Necessary Conditions for Spontaneous CP Violation

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Reference: This talk is based on H.E. Haber and Z. Surujon, *Group-theoretic Condition for Spontaneous CP Violation*, arXiv:1201.1730 [hep-ph], Physical Review **D86** (2012) in press.

Brief Review: CP in scalar field theory

Consider a scalar field theory with a set of complex scalar fields $\phi_i(\vec{\boldsymbol{x}}, t)$. The definition of the CP transformation law depends on the *basis choice* for the definition of the scalar fields. To be general, define the *generalized*-CP (GCP) transformation,

$$\phi_i(\vec{\boldsymbol{x}}, t) \longrightarrow X_{ij}\phi_j^*(-\vec{\boldsymbol{x}}, t)\,.$$

where X is an $n \times n$ unitary matrix. Such a transformation is automatically a symmetry of the free scalar field theory action.

Under a unitary change of basis, $\phi_i'(x) = U_{ij}\phi_j(x)$, the GCP-transformation in terms of the primed fields is governed by

$$X' = UXU^{\mathsf{T}}\,.$$

One can show* that if an *interacting* scalar field theory is CP-invariant, then the action is also invariant under some GCP transformation corresponding to a symmetric unitary matrix X. The latter can always be written in the form $X = U^\dagger U^*$ for some unitary U, in which case $X' = UXU^\mathsf{T} = \mathbb{1}$.

Thus, a basis exists where CP is simply complex conjugation (and inversion of the space coordinate). In this basis all coefficients of the scalar potential are real [the latter is called a *real basis*].

If a scalar field theory is CP-invariant, then one can ask whether the ground state is also CP-invariant. If $\langle\phi_i\rangle \equiv v_i$, then the vacuum is GCP-invariant if $v_i = X_{ij}v_j^*$. That is, in a basis where $X = \mathbb{1}$, all vevs are real.

- **Explicit CP Violation (XCPV)**. No real basis exists.

- **Spontaneous CP Violation (SCPV)**. A real basis exists, but there exists no real basis in which all the vevs are simultaneously real.

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*P.M. Ferreira, H.E. Haber, M. Maniatis, O. Nachtmann and J.P. Silva, Int. J. Mod. Phys. **A26**, 769 (2011).

Example: A Single Complex Scalar Field

Let ϕ be a complex scalar field. There is an associated global U$(1)_X$ symmetry, where X is the generator of global phase transformations. Write:

$$\phi(x) = \rho(x)e^{iG(x)/v}.$$

For certain potentials, the field ϕ acquires a vev, $\langle\phi\rangle = ve^{i\theta}$, breaking U$(1)_X$ spontaneously. If U$(1)_X$ is not explicitly broken, then G is an exact Goldstone boson and the phase θ is unphysical, as it can be shifted by a U$(1)_X$ transformation.

If explicit U$(1)_X$–breaking is added to the scalar potential, then G can acquire a vev, which may yield a non-zero phase. For example, consider

$$V_X = \tfrac{1}{2}b\phi^2 + \text{h.c.} = b\rho^2\cos\frac{2G}{v},$$

where b is real valued. V_X is minimized at $\theta = \langle G\rangle/v = \pi/2$. However, the phase can be removed by the field redefinition $G \to G - v\pi/2$, which is equivalent to $\phi \to -i\phi$. This transformation induces a sign flip, $b \to -b$, such that in the new basis, the minimum is at $\theta = 0$ and there is no spontaneous CP violation.

A similar conclusion would follow for any single monomial $g_k \phi^k$. However, if we introduce two terms with different powers of ϕ, then generically the minimum of the resulting potential for θ cannot be shifted to the origin.† As an example, consider

$$V_X = b\phi^2 + g\phi^4 + \mathrm{h.c.},$$

where b and g are real. The U$(1)_X$–breaking terms induce a potential for the otherwise flat θ, which is given by

$$V_X = bv^2 \cos(2\theta) + gv^4 \cos(4\theta).$$

For parameters in the range $|b| < 4gv^2$, V_X is minimized at $\cos(2\theta_{\mathrm{min}}) = -b/(4gv^2)$, generically resulting in spontaneous CP violation (SCPV).

Although V_X provides an explicit violation of the U$(1)_X$ global symmetry, we can formally make V_X neutral under U$(1)_X$ by assigning two different U$(1)_X$ charges to the coefficients b and g. Thus, in the above example, the spontaneous breaking of CP is attributed to the breaking of the U$(1)_X$ symmetry by two *spurions* whose U$(1)_X$ charges differ in magnitude.

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†Here the word "generically" should be interpreted as: "in an $\mathcal{O}(1)$ fraction of the parameter space".

Note that for any spurion with $\mathrm{U}(1)_X$ charge q, there is a complex conjugated spurion with $\mathrm{U}(1)_X$ charge $-q$. Hence, it is the *magnitude* of the charge that is relevant for determining whether SCPV is possible. Thus, we arrive at the following necessary condition for SCPV in the case of a single complex scalar field:

Spontaneous CP violation in a theory of single complex scalar field may occur only if the associated global $\mathrm{U}(1)_X$ symmetry is broken by at least two spurions whose non-zero $\mathrm{U}(1)_X$ charges differ in magnitude.

It is convenient to regard two spurions as *inequivalent* if their $\mathrm{U}(1)_X$ charges are equal in magnitude, regardless of the overall sign.

Example: Multiple Complex Scalar Fields

Given N complex fields ϕ_i $(i = 1, 2, \ldots, N)$, the independent phases correspond to the "diagonal" phases associated with the Cartan subgroup $\mathrm{U}(1)_1 \times \cdots \times \mathrm{U}(1)_N$, where each U(1) rotates the phase of one complex degree of freedom. If the scalar potential contains N_s *inequivalent* spurions,[‡] then each spurion may be labeled by an N-dimensional *charge vector* whose jth component is the charge under $\mathrm{U}(1)_j$.

Consider the $N_s \times N$ matrix whose rows are given by the charge vectors of the spurions. The rank r of this matrix is equal to the dimension of the vector space spanned by the corresponding charge vectors. Since the rank of a matrix cannot exceed the number of columns or rows, it follows that $r \leq \min\,\{N_s, N\}$.

Physical interpretation of the rank: only r independent U(1)'s are broken by the spurions, which leaves $N - r$ unbroken U(1)'s.

[‡]Two spurions will be considered to be "equivalent" if their charge vectors are equal or differ by an overall minus sign.

We can define truncated r-dimensional charge vectors with respect to the r broken U(1)'s,

$$\boldsymbol{q}^{(i)} \equiv \left(q_1^{(i)}, q_2^{(i)}, \ldots, q_r^{(i)}\right), \quad i = 1, \ldots, N_s.$$

Assemble the truncated charge vectors into an $N_s \times r$ matrix, denoted by Q, whose ith row is given by $\boldsymbol{q}^{(i)}$. By construction, $r = \mathrm{rk}\, Q$ and $N_s \geq r$.

Case 1: $\boldsymbol{N_s = r}$. This means that Q is an invertible $r \times r$ matrix. It is convenient to redefine the $\mathrm{U}(1)^r$ generators $\{X_1, \ldots, X_r\}$ by $X_i' \equiv \sum_j C_{ij} X_j$, where $C = (Q^{\mathsf{T}})^{-1}$. Relative to this new basis for the U(1) generators, the charge vectors are given by

$$\delta_j^i = \sum_{k=1}^{r} C_{jk} q_k^{(i)}, \quad i, j = 1, \ldots, r.$$

This case is equivalent to r independent copies of one complex scalar field and associated spurion (and its complex conjugate). In particular, if we denote $\langle \phi_n \rangle = v_n e^{i\theta_n}$, then

$$V_{X_1', X_2', \ldots, X_r'} = \sum_{i=1}^{r} V_i(v_n) \cos\theta_i', \qquad \text{where} \;\; \theta_i' \equiv \sum_{k=1}^{r} q_k^{(i)} \theta_k,$$

and $V_i(v_n)$ is the contribution to the potential of the ith spurion (where the complex fields ϕ_n are replaced by the v_n, respectively).

Case 2: $N_s > r$. We order the truncated r-dimensional charge vectors such that $\{\boldsymbol{q}^{(1)}, \boldsymbol{q}^{(2)}, \ldots, \boldsymbol{q}^{(r)}\}$ are linearly independent. Then, the charge vectors of the remaining spurions, $\boldsymbol{q}^{(i)}$ for $i = r+1, r+2, \ldots, N_s$, are linear combinations of the first r charge vectors. Due to the extra $N_s - r$ spurions, we are left with at least one potential physical phase. Hence,

SCPV may occur only if the number of inequivalent spurions is larger than the dimension of the vector space spanned by the corresponding charge vectors.

To determine the number of potential physical CP-violating phases, we again define new charge vectors with respect to the redefined U(1) generators $\{X'_1, \ldots, X'_r\}$,

$$\sum_{k=1}^{r} C_{jk} q_k^{(i)} = \begin{cases} \delta_j^i \,, & \text{for } i = 1, 2, \ldots, r \,, \\ q_j^{\prime\,(i)} \,, & \text{for } i = r+1, r+2, \ldots, N_s \,, \end{cases}$$

where $C = (\widetilde{Q}^{\mathsf{T}})^{-1}$ and $\widetilde{Q}$ is the $r \times r$ matrix whose rows are the first r (linearly independent) charge vectors $\{\boldsymbol{q}^{(1)}, \boldsymbol{q}^{(2)}, \ldots, \boldsymbol{q}^{(r)}\}$.

With respect to the redefined U(1)'s, we can assemble the new charge vectors into an $N_s \times r$ matrix,

$$Q' = \begin{pmatrix} \delta^i_j \\ ----- \\ q'^{(k)}_j \end{pmatrix} , \qquad (\text{column } j = 1, 2, \ldots, r) ,$$

where $i = 1, 2, \ldots, r$ and $k = r+1, r+2, \ldots, N_s$ label the N_s rows of the matrix.

One can now write out the spurion contributions to the scalar potential,

$$V_{X'_1, X'_2, \ldots, X'_r} = \sum_{i=1}^{r} V_i(v_n) \cos \theta'_i + \sum_{k=r+1}^{N_S} V_i(v_n) \cos \left(\sum_{j=1}^{r} q'^{(k)}_j \theta'_j \right) ,$$

where $\theta'_j = \sum_{k=1}^{r} q^{(j)}_k \theta_k$. The phases θ'_j that explicitly appear in the second term above are potential CP-violating phases. Generically we would expect r CP-violating phases when $N_s > r$. However, if there are r' columns of zeros below the dashed line in the matrix Q' above, i.e. $q'^{(k)}_j = 0$ for r' values of the index j [for all $k = r+1, \ldots, N_s$], then only $r - r'$ phases appear in the second term of $V_{X'_1, X'_2, \ldots, X'_r}$. The r' phases that are absent do not acquire non-trivial CP-violating expectation values since for these phases, the analysis reduces to the first vase of $N_s = r$ treated above.

Note that r' is equal to the number of charge vectors that are linearly independent of the remaining $N_s - 1$ charge vectors. Thus, we conclude that:[§]

For a scalar potential that exhibits SCPV with N_s spurion terms, the number of potential CP-odd phases is given by $d = r - r'$. That is d is equal to the difference of the dimension of the vector space spanned by the N_s charge vectors and the number of charge vectors that are linearly independent of the remaining $N_s - 1$ charge vectors.

Example 1: charge vectors $\{(1,0)\,,\,(0,1)\,,\,(-1,-1)\}$. In this example, $N_s = 3$ and $r = 2$. Since none of the charge vectors is linearly independent of the other two charge vectors, we have $r' = 0$ and conclude that $d = r - r' = 2$. Thus, in this example there are two potential CP-violating phases that characterize the vacuum.

Example 2: charge vectors $\{(1,0)\,,\,(0,1)\,,\,(0,2)\}$. In this example, $N_s = 3$ and $r = 2$. Since the first charge vector is linearly independent of the other two charge vectors, we have $r' = 1$ and conclude that $d = r - r' = 1$. Thus, in this example there is one potential CP-violating phase that characterize the vacuum.

[§]This result automatically incorporates the case of $N_S = r$ treated above, where all the charge vectors are linearly independent, in which case $r' = r$ and $d = 0$. That is, there is no SCPV when $N_S = r$.

Application 1: SCPV in the 2HDM

The 2HDM consists of two hypercharge-one, $\mathrm{SU}(2)_L$ doublets (Φ_1, Φ_2). The $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge-covariant kinetic energy terms possess an $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{SU}(2)_F$ symmetry, where the $\mathrm{SU}(2)_F$ corresponds to a "Higgs-flavor" symmetry transformation, $\Phi_i \to U_i{}^j \Phi_j$ with $U \in \mathrm{SU}(2)_F$. The generic 2HDM potential,

$$
\begin{aligned}
V &= m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left(m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right) \\
&+ \tfrac{1}{2}\lambda_1 \left(\Phi_1^\dagger \Phi_1 \right)^2 + \tfrac{1}{2}\lambda_2 \left(\Phi_2^\dagger \Phi_2 \right)^2 + \lambda_3 \Phi_1^\dagger \Phi_1 \Phi_2^\dagger \Phi_2 + \lambda_4 \Phi_1^\dagger \Phi_2 \Phi_2^\dagger \Phi_1 \\
&+ \left[\tfrac{1}{2}\lambda_5 \left(\Phi_1^\dagger \Phi_2 \right)^2 + \lambda_6 \Phi_1^\dagger \Phi_1 \Phi_1^\dagger \Phi_2 + \lambda_7 \Phi_2^\dagger \phi_2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right],
\end{aligned}
$$

breaks the $\mathrm{SU}(2)_F$ Higgs flavor symmetry completely. It is convenient to designate $\mathrm{SU}(2)_{F(L)}$ indices by Roman (Greek) indices.

Since there are four complex degrees of freedom, there are at most four physical SCPV phases, related to the four diagonal generators

$$\mathbb{1}_{ij}\mathbb{1}_{\alpha\beta}, \quad \mathbb{1}_{ij}T^3_{\alpha\beta}, \quad T^3_{ij}\mathbb{1}_{\alpha\beta}, \quad T^3_{ij}T^3_{\alpha\beta},$$

acting on the $\Phi_{i\alpha}$. The first two are SU$(2)_L\times$U$(1)_Y$ gauge symmetry generators, which can be used to remove unphysical phases. The $T^3_{ij}\mathbb{1}_{\alpha\beta}$ generate the (ungauged) Peccei-Quinn (PQ) symmetry ($\Phi_1 \to e^{i\alpha}\Phi_1$ and $\Phi_2 \to e^{-i\alpha}\Phi_2$), which is generically broken by the scalar potential. Therefore it can potentially yield SCPV. The last generator $T^3_{ij}T^3_{\alpha\beta}$ ("chiral PQ") cannot give rise to SCPV in vacua that preserve electric charge, since chiral PQ becomes degenerate with PQ when the two Higgs vevs are aligned.

Consider the following simple example:

$$m^2_{11}, m^2_{22} < 0, \quad m^2_{12} = 0,$$

$$\lambda_{1,2} > 0, \quad \lambda_{5,6} \neq 0, \quad \lambda_3 = \lambda_4 = \lambda_7 = 0,$$

where $|\lambda_{5,6}| \ll \lambda_{1,2}$.

In this case, $\langle \Phi_i^0 \rangle \simeq \sqrt{m_{ii}^2/\lambda_i}$, with small corrections of order $\mathcal{O}\left(\lambda_{5,6}/\lambda_{1,2}\right)$. We see that $\mathrm{U}(1)_{\mathrm{PQ}}$ is broken only by the terms

$$V_{\not{\mathrm{PQ}}} = \tfrac{1}{2}\lambda_5 \left(\Phi_1^\dagger \Phi_2\right)^2 + \lambda_6 (\Phi_1^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2) + \mathrm{h.c.}$$

We parametrize the two expectation values as

$$\Phi_1^0 = v_1 e^{i\theta} e^{i\varphi}, \quad \Phi_2^0 = v_2 e^{i\theta} e^{-i\varphi}.$$

The new terms induce a potential for the otherwise flat φ, which is given by

$$\Delta V = \lambda_5 v_1^2 v_2^2 \cos(4\varphi) + 2\lambda_6 v_1^3 v_2 \cos(2\varphi).$$

For parameters in the range $|\lambda_6| \tan\beta < 2\lambda_5$, this potential is minimized at

$$\cos(2\varphi_{\min}) = \frac{\lambda_6}{2\lambda_5} \tan\beta,$$

where $\tan\beta \equiv v_1/v_2$, resulting in spontaneous CP violation.

Application 2: The Chiral Lagrangian

Dashen's model of spontaneous CP violation is based on the three-flavor chiral Lagrangian,

$$\mathscr{L} = \tfrac{1}{4}f^2 \mathrm{Tr}\left(\mathcal{D}_\mu \Sigma^\dagger \mathcal{D}^\mu \Sigma\right) + \tfrac{1}{2}B_0 f^2 \mathrm{Tr}\left(M\Sigma^\dagger + \Sigma M^\dagger\right),$$

where D_μ is the gauge covariant derivative, B_0 is proportional to the quark-antiquark condensate and $M = \mathrm{diag}(m_u\,, m_d\,, m_s)$. Σ depends on the Goldstone fields via

$$\Sigma(G) = e^{iG(x)/f}\Sigma_0 e^{iG(x)/f}\,,$$

where $\Sigma_0 \equiv \langle\Sigma\rangle$ is an SU(3) matrix,

$$G(x) \equiv G^a(x)T^a = \frac{1}{\sqrt{2}}\begin{pmatrix} \tfrac{1}{\sqrt{2}}\pi^0 + \tfrac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\tfrac{1}{\sqrt{2}}\pi^0 + \tfrac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K^0} & -\tfrac{2}{\sqrt{6}}\eta \end{pmatrix}.$$

and the T^a are the SU(3) generators in the fundamental representation.

Under an $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ transformation, $\Sigma \to L\Sigma R^\dagger$. In the absence of *explicit* chiral symmetry breaking, $\Sigma_0 = \mathbb{1}$, corresponding to the spontaneous breaking of $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \longrightarrow \mathrm{SU}(3)_V$ (i.e. $L = R$). Including the explicit chiral symmetry breaking, we parameterize

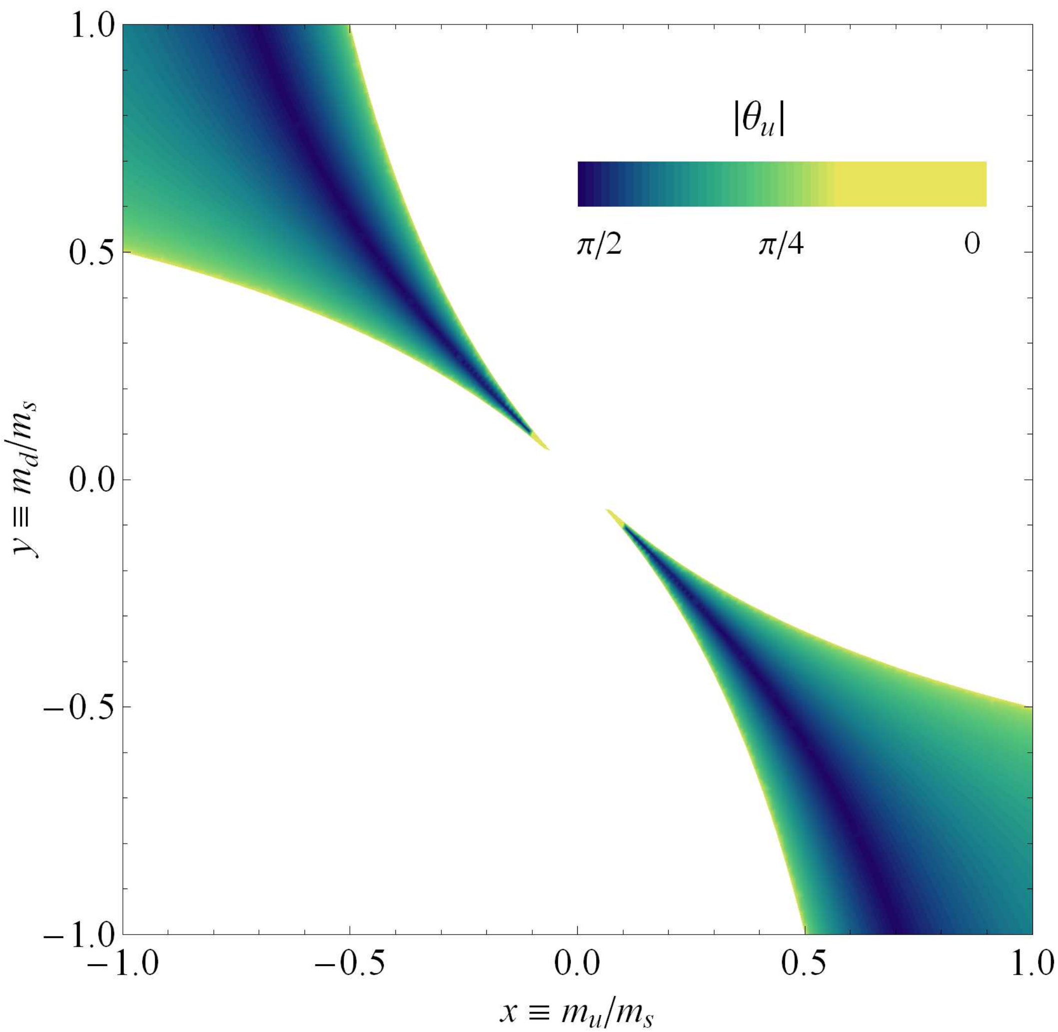
$$\Sigma_0 = \begin{pmatrix} e^{i\theta_u} & 0 & 0 \\ 0 & e^{i\theta_d} & 0 \\ 0 & 0 & e^{-i(\theta_u+\theta_d)} \end{pmatrix}.$$

Dashen's observation was that a region exists in the (m_u, m_d, m_s) parameter space where θ_u and θ_d are not minimized at the origin, thus inducing SCPV. The potential for the phases is

$$V = B_0 f^2 \left[m_u \cos\theta_u + m_d \cos\theta_d + m_s \cos(\theta_u + \theta_d) \right].$$

Provided that $m_u m_d < 0$, the potential above is minimized when

$$m_u \sin\theta_u = m_d \sin\theta_d = -m_s \sin(\theta_u + \theta_d) \;.$$



Regions of the parameter space of Dashen's Model parameter space of Dashen's model, where spontaneous CP violation occurs. A point in this parameter space corresponds to $(x, y) \equiv (m_u/m_s, m_d/m_s)$. The size of the phase θ_u is shown, with maximum values depicted in dark (blue), and minimum values in light (yellow). In these regions, θ_d also acquires a nonzero value, the value of θ_d at the point (x, y) is equal to the value of θ_u at the point (y, x).

It is convenient to introduce dimensionless mass ratios, $x \equiv m_u/m_s$ and $y \equiv m_d/m_s$. Assuming $xy < 0$, we obtain the vacuum values of θ_u and θ_d,

$$\cos\theta_u = \tfrac{1}{2}\left(\frac{y}{x^2} - \frac{1}{y} - y\right)\,, \qquad\qquad \cos\theta_d = \tfrac{1}{2}\left(\frac{x}{y^2} - \frac{1}{x} - x\right)\,,$$

under the assumption that $-1 \leq \cos\theta_{u,d} \leq 1$. If this latter assumption is false, then the minimum of the potential for the phases lies on the boundary where $|\cos\theta_{u,d}| = 1$, corresponding to a CP-conserving vacuum. Thus, SCPV can arise if and only if $xy < 0$ and $-1 < \cos\theta_{u,d} < 1$. The above inequalities yield

$$\frac{|x|}{1+|x|} < |y| < \frac{|x|}{1-|x|}\,, \qquad\qquad xy < 0\,,$$

in which case the vacuum is characterized by two independent physical phases θ_u and θ_d. Unfortunately, this range is ruled out phenomenologically using the light quark masses quoted by the Particle Data Group.

Nevertheless, we can use Dashen's model to test our SCPV conditions. Prior to turning on the explicit breaking terms (namely the spurions m_u, m_d and m_s), there are two spontaneously-broken U(1) generators that can be identified with the two diagonal SU(3) generators T^3 and T^8. In fact, it is more convenient to define linear combinations of these two generators,

$$T_u \equiv T^3 + \sqrt{3}\,T^8 = \mathrm{diag}(1, 0, -1)\,, \quad T_d \equiv -T^3 + \sqrt{3}\,T^8 = \mathrm{diag}(0, 1, -1)\,,$$

which can be used to shift the values of θ_u and θ_d, respectively. Applying T_u and T_d to the vectors $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$ yields the U(1)$_u$ and U(1)$_d$ charges of the three spurions, respectively. The corresponding charge vectors are given by:

$$m_u(1, 0), \quad m_d(0, 1), \quad m_s(-1, -1).$$

This was the previously analyzed Example 1, where we concluded that there were two potential physical phases that characterize the vacuum, in agreement with the above analysis.

Incorporating the axial anomaly

Had we considered a chiral Lagrangian based on U$(3)_L\times$U$(3)_R$ instead of SU$(3)_L\times$SU$(3)_R$, then $\Sigma_0 = \text{diag}(e^{i\theta_u}, e^{i\theta_d}, e^{i\theta_s})$, with no relation among the three phases. Prior to turning on the explicit breaking terms, there are now three spontaneously-broken U(1) generators that can be identified with T_u, T_d and T^0, where T^0 is the 3×3 identity matrix that generates an axial U$(1)_A$ transformation. The corresponding charge vectors of the spurions,

$$m_u(1,0,1), \quad m_d(0,1,1), \quad m_s(-1,-1,1)\,,$$

are linearly independent, spanning the full three-dimensional vector space, so that $N_s = \text{rk}\, Q$.

Naively, it seems that none of the three phases is physical, resulting in the absence of SCPV. However, the axial U$(1)_A$ symmetry is anomalous, and can be modeled by adding an explicit U$(1)_A$ breaking term to the chiral Lagrangian that is proportional to $(\ln\det\,\Sigma)^2$. Consequently, there is a fourth spurion so that $N_s = 4 > \text{rk}\,Q = 3$, and we again conclude that SCPV is possible. The corresponding fourth charge vector is $(0,0,1)$; hence there are three potential physical CP-violating phases θ_u, θ_d and θ_s that characterize the vacuum.

Application 3: The minimal Nelson–Barr Model

Here we will consider the model by Bento, Branco, and Parada. The model solves the strong CP problem by imposing CP as an exact symmetry, and breaking it spontaneously, thereby producing unsuppressed CKM phase, along with a suppressed strong CP phase.

The field content of the model is the SM plus a complex gauge singlet scalar S, and one pair of vector-like down quarks D_L, D_R. The new interactions of the Lagrangian are given by

$$\delta\mathscr{L} = -\mu \overline{D}_L D_R - (f_i S + f_i' S^*)\overline{D}_L d_R^i + \text{h.c.}$$

Due to the presence of terms such as S^2, S^4, etc., there is a parameter regime for which $\langle S \rangle = V e^{i\alpha}$. The SCPV phase α is physical and eventually feeds into the SM fermion mass matrices and provides the sole source of CP violation. Since the couplings f_i and f_i' are flavor-dependent, this phase can be related to the CKM phase once the scalars acquire vevs. The radiatively induced strong CP-violating parameter $\bar{\theta}$ is small and thus the strong CP problem is solved.

In terms of our SCPV conditions, the scalar sector has a single broken U(1) that is explicitly broken by more than one spurion (e.g., S^2 and S^4), such that the vacuum has one physical nonzero phase. Thus the case of one complex scalar field treated earlier in this talk applies.

Other applications and future directions

- Examples of SCPV in little Higgs models.

- Full $SU(2)_F$ spurion analysis of the 2HDM (applications beyond SCPV). This analysis is equivalent of Ivanov's group theoretical decomposition of the 2HDM scalar potential.

  This technique provides an elegant derivation of the basis-independent condition for the $U(1)_{PQ}$ symmetry of the 2HDM scalar potential.

- Applications to radiatively-generated SCPV. For example, are there examples where two inequivalent spurions are generated by higher loop processes?

- Global $U(1)$ symmetries associated with CP-violating phases may be anomalous. In this case, the anomaly is manifested by the presence of explicitly breaking terms in the Lagrangian. Do models exist in which SCPV arises solely due to the anomaly?