

# Three Higgs-doublet model with $S_3$ symmetry

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# Outline

- Motivation And Proposal
  - About S3
- The model (Machado-Pleitez arXiv: 1205.0995)
- The phenomenology (Cardenas-Machado-Pleitez-Rodriguez- work in progress)
- Final remarks

# Motivation

Glashow and Weinberg “The suppression of FCNC is natural if it depends only the symmetry and the representation content of the model”

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In a multi-Higgs models we have Flavor conservation if the fermion masses are generated by a single source.

# The proposal

- We assume that the scalar sector also has three families (there is no limit on the SM scalar sector).
- And we, also, assume that the symmetry governing the scalar sector is the  $S_3$  symmetry.

# The proposal

- Usually the discrete symmetries are used to give more predictability in flavor problem, for example reproducing ansätze.
- About S3 symmetry (arXiv:1003.3552v2)

$$\begin{aligned} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_2 \otimes \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}_2 &= (x_1 y_1 + x_2 y_2)_1 + (x_1 y_2 - x_2 y_1)_{1'} + \begin{pmatrix} x_1 y_2 + x_2 y_1 \\ x_1 y_1 - x_2 y_2 \end{pmatrix}_2, \\ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_2 \otimes (y')_{1'} &= \begin{pmatrix} -x_2 y' \\ x_1 y' \end{pmatrix}_2, \\ (x')_{1'} \otimes (y')_{1'} &= (x' y')_1. \end{aligned} \tag{41}$$

# The proposal

- Or, all permutation of  $S_3$  symmetry are represented on the reducible triplet  $(x_1, x_2, x_3)$  as:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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- We change the representation using the unitary transformation:

$$U_{\text{tribi}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} U^\dagger g U \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix},$$

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The multiplication rule is  
the same

$$3 = 2+1$$

$$3 \times 3 = (2+1)(2+1)$$



# The model

- However, we have two possible representations for the singlet and the doublet:

$$\text{A: } \mathbf{3} = \mathbf{2} + \mathbf{1} \equiv D + S,$$

$$S = \frac{1}{\sqrt{3}}(H_1 + H_2 + H_3) \sim \mathbf{1},$$

$$D \equiv (D_1, D_2) = \left[ \frac{1}{\sqrt{6}}(2H_1 - H_2 - H_3), \frac{1}{\sqrt{2}}(H_2 - H_3) \right] \sim \mathbf{2}.$$

And B:

$$S = H_1 \sim \mathbf{1}, \quad D = (H_2, H_3) \sim \mathbf{2}.$$

- All the fermions are singlets of S3 (Minimum extension of the SM).

# The model

$$SU(3) \times SU(2) \times U(1) \times S_3$$

- The most general scalar potential invariant under the symmetry is given by:

$$\begin{aligned} V(D, S) = & \mu_s^2 S^\dagger S + \mu_d^2 [D^\dagger \otimes D]_1 + \lambda_1 ([D^\dagger \otimes D]_1)^2 + \lambda_2 [(D^\dagger \otimes D)_{1'} (D^\dagger \otimes D)_{1'}]_1 \\ & + \lambda_3 [(D^\dagger \otimes D)_2 (D^\dagger \otimes D)_2] + \lambda_4 (S^\dagger S)^2 + \lambda_5 [D^\dagger \otimes D]_1 S^\dagger S + \lambda_6 S^\dagger [D^\dagger \otimes D]_1 S \\ & + \{ \lambda_7 [(S^\dagger \otimes D)_2 (D^\dagger \otimes S)_2]_1 + \lambda_8 [(S^\dagger \otimes D)_2 (D^\dagger \otimes D)_2]_1 + H.c. \} \end{aligned} \quad (1)$$

## The Yukawa Sector

$$-\mathcal{L}_{\text{Yukawa}} = \bar{L}_{iL} (G_{ij}^l l_{jR} S + G_{ij}^{\nu} \nu_{jR} \bar{S}) + \bar{Q}_{iL} (G_{ij}^u u_{jR} \bar{S} + G_{ij}^d d_{jR} S) + H.c.,$$

# Model A

## The constrain equations:

$$\begin{aligned} 18t_1 = & 6\mu_d^2(2v_1 - v_2 - v_3) + 6\mu_s^2V + 2(4\bar{\lambda} + \lambda_4 + 2\bar{\lambda}' - 2\sqrt{2}\lambda_8)v_1^3 \\ & -(4\bar{\lambda} + 6\lambda_2 - 2\lambda_4 - \bar{\lambda}' - \sqrt{2}\lambda_8)(v_2^3 + v_3^3) - 3(2\lambda_2 - 2\lambda_4 + \bar{\lambda}' + 2\sqrt{2}\lambda_8)(v_2^2v_3 + v_2v_3^2) \\ & -3(4\bar{\lambda} - 2\lambda_4 - \bar{\lambda}' - \sqrt{2}\lambda_8)v_1^2(v_2 + v_3) + 6(2\bar{\lambda} - 2\lambda_2 + \lambda_4 + 2\sqrt{2}\lambda_8)v_1(v_2^2 + v_3^2) \\ & +6(4\lambda_2 + 2\lambda_4 - \bar{\lambda}' - 2\sqrt{2}\lambda_8)v_1v_2v_3 \end{aligned}$$

$$\begin{aligned} 18t_2 = & -6\mu_d^2(v_1 - 2v_2 + v_3) + 6\mu_s^2V + 2(4\bar{\lambda} - 3\lambda_2 + \lambda_4 + 2\bar{\lambda}' - 4\sqrt{2}\lambda_8)v_2^3 \\ & -(4\bar{\lambda} - 2\lambda_4 - \bar{\lambda}' - \sqrt{2}\lambda_8)(v_1^3 + v_3^3) - 3(4\bar{\lambda} + 2\lambda_4 + \bar{\lambda}' + \sqrt{2}\lambda_8)v_2^2v_3 \\ & -3(2\lambda_2 - 2\lambda_4 + \bar{\lambda}' + 2\sqrt{2}\lambda_8)v_2^2v_1 - 3(2\lambda_2 - 2\lambda_4 + \bar{\lambda}' + 2\sqrt{2}\lambda_8)v_1v_3^2 \\ & +6(2\bar{\lambda} + \lambda_2 + \lambda_4 + \sqrt{2}\lambda_8)v_2v_3^2 + 3(4\lambda_2 + 2\lambda_4 - \bar{\lambda}' - 2\sqrt{2}\lambda_8)v_1^2v_3 \\ & -3(2\lambda_2 - 2\lambda_4 + \bar{\lambda}' + 2\sqrt{2}\lambda_8)v_1v_3^2 - 6(2\lambda_2 - 2\lambda_4 - \bar{\lambda}' + 2\sqrt{2}\lambda_8)v_1v_2v_3 \end{aligned}$$

# Model A

if  $v_1 = v_2 = v_3$ .

- The mass matrix:

$$M_n^2 = \begin{pmatrix} a_n & -b_n & -b_n \\ -b_n & a_n & -b_n \\ -b_n & -b_n & a_n \end{pmatrix} \quad a_n, b_n > 0 \text{ (or } a_n, b_n < 0 \text{)}$$

$$U_{TBM}^T M_n^2 U_{TBM} = \text{diag}(a_n - 2b_n, a_n + b_n, a_n + b_n)$$

$$U_{TBM} = \begin{pmatrix} \frac{1}{\sqrt{3}} & -\sqrt{\frac{2}{3}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

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$$U_{TBM}^T M_n^2 U_{TBM} = \text{diag}(a_n - 2b_n, a_n + b_n, a_n + b_n)$$

$$a_h = (2/3)\mu_d^2 + (2\lambda_4 + \bar{\lambda}')v^2, \quad 6b_h = 2\mu_d^2 - 3(4\lambda_4 - \bar{\lambda}')v^2$$

CP even  $m_{h_1}^2 = \frac{2}{3}\lambda_4 v_{SM}^2, \quad m_{h_2}^2 = m_{h_3}^2 \equiv m_h^2 = \mu_d^2 + \frac{1}{2}\bar{\lambda}'v_{SM}^2,$

$$a_a = (2/3)\mu_d^2 + \bar{\lambda}'v^2 \text{ and } 6b_a = 2\mu_d^2 + 3\bar{\lambda}'v^2$$

CP odd  $m_{a_1}^2 = 0, \quad m_{a_2}^2 = m_{a_3}^2 \equiv m_a^2 = \mu_d^2 + \frac{1}{6}\bar{\lambda}'v_{SM}^2$

Charged  $6a_c = 2\mu_d^2 + 3\lambda_5 v^2 \text{ and } 12b_c = 2\mu_d^2 + 3\lambda_5 v^2$

$$m_{c_1}^2 = 0, \quad m_{c_2}^2 = m_{c_3}^2 \equiv m_c^2 = \frac{1}{2}\mu_d^2 + \frac{\lambda_5}{12}v_{SM}^2.$$

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SM Higgs  $m_{h_1}^2 = \frac{2}{3}\lambda_4 v_{SM}^2, \quad m_{h_2}^2 = m_{h_3}^2 \equiv m_h^2 = \mu_d^2 + \frac{1}{2}\bar{\lambda}'v_{SM}^2,$

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Goldstones

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This parameter may be  $> 0$  or  $< 0$ , and since it is not protected by any symmetry.

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# Model A

if  $v_1 = v_2 = v_3$ .

The constrain equations becomes:

$$t_1 = t_2 = t_3 = v(\mu_s^2 + 3\lambda_4 v^2),$$

$$\mu_s^2 = -3\lambda_4 v^2 < 0, \lambda_4 > 0.$$

We have to check if this choice is a minimum:  
to do so first random values are assigns for the lambdas

```
L1 = RandomReal[{-10, 10}, 100];  
L2 = RandomReal[{-10, 10}, 100];  
L3 = RandomReal[{-10, 10}, 100];  
L4 = RandomReal[{-10, 10}, 100];  
L5 = RandomReal[{-10, 10}, 100];  
L6 = RandomReal[{-10, 10}, 100];  
L7 = RandomReal[{-10, 10}, 100];  
L8 = RandomReal[{-10, 10}, 100];  
mud2 = RandomReal[{-40, 200}, 100];
```

therefore, the potential now is a function of  
 $V(\mu_s^2, v_1, v_2, v_3)$



# Model A

if  $v1 = v2 = v3$ .

Second, we asked for the program to find the minimum of the function

$$\{-3.52665 \times 10^7, \{v1 \rightarrow 45.715, v2 \rightarrow 45.715, v3 \rightarrow 45.715\}\}$$

$$\{-1.836967849326120 \times 10^{408}, \{v1 \rightarrow 1.34603 \times 10^{102}, v2 \rightarrow 5.9412 \times 10^{101}, v3 \rightarrow 5.9412 \times 10^{101}\}\}$$

$$\{-1.38087 \times 10^7, \{v1 \rightarrow 28.6058, v2 \rightarrow 28.6058, v3 \rightarrow 28.6058\}\}$$

$$\{-1.53611 \times 10^7, \{v1 \rightarrow 30.1709, v2 \rightarrow 30.1709, v3 \rightarrow 30.1709\}\}$$

$$\{-2.78463 \times 10^7, \{v1 \rightarrow 40.622, v2 \rightarrow 40.622, v3 \rightarrow 40.622\}\}$$

$$\{-2.29306195651464 \times 10^{415}, \\ \{v1 \rightarrow 7.01408 \times 10^{103}, v2 \rightarrow -3.39999 \times 10^{103}, v3 \rightarrow -3.39999 \times 10^{103}\}\}$$

$$\{-6.092597713811284 \times 10^{393}, \{v1 \rightarrow -4.5239 \times 10^{83}, v2 \rightarrow 1.55581 \times 10^{98}, v3 \rightarrow -1.55581 \times 10^{98}\}\}$$

$$\{-6.96637 \times 10^{205}, \{v1 \rightarrow 4.22274 \times 10^{51}, v2 \rightarrow 1.72579 \times 10^{51}, v3 \rightarrow 1.72579 \times 10^{51}\}\}$$

$$\{-1.021185565877198 \times 10^{420}, \{v1 \rightarrow 4.62458 \times 10^{104}, v2 \rightarrow 4.62458 \times 10^{104}, v3 \rightarrow 4.62458 \times 10^{104}\}\}$$

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implies restrictions to the values of lambdas or mud2

Almost all cases meet to  $v1 = v2 = v3$ .

This result is a global minimum, we do not know if it's the only choice, probably not.

# Model A

- After diagonalize the mass matrix we obtain the following mass eigenstates:

$$S \equiv h_1 = \begin{pmatrix} h_1^\dagger \\ \frac{1}{\sqrt{2}}(3v + h_1^0 + ia_1^0) \end{pmatrix}, \quad D \equiv -(h_2, h_3), \quad h_k = \begin{pmatrix} h_k^\dagger \\ \frac{1}{\sqrt{2}}(h_k^0 + ia_k^0) \end{pmatrix}$$

$$\begin{aligned} V(h_i) = & 3\lambda_4 v^2 h_1^\dagger h_1 + \mu_d^2 (h_2^\dagger h_2 + h_3^\dagger h_3) + \lambda_1 (h_2^\dagger h_2 + h_3^\dagger h_3)^2 + \lambda_2 (h_2^\dagger h_3 - h_3^\dagger h_2)^2 \\ & + \lambda_3 [(h_2^\dagger h_3 + h_3^\dagger h_2)^2 + (h_2^\dagger h_2 - h_3^\dagger h_3)^2] + \lambda_4 (h_1^\dagger h_1)^2 + \lambda_5 h_1^\dagger h_1 (h_2^\dagger h_2 + h_3^\dagger h_3) \\ & + (\lambda_6 + \lambda_7) h_1^\dagger (h_2^\dagger h_2 + h_3^\dagger h_3) h_1 + [\lambda_8 h_1^\dagger h_2 (h_2^\dagger h_3 + h_3^\dagger h_2) \\ & + h_1^\dagger h_3 (h_2^\dagger h_2 + h_1^\dagger h_1) + H.c.]. \end{aligned}$$

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SM Higgs Goldstones

$$\begin{aligned}
 V(h_i) = & 3\lambda_4 v^2 h_1^\dagger h_1 + \mu_d^2 (h_2^\dagger h_2 + h_3^\dagger h_3) + \lambda_1 (h_2^\dagger h_2 + h_3^\dagger h_3)^2 + \lambda_2 (h_2^\dagger h_3 - h_3^\dagger h_2)^2 \\
 & + \lambda_3 [(h_2^\dagger h_3 + h_3^\dagger h_2)^2 + (h_2^\dagger h_2 - h_3^\dagger h_3)^2] + \lambda_4 (h_1^\dagger h_1)^2 + \lambda_5 h_1^\dagger h_1 (h_2^\dagger h_2 + h_3^\dagger h_3) \\
 & + (\lambda_6 + \lambda_7) h_1^\dagger (h_2^\dagger h_2 + h_3^\dagger h_3) h_1 + [\lambda_8 h_1^\dagger h_2 (h_2^\dagger h_3 + h_3^\dagger h_2) \\
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 & + \lambda_3 [(h_2^\dagger h_3 + h_3^\dagger h_2)^2 + (h_2^\dagger h_2 - h_3^\dagger h_3)^2] + \lambda_4 (h_1^\dagger h_1)^2 + \lambda_5 h_1^\dagger h_1 (h_2^\dagger h_2 + h_3^\dagger h_3) \\
 & + (\lambda_6 + \lambda_7) h_1^\dagger (h_2^\dagger h_2 + h_3^\dagger h_3) h_1 + [\lambda_8 h_1^\dagger h_2 (h_2^\dagger h_3 + h_3^\dagger h_2) \\
 & + h_1^\dagger h_3 (h_2^\dagger h_2 + h_1^\dagger h_1) + H.c.].
 \end{aligned}$$

- It is possible to note that there are still a residual Z2 symmetry, it is the exchange of the doublets h2 and h3 still allowed.

# Model A

- Soft terms break the  $Z_2$  symmetry to avoid the mass degeneracy:

$$\mu_{nm}^2 H_n^\dagger H_m, \quad n, m = 2, 3$$

$$M_n^2 = \begin{pmatrix} a_n & -b_n & -b_n \\ -b_n & a_n + \mu_{22}^2 & -b_n + \mu_{23}^2 \\ -b_n & -b_n + \mu_{23}^2 & a_n + \mu_{33}^2 \end{pmatrix}$$

$$\mu_{22}^2 = \mu_{33}^2 = -\mu_{23}^2 \equiv \mu^2.$$

$$\mu_{22}^2 = \mu_{33}^2 = \nu^2 ; \quad \mu_{23}^2 = \mu^2$$

$$(2a_n - b_n, a_n + b_n, a_n + b_n + \mu^2)$$

"this mass matrix is good for neutrinos but not for Higgs scalars sector all eigenvalues are different from zero: there are no Goldstone bosons"

# Model A

- It is possible to explain, for example, three bosons with mass equal 125 GeV and the charged with 308 GeV.

```
V = 246 / Sqrt[2];  
Mh1 = Sqrt[(2 / 3) * L4 * V^2];  
Mh2[mud2_] := Sqrt[mud2 + (1 / 2) * L * V^2];  
Mc2[mud2_] := Sqrt[(1 / 2) mud2 + (1 / 12) * L5 * V^2];  
NSolve[Mh1 == 125, L4]  
{{L4 → 0.774589}}  
NSolve[Mh2[- 20] == 125, L]  
{{L → 1.03411}}  
NSolve[Mh2[20] == 125, L]  
{{L → 1.03146}}  
NSolve[Mh2[- 40] == 125, L]  
{{L → 1.03543}}  
NSolve[Mh2[- 140] == 125, L]  
{{L → 1.04204}}
```

```
NSolve[Mc2[- 20] == 308, L5]
```

```
{{L5 → 37.626}}
```

```
NSolve[Mc2[20] == 308, L5]
```

```
{{L5 → 37.6181}}
```

```
NSolve[Mc2[- 140] == 308, L5]
```

```
{{L5 → 37.6498}}
```

```
NSolve[Mc2[40] == 308, L5]
```

```
{{L5 → 37.6141}}
```

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```
{{L → 1.03543}}
```

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NSolve[Mh2[- 140] == 125, L]
```

```
{{L → 1.04204}}
```

```
L1 = - 1;  
L2 = - 1;  
L3 = - 1;  
L4 = 0.7746;  
L5 = 37.6;  
L6 = - 6.6;  
L7 = - 15;  
L8 = 0;  
mud2 = - 20;
```

```
FindMinimum[VA[- 23 438, v1, v2, v3], {v1, 53}, {v2, 53}, {v3, 53}]
```

```
{- 1.77298 × 108, {v1 → 100.429, v2 → 100.429, v3 → 100.429}}
```

```
NSolve[Mc2[- 20] == 308, L5]
```

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NSolve[Mh2[- 20] == 125, L]
{{L -> 1.03411}}

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{{L -> 1.03146}}

NSolve[Mh2[- 40] == 125, L]
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{{L -> 1.04204}}

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{{L5 -> 37.6181}}

NSolve[Mc2[- 140] == 308, L5]
{{L5 -> 37.6498}}

NSolve[Mc2[40] == 308, L5]
{{L5 -> 37.6141}}

L1 = -1;
L2 = -1;
L3 = -1;
L4 = 0.7746;
L5 = 37.6;
L6 = -6.6;
L7 = -15;
L8 = 0;
mud2 = -20;
FindMinimum[VA[-23 438, v1, v2, v3], {v1, 53}, {v2, 53}, {v3, 53}]
{-1.77298 * 10^8, {v1 -> 100.429, v2 -> 100.429, v3 -> 100.429}}

```

$\mu_s^2 = -\lambda_4 v_{SM}^2 = -0.7746 * (246/\text{Sqrt}[2])^2$

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```

```
{{L5 → 37.6498}}
```

```
NSolve[Mc2[40] == 308, L5]
```

```
{{L5 → 37.6141}}
```

$v_1 = v_2 = v_3 = v_{sm} / \text{Sqrt}[3] = 246 / \text{Sqrt}[6] = 100.429$



# Model A: phenomenology

$$\begin{aligned}
 -\mathcal{L}_l &= \bar{\nu}_{iL} \frac{\hat{M}_i^l}{v_{SM}} (V_{PMNS})_{ij} l_{jR} h_1^+ + \bar{l}_{iL} \frac{\hat{M}_i^l}{v_{SM}} l_{jR} \left[ 1 + \frac{h_1^0 + ia_1^0}{\sqrt{2}} \right] \\
 &+ \bar{l}_{iL} \frac{\hat{M}_i^\nu}{v_{SM}} (V_{PMNS})_{ij} \nu_{jR} h_1^- + \bar{\nu}_{iL} \frac{\hat{M}_i^\nu}{v_{SM}} \nu_{iR} \left[ 1 + \frac{h_1^0 + ia_1^0}{\sqrt{2}} \right] + H.c.,
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{L}_{gauge} &= (\mathcal{D}_\mu S)^\dagger (\mathcal{D}^\mu S) + (\mathcal{D}_\mu D)^\dagger (\mathcal{D}^\mu D) \\
 &= (\mathcal{D}_\mu H_1)^\dagger (\mathcal{D}^\mu H_1) + (\mathcal{D}_\mu H_2)^\dagger (\mathcal{D}^\mu H_2) + (\mathcal{D}_\mu H_3)^\dagger (\mathcal{D}^\mu H_3)
 \end{aligned}$$

Interactions with h20 and the charged ones:

$$\begin{aligned}
 &(\lambda_6 + \lambda_7) h_1^- h_2^+, \quad -\lambda_8 h_2^- h_2^+, \quad \lambda_8 h_3^- h_3^+, \\
 &-(\lambda_6 + \lambda_7) (h_1^0 h_2^0 + a_1^0 a_2^0), \quad i(\lambda_6 + \lambda_7) (-h_1^0 a_2^0 + h_2^0 a_1^0), \\
 &-\lambda_8 h_2^0 h_2^0, \quad -\lambda_8 a_2^0 a_2^0, \quad \lambda_8 h_3^0 h_3^0, \quad \lambda_8 a_3^0 a_3^0.
 \end{aligned}$$

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 &+ \bar{l}_{iL} \frac{\hat{M}_i^\nu}{v_{SM}} (V_{PMNS})_{ij} \nu_{jR} h_1^- + \bar{\nu}_{iL} \frac{\hat{M}_i^\nu}{v_{SM}} \nu_{iR} \left[ 1 + \frac{h_1^0 + ia_1^0}{\sqrt{2}} \right] + H.c.,
 \end{aligned}$$

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 \mathcal{L}_{gauge} &= (\mathcal{D}_\mu S)^\dagger (\mathcal{D}^\mu S) + (\mathcal{D}_\mu D)^\dagger (\mathcal{D}^\mu D) \\
 &= (\mathcal{D}_\mu H_1)^\dagger (\mathcal{D}^\mu H_1) + (\mathcal{D}_\mu H_2)^\dagger (\mathcal{D}^\mu H_2) + (\mathcal{D}_\mu H_3)^\dagger (\mathcal{D}^\mu H_3)
 \end{aligned}$$

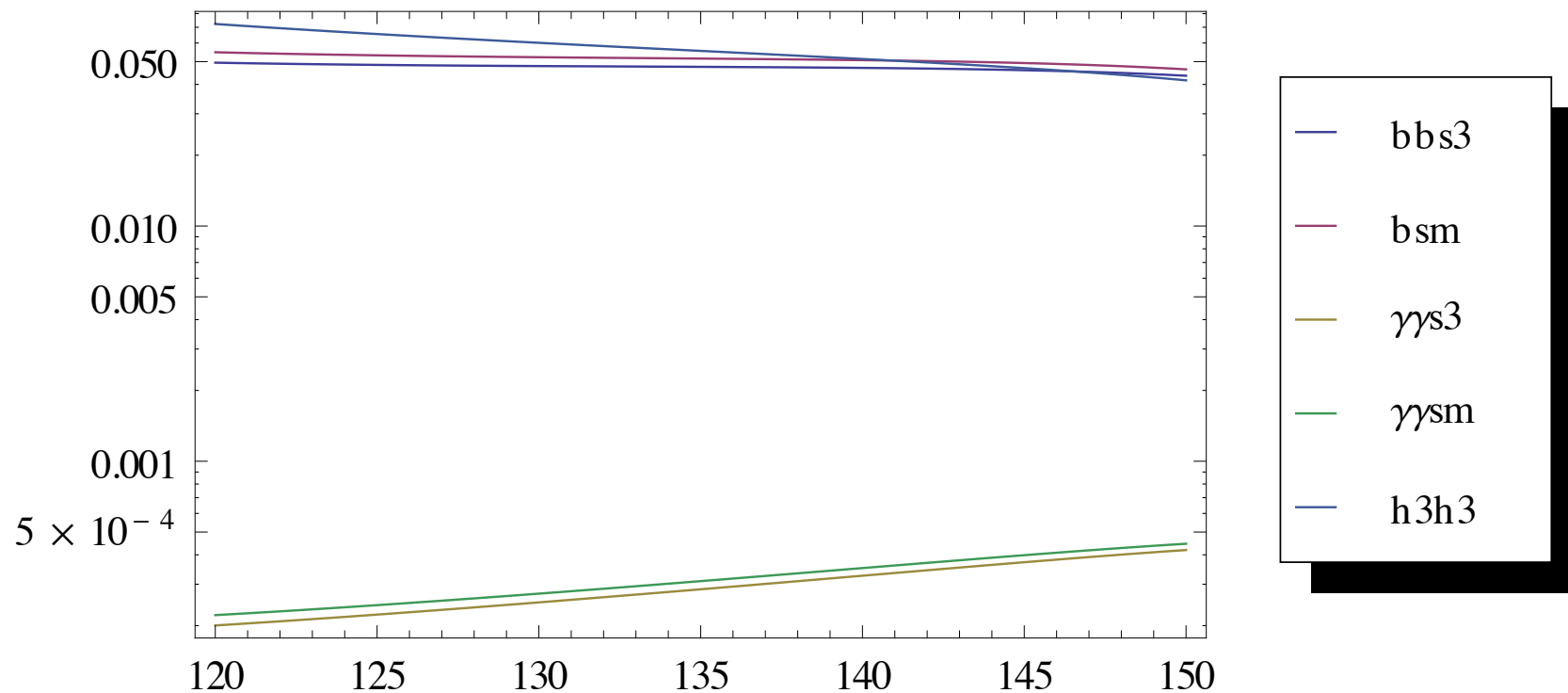
Interactions with h20 and the charged ones:

$$\begin{aligned}
 &(\lambda_6 + \lambda_7) h_1^- h_2^+, \quad -\lambda_8 h_2^- h_2^+, \quad \lambda_8 h_3^- h_3^+, \\
 &-(\lambda_6 + \lambda_7) (h_1^0 h_2^0 + a_1^0 a_2^0), \quad i(\lambda_6 + \lambda_7) (-h_1^0 a_2^0 + h_2^0 a_1^0), \\
 &-\lambda_8 h_2^0 h_2^0, \quad -\lambda_8 a_2^0 a_2^0, \quad \lambda_8 h_3^0 h_3^0, \quad \lambda_8 a_3^0 a_3^0.
 \end{aligned}$$

the unitary gauge

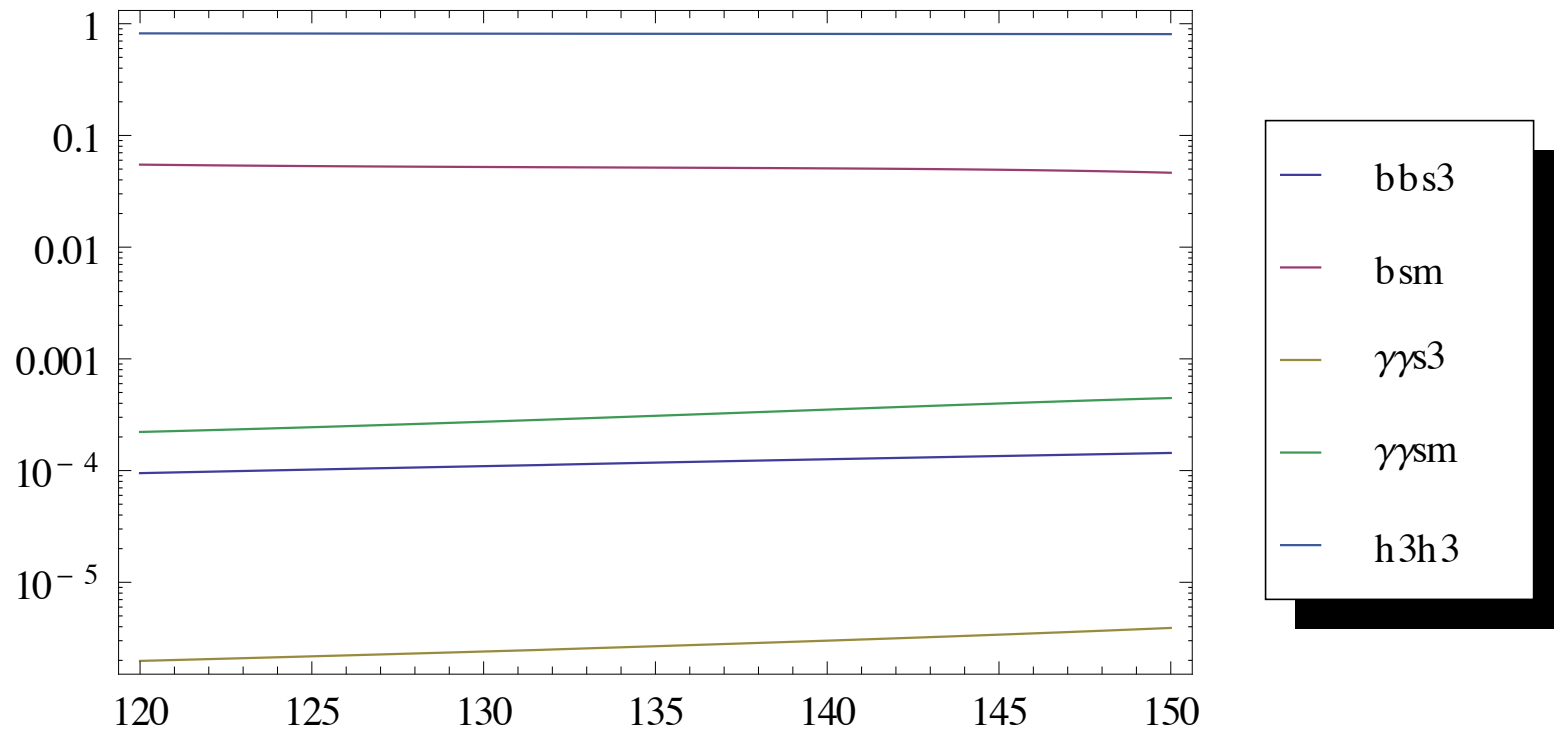
# Model A: phenomenology

$L5=0.001$  and  $L6=L7=0.02$  and  $m_{h2} = 350$  GeV  
main decays



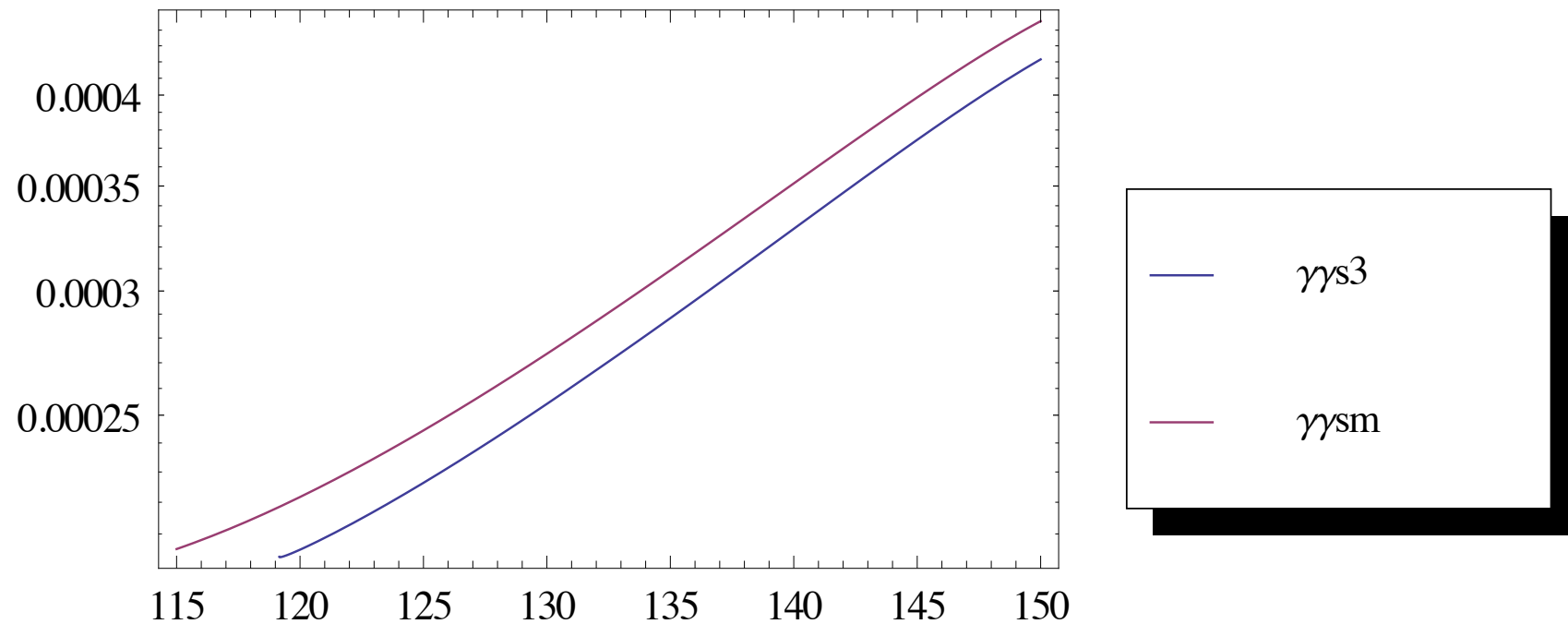
# Model A

$L5=L6=L7=1$  and  $m_{h2} = 350$  Gev  
maindecays



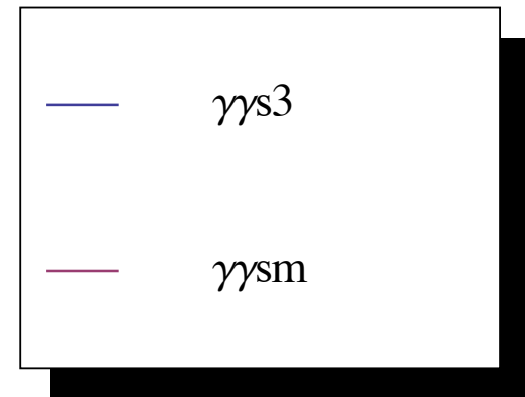
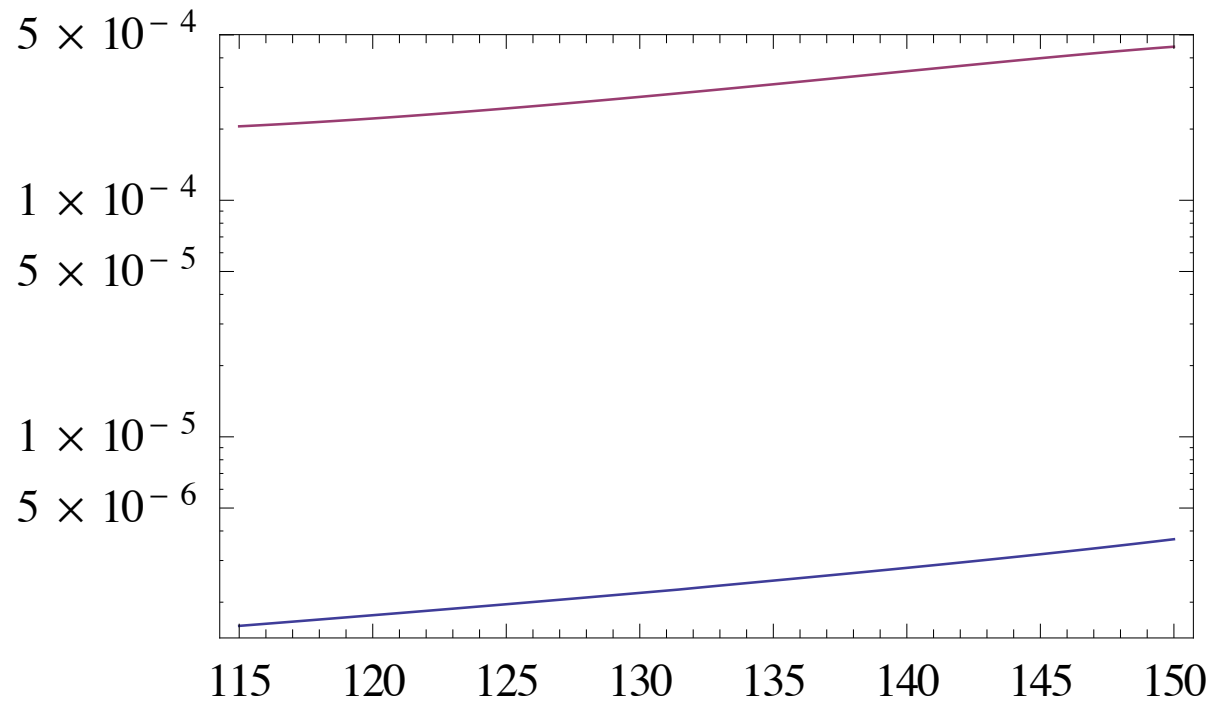
# Model A

$L5=0.001$  and  $L6=L7=0.02$  and  $m_{h2} = 350$  Gev  
comparing with the standard Higgs



# Model A

$L5=L6=L7=1$  and  $m_{h2} = 350$  Gev  
comparing with the standard Higgs





# Model A

- So we can see that:
  - The decays depend on the value of the parameters  $\lambda$ .
  - is possible to obtain a scalar potential that satisfies all the theoretical conditions of spontaneous symmetry breaking to give mass to the model spectrum.
  - We can obtain decay rates similar to SM for the S3 model.
  - We are analyzing the two-photon channel and it seems that it is possible to explain the results of the LHC...

# Model B

$$S = H_1 \sim \mathbf{1}, \quad D = (H_2, H_3) \sim \mathbf{2}. \quad (3)$$

$$v_1 \equiv v_{SM}, \quad v_2 = v_3 = 0$$

The constrain equation implies:

$$\mu_s^2 = -\lambda_4 v_{SM}^2$$

L1 = 1;

L2 = 1;

L3 = 1;

L4 = 1;

L5 = 1;

L6 = 1;

L7 = 1;

L8 = 0;

mud2 = -10;

FindMinimum[VB[-100, v1, v2, v3], {v1, 5}, {v2, 0}, {v3, 0}]

{-2500., {v1 → 10., v2 → 0., v3 → 0.}}

This is a Minimum Global, is easy to check  
that for any numerical values is possible to  
obtain this vacuum alignment

# Model B

- The mass matrix is diagonal in this case, but the masses are the same as in case of the Model A

$$m_{h_1}^2 = \frac{2}{3}\lambda_4 v_{SM}^2, \quad m_{h_2}^2 = m_{h_3}^2 \equiv m_h^2 = \mu_d^2 + \frac{1}{2}\bar{\lambda}' v_{SM}^2,$$

$$m_{a_1}^2 = 0, \quad m_{a_2}^2 = m_{a_3}^2 \equiv m_a^2 = \mu_d^2 + \frac{1}{6}\bar{\lambda}' v_{SM}^2$$

$$m_{c_1}^2 = 0, \quad m_{c_2}^2 = m_{c_3}^2 \equiv m_c^2 = \frac{1}{2}\mu_d^2 + \frac{\lambda_5}{12}v_{SM}^2.$$

- However the term below breaks S2 symmetry between v2 and v3 which meaning that this term should be forbidden, a additional Z2 symmetry is able to do so, if D -> - D.

$$\lambda_8[(S^\dagger \otimes D)_2(D^\dagger \otimes D)_2]_1 + H.c.$$

# Model B

```
L1 = -1;  
L2 = -1;  
L3 = -1;  
L4 = 0.7746;  
L5 = 37.6;  
L6 = -6.6;  
L7 = -15;  
L8 = 0;  
mud2 = -20;
```

```
FindMinimum[VB[-23438, v1, v2, v3], {v1, 5}, {v2, 0}, {v3, 0}]
```

```
{-1.77298 × 108, {v1 → 173.949, v2 → 0., v3 → 0.}}
```

# Model B

- To avoid the mass degeneration we will adding quadratic terms, like we did in model A.

$$\mu_{nm}^2 H_n^\dagger H_m, \quad n, m = 2, 3$$

$$M_n^2 = \begin{pmatrix} m_{n1}^2 & 0 & 0 \\ 0 & m_{n2}^2 + \nu^2 & \mu^2 \\ 0 & \mu^2 & m_{n2}^2 + \nu^2 \end{pmatrix} \quad \mu_{22}^2 = \mu_{33}^2 = -\mu_{23}^2 \equiv \mu^2.$$

Do nothing

$$\mu_{22}^2 = \mu_{33}^2 = \nu^2 \quad ; \quad \mu_{23}^2 = \mu^2$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$S \equiv h_1 = \begin{pmatrix} h_1^+ \\ \frac{1}{\sqrt{2}}(3v + h_1^0 + ia_1^0) \end{pmatrix},$$

$$D_1 = \begin{pmatrix} \frac{1}{\sqrt{2}}(-h_2^+ + h_3^+) \\ \frac{1}{2}[-h_2^0 + h_3^0 + i(-a_2^0 + a_3^0)] \end{pmatrix}, \quad D_2 = \begin{pmatrix} \frac{1}{\sqrt{2}}(h_2^+ + h_3^+) \\ \frac{1}{2}[h_2^0 + h_3^0 + i(a_2^0 + a_3^0)] \end{pmatrix}.$$

# Final Thoughts

- We have a three higgs doublets model that have a simple scalar potential.
- This model remain closer to the standard model than a 3 - higgs model without  $s_3$  symmetry.
- $h_2$  and  $h_3$  could be possible candidates to explain the photon counts in the LHC.
- Models A and B are similar or not?



Thank you