



Radiative corrections to Higgs coupling constants in two Higgs doublet models

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1. S. Kanemura, M. K., K. Yagyu, Physics Letters B731 (2014) 27.
2. S. Kanemura, M.K., K. Yagyu, In preparation

Multi-Higgs Models at the Complexo Interdisciplinar da UL
2014/09/02-05

Higgs Questions

LHC discovered h , and SM was confirmed to be a good approximation.

However, most of non-minimal Higgs sector can explain the current LHC data as well.

If extended Higgs sectors, then many questions:

- Shape 2HDM? Which type? Singlet? Triplet?
- Mass of the second Higgs boson
- Decoupling/Non-decoupling property
- ...

Explore Higgs by
bottom-up approach
with future data



- Direct search
- Indirect search

Physics of h ($m_h = 126$ GeV)

Can we reconstruct “Extended” Higgs sector from precision measurements of the discovered h ?

h -couplings !

$hZZ, hWW, hbb, h\tau\tau, htt, hcc, h\chi\chi, h\chi Z, hhh, \dots$

They will be measured with high precision at future colliders!! (HL-LHC, ILC, $\dots$)

Theoretical predictions
at loop level



Precision measurements
of Higgs couplings



New Physics !



***h*-couplings**

$$\Delta g_X = \frac{g_{hXX}^{\text{THDM}} - g_{hXX}^{\text{SM}}}{g_{hXX}^{\text{SM}}}$$

hWW, hZZ
(weak gauge
couplings)

Verification of Higgs mechanism
Accuracy; 2-5 % at HL-LHC
0.3-0.5 % at ILC(500)

h $\gamma\gamma$

It's sensitive to new physics effect because
of loop induce diagram.
Accuracy; 2-5 %, 4-8 %

hbb, h $\tau\tau$, htt, ...

The origin of masses, Flavor structure
The pattern of deviations are sensitive
to the detail of the Higgs sector.

Accuracy; *hbb* 4-7%, 0.6-1.0% *h $\tau\tau$* 2-5%, 1-2% *htt* 7-10 %, 1-3 %

hhh
(self coupling)

Structure of the Higgs potential
Large corrections by the loop contributions
Accuracy; 50 % , 13-20 %



In this talk

- We consider Two Higgs doublet models (2HDMs) as an example.
- We impose 2HDM a softly broken Z_2 symmetry in order to suppress FCNC at the tree level.
 - 4 types of different Yukawa interactions appear.
- We calculate a full set of 1-loop corrected h -couplings in the on-shell scheme.
- We discuss how we can obtain information of the Higgs sector by using future precision data and theory predictions with radiative corrections.

Higgs potential

- CP invariance & softly broken Z_2

Soft-breaking scale of Z_2 sym.

$$M^2 = \frac{m_3^2}{\sin\beta\cos\beta}$$

$$V_{\text{THDM}} = m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 - m_3^2(\Phi_1^\dagger\Phi_2 + \text{h.c.}) + \frac{1}{2}\lambda_1|\Phi_1|^4 + \frac{1}{2}\lambda_2|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \frac{1}{2}\lambda_5[(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}]$$

- Mass eigenstates

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}, \quad \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} z \\ A \end{pmatrix},$$

$$\begin{pmatrix} \omega_1^+ \\ \omega_2^+ \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \omega^+ \\ H^\pm \end{pmatrix}.$$

h ,

SM-like Higgs

$H, A, H^\pm$

Extra Higgs

- Parameters(8)

$$v \simeq 246 \text{ GeV} \quad m_h \simeq 126 \text{ GeV}$$

$$v^2 = v_1^2 + v_2^2 \sim (246 \text{ GeV})^2$$

$$\underline{m_H \quad m_A \quad m_{H^\pm} \quad a \quad \beta \quad M^2}$$

$$\tan\beta = \frac{v_2}{v_1}$$

6 free parameters

Higgs couplings at the tree level

- hWW, hZZ

$$g_{hVV(2HDM)} = \sin(\beta - \alpha) \frac{2m_V^2}{v}$$

$$\kappa_V \equiv \frac{g_{hVV(2HDM)}}{g_{hVV(SM)}} = \sin(\beta - \alpha)$$

SM-like limit; $\kappa_V = \sin(\beta - \alpha) \rightarrow 1$

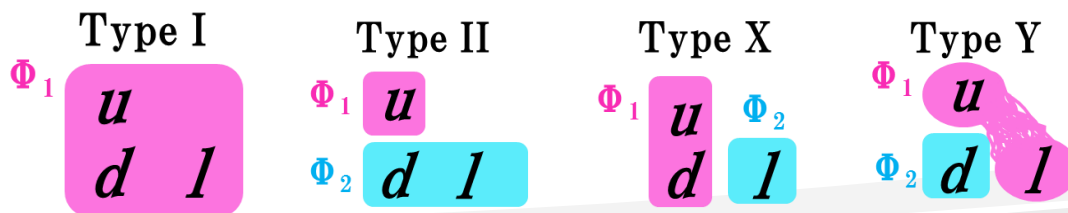
- Yukawa couplings ($htt, hbb, h\tau\tau, \dots$)

If f couples to Φ_1

$$\kappa_f = \sin(\beta - \alpha) + \cot\beta \cos(\beta - \alpha)$$

If f couples to Φ_2

$$\kappa_f = \sin(\beta - \alpha) - \tan\beta \cos(\beta - \alpha)$$



The pattern of deviations in Yukawa couplings depends on the Type of the 2HDM.

Coupling constants at 1-loop level

We calculate 1-loop corrections to h -couplings in the on-shell renormalization scheme.
(No QCD correction included. They can be factorized)

hZZ hWW hbb $h\tau\tau$ htt hcc hhh

hZZ
 hWW

*Hollik, Penaranda, Eur. Phys. J. C23 (2002) [in the MSSM]
Kanemura, Kiyoura, Okada, Senaha, Yuan PLB558, (2003);
Kanemura, Okada, Senaha, Yuan, PRD70 (2004).*

hff

*Guasch, Hollik, Penaranda, PLB515 (2001) [in the MSSM]
Guasch, Hafliger, Spira, PRD68 (2003) [in the MSSM]
Kanemura, Kikuchi, Yagyu, PLB731 (2014)*

hhh

Kanemura, Okada, Senaha, Yuan, PRD70 (2004).

Renormalization of Higgs sector

Parameters in $V(\phi_1, \phi_2)$; m_1^2 m_2^2 M^2 λ_1 λ_2 λ_3 λ_4 λ_5

Fields ; h_1 h_2 z_1 z_2 $\omega_1^\pm$ $\omega_2^\pm$

EWSB



Parameters ; m_h m_H m_A $m_{H^\pm}$ M^2 $\sin(\beta - \alpha)$ $\tan\beta$ v 8

Tadpole ; T_1 T_2 2

Eigenstate ; h H A $H^\pm$, G^0 $G^\pm$ 6

Field mixing ;

h - H , G^0 - A , $G^\pm$ - $H^\pm$ 3

Counter terms; δm_h δm_H δm_A $\delta m_{H^\pm}$ δa $\delta \beta$ δM δv δT_h δT_H
 δZ_h δZ_H δZ_A $\delta Z_{H^\pm}$ δZ_{G^0} $\delta Z_{G^\pm}$ δC_{hH} δC_{GA} $\delta C_{G^\pm H^\pm}$

8(parameters) + 2(Tadpole) + 6(fields) + 3(field mixing) = 19

Renormalization conditions

Tadpole

$$\text{---}\phi\text{---}\bigcirc_{1\text{PI}} + \text{---}\bigotimes = 0$$

$$\delta T_h \quad \delta T_H$$

Mass

$$\text{---}\phi\text{---}\bigcirc_{1\text{PI}}\text{---}\phi + \text{---}\bigotimes\text{---} = 0 \quad (\text{On shell})$$

$\leftarrow p$ $@ p^2 = m_a^2$

$$\delta m_h^2 \quad \delta m_H^2$$

$$\delta m_A^2 \quad \delta m_{H^\pm}^2$$

mixing

$$\text{---}\phi\text{---}\bigcirc_{1\text{PI}}\text{---}\phi' + \text{---}\bigotimes\text{---} = 0 \quad (\text{On shell})$$

$\leftarrow p$ $@ p^2 = m_\phi^2 = m_{\phi'}^2$

$$\delta\alpha \quad \delta\beta \quad \delta C_{hH}$$

$$\delta C_{ZA} \quad \delta C_{W+H^-}$$

Wave function renormalization

$$\frac{d}{dp^2}\Pi_{hh}(m_h^2) = \frac{d}{dp^2}\Pi_{hh}(m_h^2) = \dots = 0 \quad (\text{On shell})$$

$$\delta Z_h \quad \delta Z_H \quad \delta Z_A \quad \delta Z_{H^\pm} \quad \delta Z_{G^0} \quad \delta Z_{G^\pm}$$

Renormalization in gauge sector δv

Minimal subtraction δM^2

All 19 counter terms are determined
 by putting 19 conditions



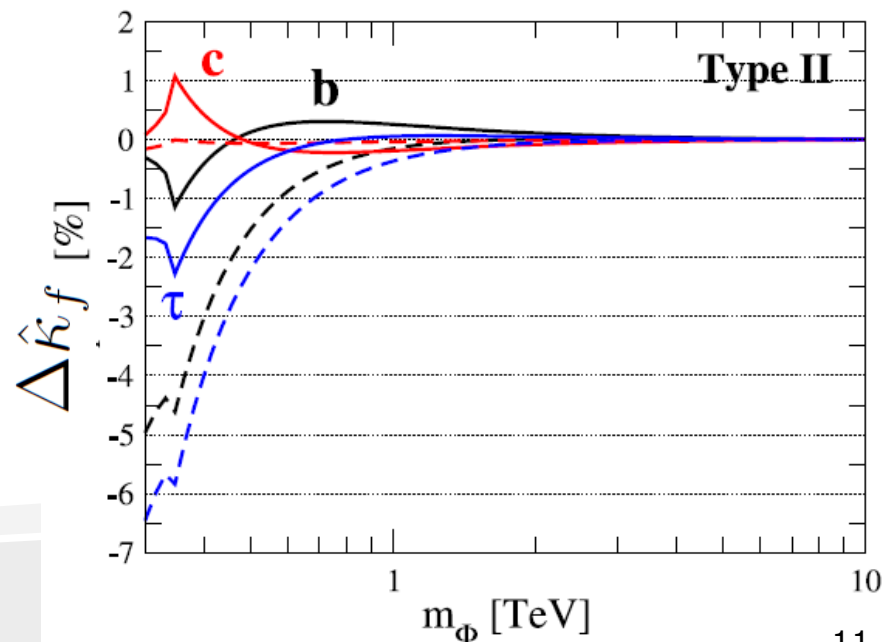
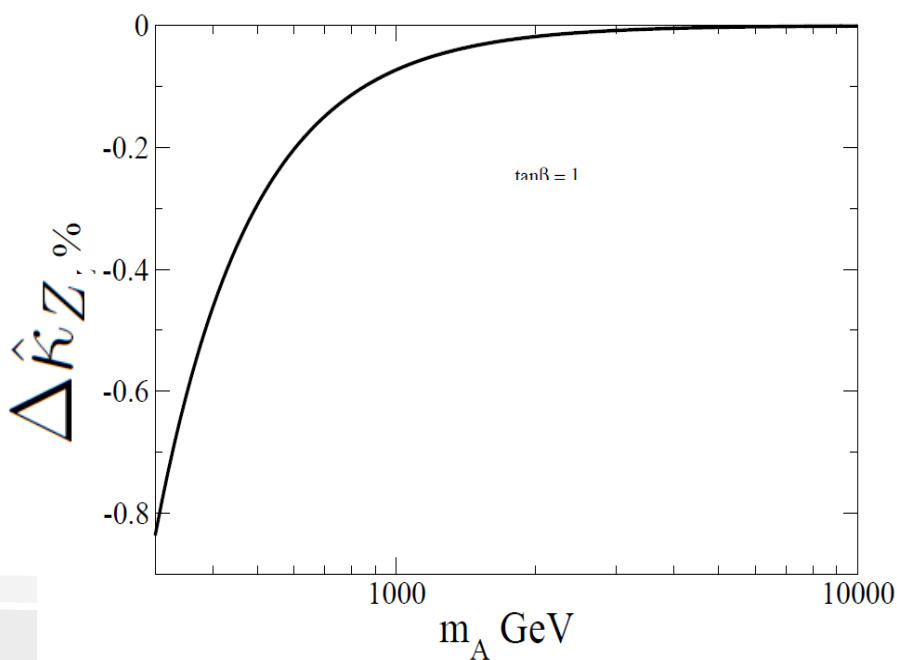
Check of our one-loop calculation code

Behavior of large mass limit

$$\Delta \hat{\kappa}_X = \frac{\hat{g}_{hXX}^{2\text{HDM}} - \hat{g}_{hXX}^{\text{SM}}}{\hat{g}_{hXX}^{\text{SM}}}$$

$$m_A^2 = M^2 + (300\text{GeV})^2$$

We can check behavior of decoupling limit.



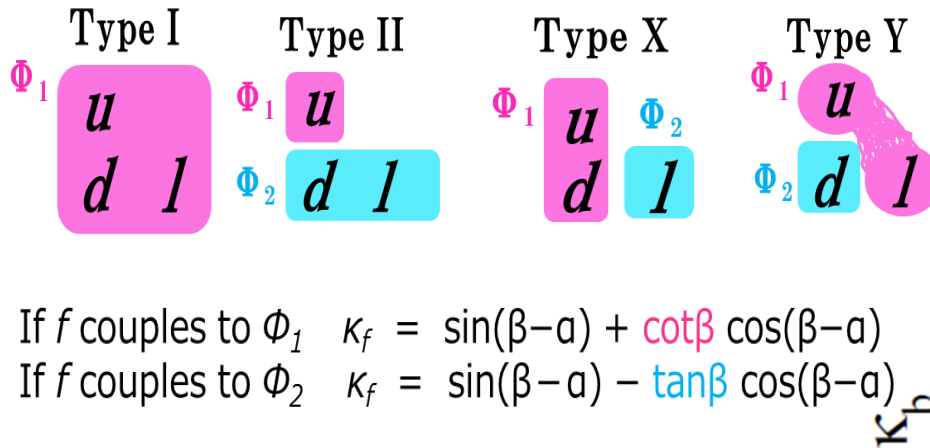


Questions

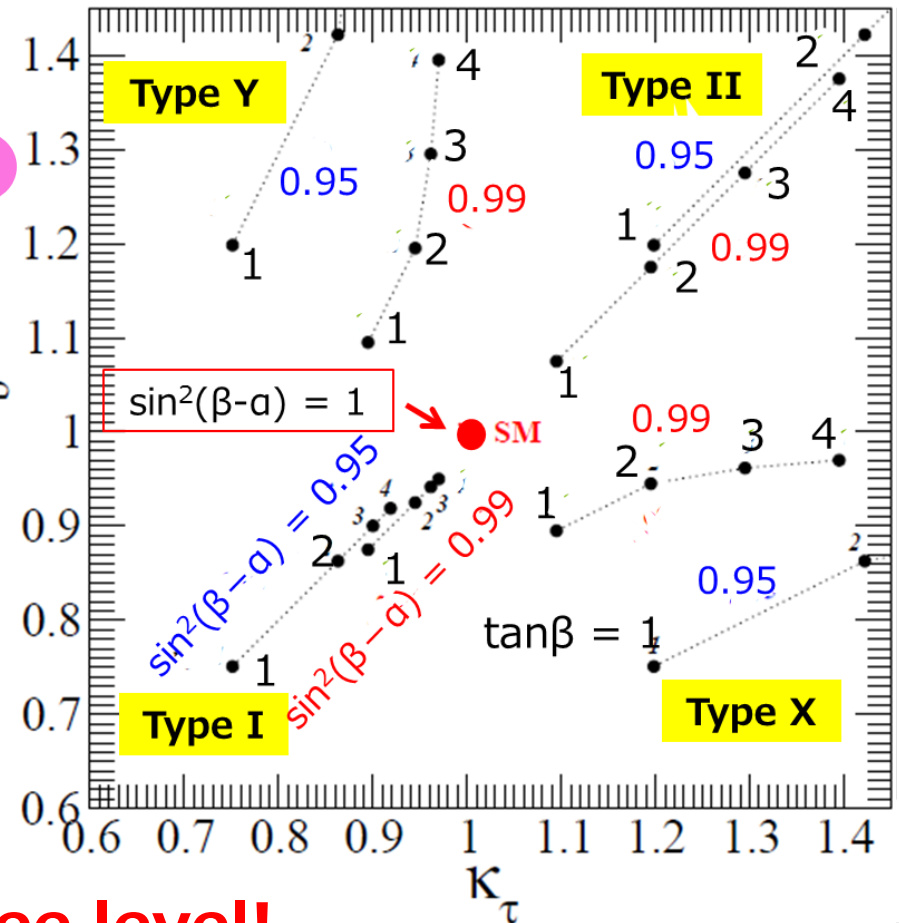
- Shape SM? 2HDM? Yukawa type then? Singlet? Triplet?
- Mass scale of the 2nd Higgs boson
- Decoupling property of extra Higgs bosons
Decoupling? or Non-decoupling?

Can we explore these questions by the study of radiative correction to h -couplings with future precision data?

Which Yukawa Type is it ? (tree)

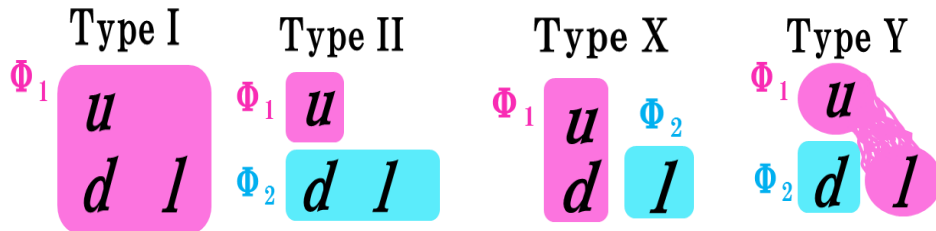


Even if only κ_V slightly differ from 1 (as large as 0.99 %), the type of Yukawa interactions can be discriminate by precision measurements at the ILC.



This is the conclusion at tree level!
What is going on with radiative correction?

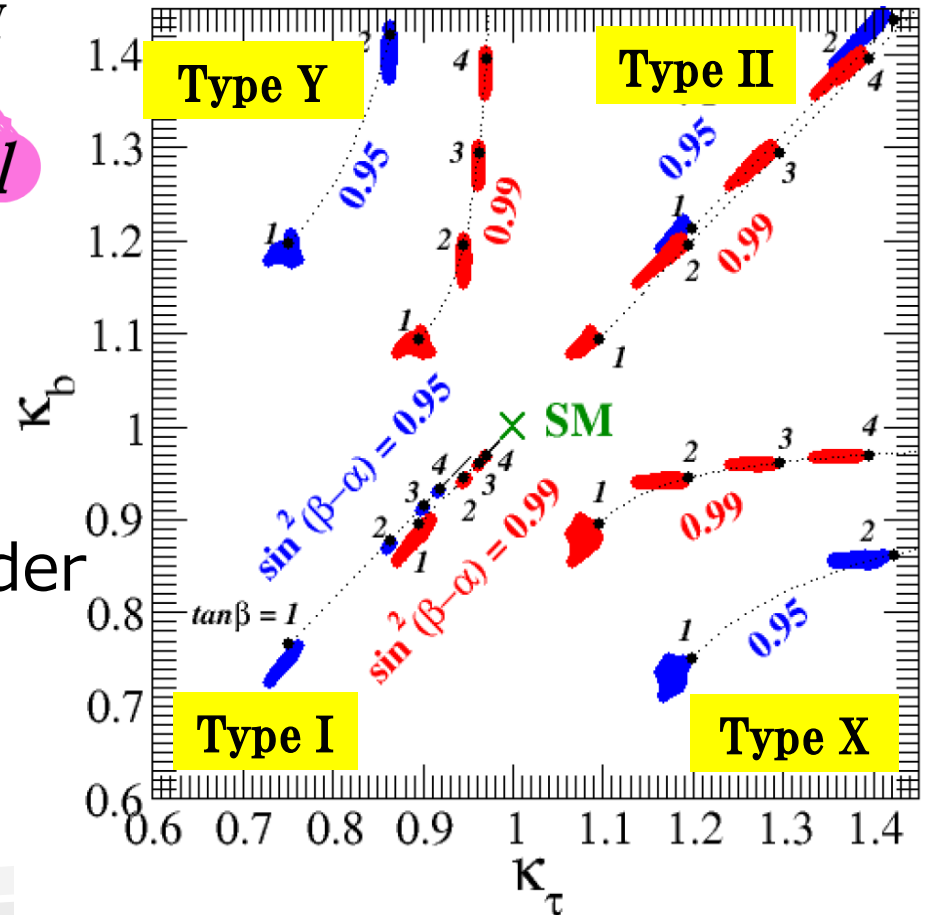
Which Yukawa Type is it ? (loop)



If f couples to Φ_1 $\kappa_f = \sin(\beta - \alpha) + \cot\beta \cos(\beta - \alpha)$
 If f couples to Φ_2 $\kappa_f = \sin(\beta - \alpha) - \tan\beta \cos(\beta - \alpha)$

Evaluation at one-loop

Scan of inner parameters under theoretical and experimental constraints (for each $\tan\beta$)



The separation of type can also be done at loop level!



Rest questions

- Decoupling or Non-decoupling ?
- Mass scale of 2nd Higgs bosons
- ...

Can we extract information for these issues
by using h -couplings?



Mixing or Loop effect?

$$\Delta \hat{\kappa}_X = \frac{\hat{g}_{hXX}^{2\text{HDM}} - \hat{g}_{hXX}^{\text{SM}}}{\hat{g}_{hXX}^{\text{SM}}}$$

Scale factor of
 hVV coupling
at 1-loop level

Deviation from unity

$$\Delta \hat{\kappa}_V \simeq -\frac{1}{2}x^2 - A(m_\Phi^2, M^2)$$

Mixing

Loop

Mixing parameter

$$x = \cos(\beta - \alpha) \quad \left[\sin(\beta - \alpha) = 1 - \frac{x^2}{2} \right]$$

SM-like
 $x \ll 1$

Loop Effect

$$A(m_\Phi, M) = \frac{1}{16\pi^2} \frac{1}{6} \sum_{\Phi} c_{\Phi} \frac{m_{\Phi}^2}{v^2} \left(1 - \frac{M^2}{m_{\Phi}^2} \right)^2 \quad (\Phi = H^\pm, A, H)$$

$$m_{\Phi}^2 \left(1 - \frac{M^2}{m_{\Phi}^2} \right)^2 \left\{ \begin{array}{ll} \propto \frac{1}{m_{\Phi}^2} & (M \gg v) \text{ Decoupling!} \\ \propto m_{\Phi}^2 & (M \sim v) \text{ Non-decoupling!} \end{array} \right.$$

Scale factors of loop-corrected Yukawa interaction

$$x \ll 1 \quad \sin(\beta - \alpha) = 1 - \frac{1}{2}x^2 \quad (\cos(\beta - \alpha) = x)$$

$$\Delta \hat{\kappa}_V \simeq -\frac{1}{2}x^2 - A(m_\Phi^2, M^2)$$

	ξ_u	ξ_d	ξ_ℓ
Type-I	$\cot \beta$	$\cot \beta$	$\cot \beta$
Type-II	$\cot \beta$	$-\tan \beta$	$-\tan \beta$
Type-X	$\cot \beta$	$\cot \beta$	$-\tan \beta$
Type-Y	$\cot \beta$	$-\tan \beta$	$\cot \beta$

$$\Delta \hat{\kappa}_\tau - \Delta \hat{\kappa}_V \simeq \xi_\ell x$$

$$\Delta \hat{\kappa}_c - \Delta \hat{\kappa}_V \simeq \xi_u x$$

$$\Delta \hat{\kappa}_b - \Delta \hat{\kappa}_V \simeq \xi_d x - \xi_u \xi_d F_b(m_{H^\pm}, M)$$

$$\Delta \hat{\kappa}_t - \Delta \hat{\kappa}_V \simeq \xi_u x - \xi_u^2 F_t(m_\Phi)$$



$$\Delta \hat{\kappa}_f - \Delta \hat{\kappa}_V$$

$$\Delta \hat{\kappa}_\tau - \Delta \hat{\kappa}_V \simeq \xi_l x,$$

$$\Delta \hat{\kappa}_c - \Delta \hat{\kappa}_V \simeq \xi_u x,$$

We assume that Yukawa type has been determined.

Even at one-loop, $\Delta \hat{\kappa}_{\tau,c} - \Delta \hat{\kappa}_V$ do not change the tree-level formulae, because main loop effects are canceled between $\Delta \hat{\kappa}_{\tau,c}$ and $\Delta \hat{\kappa}_V$.

1. Using the future data, $\xi_l x$ and $\xi_u x$ can be determined.

If it is Type I or II or X,
 $\xi_d x$ also can be determined
 because either
 $\xi_d x = \xi_u x$ or $\xi_d x = \xi_l x$

Ex) $\xi_d x = \xi_l x = -\tan \beta$ for Type II

	ξ_u	ξ_d	ξ_l
Type-I	$\cot \beta$	$\cot \beta$	$\cot \beta$
Type-II	$\cot \beta$	$-\tan \beta$	$-\tan \beta$
Type-X	$\cot \beta$	$\cot \beta$	$-\tan \beta$
Type-Y	$\cot \beta$	$-\tan \beta$	$\cot \beta$

Determination of mass scale etc

2. Once $(\xi_{u,d,l} x)$ are determined, we can extract information for x from $\Gamma(h \rightarrow gg)$

$$\Gamma[h \rightarrow gg] = \frac{\sqrt{2}G_F\alpha_s^2 m_h^3}{128\pi^3} \left| \left(1 + \xi_u x - \frac{1}{2}x^2 \right) I_t + \left(1 + \xi_d x - \frac{1}{2}x^2 \right) I_b \right|^2,$$

3. Then $\tan\beta$ and loop-effect A can be determined by x .

4. Loop effects F_b and F_t can also be determined!

$$\Delta\hat{\kappa}_b - \Delta\hat{\kappa}_V \simeq \xi_d x - \xi_u \xi_d F_b(m_{H^\pm}, M)$$

$$\Delta\hat{\kappa}_t - \Delta\hat{\kappa}_V \simeq \xi_u x - \xi_u^2 F_t(m_\Phi)$$

5. Finally m_ϕ and M can be extracted from A F_b F_t

Consistency can be tested using $h\gamma\gamma$ and hhh



How Accurately?

- For the determination of the Type of Yukawa, Tree-level study is enough.
- We here numerically study how accurately inner parameters can be determined
- Example: suppose that κ_i are measured like

$$\Delta\hat{\kappa}_V = -2.0 \pm 0.4\%$$

$$\Delta\hat{\kappa}_\tau = +18 \pm 1.9\%$$

$$\Delta\hat{\kappa}_b = +18 \pm 0.9\%$$

**Errors are from ILC(500)
in Snowmass 2014 Rep.**

**These data indicate that
it is **Type-II** in 2HDM**

**How we can know the
inner parameters?**

$x, m_\phi, \tan\beta, M$

Example 1

$$\Delta\hat{\kappa}_\tau - \Delta\hat{\kappa}_V \simeq -\tan\beta \times$$

$$\Delta\hat{\kappa}_V = -2.0 \pm 0.4\%$$

$$\Delta\hat{\kappa}_\tau = +18 \pm 1.9\%$$

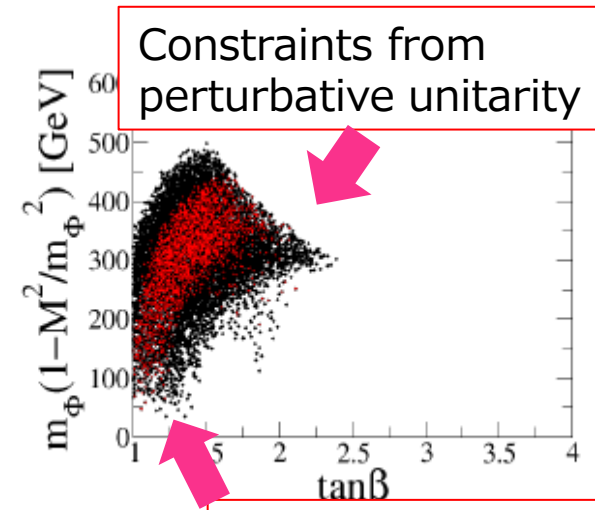
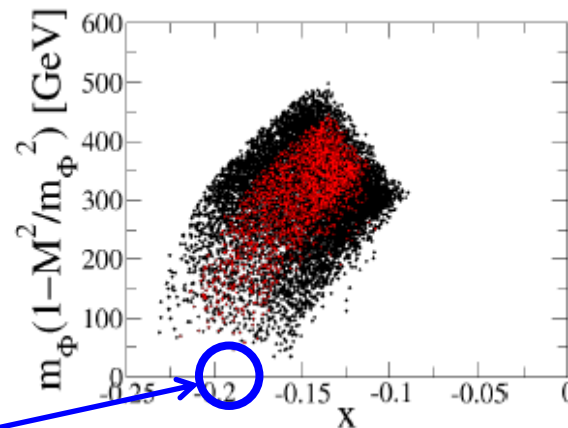
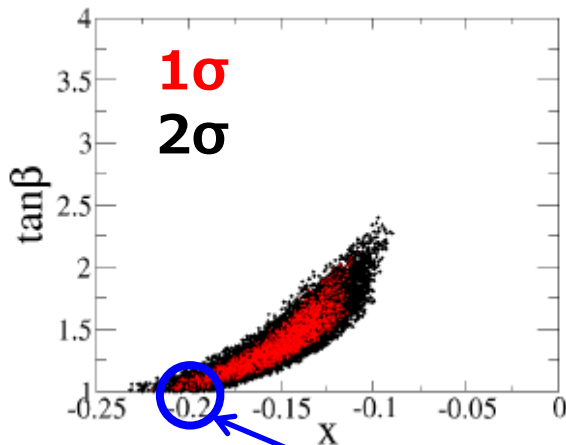
$$\Delta\hat{\kappa}_b = +18 \pm 0.9\%$$

Type-II

$$\Delta\hat{\kappa}_V \simeq -\frac{1}{2}x^2 - A(m_\Phi^2, M^2)$$

$$A(m_\Phi, M) = \frac{1}{16\pi^2} \frac{1}{6} \sum_\Phi c_\Phi \frac{1}{v^2} \left\{ m_\Phi \left(1 - \frac{M^2}{m_\Phi^2} \right) \right\}^2$$

($\Phi = H^\pm, A, H$)



Point by tree-level analysis

New mass scale can be extracted!

In addition to the type, tree parameters x and $\tan\beta$!!

Example 2

$$\begin{aligned}\Delta\hat{\kappa}_V &= -2.0 \pm 0.4\% \\ \Delta\hat{\kappa}_\tau &= +10 \pm 1.9\% \\ \Delta\hat{\kappa}_b &= +10 \pm 0.9\%\end{aligned}$$

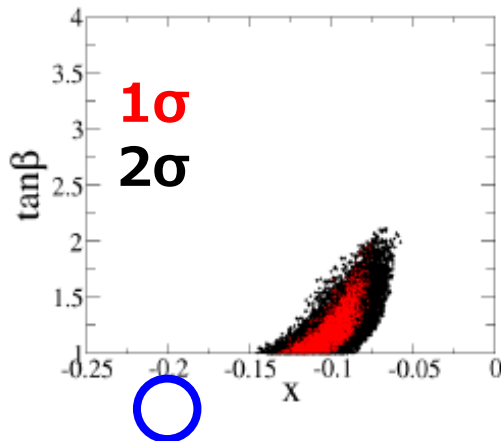
Type-II

$$\Delta\hat{\kappa}_\tau - \Delta\hat{\kappa}_V \simeq -\tan\beta \times$$

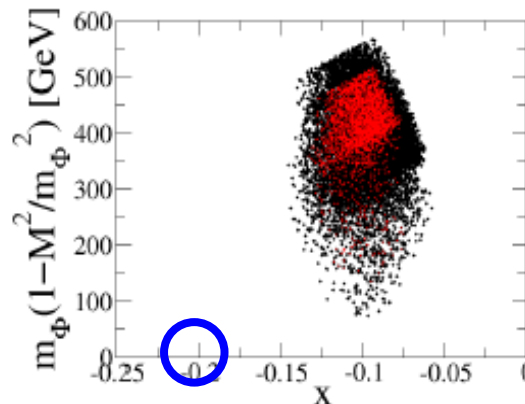
$$\Delta\hat{\kappa}_V \simeq -\frac{1}{2}x^2 - A(m_\Phi^2, M^2)$$

$$A(m_\Phi, M) = \frac{1}{16\pi^2} \frac{1}{6} \sum_\Phi c_\Phi \frac{1}{v^2} \left\{ m_\Phi \left(1 - \frac{M^2}{m_\Phi^2} \right) \right\}^2$$

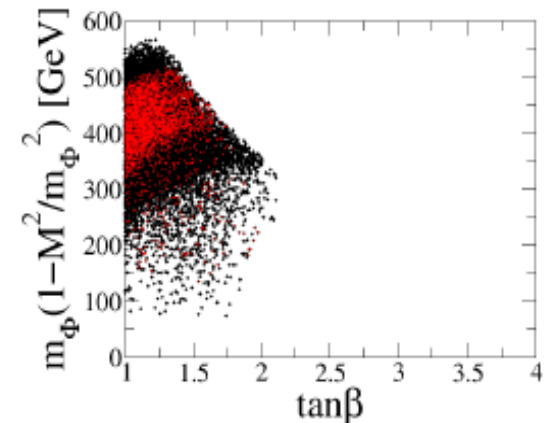
($\Phi = H^\pm, A, H$)



Point by tree-level analysis



Larger non-decoupling effect is extracted!





Summary

- We calculate a full set of **one-loop corrected h -couplings** in on-shell scheme in 2HDM
- We discussed how extended Higgs can be indirectly explored by precisely measuring h -couplings

Shape, mass scale, decoupling/non-decoupling

- If κ_V turns out to be slightly less than unity, the inner parameters **x , $\tan\beta$, m_ϕ** and **M** can be determined to the considerable extent from the study of h -couplings
- In conclusion, **h -couplings will be definitely a good probe of Higgs sector**



Renormalization

➤ Kinetic term

- Parameters in Lagrangian $\cdots g, g', v$
- Physical parameters $\cdots m_W, m_Z, \sin\theta_W, G_F, a_{em}$
- Counter-terms $\cdots \delta m_W, \delta m_Z, \delta s_W, \delta G_F, \delta a_{em}, \cdots$
- Renormalized conditions $\cdots$

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} = 1$$

$$\sin^2 \theta_W = 1 - \frac{m_W^2}{m_Z^2},$$

$$G_F = \frac{\pi \alpha_{em}}{\sqrt{2} m_W^2 \sin^2 \theta_W}$$

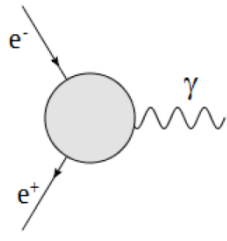
$$Re\Pi_{WW}(p^2)|_{p^2=m_W^2} = 0,$$

$$\delta m_W^2 = Re\Pi_{WW}^{1PI}(m_W^2),$$

$$Re\Pi_{ZZ}(p^2)|_{p^2=m_Z^2} = 0,$$

$$\delta m_Z^2 = Re\Pi_{ZZ}^{1PI}(m_Z^2),$$

On-shell conditions



$$= -ie\gamma^\mu$$

$$\frac{\delta \alpha_{em}}{\alpha_{em}} = \frac{d}{dp^2} \Pi_{\gamma\gamma}^{1PI}(p^2)|_{p^2=0} - \frac{2s_W}{c_W} \frac{\Pi_{\gamma Z}^{1PI}(0)}{m_Z^2}$$

- Counter term of v

$$v^2 = \frac{m_W^2 \sin^2 \theta_W}{\pi a_{em}}$$



$$\frac{\delta v}{v} = \frac{1}{2} \left(\frac{\delta m_W^2}{m_W^2} - \frac{\delta \alpha_{em}}{\alpha_{em}} + \frac{\delta s_W^2}{s_W^2} \right)$$

Approximately formulae

$x \ll 1$

$$\Delta\hat{\kappa}_V \simeq -\frac{1}{2}x^2 - \frac{1}{16\pi^2} \frac{1}{6} \sum_{\Phi=A,H,H^\pm} c_\Phi \frac{m_\Phi^2}{v^2} \left(1 - \frac{M^2}{m_\Phi^2}\right)^2$$

$$\Delta\hat{\kappa}_\tau \simeq \Delta\hat{\kappa}_V + \xi_\ell x,$$

$$\Delta\hat{\kappa}_c \simeq \Delta\hat{\kappa}_V + \xi_u x,$$

$$\Delta\hat{\kappa}_b \simeq \Delta\hat{\kappa}_V + \xi_d x - \frac{1}{16\pi^2} \xi_u \xi_d \frac{2m_t^2}{v^2} \left(1 - \frac{m_t^2}{m_{H^\pm}^2} - \frac{M^2}{m_{H^\pm}^2}\right) - \frac{1}{16\pi^2} \frac{1}{6} \xi_d^2 \sum_{\Phi=A,H,H^\pm} \frac{m_b^4}{v^2 m_\Phi^2},$$

$$\Delta\hat{\kappa}_t \simeq \Delta\hat{\kappa}_V + \xi_u x - \frac{1}{16\pi^2} \frac{1}{6} \left[\xi_u^2 \sum_{\Phi=A,H,H^\pm} \frac{m_t^4}{v^2 m_\Phi^2} + \xi_d^2 \frac{m_b^2 m_t^2}{v^2 m_{H^\pm}^2} \right]$$

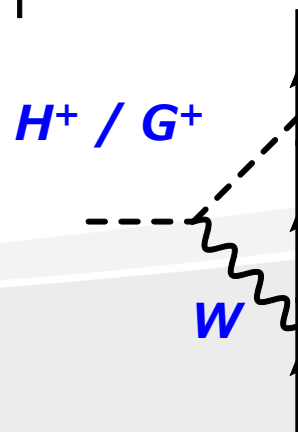
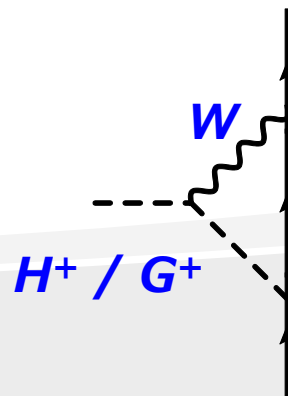
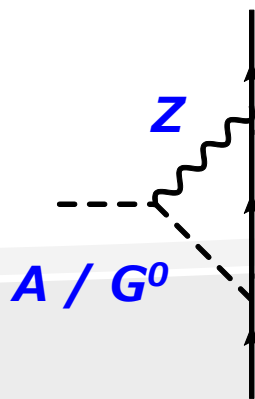
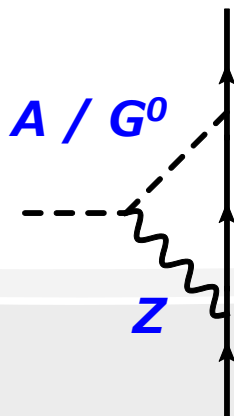
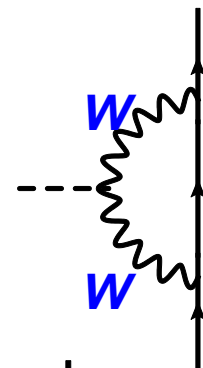
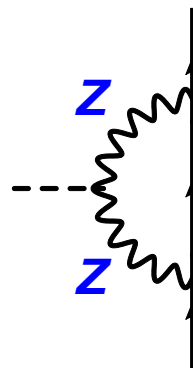
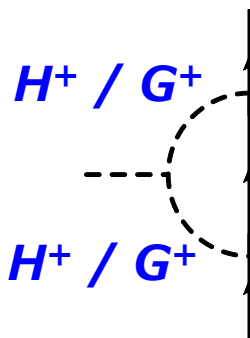
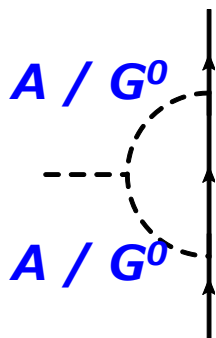
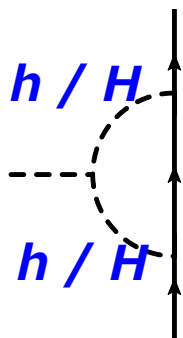
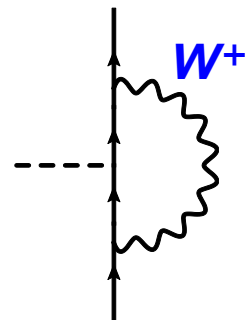
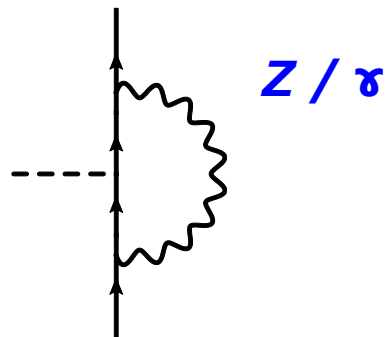
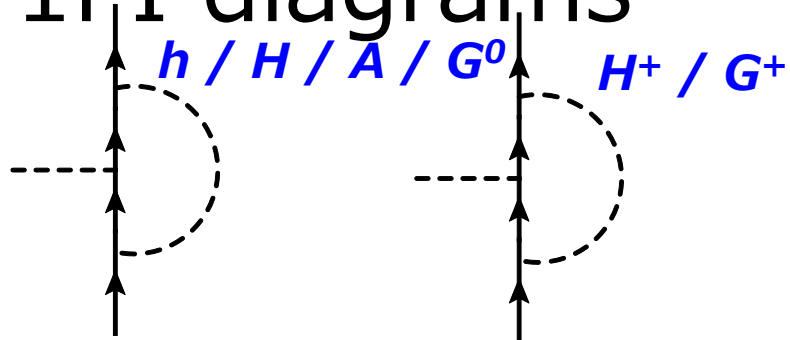
$$\Delta\hat{\kappa}_h \simeq \left(\frac{3}{2} - \frac{2M^2}{m_h^2}\right) x^2 + \frac{1}{16\pi^2} \sum_{\Phi=A,H,H^\pm} c_\Phi \frac{4}{3} \frac{m_\Phi^4}{m_h^2 v^2} \left(1 - \frac{M^2}{m_\Phi^2}\right)^3.$$

$$\Gamma(h \rightarrow \gamma\gamma) \simeq \frac{G_F \alpha_{\text{em}}^2 m_h^3}{128\sqrt{2}\pi^3} \left| -\frac{1}{3} \left(1 - \frac{M^2}{m_{H^\pm}^2}\right) + Q_t N_c \left(1 + \xi_u x - \frac{x^2}{2}\right) I_t + Q_b N_c \left(1 + \xi_d x - \frac{x^2}{2}\right) I_b + \left(1 - \frac{x^2}{2}\right) I_W \right|^2.$$

$$\Gamma(h \rightarrow gg) \simeq \frac{G_F \alpha_s^2 m_h^3}{64\sqrt{2}\pi^3} \left| \left(1 + \xi_u x - \frac{x^2}{2}\right) I_t + \left(1 + \xi_d x - \frac{x^2}{2}\right) I_b \right|^2$$



1PI diagrams





Example Type II

$$\Delta\hat{\kappa}_V = -2.0 \pm 0.4\%$$

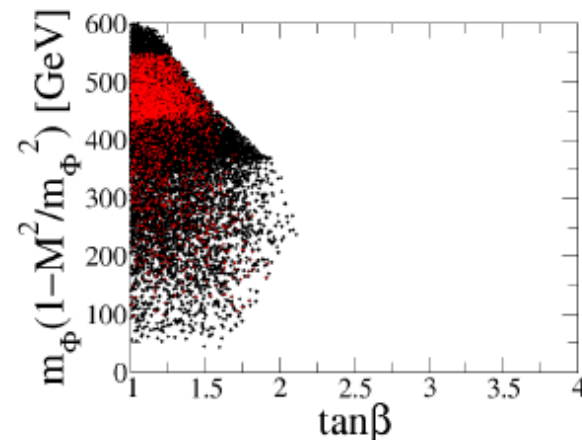
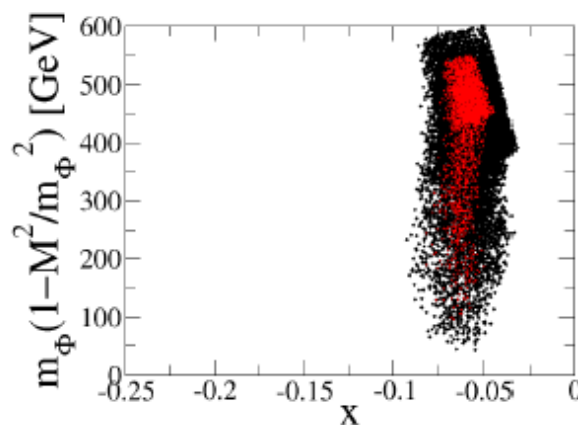
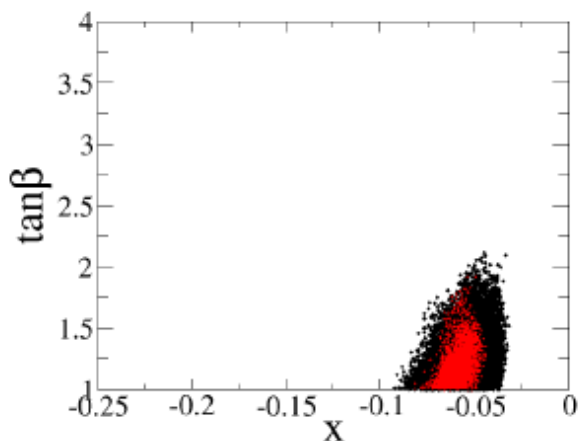
$$\Delta\hat{\kappa}_\tau = +5 \pm 1.9\%$$

$$\Delta\hat{\kappa}_b = +5 \pm 0.9\%$$

$$\Delta\hat{\kappa}_\tau - \Delta\hat{\kappa}_V \simeq -\tan\beta \times$$

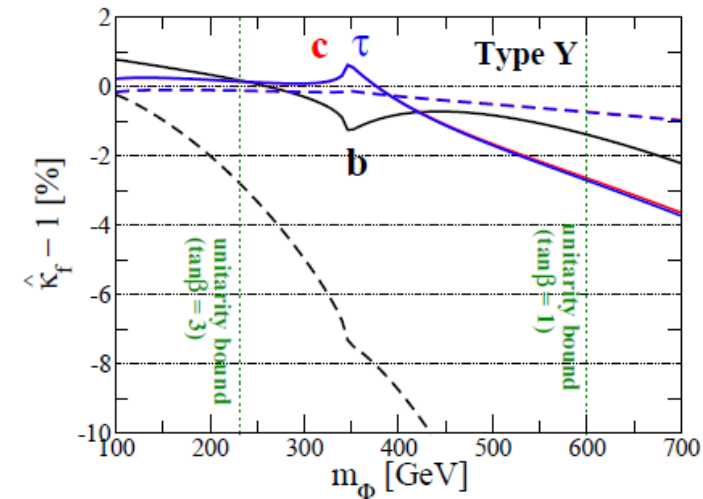
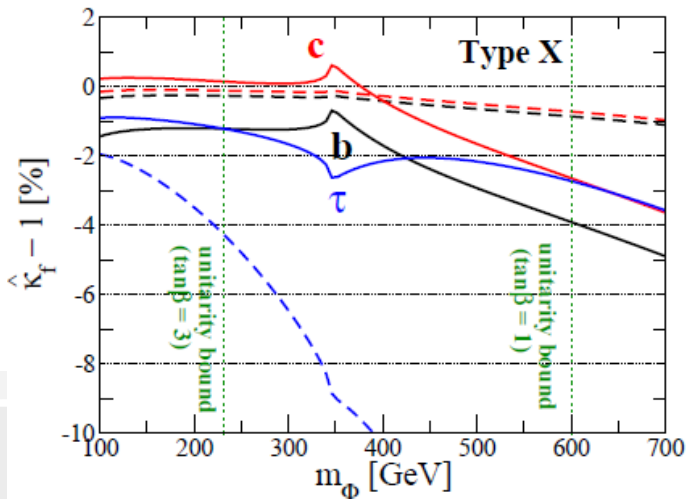
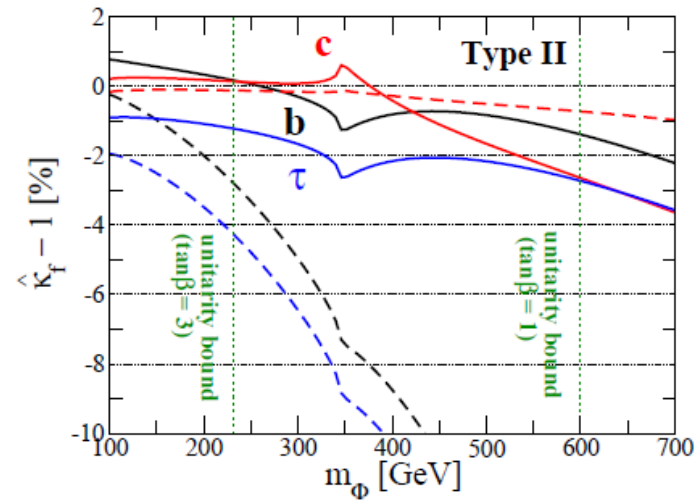
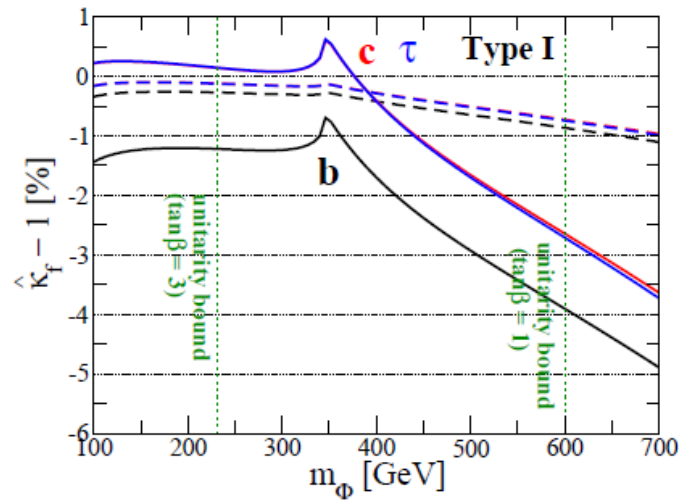
$$\Delta\hat{\kappa}_V \simeq -\frac{1}{2}x^2 - A(m_\Phi^2, M^2)$$

$$A(m_\Phi^2, M^2) = -\frac{1}{16\pi^2} \frac{1}{6} \sum_{\Phi=H^+, m_A, m_H} c_\Phi \frac{1}{v^2} \left\{ m_\Phi \left(1 - \frac{M^2}{m_\Phi^2} \right) \right\}^2$$



Deviations in hff

$M = 0$

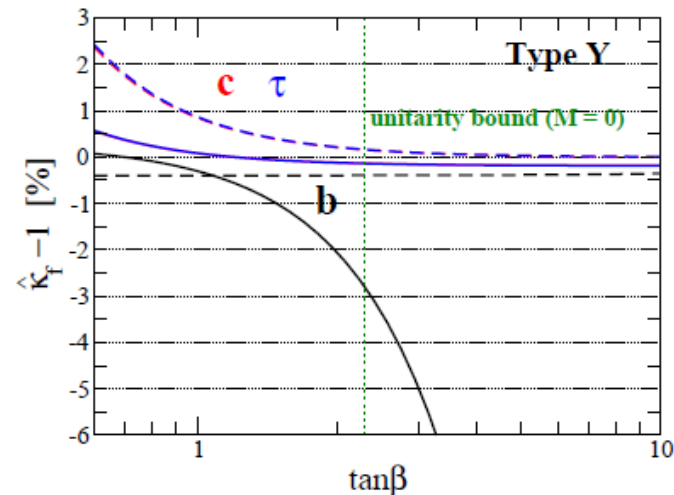
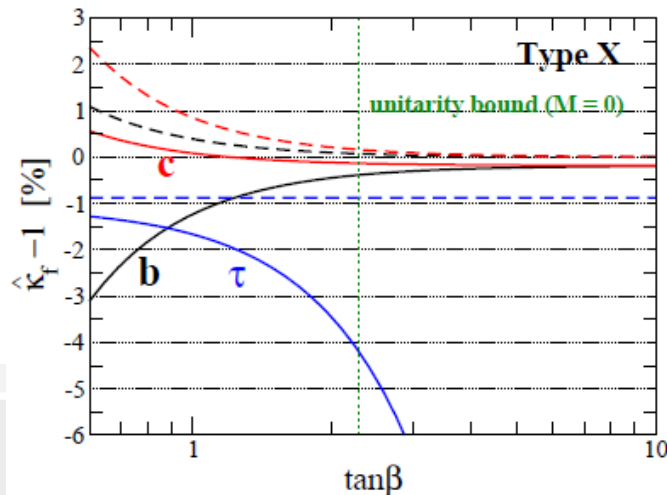
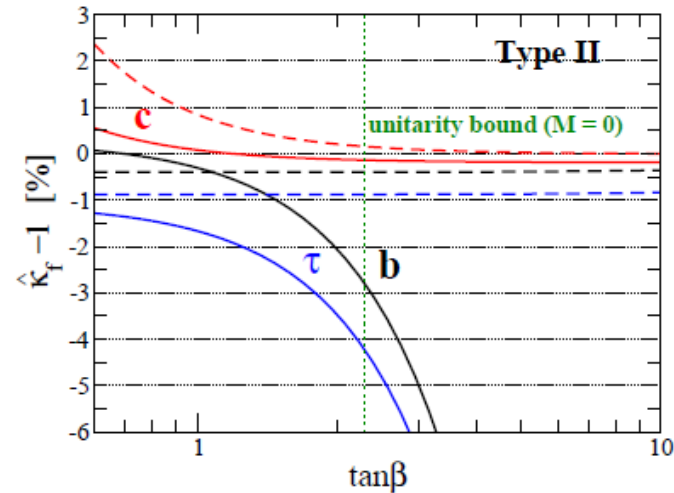
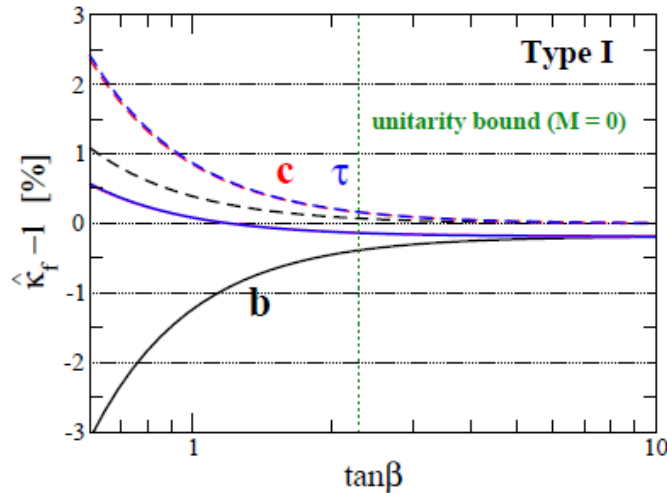


$$\sin^2(\beta - \alpha) = 1$$

— $\tan\beta=1$
 --- $\tan\beta=3$

Deviations in hff

$$m_\phi = 300 \text{ GeV}, M = 0$$



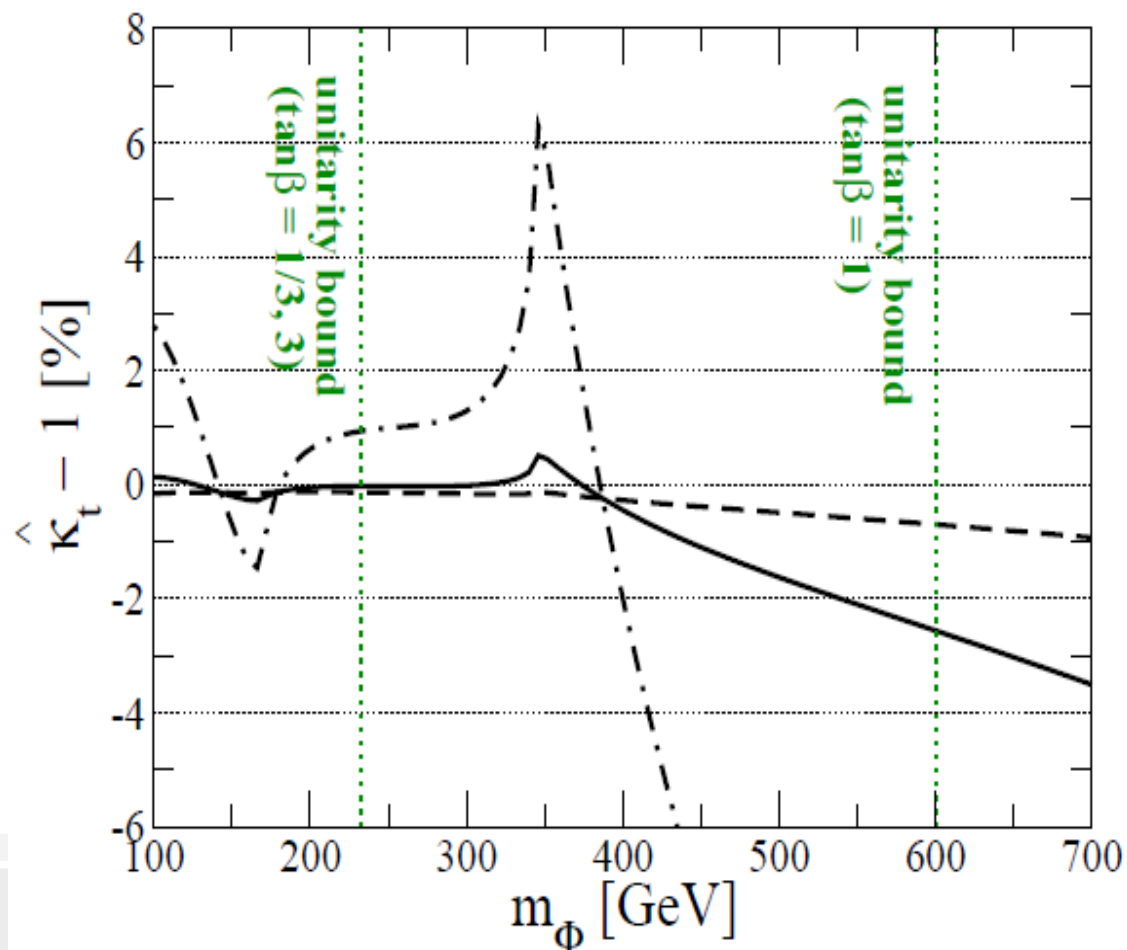
Deviations in htt

$$\sin^2(\beta - \alpha) = 1$$

- $\tan\beta=1$
- $\tan\beta=3$

$$M = 0$$

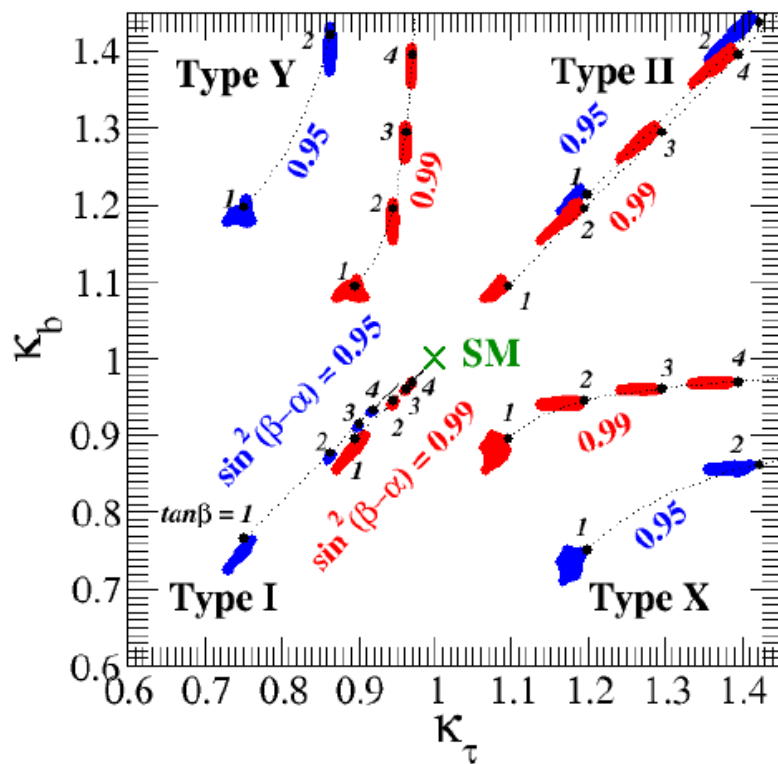
Kanemura, Kikuchi, Yagyu(2013)



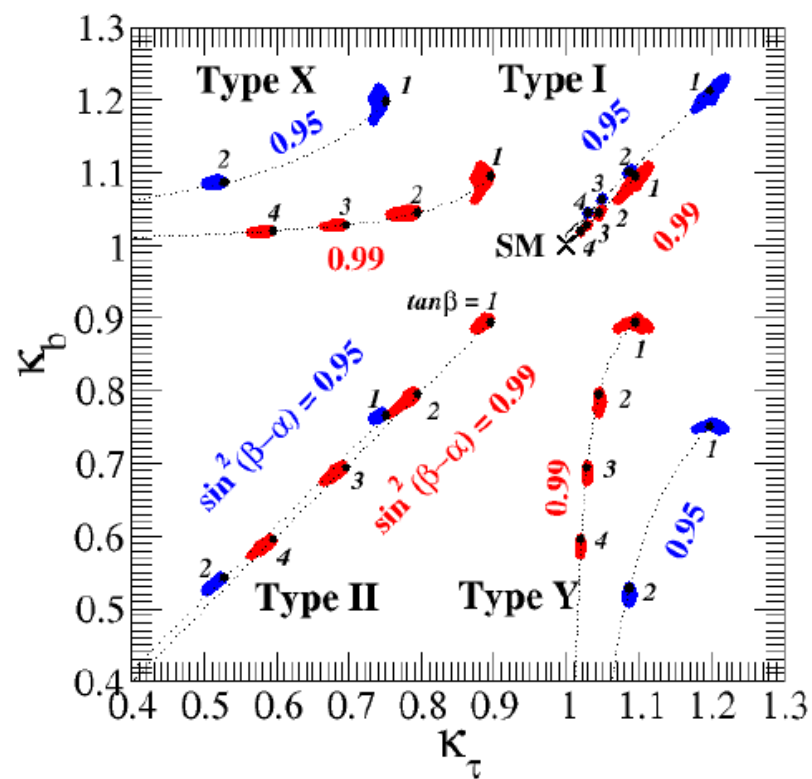


κ_b VS κ_τ

$\cos(\beta-\alpha) < 0$

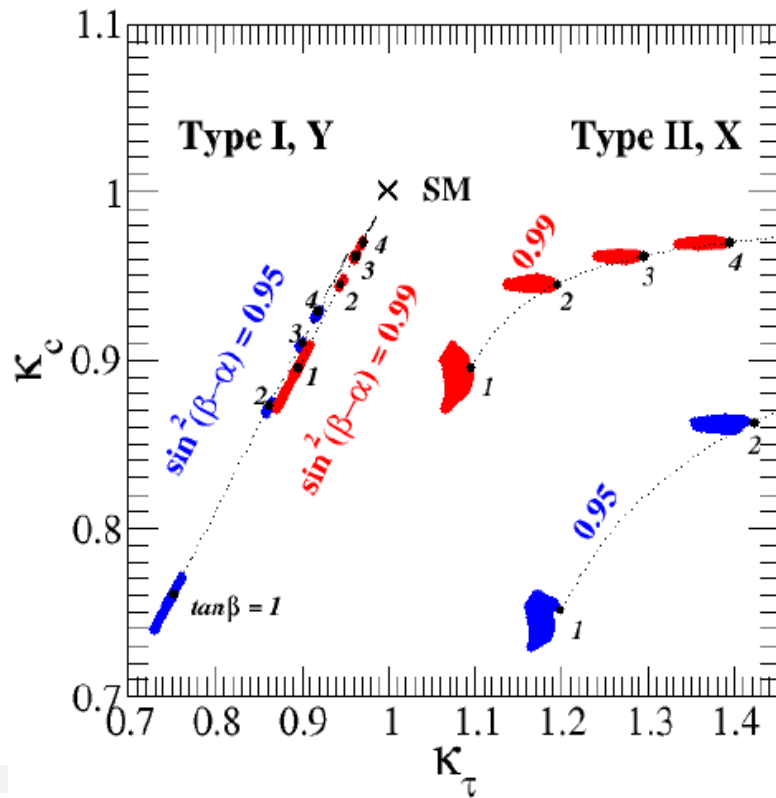


$\cos(\beta-\alpha) > 0$

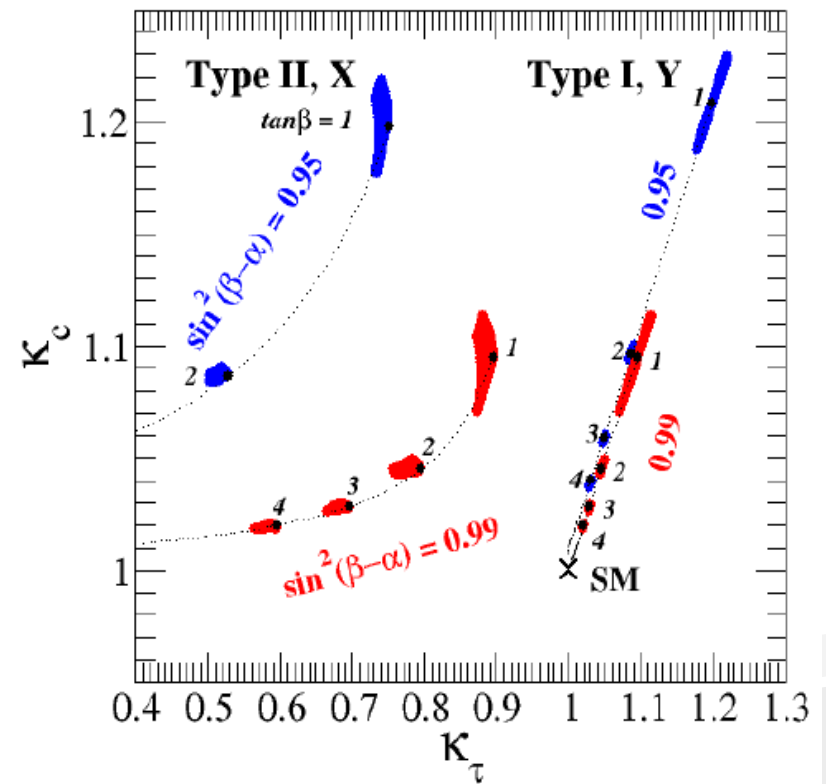


κ_τ VS κ_c

$\cos(\beta-\alpha) < 0$

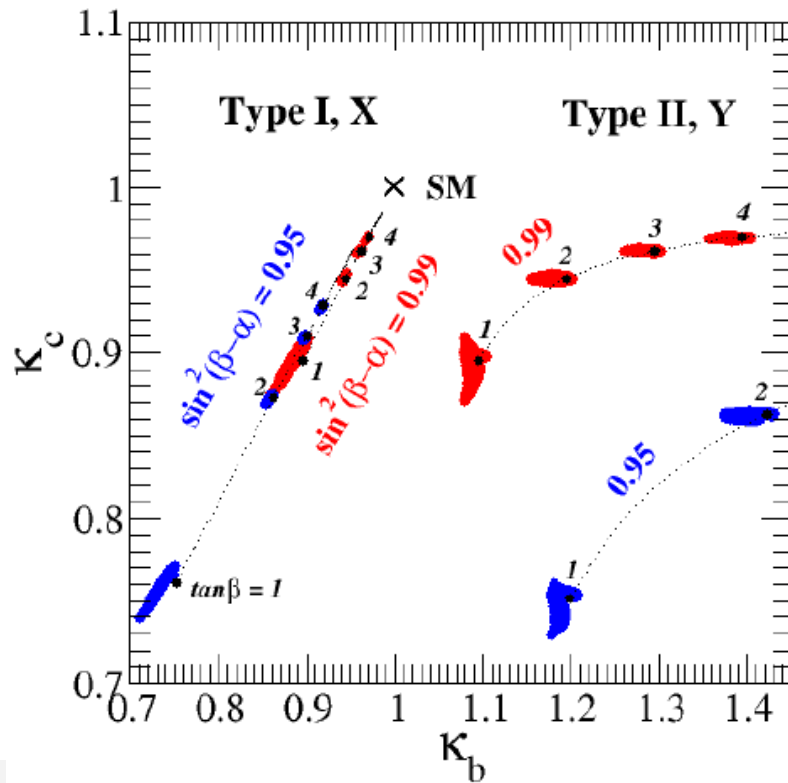


$\cos(\beta-\alpha) > 0$

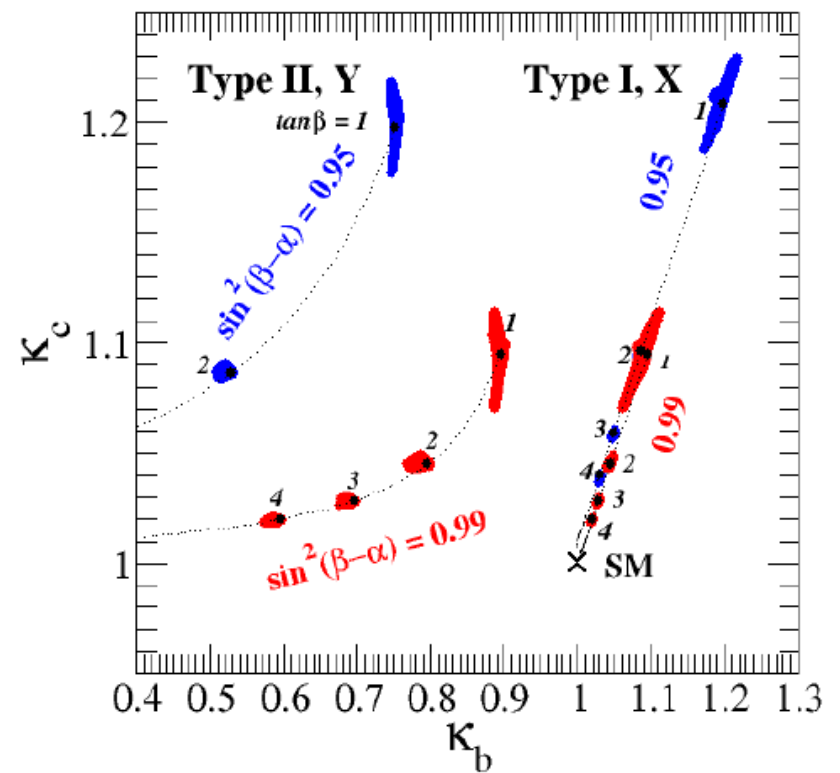


κ_b VS κ_c

$\cos(\beta-\alpha) < 0$



$\cos(\beta-\alpha) > 0$







Radiative corrections

- Decoupling Theorem

Effects of heavy particle appear as inverse powers of mass.

$$\sim \frac{1}{m^2} \rightarrow 0$$

- Exceptional Case (Non-decoupling effects)

- Loop effects of chiral fermion

$$m_f \sim y_f v \quad \Delta\rho \sim c \frac{N_c}{(4\pi)^2} \frac{m_t^2}{v^2} + \dots$$

- Loop effects of scalar bosons

$$m_\phi^2 \sim \lambda_i v^2 + M^2 \quad (M^2 \ll m_\phi^2)$$

$$\Delta\rho \sim c' \frac{1}{(4\pi)^2} \frac{(m_A - m_{H^\pm})^2}{v^2} +$$

...

Decoupling property of extended Higgs sector

Loop corrections of extended Higgs models can be both

- decoupling
- non-decoupling

Ex) Triple Coupling hhh in 2HDM

$$\Delta\lambda_{hhh} \simeq -\frac{N_c}{16\pi^2} \frac{m_t^4}{m_h^2 v^2} + \frac{1}{16\pi^2} \frac{m_\Phi^4}{m_h^2 v^2} \left(1 - \frac{M^2}{m_\Phi^2}\right)^4$$

Kanemura, Okada, Senaha, Yuan (2004)

$$m_\Phi^2 \sim \lambda_i v^2 + M^2$$

- If $M \gg v$

$$m_\Phi^4 \left(1 - \frac{M^2}{m_\Phi^2}\right)^3 = m_\Phi^4 \left(\frac{\lambda_i v^2}{m_\Phi^2}\right)^3 \rightarrow \frac{1}{m_\Phi^2} \quad \text{Decoupling!}$$

- If $M \sim v$

$$m_\Phi^4 \left(1 - \frac{M^2}{m_\Phi^2}\right)^3 \rightarrow m_\Phi^4$$

Non-decoupling!

Ex. EW Baryogenesis

Ex. SUSY

Higgs coupling measurements

h-couplings can be measured with high precision at future collider experiments!!

Facility	LHC	HL-LHC	ILC500	ILC500-up	ILC1000	ILC1000-up	CLIC	TLEP (4 IPs)
$\sqrt{s}$ (GeV)	14,000	14,000	250/500	250/500	250/500/1000	250/500/1000	350/1400/3000	240/350
$\int \mathcal{L} dt$ (fb ⁻¹)	300/expt	3000/expt	250+500	1150+1600	250+500+1000	1150+1600+2500	500+1500+2000	10,000+2600
κ_γ	5 – 7%	2 – 5%	8.3%	4.4%	3.8%	2.3%	–/5.5/<5.5%	1.45%
κ_g	6 – 8%	3 – 5%	2.0%	1.1%	1.1%	0.67%	3.6/0.79/0.56%	0.79%
κ_W	4 – 6%	2 – 5%	0.39%	0.21%	0.21%	0.2%	1.5/0.15/0.11%	0.10%
κ_Z	4 – 6%	2 – 4%	0.49%	0.24%	0.50%	0.3%	0.49/0.33/0.24%	0.05%
κ_ℓ	6 – 8%	2 – 5%	1.9%	0.98%	1.3%	0.72%	3.5/1.4/<1.3%	0.51%
$\kappa_d = \kappa_b$	10 – 13%	4 – 7%	0.93%	0.60%	0.51%	0.4%	1.7/0.32/0.19%	0.39%
$\kappa_u = \kappa_t$	14 – 15%	7 – 10%	2.5%	1.3%	1.3%	0.9%	3.1/1.0/0.7%	0.69%

Higgs Working Group Report (2014)

To compare with future precision measurements, it is essentially important to evaluate loop contributions.

Formula of renormalized couplings

$$\Gamma_{hVV}^{\text{reno}}[p_1^2, p_2^2, q^2] = \frac{2m_V^2}{v^2} s_{\beta-\alpha} \left(1 + \frac{\delta m_V^2}{m_V^2} - \frac{\delta v}{v} + \delta Z_V + \frac{1}{2} \delta Z_h + \frac{c_{\beta-\alpha}}{s_{\beta-\alpha}} (\delta\beta + \delta C_{hH}) \right) + \Gamma_{hVV}^{\text{1PI}}[p_1^2, p_2^2, q^2],$$

$$\Gamma_{hff}^{\text{reno}}[p_1^2, p_2^2, q^2] = -\frac{m_f}{v} \xi_f^h \left(1 + \frac{\delta m_f}{m_f} - \frac{\delta v}{v} + \frac{\xi_f^H}{\xi_f^h} \delta C_{Hh} - \xi_f^A \delta\beta + \frac{1}{2} \delta Z_h + \delta Z_f \right) + \Gamma_{hff}^{\text{1PI},S}[p_1^2, p_2^2, q^2],$$