

# Dark Scalar Doublet and Neutrino Mass

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# Dark Scalar Doublet

After July 4, 2012, we are now confident that something very close to the standard-model **Higgs boson** has been discovered. Its presence explains how all known fundamental bosons ( $W^\pm, Z^0$ ) and fermions (quarks and leptons) acquire mass. The only possible exception is the neutrino, which I will discuss later.

This still leaves dark matter and that is where the **dark scalar doublet** comes in. It is simply a second scalar doublet ( $\eta^+, \eta^0$ ) which is odd under an exactly conserved  $Z_2$  symmetry, and interacts with the  $SU(2)_L \times U(1)_Y$

gauge bosons as well as the standard-model Higgs doublet  $(\phi^+, \phi^0)$ . It was postulated in 1978 (Deshpande/Ma) as a possible addition to the Higgs sector of the standard model from symmetry considerations. Its utility as **dark matter** was not realized until 2006, when it was first used to obtain **radiative neutrino masses** (Ma) as well. Two months later, it was considered alone (Barbieri/Hall/Rychkov) as a means of modifying the oblique  $S, T, U$  parameters in precision electroweak measurements. It has become known also as the 'inert' Higgs doublet and inspired many studies.

Two bonuses of having  $(\eta^+, \eta^0)$ :

(1) Changing  $\Gamma(H \rightarrow \gamma\gamma)$ . The term  $\lambda_3(\Phi^\dagger\Phi)(\eta^\dagger\eta)$  allows  $\eta^+$  to contribute to  $\Gamma(H \rightarrow \gamma\gamma)$ . If  $\lambda_3 < 0$ , then  $\Gamma(H \rightarrow \gamma\gamma)$  is enhanced, which may be indicated by LHC data. [Posch(2011), Arhib/Benbrik/Gaur(2012)]

(2) Changing  $V_{eff}(H)$  at finite temperature to allow for **electroweak baryogenesis**.

[Chowdhury/Nemevsek/Senjanovic/Zhang(2011), Borah/Cline(2012), Gil/Chankowski/Krawczyk(2012)]

# Radiative Neutrino Mass

In 2006 [E. Ma, Phys. Rev. D 73, 077301 (2006)], it was proposed that neutrino masses are one-loop quantum effects due to the existence of dark matter, i.e.

**scotogenic** from the Greek 'scotos' meaning darkness.

The standard model of particle interactions is extended to include 3 singlet Majorana neutral fermions  $N_{1,2,3}$  (analogs of  $\nu_R$ ) + one extra scalar doublet  $(\eta^+, \eta^0)$  in addition to the usual  $(\phi^+, \phi^0)$ . An exactly conserved  $Z_2$  (odd-even) symmetry is imposed so that  $N_{1,2,3}$  (**scotinos**) and  $(\eta^+, \eta^0)$  are odd and all other particles are even.

The  $Z_2$  symmetry forbids  $\nu N$  coupling to  $\phi^0$ , so there is no Dirac mass linking  $\nu$  to  $N$ , i.e.

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & 0 \\ 0 & m_N \end{pmatrix}$$

at the classical (tree) level. However, at the quantum level, a one-loop diagram induces a Majorana mass for  $\nu$ , so that

$$\mathcal{M}_\nu = \begin{pmatrix} m_\nu & 0 \\ 0 & m_N \end{pmatrix}.$$

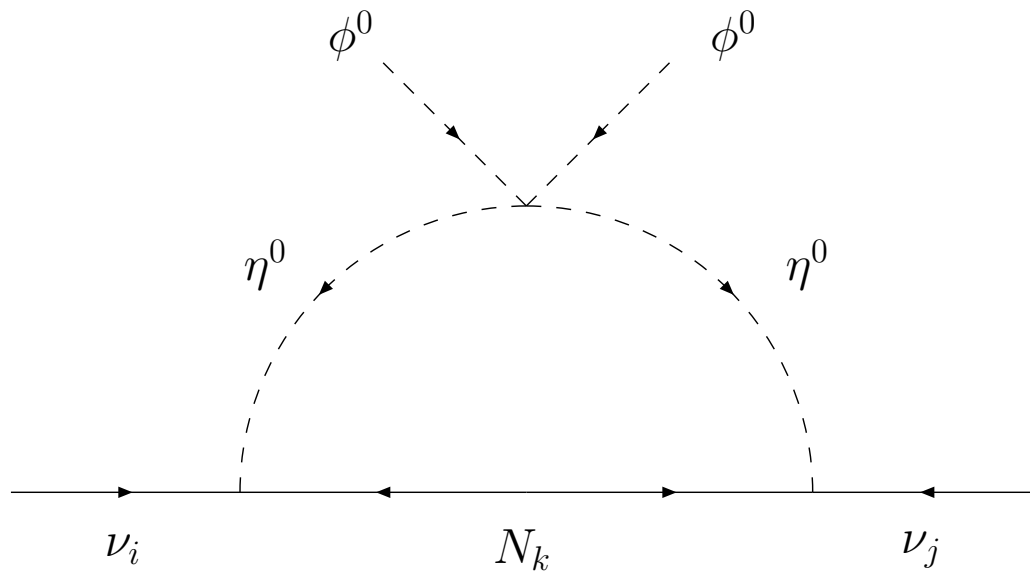


Figure 1: One-loop generation of neutrino mass with  $Z_2$  dark matter.



The  $(1/2)\lambda_5(\eta^\dagger\Phi)^2 + H.c.$  term allowed by  $Z_2$  implies that  $\eta_R^0$  and  $\eta_I^0$  are split by  $\langle\phi^0\rangle = v$  to have different physical masses.

The one-loop diagram can then be exactly calculated, i.e.

$$(\mathcal{M}_\nu)_{ij} =$$

$$\sum_k \frac{h_{ik}h_{jk}M_k}{16\pi^2} \left[ \frac{m_R^2}{m_R^2 - M_k^2} \ln \frac{m_R^2}{M_k^2} - \frac{m_I^2}{m_I^2 - M_k^2} \ln \frac{m_I^2}{M_k^2} \right].$$

The lightest particle among  $\eta_R^0, \eta_I^0, N_{1,2,3}$  is absolutely stable and is a good dark matter candidate.

The **prejudice** in neutrino physics is that neutrino mass comes from new physics beyond the electroweak scale, i.e.  $m_R, m_I \ll M_k$ , so

$$(\mathcal{M}_\nu)_{ij} = \sum_k \frac{h_{ik}h_{jk}}{16\pi^2 M_k} \left( m_R^2 \ln \frac{M_k^2}{m_R^2} - m_I^2 \ln \frac{M_k^2}{m_I^2} \right).$$

This expression is inversely proportional to  $M_k$ , as is in the canonical seesaw mechanism. In this case,  $\eta_R^0$  or  $\eta_I^0$  is **cold** dark matter. Many studies of this and other related scenarios have been made.

# Neutrino Mass Without Seesaw

It has been known since 1979 (Weinberg) that given the particle content of the standard model, a unique dimension-five operator exists for Majorana neutrino mass, i.e.

$$\mathcal{L}_5 = \frac{f_{ij}}{\Lambda} (\nu_i \phi^0 - l_i \phi^+) (\nu_j \phi^0 - l_j \phi^+) + H.c.$$

All models which claim to have 'naturally' small neutrino masses obey this 'seesaw' formula.

However, it was recently (E. Ma, arXiv:1206.1812) noticed that if  $M_k \ll m_R, m_I$ , a radically new formula for neutrino mass is obtained, i.e.

$$(\mathcal{M}_\nu)_{ij} = \frac{\ln(m_R^2/m_I^2)}{16\pi^2} \sum_k h_{ik} h_{jk} M_k,$$

which is **directly proportional** to  $M_k$  !!

Comment on the canonical seesaw with  $\nu_R$  singlets.

The usual statement is that in the presence of  $\nu_R$ , a Majorana mass  $m_R$  is allowed so that lepton number is

broken. The  $2 \times 2$  mass matrix spanning  $(\nu_L, \nu_R)$  is then of the form

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix}.$$

To get a seesaw mass,  $m_D \ll m_R$  is required, but  $m_R$  breaks lepton number and  $m_D$  does not, so 'naturalness' would imply that  $m_R \ll m_D$ .

In my new formula,  $M_k$  breaks lepton number, and the lepton-number conserving mass to compare it with is the electron mass, so it may well be 10 keV which would be good for **warm dark matter**.

# Warm Dark Matter

Let  $M_N \sim 10$  keV, then  $m_\nu \sim 0.1$  eV implies  $h_{ik}^2 \sim 10^{-3}$ . Since the lightest  $N$  (call it  $N_1$ ) is absolutely stable, there is no  $N_1 \rightarrow \nu\gamma$  decay which would put an upper bound of 2.2 keV on its mass if it were the usual sterile neutrino which is produced nonresonantly through its mixing with the active neutrinos (Dodelson-Widrow).

The stability of  $N_1$  removes the **tension** between this would-be upper bound and the lower bound of perhaps 5.6 keV from Lyman- $\alpha$  forest observations.

Implications for particle physics:

$$(1) \quad B(\mu \rightarrow e\gamma) = \frac{\alpha}{768\pi} \frac{|\sum_k h_{\mu k} h_{ek}^*|^2}{(G_F m_{\eta^+}^2)^2} < 2.4 \times 10^{-12}$$

implies  $m_{\eta^+} > 310 \text{ GeV} (|\sum_k h_{\mu k} h_{ek}^*|/10^{-3})^{1/2}$ .

(2) Anomalous magnetic moment of muon is given by

$$\Delta a_\mu = -\frac{m_\mu^2}{96\pi^2 m_{\eta^+}^2} \sum_k |h_{\mu k}|^2 < 1.23 \times 10^{-13} \frac{\sum_k |h_{\mu k}|^2}{|\sum_k h_{\mu k} h_{ek}^*|},$$

which is much below the experimental uncertainty of  $6 \times 10^{-10}$ .

(3) Since  $N_k$  are light, muon decay also proceeds at tree level through  $\eta^+$  exchange. The inclusive rate is given by

$$\Gamma(\mu \rightarrow N_\mu e \bar{N}_e) = \frac{(\sum_k |h_{\mu k}|^2)(\sum_k |h_{ek}|^2)m_\mu^5}{6144\pi^3 m_{\eta^+}^4}$$

$$< 2.5 \times 10^{-8} \frac{(\sum_k |h_{\mu k}|^2)(\sum_k |h_{ek}|^2)}{|\sum_k h_{\mu k} h_{ek}^*|^2} \Gamma(\mu \rightarrow \nu_\mu e \bar{\nu}_e),$$

which is much below the experimental uncertainty of  $10^{-5}$  in the determination of  $G_F$ .



Implications for cosmology:

(1) Whereas  $N_1$  is absolutely stable,  $N_{2,3}$  will decay.

$$\Gamma(N_2 \rightarrow N_1 \bar{\nu}_i \nu_j) = \frac{|h_{i2} h_{j1}^*|^2}{256\pi^3 M_2} \left( \frac{1}{m_R^2} + \frac{1}{m_I^2} \right)^2$$

$$\times \left( \frac{M_2^6}{96} - \frac{M_1^2 M_2^4}{12} + \frac{M_1^6}{12} - \frac{M_1^8}{96 M_2^2} + \frac{M_1^4 M_2^4}{8} \ln \frac{M_2^2}{M_1^2} \right)$$

$$\simeq \frac{|h_{i2} h_{j1}^*|^2 (\Delta M)^5}{1920\pi^3} \left( \frac{1}{m_R^2} + \frac{1}{m_I^2} \right)^2,$$

if  $M_2 - M_1 = \Delta M \ll M_{1,2}$ .

As an example, let  $\Delta M = 1$  keV,  $|h_{i2}h_{j1}^*|^2 = 10^{-6}$ ,  $m_R = 240$  GeV,  $m_I = 150$  GeV, then  $\Gamma = 6.42 \times 10^{-50}$  GeV, corresponding to a decay lifetime of  $3.25 \times 10^{17}$  y, which is much longer than the age of the Universe, i.e.  $13.75 \pm 0.11 \times 10^9$  y. This means that  $N_{1,2,3}$  may all be components of dark matter today.

Note that  $N_2 \rightarrow N_1\gamma$  is now possible with  $E_\gamma \simeq \Delta M$ , but since  $\Delta M$  may be small, whereas  $M_{1,2,3} \sim 10$  keV, the **tension** between galactic X-ray data and Lyman- $\alpha$  forest observations is easily relaxed.

(2) The effective  $N\bar{N} \rightarrow l\bar{l}, \nu\bar{\nu}$  interactions are of order  $h^2/m_\eta^2 \sim 10^{-8} \text{ GeV}^{-2}$ , hence they remain in thermal equilibrium in the early Universe until a temperature of a few GeV. Their number density  $n_N$  is given by

$$\frac{n_N}{n_\gamma} = \left( \frac{43/4}{g_{dec}^*} \right) \left( \frac{2}{11/2} \right)^{3/2},$$

where  $g_{dec}^* = 16$ , counting  $N_{1,2,3}$  in addition to photons, electrons, and the three neutrinos. Their relic abundance at present would then be

$$\Omega_N h^2 \simeq \frac{115}{16} \left( \frac{\sum_i M_i}{\text{keV}} \right).$$

Since  $\Omega_N h^2$  should be  $0.1123 \pm 0.0035$ , a dilution factor of about  $1.9 \times 10^3$  is needed for  $\sum_i M_i \sim 30$  keV.

(3) The dilution factor may be accomplished by a particle which decouples after  $N_1$  and decays later as it becomes nonrelativistic, with a large release of entropy. It is a well-known mechanism. Some recent papers: Bezrukov/Hettmansperger/Lindner(2010), Nemevsek/Senjanovic/Zhang(2012), King/Merle(2012).

(4) Another solution is to assume that the reheating temperature of the Universe is below a few GeV, so that  $N_i$  are not thermally produced. Instead, they come from the decay of a scalar singlet  $S$  with the interaction  $(f_{ij}/2)SN_iN_j$  where  $f_{ij} < 10^{-4}$  for  $m_S \simeq 2$  GeV. The interaction  $\sqrt{2}\lambda_3HS^2$  allows  $S$  to be thermally produced and to decouple as it becomes nonrelativistic with  $\langle\sigma v_{rel}\rangle \sim 10^{-5}$  pb for  $\lambda_3 \sim 10^{-3}$ . Now  $S$  decays to  $NN$ , so the relic density of  $N$  is reduced by  $2M/m_S \simeq 10^{-5}$ . Since  $\langle\sigma v_{rel}\rangle$  is inversely proportional to relic density, this would yield the correct observed value.

Implications at the Large Hadron Collider:

$\Gamma(H \rightarrow SS) = \lambda_3^2 v^2 / 4\pi m_H \sim 0.02$  MeV. Since the total width of  $H$  is about 4.3 MeV, this invisible mode is very hard to check. However,  $\eta^+ \rightarrow l_i^+ N_j$  and  $\eta^+ \rightarrow \eta_{R,I} W^+$  as well as  $\eta_R \rightarrow \eta_I Z$  are possible signatures.

Negative impact on the search of dark matter:

- (1) Nonobservation of any dark-matter signal at underground experiments. Latest news from XENON100: 50 GeV dark matter is excluded at  $2.0 \times 10^{-45}$  cm<sup>2</sup>.
- (2) Nonobservation of dark-matter annihilation products (gamma rays, etc.) from space.

# Conclusion

The dark scalar doublet  $(\eta^+, \eta^0)$  is useful for many things ( $S, T, U, \Gamma(H \rightarrow \gamma\gamma)$ , electroweak baryogenesis). If it is also used for radiative neutrino mass, then a new formula is obtained:

$$(\mathcal{M}_\nu)_{ij} = \frac{\ln(m_R^2/m_I^2)}{16\pi^2} \sum_k h_{ik} h_{jk} M_k.$$

Now  $N_{1,2,3}$  with masses  $\sim 10$  keV may be warm dark matter and  $\eta^\pm, \eta_R, \eta_I$  may be easier to find at the LHC.