Geometrical CP violation in multi-Higgs models

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Outline

Spontaneous CP violation

Complex VEVs not sufficient. CP conserved if:

$$H_i \longrightarrow H'_i = U_{ij} H_j \,, \tag{1}$$

$$U_{ij} \langle H_j \rangle^* = \langle H_i \rangle \,, \tag{2}$$

while U leaves the Lagrangian invariant.

Calculable phases

Branco, Gérard, Grimus (1979) PLB

- VEV phases have geometrical values independent of arbitrary couplings.
- Require > 2 Higgs doublets and non-Abelian symmetries.
- Interesting $\Delta(27) \equiv C_3 \ltimes (C_3 \times C_3)$ example found.

Potential with 3 scalars

$$\begin{aligned}
V(H) \sim &+ \sum_{i<j} \left[\frac{\gamma_i}{2} (H_i^\dagger H_j + \text{H.c.}) + \frac{D_i}{2} \left((H_i^\dagger H_j)^2 + \text{H.c.} \right) \right] \\
&+ \frac{1}{2} \sum_{i \neq j} \left[E_{1ij} (H_i^\dagger H_i)(H_i^\dagger H_j) + \text{H.c.} \right] \\
&+ \frac{1}{2} \sum_{\substack{i \neq j \neq k \\ j<k}} \left[E_{2i} (H_i^\dagger H_j)(H_k^\dagger H_i) + E_{3i} (H_i^\dagger H_i)(H_k^\dagger H_j) \right. \\
&\left. + E_{4i} (H_i^\dagger H_j)(H_i^\dagger H_k) \; + \; \text{H.c.} \right] ,
\end{aligned} \tag{3}$$

S_3 potential

Relating coefficients

Smallest non-Abelian group S_3

$$\gamma = \gamma_i \,, D = D_i \,, E_1 = E_{1ij} \,, E_\alpha = E_{\alpha i} \,,\; i,j = 1,2,3 \,, \alpha = 2,3,4 \,. \tag{4}$$

Only E_4: potential of $\Delta(27)$.

$\Delta(3n^2)$ or $\Delta(6n^2)$

IdMV, Emmanuel-Costa (2010) PLB

- Replace C_3 with S_3, $\Delta(54) \equiv S_3 \ltimes (C_3 \times C_3)$ ...
- Or any C_n: $\Delta(3n^2)$ or $\Delta(6n^2)$ as long as n is multiple of 3.

VEVs depend on the sign of E_4 ($\omega = e^{i2\pi/3}$, $\omega^3 = 1$):

$$\langle H \rangle^{\mathrm{T}} = \frac{v}{\sqrt{3}} \left(1, \omega, \omega^2\right), \tag{5a}$$

$$\langle H \rangle^{\mathrm{T}} = \frac{v}{\sqrt{3}} \left(\omega, 1, 1\right), \tag{5b}$$

Fermions: $\Delta(27)$

$QH^\dagger u^c$, QHd^c and Q_i as...

- triplet: 1 sector $\mathbf{3_{0i}} \times \mathbf{3_{0i}} \times \mathbf{3_{0i}}$.
  Already pointed out as not viable.
- singlets: Both sectors $\mathbf{3_{01}} \times \mathbf{3_{02}} \times \mathbf{1_{rs}}$
  can get:
  rank 1 mass matrices or
  one generation decoupled or
  diagonal matrices with three distinct eigenvalues.

Fermions: $\Delta(54)$

$QH^\dagger u^c$, QHd^c and Q_i as...

- triplet: 1 sector 3-triplet (e.g. $\mathbf{3_1^a} \times \mathbf{3_1^a} \times \mathbf{3_1^a}$) has two degenerate eigenvalues.
- combination of doublet and singlet: again degenerate eigenvalues.
- singlets: the structures are rank 1.

Potentially interesting for leptonic sector:
leading order structure with rank 1 charged lepton + 2 degenerate neutrinos.

Parametrisations

$$\langle H^1 \rangle = v_1 e^{i\varphi_1}\,, \quad \langle H^2 \rangle = v_2 e^{i\varphi_2}\,, \quad \langle H^3 \rangle = v_3 e^{i\varphi_3}\,, \tag{6}$$

$$\begin{aligned} v_1 &= \sin(\alpha \cdot \pi)\cos(\beta \cdot \pi)\,, \\ v_2 &= \sin(\alpha \cdot \pi)\sin(\beta \cdot \pi)\,, \\ v_3 &= \cos(\alpha \cdot \pi)\,. \end{aligned} \tag{7}$$

(numerical coincidence: $\cos(0.3 \cdot \pi) - 1/\sqrt{3} < 0.0105$)

Phase-independent terms and direction

IdMV, Leser, Emmanuel-Costa (2012) PLB

	+	-
$v_1^n + c.p.$	$(1,1,1)$	$(0,0,1)$
$v_1^m v_2^n + c.p.$	$(0,0,1)$	$(0,1,1)$
$v_1^l v_2^m v_3^n + c.p.$	$(0,0,1)/(0,1,1)$	$(1,1,1)$

Table : Types of phase-independent combinations and preferred VEVs according to the sign of their coefficient.

(1,1,1) is natural

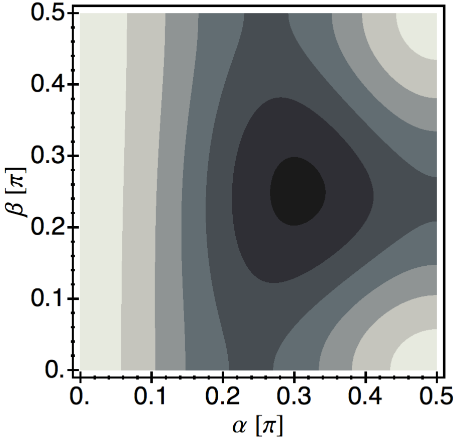

Figure : VEV-type $(1, 1, 1)$ arises from cooperating terms.

(1,1,1) is really natural

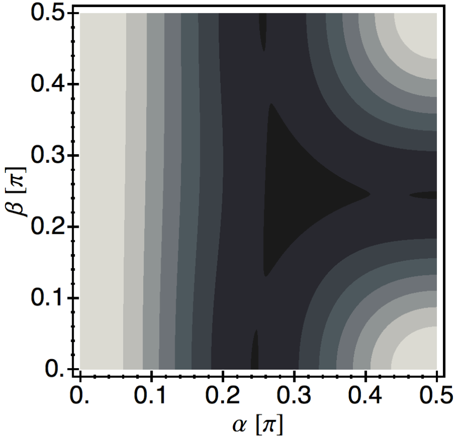

Figure : VEV-type $(1,1,1)$ arises from dominant term.

Phase-dep. terms

$$\left(H_i^2 (H_j H_k)^\dagger\right)^m \to \theta_i^m \equiv -2m\varphi_i + m\varphi_j + m\varphi_k \,, i \neq j \neq k \,. \quad (8)$$

$$\left(H_i H_j^\dagger\right)^{3m} \to \rho_i^m \equiv 3m\varphi_i - 3m\varphi_j + 0\varphi_k \,, i \neq j \neq k \,. \quad (9)$$

Reparametrisations

IdMV (2012) JHEP

- Any given phase-dependence is expressed as $A_1 = \sum a_i \varphi_i$ (and c.p.)
- Not all independent: $\sum A_i = \left(\sum a_i\right)\left(\sum \varphi_i\right)$
- With doublets we have: $\sum a_i = 0 \rightarrow \sum A_i = 0$

Triangle phases

$$V_3 = \left(e^{iA_1} + e^{iA_2} + e^{iA_3}\right) + h.c.\,, \tag{10}$$

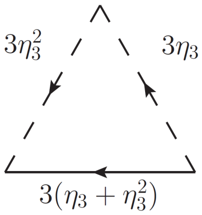

Figure : Minimising V_3 with $A_i = \pm(2\pi/3)$ (and $\eta_n^n = 1$).

Even N

Can always have all $A_i = \pm\pi$ (but still interesting).

Odd N

$$V_5 = \left(e^{iA_1} + e^{iA_2} + e^{iA_3} + e^{iA_4} + e^{iA_5}\right) + h.c.\,, \qquad (11)$$

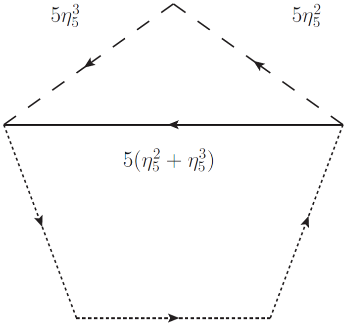

Figure : Minimising V_5 with $A_i = \pm 2(2\pi/5)$ (and $\eta_n^n = 1$).

$N \neq n$

Groups with N dim. irreps. with a C_N factor and C_n factors:

$n = kN$, invariant is "multiplied" by k, phases divided by k.

For $\Delta(108)$ ($n = 6$), a particular irrep. has the $H_1^4 (H_2^2 H_3^2)^\dagger + c.p.$ invariant to get e.g. $2\pi/6$.

Optimal primes

Groups with N dim. irreps. with a C_N factor and C_n factors:

If N and n do not share a prime factor, invariant only of $(H_1 H_2^\dagger)^n + c.p.$ type, phases divided by n.

For $\Delta(3n^2)$ with $n = 4, 5$, you can get e.g. $2\pi/12$, $2\pi/15$.

Table of new cases

Case	a_i vector	Phase
Even N	$(N-1,-1,(...),-1)$	$\pi/2$
Even N	$(N/2,-N/2,(...),N/2,-N/2)$	$\pm l\frac{2\pi}{N}$ (l odd)
$n=kN$	$k(N-1,-1,(...),-1)$	$\pm l\frac{\pi}{k}$ (l odd)
$n\neq kN$	$(n,-n,0,(...),0)$	$\pm l\frac{2\pi}{2N}$
Odd N	$(N-1,-1,(...),-1)$	$\mp\frac{N-1}{2}\frac{2\pi}{N}$
$n=kN$	$k(N-1,-1,(...),-1)$	$\mp\frac{(2l-1)N-1}{2}\frac{2\pi}{kN}$
$N=lp$, $n=mp$	$m([p-1]_l,-1,(...),-1)$	$\mp\frac{(lp-1)\pi}{mlp}$
$N\neq kp$, $n=mp$	$(n,-n,0,(...),0)$	$\mp\frac{(2l-1)N-1}{2}\frac{2\pi}{nN}$

Conclusions

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- Geometrical CP violation with promising fermion masses.
- The calculability can survive to higher orders.
- New calculable phases have been found.