, $\gamma\gamma$, ZZ, WW.

Higgs phenomenology and LHC

- Rich low mass region: 5 decay modes can be exploited $\tau\tau$, $b\bar{b}$, $\gamma\gamma$, ZZ, WW.



Higgs phenomenology and LHC

- To look for any excess in these decay channels and compare with LHC results, we define:

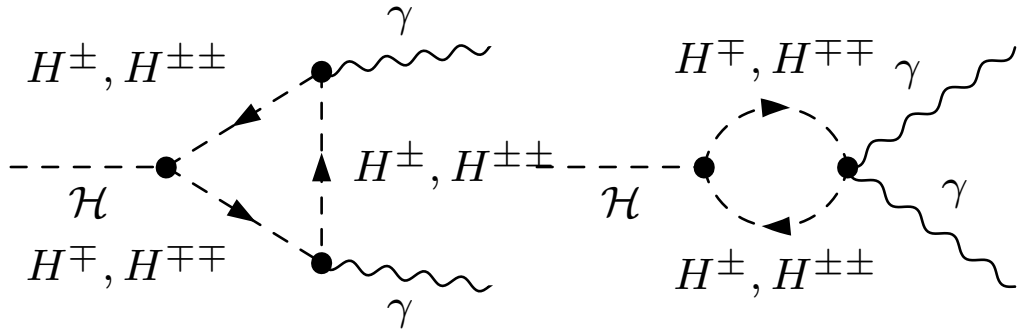
$$R_{XX} = \frac{(\Gamma(h \to gg) \times Br(h \to XX))_{HTM}}{(\Gamma(h \to gg) \times Br(h \to XX))_{SM}}$$

$h \to \gamma\gamma$ in HTM

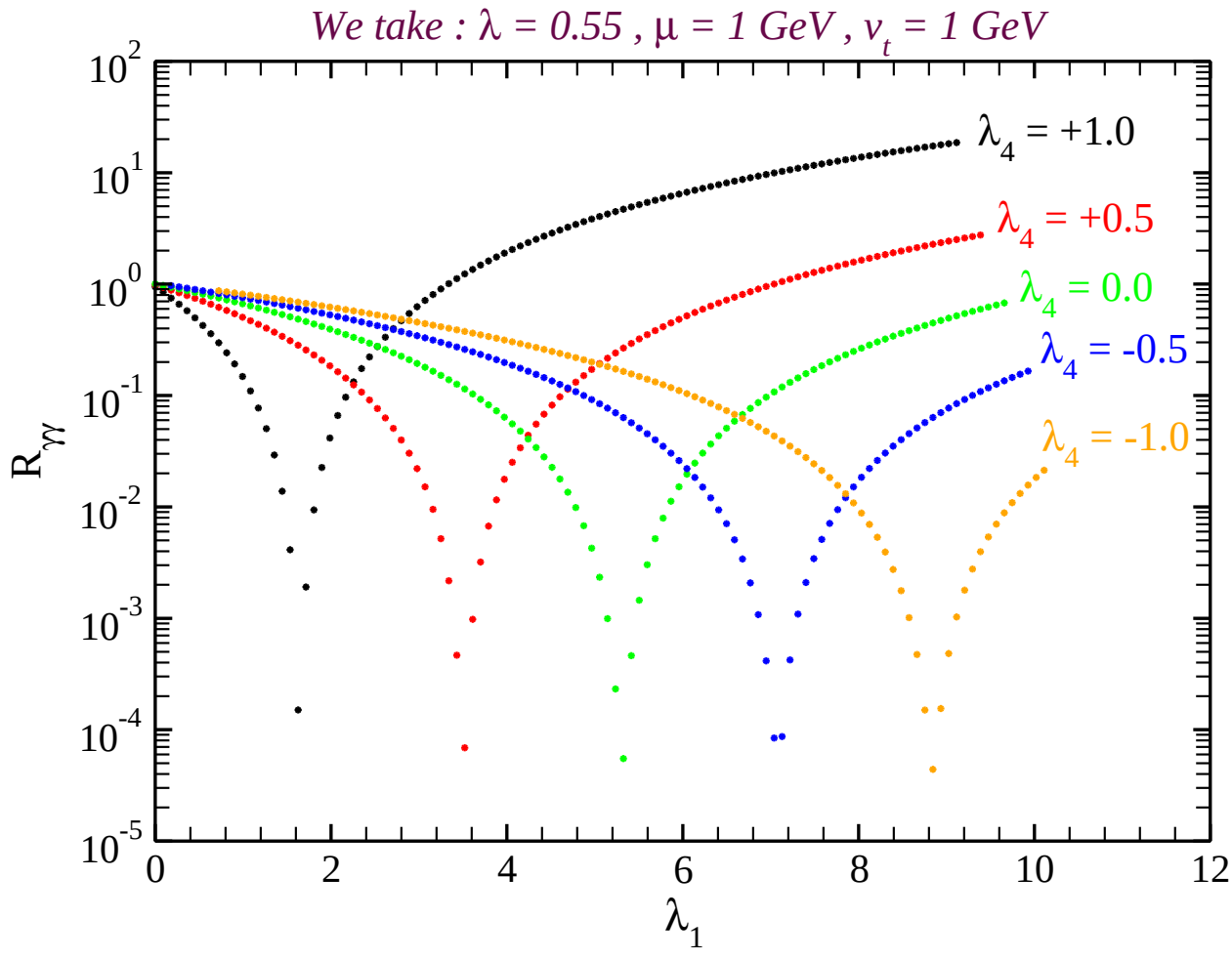
- $Higgs \to \gamma\gamma$ decay mode of special interest:
  Tool to constrain parameter space of New Physics.

$$g_{h^0 H^{++} H^{--}} = -\{2\lambda_2 v_t s_\alpha + \lambda_1 v_d c_\alpha\} \approx -\lambda_1 v_d c_\alpha$$

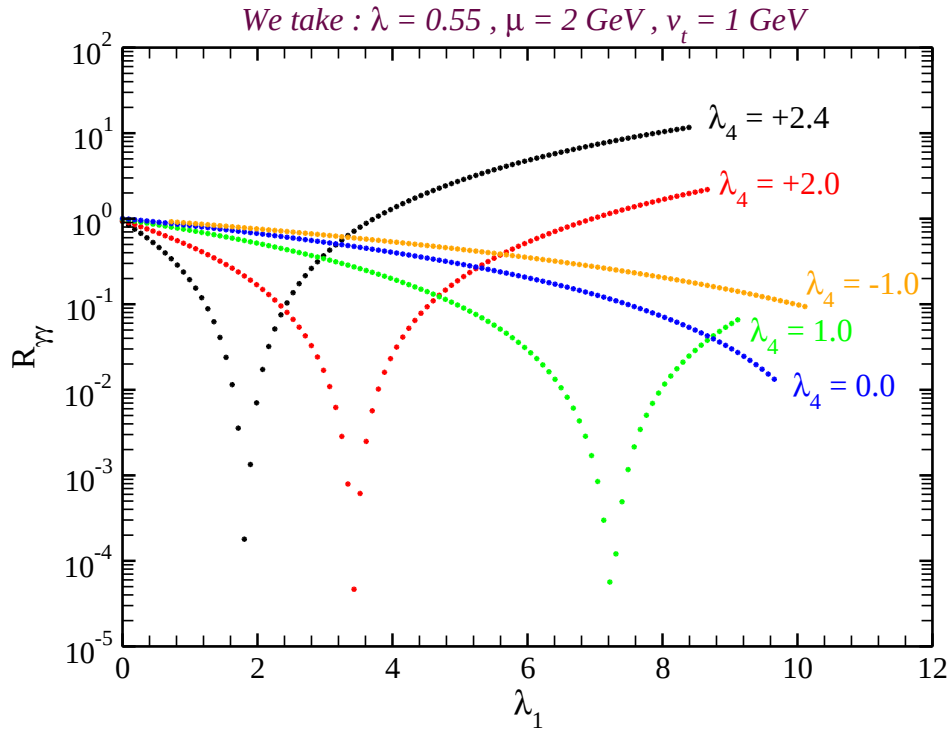
$$g_{h^0 H^{+} H^{-}} = -\frac{1}{2}\Big\{\{\lambda\, v_d s^2_{\beta'} + \tilde{\lambda}^{+}_{14} v_d c^2_{\beta'} + (4\mu - \sqrt{2}\lambda_4 v_t) c_{\beta'} s_{\beta'}\} c_\alpha\Big\}$$



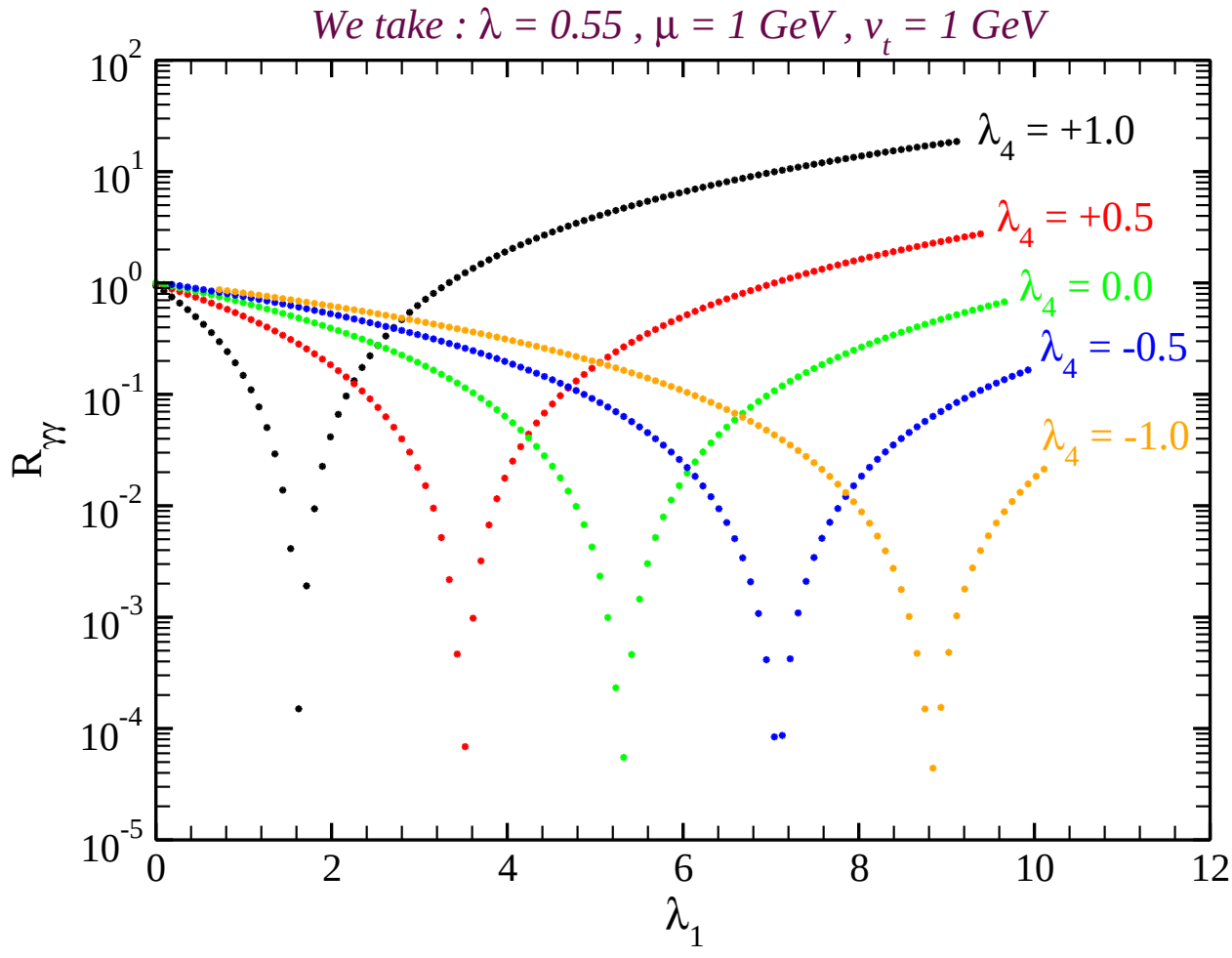
$h \to \gamma\gamma$

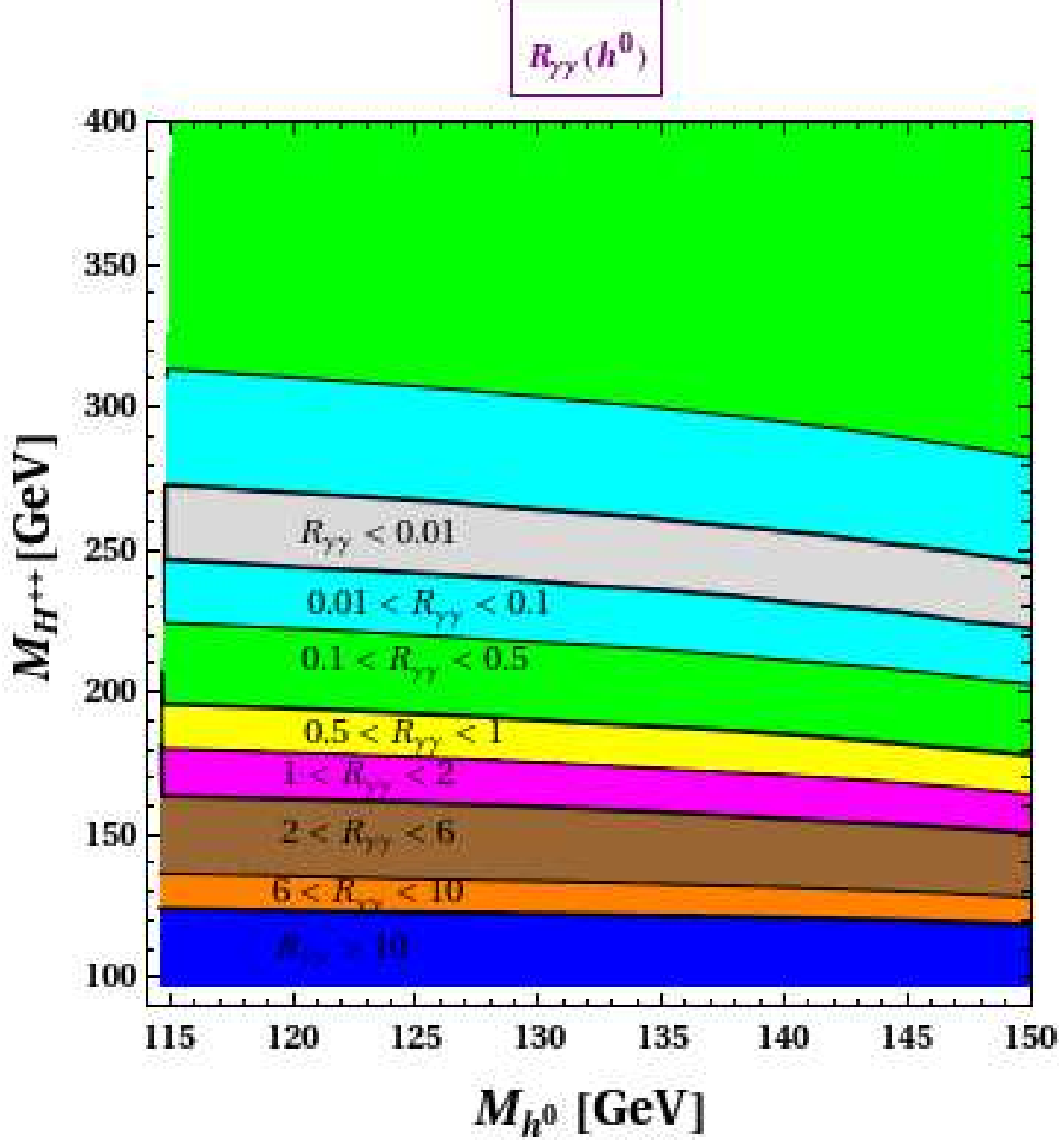


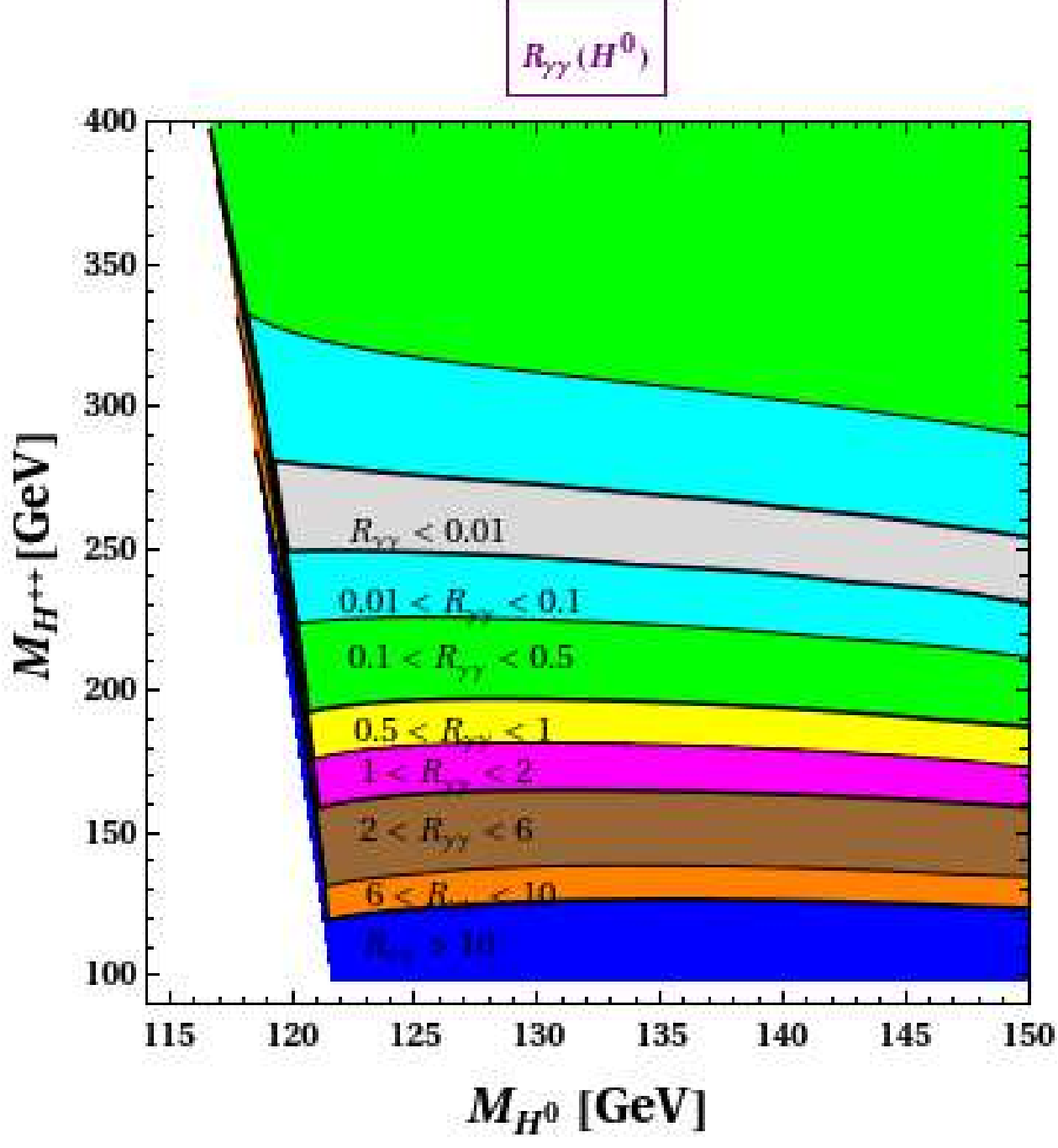
$h \to \gamma\gamma$



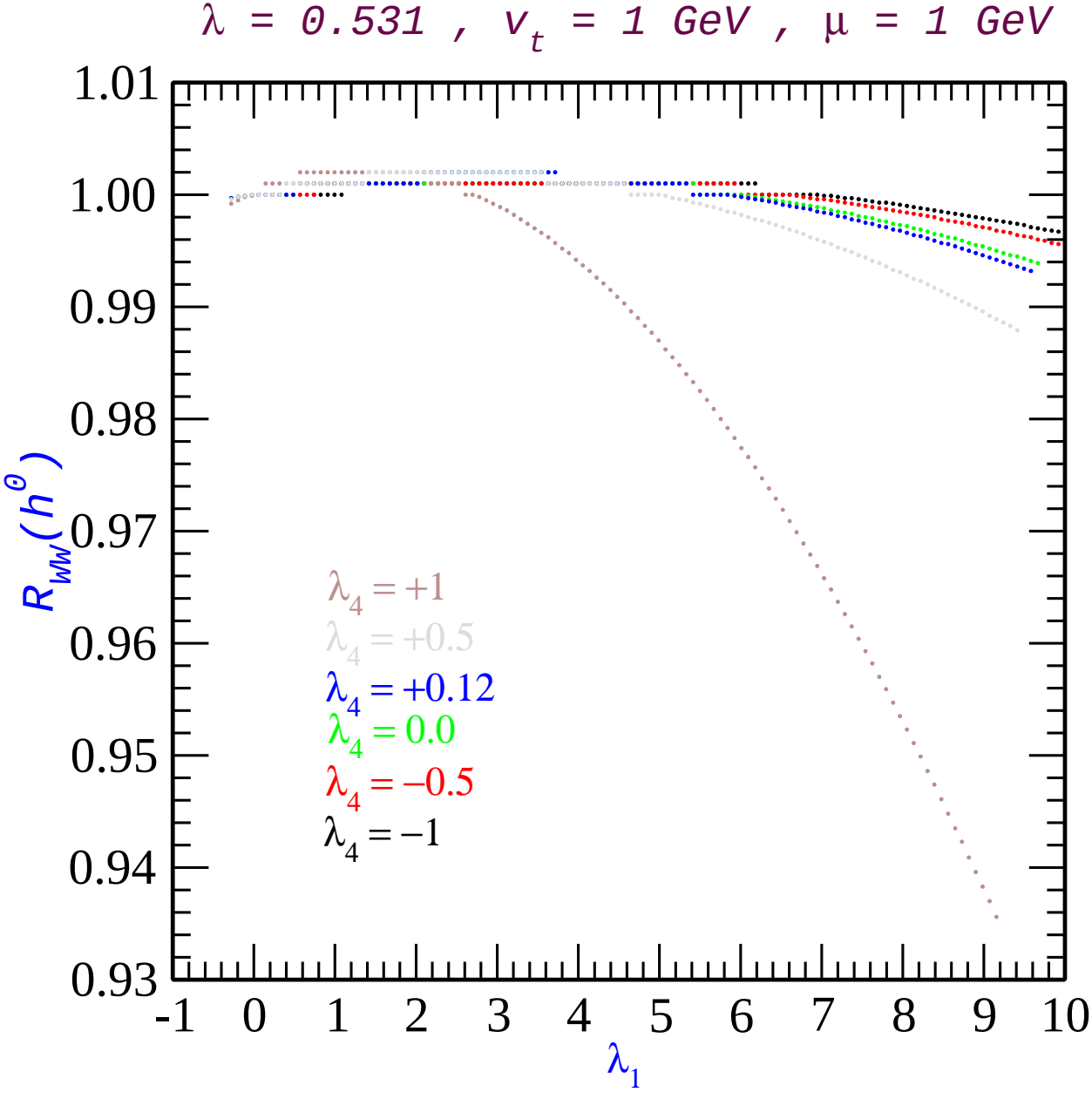
$h \to \gamma\gamma$



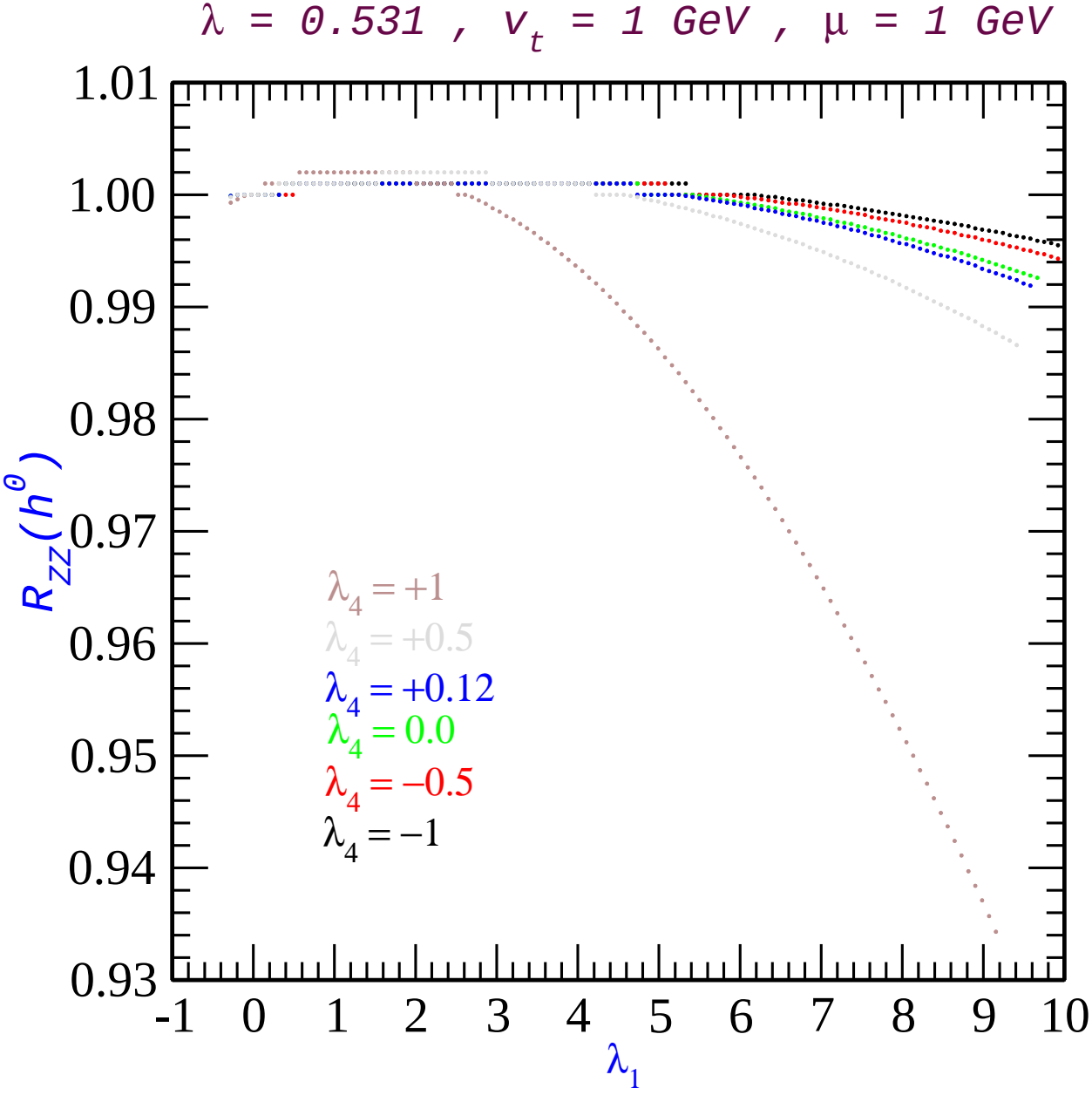

$R_{\gamma\gamma}(h^0)$
400
350
300
250
200
150
100
$M_{H^{++}}$ [GeV]
$R_{\gamma\gamma} < 0.01$
$0.01 < R_{\gamma\gamma} < 0.1$
$0.1 < R_{\gamma\gamma} < 0.5$
$0.5 < R_{\gamma\gamma} < 1$
$1 < R_{\gamma\gamma} < 2$
$2 < R_{\gamma\gamma} < 6$
$6 < R_{\gamma\gamma} < 10$
$R_{\gamma\gamma} > 10$
115
120
125
130
135
140
145
150
M_{h^0} [GeV]


$R_{\gamma\gamma}(H^0)$
400
350
300
250
200
150
100
115
120
125
130
135
140
145
150
$M_{H^{++}}$ [GeV]
M_{H^0} [GeV]
$R_{\gamma\gamma} < 0.01$
$0.01 < R_{\gamma\gamma} < 0.1$
$0.1 < R_{\gamma\gamma} < 0.5$
$0.5 < R_{\gamma\gamma} < 1$
$1 < R_{\gamma\gamma} < 2$
$2 < R_{\gamma\gamma} < 6$
$6 < R_{\gamma\gamma} < 10$
$R_{\gamma\gamma} > 10$

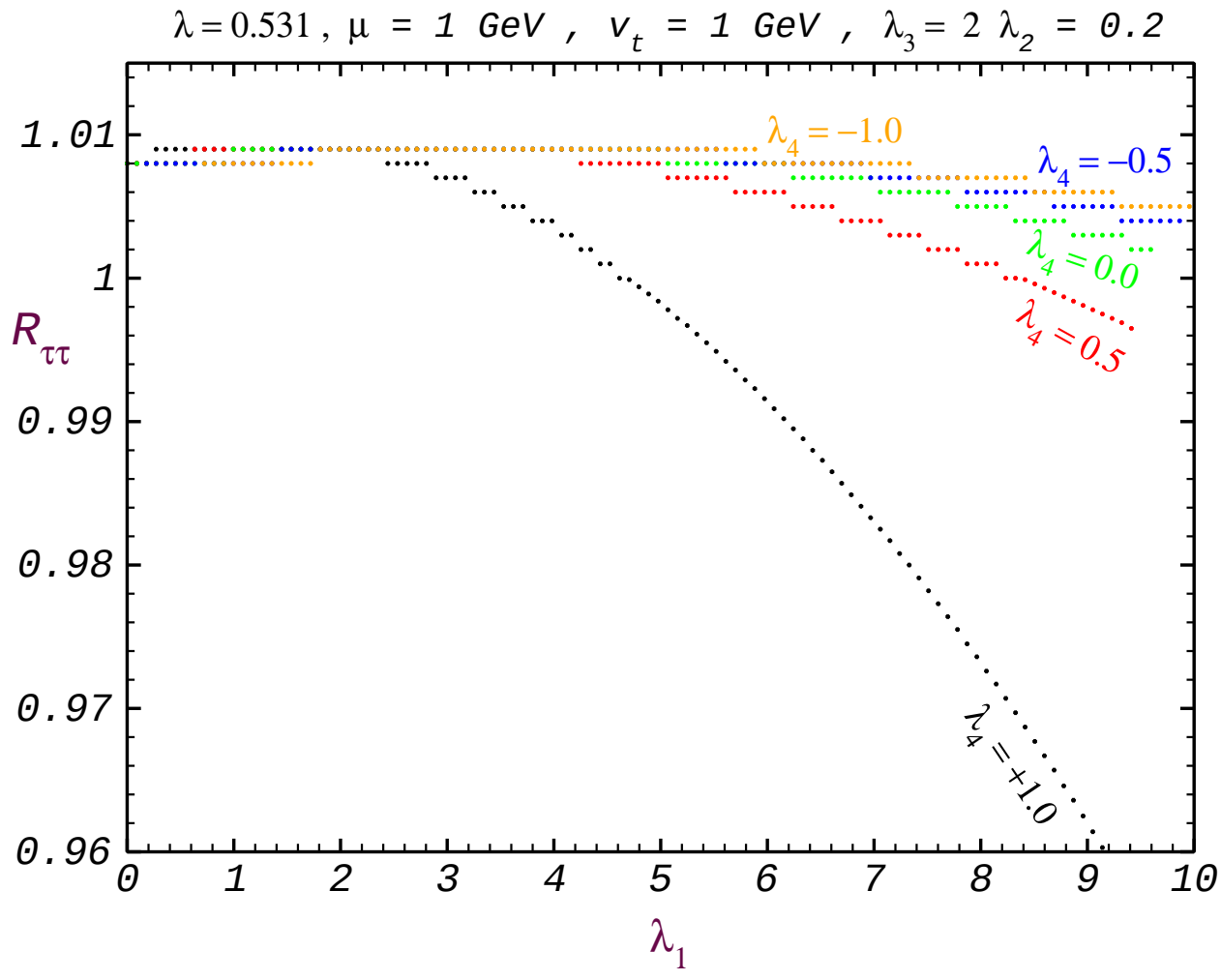
$h \to ZZ, WW, \tau\tau, b\bar{b}$



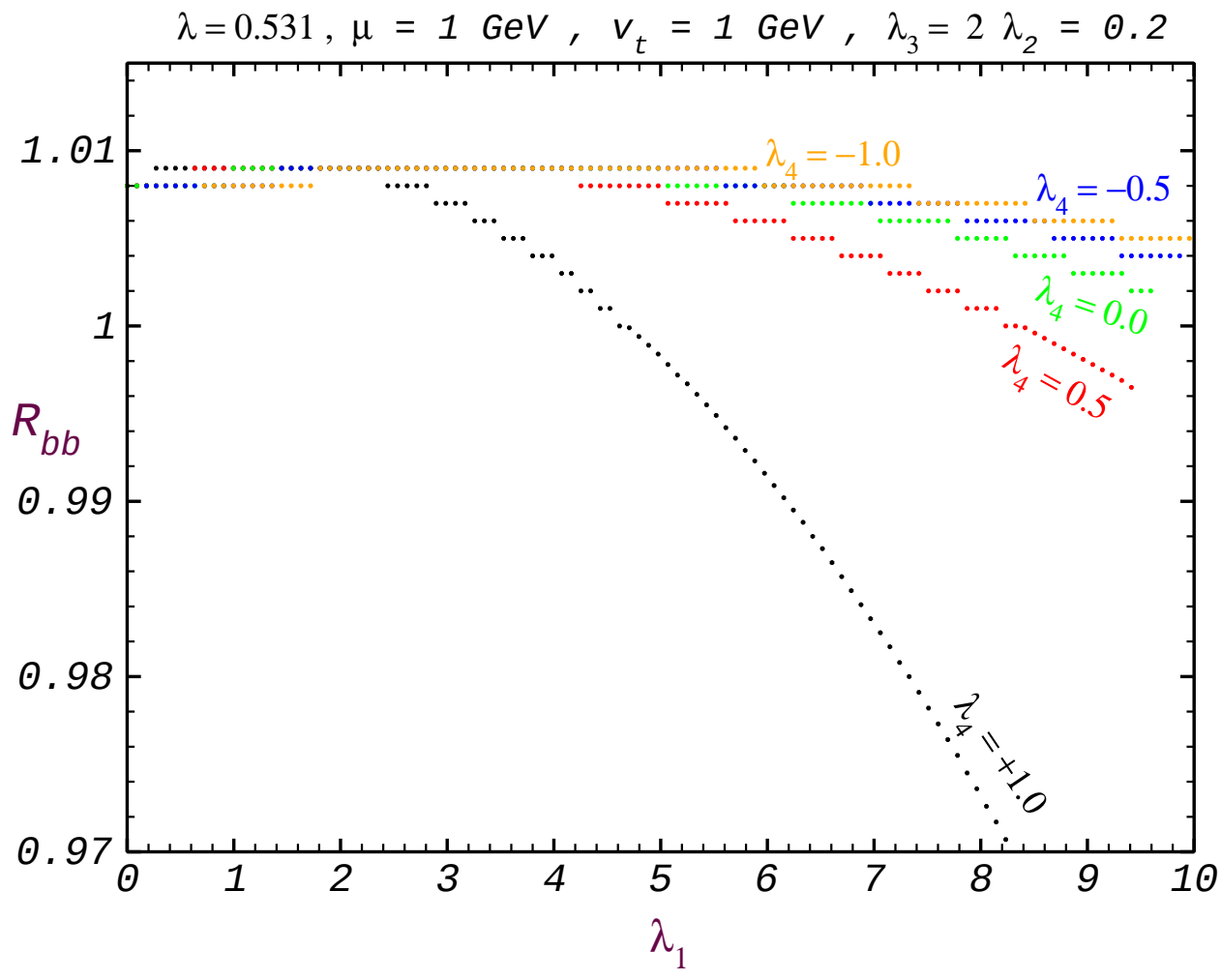
$h \longrightarrow ZZ, WW, \tau\tau, b\bar{b}$



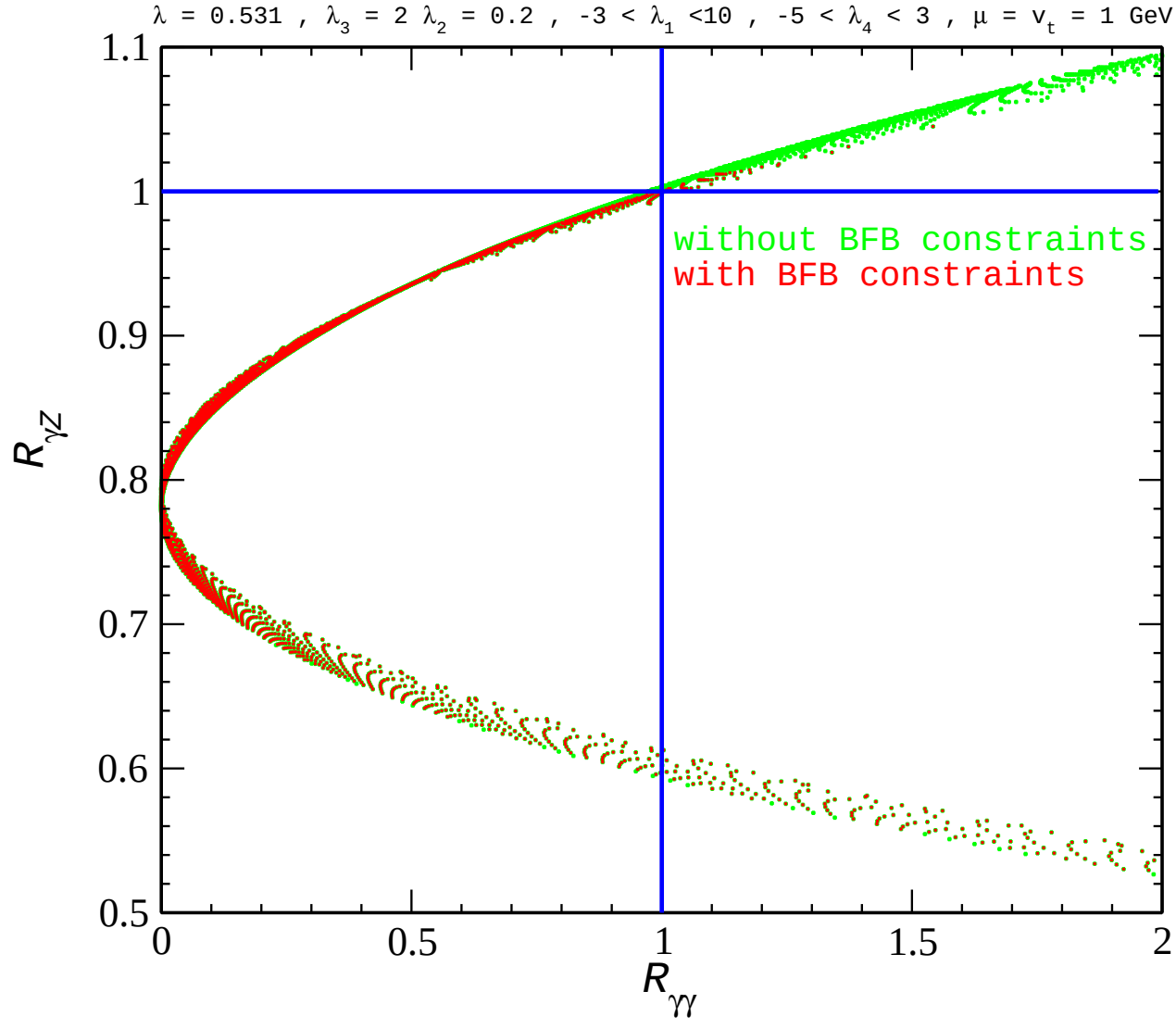
$h \to ZZ, WW, \tau\tau, b\bar{b}$



$h \to ZZ, WW, \tau\tau, b\bar{b}$



$h \to \gamma Z$


λ = 0.531 , λ3 = 2 λ2 = 0.2 , -3 < λ1 <10 , -5 < λ4 < 3 , μ = vt = 1 GeV
without BFB constraints
with BFB constraints
1.1
1
0.9
0.8
0.7
0.6
0.5
0
0.5
1
1.5
2
Rγγ
RγZ

Summary and Conclusions

- HTM Higgs potential described by 7 parameters.
- Full set of tree level unitarity and BFB constraints is derived.
- Unitarity constraint on the SM Higgs slightly modified (tiny region with small μ only)
- Large splitting between Higgs exists and might allow Higgs to Higgs decays: $H^{\pm} \to W^{\pm} A^0$, $H^{\pm} \to W^{\pm} H^{\pm\pm}$...
- Enhancement of diphoton event rate ($h \to \gamma\gamma$) observed at LHC: accommodated in allowed parameter space.
- $h \to \gamma\gamma$ can constrain the HTM parameter space.
- Diphoton decay is sensitive to $H^{\pm\pm}$ mass.
- The other decay channels remain SM - like as reported by ATLAS/CMS.