A Systematical Renormalization of the Type I/II 2HDM and the Automated Generation of NLO Amplitudes

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In Collaboration with
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Tests for the scalar sector

Needed:
Accurate experimental measurements & precise theoretical predictions
↑
our contribution

1. **Type I/II Two-Higgs Doublet Model**
   - Define parameters at higher order
2. **Renormalization**
   - Consistent set of renormalization conditions
3. **Implementation**
   - Automatization to be able to calculate many processes in an efficient way
4. **Outlook & Application**
   - Higgs decay into 4 fermions

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Type I/II Two-Higgs-Doublet model

Bare Higgs Lagrangian:

$$\mathcal{L}_{\text{Higgs,bare}} = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2) - V,$$

with

$$\begin{aligned} V =& m_{11} \Phi_1^\dagger \Phi_1 + m_{22} \Phi_2^\dagger \Phi_2 - [m_{12} \Phi_1^\dagger \Phi_2 + h.c.] \\ &+ \frac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\ &+ \frac{1}{2}\lambda_5 [(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2]. \end{aligned}$$

Constraints:

- CP conservation
- No FCNCs at tree level

Expand the fields around the vevs: $\Phi_i = \begin{pmatrix} \phi_i^+ \\ \frac{1}{\sqrt{2}}(\eta_i + i c_i^0) + v_i \end{pmatrix}$

LO Mass eigenstates:

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} \cos\beta_{\mathrm{LO}} & -\sin\beta_{\mathrm{LO}} \\ \sin\beta_{\mathrm{LO}} & \cos\beta_{\mathrm{LO}} \end{pmatrix} \begin{pmatrix} G_0 \\ A_0 \end{pmatrix}$$

$$\begin{pmatrix} \phi_1^{\pm} \\ \phi_2^{\pm} \end{pmatrix} = \begin{pmatrix} \cos\beta_{\mathrm{LO}} & -\sin\beta_{\mathrm{LO}} \\ \sin\beta_{\mathrm{LO}} & \cos\beta_{\mathrm{LO}} \end{pmatrix} \begin{pmatrix} G^{\pm} \\ H^{\pm} \end{pmatrix}$$

$$\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha_{\mathrm{LO}} & -\sin\alpha_{\mathrm{LO}} \\ \sin\alpha_{\mathrm{LO}} & \cos\alpha_{\mathrm{LO}} \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix},$$

with

$$\tan\beta = v_2/v_1$$

Mass mixing terms occur at $\mathcal{O}(\alpha)$:

$$\begin{aligned}\mathcal{L}_{\mathrm{mass}} =& \delta M_{Hh}\, H\, h + \delta M_{AG}\, A_0\, G_0 + \delta M_{H^\pm G^\pm} (H^+ G^- + G^+ H^-) \\ &+ \text{ordinary mass terms}\end{aligned}$$

Tadpole terms do not vanish in $\mathcal{O}(\alpha_s)$:

$$\left.\frac{\partial V}{\partial H}\right|_{\text{Fields}=0} = \delta t_H, \qquad \left.\frac{\partial V}{\partial h}\right|_{\text{Fields}=0} = \delta t_h$$

Yukawa Couplings to Fermions:

- So far: Neglect CKM-mixing

Downtype fermions: $\mathcal{L}_{\text{Yukawa}} = \sqrt{2}\, m_f / v_{1,2}\, \overline{F}^L f^R\, \Phi_{1,2}$

Uptype fermions: $\mathcal{L}_{\text{Yukawa}} = \sqrt{2}\, m_f / v_{1,2}\, \overline{F}^L f^R\, \tilde{\Phi}_{1,2}$

	u_i	d_i	e_i
Type I	Φ_2	Φ_2	Φ_2
Type II	Φ_2	Φ_1	Φ_1

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Content of the Higgs sector at LO

THDM

Fields:
$H, h, A_0, H^\pm, G, G^\pm$
parameters:
"input" parameters:
$m_{11}, m_{22}, m_{12}, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$
In terms of "physical parameters"
$M_H, M_h, M_{A_0}, M_{H^\pm}, M_W, \tan\beta, \sin\alpha_{\mathrm{LO}}, \lambda_5$

- Method 1: Renormalization of the "input" parameters and fields

$$p_i \to p_i + \delta p_i, \qquad \Phi_j \to \Phi_j + \delta Z_{H_j} \Phi_j, \qquad v_j \to \delta Z_{H_j}(v_j - \delta v_j)$$

- Method 2: Renormalization of the "physical parameters"

$$p_p \to p_p + \delta p_p, \qquad \delta t_{H/h}, \qquad \delta M_{Hh}(\delta\lambda_3), \delta M_{AG}, \delta M_{H^\pm G^\pm}$$

and fields

$$\begin{pmatrix} H \\ h \end{pmatrix} \to \begin{pmatrix} 1 + \frac{1}{2}\delta Z_H & \frac{1}{2}\delta Z_{Hh} \\ \frac{1}{2}\delta Z_{hH} & 1 + \frac{1}{2}\delta Z_h \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix},$$
$$\begin{pmatrix} A_0 \\ G_0 \end{pmatrix} \to \begin{pmatrix} 1 + \frac{1}{2}\delta Z_{A_0} & \frac{1}{2}\delta Z_{AG} \\ \frac{1}{2}\delta Z_{GA} & 1 + \frac{1}{2}\delta Z_G \end{pmatrix} \begin{pmatrix} A_0 \\ G_0 \end{pmatrix},$$
$$\begin{pmatrix} H^\pm \\ G^\pm \end{pmatrix} \to \begin{pmatrix} 1 + \frac{1}{2}\delta Z_{H^+} & \frac{1}{2}\delta Z_{HG^+} \\ \frac{1}{2}\delta Z_{GH^+} & 1 + \frac{1}{2}\delta Z_{G^+} \end{pmatrix} \begin{pmatrix} H^\pm \\ G^\pm \end{pmatrix},$$

- Both approaches are equivalent for the parameter renormalization
- Second approach allows for on-shell scheme in the field renormalization

Renormalization conditions

As in SM:

- Masses δM_i: Poles of the propagators have to be at the mass

$$\hat{\Gamma}^a(k^2 = M_a^2) =$$  $$\Big|_{k^2=M_a^2} = 0$$

- Fields δZ_i: The residuum of the propagators have to be i

$$\lim_{k^2 \to M_a^2} \frac{\hat{\Gamma}^a(k)}{k^2 - M_a^2} = \lim_{k^2 \to M_a^2} \frac{1}{k^2 - M_a^2}$$  $$\Big|_{k^2=M_a^2} = \mathrm{i}$$

- Field mixing δZ_{ij}: The mixings of the fields have to vanish on-shell

$$\hat{\Gamma}^{ab}(k^2 = M_a^2 \text{ or } M_b^2) = \Big|_{k^2=M_a^2 \text{ or } M_b^2} = 0$$

- Tadpoles $\delta t_{H,h}$: All tadpole terms have to vanish

$$\hat{T}_H = \quad = 0$$

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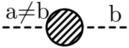

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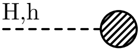


$$= 0$$

Renormalization conditions

- $\delta \tan\beta$: $\overline{MS}$ scheme

Condition for the vevs: [Sperling, Stöckinger, Voigt 13]

$$\delta v_1/v_1\big|_{\mathrm{UV-div}} = \delta v_2/v_2\big|_{\mathrm{UV-div}}$$

$\Rightarrow$ $\delta \tan\beta$ includes only contributions from the field renormalization
$\Rightarrow$ UV-Divergences can be calculated from field renormalization constants:

$$\delta \tan\beta = \frac{\delta Z_h - \delta Z_H}{c_\alpha^2 - s_\alpha^2}\,\frac{\tan\beta}{2}\bigg|_{\mathrm{UV-div}}$$

- $\delta\lambda_3$: $\overline{MS}$-Scheme

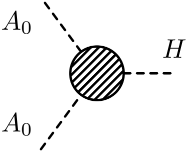
$\delta\lambda_3 = \delta\lambda_3(\delta M_{Hh}, \{\delta ...\})$ with

$$\delta M_{Hh}\Big|_{\mathrm{UV-div}} = -\frac{s_\alpha c_\alpha}{\tan\beta}\delta\tan\beta(M_{\mathrm{H}}^2 - M_h^2) - \Sigma^{Hh}(M_{\mathrm{H}}^2)\Big|_{\mathrm{UV-div}},$$

Renormalization conditions

- $\delta\lambda_5$: $\overline{MS}$ scheme

$$\hat{\Gamma}^{HA_0A_0}\Big|_{\mathrm{UV-div}} =$$



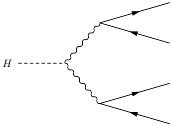
$$\Big|_{\mathrm{UV-div}} = 0$$

- MATHEMATICA code to calculate the counterterm potential in dependence of "physical" parameters and renormalization constants
- Implemented the whole Lagrangian in FEYNRULES [Alloul, Christensen, Degrande, Duhr, Fuks 09], for generating Feynman Rules and a FEYNARTS [Hahn 99] model file at NLO level
- A MATHEMATICA routine, which includes the renormalization conditions consistently into the FEYNARTS model file
- Generation and Calculation of amplitudes and squared matrix elements with FEYNARTS & FORMCALC [Hahn 99]

$\Rightarrow$ Applicable to arbitrary processes! NLO EW & QCD corrections in general possible

Application: The Higgs decay into 4 fermions

Higgs decay into 4 fermions:



Significant effects in the THDM with a SM like h?

For SM calculation: Programm PROPHECY4F [Bredestein, Denner, Dittmaier 06]

- Computation of EW & QCD corrections
- Arbitrary light final states
- Subtraction and Slicing method for the cancellation of IR divergences
- Full off-shell effects
- Complex mass scheme
- Generation of unweighted events

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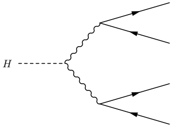

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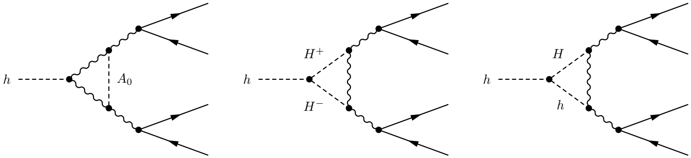
- Use as much of the PROPHECY4F code as possible!
- LO & photon/gluon emission: Change of the HVV coupling
- virtual corrections: New diagrams



$\Rightarrow$ Calculation with FEYNARTS & FORMCALC

- Interesting observable: $\Delta\phi_{ij}$, azimuthal angle of the momenta of the fermion pairs

We expect results soon!

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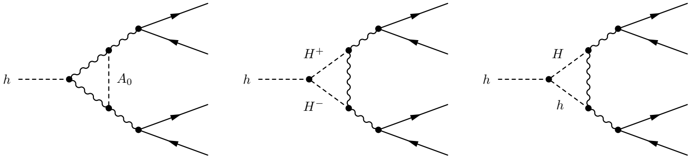

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Thank you for your Attention!

Backup

renormalization conditions: On-shell scheme whenever it's possible, otherwise $\overline{MS}$ scheme

parameters	
Electroweak sector:	$\delta M_{\mathrm{Z}}^2, \delta M_{\mathrm{W}}^2, \delta e$
Fermion masses:	$\delta m_{f,i}, \qquad f = l, u, d$
Higgs masses:	$\delta M_{\mathrm{H}}^2, \delta M_{\mathrm{h}}^2, \delta M_{\mathrm{A_0}}^2, \delta M_{H^+}^2$
Higgs potential:	$\delta\lambda_3(\delta M_{Hh}), \delta\lambda_5, \delta\tan\beta$
Tadpoles:	$\delta t_H, \delta t_h$
fields	
Electroweak sector:	$\delta Z_W, \delta Z_{ZZ}, \delta Z_{ZA}, \delta Z_{AZ}, \delta Z_{ZZ}$
Left fermions:	$\delta Z_i^{f,L}, \qquad f = \nu, l, u, d$
Right fermions:	$\delta Z_i^{f,R}, \qquad f = l, u, d$
Higgs:	$\delta Z_H, \delta Z_{Hh}, \delta Z_{hH}, \delta Z_h$
	$\delta Z_{A_0}, \delta Z_{AG}, \delta Z_{GA}, \delta Z_G$
	$\delta Z_{H^+}, \delta Z_{HG^+}, \delta Z_{GH^+}, \delta Z_{G^+}$

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- electric charge δe: Corrections to $\gamma \bar{f} f$-vertex in the Thomson limit have to vanish

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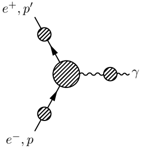

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