Phase evolution of earlier Universe in the Inert Doublet Model. Possible degeneracy of intermediate phase state

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Inert Doublet Model. Brief review

SM with standard Higgs field ϕ_S is supplemented by Higgs field ϕ_D, having no interaction with matter fields and v.e.v. $\langle\phi_D\rangle = 0$.
G. Despande, L. Ma *Phys.Rev.* **D18** (1978) 2574; many papers now

Lagrangian: $$\mathcal{L} = \mathcal{L}_{gf}^{SM} + \mathcal{L}_Y + \frac{1}{2}(D_\mu\phi_S D_\mu\phi_S^\dagger + D_\mu\phi_D D_\mu\phi_D^\dagger) - V\,.$$
$\mathcal{L}_{gf}^{SM}$: $SU(2)\times U(1)$ SM interaction of gauge bosons and fermions;
$\mathcal{L}_Y$: Yukawa interaction of fermions with Higgs field ϕ_S only.

In some points I repeat our paper I.F. G., K.A. Kanishev, M. Krawczyk, D. Sokolowska. Phys. Rev. **D 82**, 123533 (2010), hep-ph/1009.4593

Higgs potential, forbidding (ϕ_S, ϕ_D) mixing:

$$V = -\frac{1}{2}\left(m_{11}^2(\phi_S^\dagger\phi_S)+m_{22}^2(\phi_D^\dagger\phi_D)\right)+ \\ +\frac{1}{2}\left(\lambda_1(\phi_S^\dagger\phi_S)^2+\lambda_2(\phi_D^\dagger\phi_D)^2\right)+\lambda_3(\phi_S^\dagger\phi_S)(\phi_D^\dagger\phi_D)+ \\ +\lambda_4(\phi_S^\dagger\phi_D)(\phi_D^\dagger\phi_S)+\frac{\lambda_5}{2}\left((\phi_S^\dagger\phi_D)^2+(\phi_D^\dagger\phi_S)^2\right)+V_0.$$

We fix $\lambda_5 < 0$, real – without loss of generality.

Potential is positive at large quasi-classical values for fields ϕ_i and gives neutral DM if only

$$\lambda_1 > 0\,,\quad \lambda_2 > 0,\quad R+1 > 0;\quad R = \lambda_{345}/\sqrt{\lambda_1\lambda_2},\quad \lambda_{345} = \lambda_3+\lambda_4+\lambda_5.$$

To describe IDM, parameters m_{ii}^2, λ_i should lie in some bounded area.

Temperature dependence

At the finite temperature the ground state of system is given by minimum of the Gibbs potential $V_G = Tr\left(V e^{-\hat{H}/T}\right)/Tr\left(e^{-\hat{H}/T}\right)$. At high enough temperatures in the main approximation V_G has the same form as V with the same λ_i, and mass terms varying with temperature

$$m_{11}^2(T) = m_{11}^2(0) - c_1 T^2\,, \quad m_{22}^2(T) = m_{22}^2(0) - c_2 T^2\,,$$

$$c_1 = c_{H1} + c_{SM} + c_Y\,, \quad c_2 = c_{H2} + c_{SM};$$

$$c_{Hi} = \frac{3\lambda_i + 2\lambda_3 + \lambda_4}{12} \ (i = 1,\, 2); \ c_{SM} = \frac{3g^2 + g'^2}{32}; \ c_Y = \frac{g_t^2 + g_b^2}{8}.$$

Here g and g' are coupling constants of gauge EW interaction; the Yukawa couplings for t and b quarks are $g_t \approx 1$ and $g_b \approx 0.03$.

Due to this variation of potential,

position and **properties** of ground state vary with temperature.

Extrema of potential

The extrema of the potential define the values $\langle\phi_{S,D}\rangle$ of the fields $\phi_{S,D}$ via equations: $\partial V/\partial\phi_i|_{\phi_i=\langle\phi_i\rangle} = 0$. For each extremum with $\langle\phi_S\rangle \neq 0$ we choose the z axis in the weak isospin space so that $\langle\phi_S\rangle = \begin{pmatrix} 0 \\ v_S \end{pmatrix}$ with real $v_S > 0$ ("neutral direction"). After this choice the most general form extremum is $\langle\phi_S\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_S \end{pmatrix}$, $\quad \langle\phi_D\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u \\ v_D \end{pmatrix}$.

Useful abbreviations:

$$\mu_1 = \frac{m_{11}^2}{\sqrt{\lambda_1}}, \quad \mu_2 = \frac{m_{22}^2}{\sqrt{\lambda_2}}, \quad \tilde{c}_1 = \frac{c_1}{\sqrt{\lambda_1}}, \quad \tilde{c}_2 = \frac{c_2}{\sqrt{\lambda_2}}.$$

Complete set of extrema with their v.e.v.'s and energies $\bar{\mathcal{E}}_a = \mathcal{E}_a - V_0$.

I. The electroweak symmetry preserving extremum $\boldsymbol{EWs}$

$$v_D = 0, \quad v_S = 0, \quad \bar{\mathcal{E}}_{EWs} = 0.$$

Electroweak symmetry violating extrema:

II. Inert extremum $\boldsymbol{I_1}$: $v_D = 0$, $v_S^2 = \dfrac{m_{11}^2}{\lambda_1}, \quad \bar{\mathcal{E}}_{I_1} = -\dfrac{\mu_1^2}{8}$.

III. Inert-like extremum $\boldsymbol{I_2}$: $v_S = 0$, $v_D^2 = \dfrac{m_{22}^2}{\lambda_2}, \quad \bar{\mathcal{E}}_{I_2} = -\dfrac{\mu_2^2}{8}$.

IV. Mixed extremum $\boldsymbol{M}$:

$$v_S^2 = \frac{\mu_1 - R\mu_2}{\sqrt{\lambda_1}(1-R^2)}, \quad v_D^2 = \frac{\mu_2 - R\mu_1}{\sqrt{\lambda_2}(1-R^2)}; \quad \bar{\mathcal{E}}_M = -\frac{\mu_1^2 + \mu_2^2 - 2R\mu_1\mu_2}{8(1-R^2)}.$$

The vacuum with $u \neq 0$ is excluded in the IDM with neutral DM particle.

If some v_a^2, given by these equations, are negative, corresponding extremum is absent.

In the inert state I_1 we have ($G^\pm$, G – Goldstone modes)

$$\phi_S = \begin{pmatrix} G^+ \\ \dfrac{v+h+iG}{\sqrt{2}} \end{pmatrix}, \quad \phi_D = \begin{pmatrix} D^+ \\ \dfrac{D+iD_A}{\sqrt{2}} \end{pmatrix}.$$

We denote by M_h, M_D, M_A, $M_\pm \equiv M_+$ masses of h, D, D_A and $D^\pm$. Scalar h interacts with the fermions and gauge bosons just as Higgs boson in the SM. As in SM, $M_h^2 = \lambda_1 v^2$. D-scalars D, D_A, $D^\pm$ don't couple to fermions. The lightest from these D-scalars can play a role of DM particle, at $\lambda_4+\lambda_5<0$ it is neutral:

$$M_D^2 = \sqrt{\lambda_2}\frac{R\mu_1-\mu_2}{2}, \;\; M_A^2 = M_D^2 - v^2\lambda_5, \;\; M_\pm^2 = M_D^2 - v^2\frac{\lambda_4+\lambda_5}{2}.$$

This state can be ground state (vacuum) if only

$$\boxed{\begin{gathered} m_{11}^2 > 0 \;\; at \;\; any \;\; R, \\ \mu_1 > \mu_2 \;\; at \;\; R>1, \quad R\mu_1 > \mu_2 \;\; at \;\; |R|<1. \end{gathered}}$$

We assume that our world is described by I_1 (inert phase).
In this case $v(T=0)=246\ GeV$.
Recent LHC data give $M_h = 125$ GeV $\Rightarrow$

$$\lambda_1 = 0.25, \qquad \mu_1(T=0) = 3.125 \cdot 10^4 \text{ Gev}^2.$$

Inert-like phase I_2 looks similar to the inert phase.

Field ϕ_D **looks** similar to Higgs field in SM. It splits into 3 Goldstone modes + observable Higgs boson D_h with mass $M^2_{D_h} = \lambda_2 v^2$. D_h don't couple with fermions. The ϕ_S field is realized as physical fields S_H, S_A, $S_\pm$, coupled to massless fermions, and with masses

$$M^2_{S_H} = \sqrt{\lambda_1}\frac{R\mu_2 - \mu_1}{2}, \; M^2_{S_A} = M^2_{S_H} - v^2\lambda_5, \;\; M^2_{S_\pm} = M^2_{S_H} - v^2\frac{\lambda_4 + \lambda_5}{2}.$$

If this state is vacuum, we see no candidates for DM particle.

This state can be ground state (vacuum) if simultaneously

$$\boxed{\begin{array}{c} m^2_{22} > 0 \;\; at \;\; any \;\; R\,, \\ \mu_2 > \mu_1 \;\; at \;\; R > 1, \quad R\mu_2 > \mu_1 \;\; at \;\; |R| < 1. \end{array}}$$

M (mixed phase) is similar to that in 2HDM with Model I for Yukawa interaction. Scalars h, H, A, $H^{\pm}$ + 3 Goldstones mix components of ϕ_D and ϕ_S.

This extremum can be minimum if only $|R| = |\lambda_{345}/\sqrt{\lambda_1\lambda_2}| < 1$.

Since symmetry $\phi_D \leftrightarrow -\phi_D$ of entire Lagrangian (including Yukawa term), this extremum is degenerated in the sign of $\langle\phi_D\rangle$,

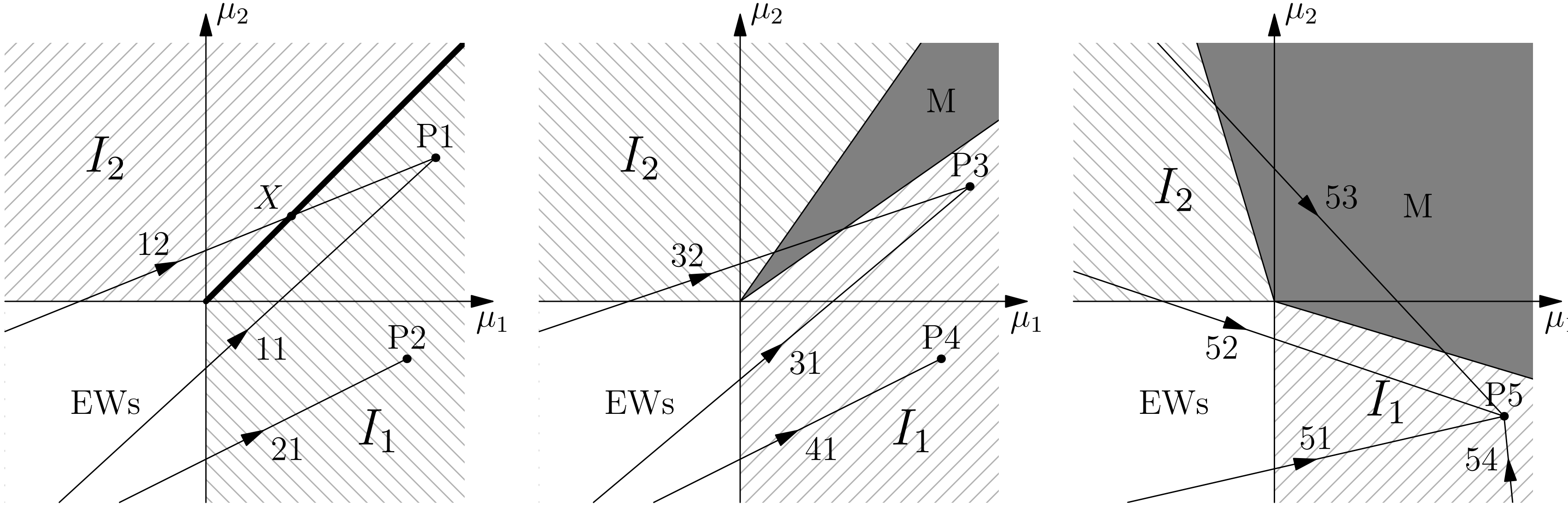
there are 2 mixed type states $M_{\pm}$, with $\langle\phi_D\rangle = v_D$ and $\langle\phi_D\rangle = -v_D$.

These vacua can be distinguished by the value of couplings of h and H to gauge bosons $\propto \cos(\beta \pm \alpha)$, etc.

The actual mixed phase consists of domains M_+ and M_-.

Thermal evolution of Universe

During cooling of the Universe mass terms in V were changed $\Rightarrow$ phase states were changed. Possible ways of evolution are shown here:



. $R > 1$ $\quad$ $1 > R > 0$ $\quad$ $0 > R > -1$

Evolution through mixed phase

Ray 32 $\left(0 < \dfrac{\tilde{c}_2}{\tilde{c}_1} < \dfrac{\mu_2(0)}{\mu_1(0)}\right)$.

I. Starting from EWs state the Universe comes to the Inert-like phase I_2 at $T_{EWs,2} = \sqrt{\mu_2(0)/\tilde{c}_2}$ (2-nd order phase transition (PT) with the order parameter $\eta_{EW,2} \propto \langle\phi_2\rangle \equiv v_D$).

(*At some values of parameters more accurate calculation of the Gibbs potential can transform this PT to the 1-st order PT*).

II. The inert like phase I_2 has no candidates for DM particles.

The Universe cooled in this phase up to a temperature

$$T = T_{2,M} = \sqrt{(\mu_1(0) - R\mu_2(0))/(\tilde{c}_1 - R\tilde{c}_2)}.$$

III. At the temperature $T = T_{2,M}$ the Universe comes to the mixed phase M with domains M_+, M_-. That is 2-nd order PT with order parameter $\eta_{2,M} \propto \langle\phi_S\rangle \propto \sqrt{|\mu_1 - R\mu_2|}$, which is represented by mass M_{SH} in the phase I_2 and mass M_h in the mixed phases $M_\pm$. At the transition point these masses vanish.

The height of domain wall is given by position of lowest saddle extremum between M_+ and M_-.

Here $\mu_2 > \mu_1$, and lowest saddle extremum is inert-like I_2 with

$$\text{height of domain wall } E_b = \mathcal{E}_{I2} - \mathcal{E}_M = \frac{(\mu_2 R - \mu_1)^2}{8(1-R^2)} \quad .$$

Near the PT $I_2 \to M$ (at $T \sim T_{2,M}$ – small $\eta_{2,M}$) we have $(\mu_2 R - \mu_1) = A_2(T^2 - T_{2,M}^2)$ with $A_2 > 0$. At these temperatures the system is highly non homogeneous, with the domains of I_2 phase (obliged by fluctuation of temperature and density), and phases M_+ and M_- with height of wall between domains $E_b \propto \eta_{2,M}^4$. The distribution of these domains in the space is constantly changing. The characteristic correlation radius is

$$R_c(T) \propto 1/\eta_{2,M} \propto 1/\sqrt{|T^2 - T_{2,M}^2|}.$$

With decreasing of temperature the domains of I_2 become energetically unfavourable. The domains M_+ and M_- are hardened, since the height of walls between them increases. The correlation radius decreases, domains become bubbles with surface tension $\sigma_s \sim E_b R_c$. The curved surface of this bubble is under pressure $\sim \sigma_s/r$, where r is the local radius of curvature. This pressure leads to the absorption of small domains by larger.
The local velocity of motion of domain walls is $\sim c$ – speed of light. But the global merging process is slow diffuse process with characteristic time $\sim (R/c)\sqrt{R/R_c}$, where R is characteristic radius of Universe inhomogeneity.
This temp must be compared with the temp of cooling of Universe.

At the cooling of Universe below $T = T_{2,1} = \sqrt{(\mu_1(0) - \mu_2(0))/(\tilde{c_1} - \tilde{c_2})}$, we come to the region $\mu_1 > \mu_2$, and the wall between domains M_+ and M_- is given by inert extremum I_1 with $E_b = \mathcal{E}_{I1} - \mathcal{E}_M = \dfrac{(\mu_1 R - \mu_2)^2}{8(1 - R^2)}$.
With subsequent cooling the Universe passes to the inert phase I_1 at the temperature $T = T_{M,1} = \sqrt{(R\mu_1(0) - \mu_2(0))/(R\tilde{c}_1 - \tilde{c}_2)}$. That is 2-nd order PT with order parameter
$\eta_{M,1} \propto \langle \phi_D \rangle \propto \sqrt{|R\mu_1 - \mu_2|} \propto (T^2 - T^2_{M,1})$, representing by mass M_H in the mixed phases $M_\pm$ and mass of DM particle M_D in the phase I_1. At the transition point these masses vanish.
The evolution of fluctuations (domains) near this transition in the mixed phase is similar to that discussed for the transition $I_2 \to M$.
It looks very probable, that after these transitions the Universe become strongly inhomogeneous.

IV. Below $M \to I_1$ transition the system has fluctuations of type M_+ and M_-, obliged by fluctuations of temperature and density.
The previous history of fluctuations in the mixed phase can enhance size of these latest fluctuations.
Depending on parameters of model, the temperature of this transition $T_{M,1}$ can be low enough. It can results inhomogeneities in the modern Universe. In this case these fluctuations can influence for history of baryogenesis. In this case our approximation can give only qualitative picture, lattice calculations can be useful.

□ The considered IDM can be denoted as $(1+1)$ IDM (1–Dark, 1–Standard). More complex IDM with 2 "standard" Higgs field ϕ_{S1}, ϕ_{S2} and one "dark" doublet ϕ_D – $(1+2)$ IDM can be treated also (see e.g. B. Grzadkowski, O.M. Ogreid, P. Osland, A. Pukhov, M. Purmohammadi). Complete description of temperature evolution of Universe in this model is absent to-day. Perhaps, the most interesting new physical phenomenon in the evolution of Universe in this case would be intermediate stage with charged vacuum, without massless photons and electric charge conservation and with very strong winds after transition from this phase.

The intermediate mixed phase similar to that described above with the same degeneracy and similar fluctuations takes place in such model as well.

The end