Scalar Dark Matter Candidates in Two Inert Higgs Doublet Model

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Based on arxiv:1407.4749 and arxiv:1205.0995

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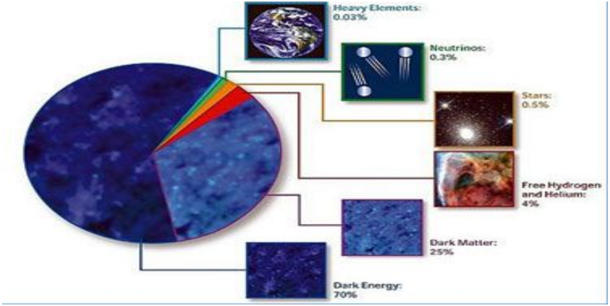
Outline

The Universe Content



http://www.conservapedia.com

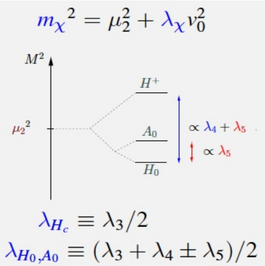
$$\Omega h^2 = 0.112 \pm 0.0009.$$

The IDM

SM + one $SU(2)_L$ n-uplet
Impose Z_2 parity: SM particles plus an extra Higgs

$$H_1 = H_{SM} \quad ; \quad H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(h + iA) \end{pmatrix} \tag{1}$$

Both h or A may be dark matter candidates.



from Laura Lopez seminar

The IDM

L. Lopez Honorez, E. Nezri , J.Oliver(JCAP 0702:028,2007)
S.Andreas, Th.Hambye(JCAP 0810:034,2008)
S.Andreas, Q.Swillens (JCAP0904:004,2009)
E. Nezri, G.Vertongen (JCAP 0904:014,2009)
Ch.Arina, F.S. Ling (JCAP 0910:018,2009)
M. Krawczyk, D. Sokolowska, B. Swiezewska (J.Phys.Conf.Ser. 447 (2013) 012050)
and go on...

IDM history

It is well known that in the one inert doublet model there exists a set of allowed parameters in which we have a dark matter candidate:

- $\lesssim$ 10 GeV;
- 40-150 GeV;
- $\gtrsim$ 500 GeV.

DM and IDM

WMAP

$$\Omega h^2 = 0.112 \pm 0.0009.$$

Question

Why are there three families of quarks and leptons?

proposal

Why not three families of scalars?

V. Keus at all in arXiv:1407.7859

The Model

Matter Content

- The SM particles *plus* H_2 and H_3.

The Model Symmetry

$$SU(3)_c \times SU(2)_L \times U(1)_Y \times S_3$$

The Fields Transformation

SM particles are Singlets under S_3 and H_2 and H_3 form a doublet of S_3

$$S = H_1 \sim \mathbf{1}, \quad D = (H_2, H_3) \sim \mathbf{2}. \tag{2}$$

The Yukawa interactions

If in the lepton and quark sectors all fields are transformed as singlet under S_3, they only interact with the singlet S as following:

The Yukawa

$$-\mathcal{L}_{yukawa} = \bar{L}_{iL}(G^{l}_{ij} l_{jR} S + G^{\nu}_{ij} \nu_{jR} \tilde{S}) + \bar{Q}_{iL}(G^{u}_{ij} u_{jR} \tilde{S} + G^{d}_{ij} d_{jR} S) + H.c., \quad (3)$$

$\tilde{S} = i\tau_2 S^*$ and we have included right-handed neutrinos.

Three Higgs-scalar doublet models

H_i with $i = 1, 2, 3$ are $SU(2)$ doublets having $Y = +1$.

The most general scalar potential invariant under $SU(2) \otimes U(1)_Y \otimes S_3$

The Potencial

$$\begin{aligned} V(D,S) &= \mu_s^2 S^\dagger S + \mu_d^2 [D^\dagger \otimes D]_1 + \lambda_1 ([D^\dagger \otimes D]_1)^2 \\ &+ \lambda_2 [(D^\dagger \otimes D)_{1'} (D^\dagger \otimes D)_{1'}]_1 + \lambda_3 [(D^\dagger \otimes D)_2 (D^\dagger \otimes D)_2]_1 \\ &+ \lambda_4 (S^\dagger S)^2 + \lambda_5 [D^\dagger \otimes D]_1 S^\dagger S + \lambda_6 [[S^\dagger D]_2 [S^\dagger D]_2]_1 \\ &+ \lambda_7 S^\dagger [D \otimes D^\dagger]_1 S + \lambda_8 [(S^\dagger \otimes D)_2 (D^\dagger \otimes D)_2]_1 + H.c. \quad (4) \end{aligned}$$

With $D \equiv \mathbf{2} = (x_1, x_2)$, we have $\mathbf{2} \otimes \mathbf{2} = \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{2}'$ in wich
$\mathbf{1} = x_1 y_1 + x_2 y_2$, $\mathbf{1}' = x_1 y_2 - x_2 y_1$,
$\mathbf{2}' = (x_1 y_2 + x_2 y_1, x_1 y_1 - x_2 y_2)$, and $\mathbf{1}' \otimes \mathbf{1}' = \mathbf{1}$

The Model

H_i forms a triplet under S_3 $\mathbf{3} = (H_1, H_2, H_3)$ since it is a reducible representation it is broken down to:

Case A

$$S = \frac{1}{\sqrt{3}}(H_1 + H_2 + H_3) \sim \mathbf{1},$$

$$D \equiv (D_1, D_2) = \left[\frac{1}{\sqrt{6}}(2H_1 - H_2 - H_3), \frac{1}{\sqrt{2}}(H_2 - H_3)\right] \sim \mathbf{2}(5)$$

Choosing:

Case A

$$v_1 = v_2 = v_3 = \frac{v_{SM}}{\sqrt{3}}$$

The Model: After the symmetry breaking

CP-even neutral real scalars

$$m_{h_1}^2 \equiv m_h^2 = 2\lambda_4 v_{SM}^2, \qquad m_{h_2}^2 = m_{h_3}^2 \equiv m_H^2 = \mu_d^2 + \frac{1}{2}\bar{\lambda}' v_{SM}^2, \tag{6}$$

$$\bar{\lambda}' = (\lambda_5 + \lambda_6 + 2\lambda_7)$$

CP-add neutral real scalars

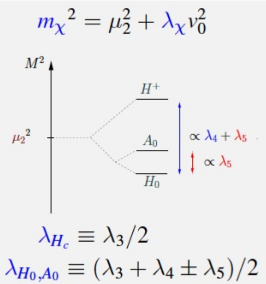
$$m_{A_1}^2 = 0, \qquad m_{A_2}^2 = m_{A_3}^2 \equiv m_A^2 = \mu_d^2 + \frac{1}{2}\bar{\lambda}'' v_{SM}^2. \tag{7}$$

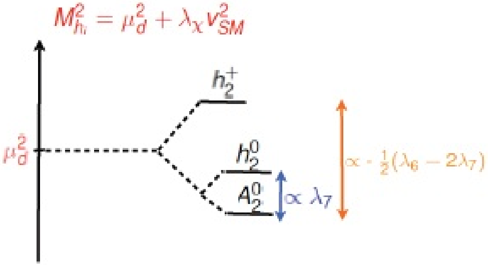
$$\bar{\lambda}'' = (\lambda_5 + \lambda_6 - 2\lambda_7)$$

Charged scalars

$$m_{c_1}^2 = 0, \qquad m_{c_2}^2 = m_{c_3}^2 \equiv m_c^2 = \mu_d^2 + \frac{1}{2}\lambda_5 v_{SM}^2 \tag{8}$$

The Model: After the symmetry breaking





Because of the degeneracy we have a IDM replica

To break the symmetry we can add soft terms, like $\nu_i^2 H_i^\dagger H_j$

The Fields mass eigenstates and The scalar potencial as a function of it:

$$S = \begin{pmatrix} h_1^+ \\ \frac{1}{\sqrt{2}}(v_{SM} + h_1^0 + iA_1^0) \end{pmatrix}, \quad (9)$$

$$D = -\left[\begin{pmatrix} h_2^+ \\ \frac{1}{\sqrt{2}}(h_2^0 + iA_2^0) \end{pmatrix}, \begin{pmatrix} h_3^+ \\ \frac{1}{\sqrt{2}}(h_3^0 + iA_3^0) \end{pmatrix}\right]$$

The potential in terms of the mass eigenstates given above:

$$\begin{aligned} V(h_i) &= 3\lambda_4 v^2 h_1^\dagger h_1 + \mu_d^2(h_2^\dagger h_2 + h_3^\dagger h_3) + \lambda_1(h_2^\dagger h_2 + h_3^\dagger h_3)^2 \\ &+ \lambda_2(h_2^\dagger h_3 - h_3^\dagger h_2)^2 + \lambda_3[(h_2^\dagger h_3 + h_3^\dagger h_2)^2 + (h_2^\dagger h_2 - h_3^\dagger h_3)^2] \\ &+ \lambda_4(h_1^\dagger h_1)^2 + \lambda_5 h_1^\dagger h_1(h_2^\dagger h_2 + h_3^\dagger h_3) + \lambda_6[|h_1^\dagger h_2|^2 + |h_1^\dagger h_3|^2] \\ &+ \{\lambda_7[(h_1^\dagger h_2)^2 + (h_3^\dagger h_1)^2] + \lambda_8[h_1^\dagger h_2(h_2^\dagger h_3 + h_3^\dagger h_2) \\ &+ h_1^\dagger h_3(h_3^\dagger h_3 - h_2^\dagger h_2)] + H.c.\}. \quad (10) \end{aligned}$$

A change of basis

Case B

$$S = H_1 \sim \mathbf{1}, \quad D = (H_2, H_3) \sim \mathbf{2}$$

Vacuum Alignment

$$v_1 = v_{SM} \text{ and } v_2 = v_3 = 0$$

The two cases are related by a change of basis, and the matrix that changes the basis is the well known Tribimaximal one:

$$\begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix} \tag{11}$$

A new proposal

From linear Algebra we know that every quadratic form $ax^2 + cy^2 + 2bxy$ has a stationary point at the origin, and with $ax^2 + cy^2 + 2bxy > 0 \forall (x, y) \neq (0, 0)$ it has a minimum. Rewriting this equation we have:

$$ax^2 + cy^2 + 2bxy = a\left[x + \frac{b}{a}y\right]^2 + y^2\left[c - \frac{b^2}{a}\right]^2 \tag{12}$$

therefore, for this equation be positive we must have:

$$a > 0 \;\; ; \;\; ac > b^2, \tag{13}$$

A new proposal

We can also obtain this conditions in a matrix way, we can writ:

$$ax^2 + cy^2 + 2bxy = \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} a & b \\ b & c \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad (14)$$

Setting $V = \begin{pmatrix} x & y \end{pmatrix}^T$ and A a 2×2 matrix we have:

$$ax^2 + cy^2 + 2bxy = V^T A V \quad (15)$$

then if $V^T A V > 0$ for $\forall V \neq 0$ the matrix A will be positive definite.

A new proposal

So, for a matrix A Hermitian statements are equivalent the following statements are equivalent

- For a symmetric matrix A of order 2: $a_{ii} \geqslant 0$ and $a_{12} + \sqrt{a_{11}a_{22}} \geqslant 0$
- For a symmetric matrix A of order 3: $a_{ii} \geqslant 0$ and $v_{ij} = a_{ij} + \sqrt{a_{ii}a_{jj}} \geqslant 0$ and $\sqrt{a_{11}a_{22}a_{33}} + a_{12}\sqrt{a_{33}} + a_{13}\sqrt{a_{22}} + a_{23}\sqrt{a_{11}} + \sqrt{v_{12}v_{13}v_{23}} \geqslant 0$

A new proposal

Applying the definitions for the matrix of order 2 in Eq.(12) we obtain:

$$a \geqslant 0, \; c \geqslant 0; \; b + \sqrt{ac} \geqslant 0 \tag{16}$$

which are exactly the same constraints that we obtained previously in Eq.(13).

A new proposal

The scalar potential has to be bounded from below to ensure it stability.

A scalar potential has a quadratic form in the quartic couplings in the form $\lambda_{ab}\phi_a^2\phi_b^2$ if the matrix λ_{ab} is copositive is possible to ensure that the potential has a global minimum, this idea was first pointed out in The European Physical Journal C July 2012, 72:2093.

Let us apply this analysis to our case.

A new proposal

Considering the scalar potential given in Eq.(4), and defining:

$$|H_i|^2 = h_i^2 \quad (17)$$

$$H_i^\dagger H_j = h_i h_j \rho_i e^{i\phi_i} \quad (18)$$

where ρ_i and ϕ_i are not physical parameters. After we open the scalar potential of Eq.(4) for the S_3 products and substitute the definitions of Eq.(10) we obtain the λ matrix in the base (h_1^2, h_2^2, h_3^2) the matrix elements are given by:

A new proposal

$$
\begin{aligned}
V(h_i) &= \lambda_1(h_2^2+h_3^2)^2 \\
&+ \lambda_2[-4h_2^2h_3^2\rho_3^2\sin(\phi_3)] \\
&+ \lambda_3[h_2^4+h_3^4+2h_2^2h_3^2(-1+\rho_3^2)+2h_2^2h_3^2\rho_3^2\cos(2\phi_3)] \\
&+ \lambda_4h_1^4+\lambda_5h_1^2(h_2^2+h_3^2)+\lambda_6[h_1^2(h_2^2\rho_1^2+h_3^2\rho_2^2)] \\
&+ \lambda_7[2h_1^2(h_2^2\rho_1^2\cos(2\phi_1)+h_3^2\rho_2^2\cos(2\phi_2))] \\
&+ \lambda_8[2h_1h_2((-h_2^2+h_3^2)\rho_1\cos(\phi_1)] \\
&+ 2h_3^2\rho_2\rho_3\cos(\phi_2)\cos(\phi_3))
\end{aligned} \tag{19}
$$

setting $\lambda_8=0$

A new proposal

The matrix elements are:

$$\begin{aligned}
A_{11} &= \lambda_4\\
A_{22} &= \lambda_1 + \lambda_3\\
A_{33} &= A_{22}\\
A_{12} &= A_{21} = \frac{1}{2}(\lambda_5 + \rho_1^2(\lambda_6 + 2\lambda_7\cos(2\phi_1))\\
A_{13} &= A_{31} = \frac{1}{2}(\lambda_5 + \rho_2^2(\lambda_6 + 2\lambda_7\cos(2\phi_2))\\
A_{23} &= A_{32} = \lambda_1 + \lambda_3[1 - \rho_3^2(1 - \cos(2\phi_3))] + \lambda_2(1 - \rho_3^2\cos(2\phi_3))
\end{aligned}$$

For the terms $2\lambda_7\cos(2\phi_1)$ and $2\lambda_7\cos(2\phi_2)$ it is obvious that the minimum will be when $\cos(2\phi_1) = \cos(2\phi_2) = -1$, for the element A_{23} to the minimum occurs for $\rho_3 = 1$ and $\cos(2\phi_3) = -1$.

A new proposal

Finally we have the following expressions for the matrix elements, which leaves us with:

$$\begin{aligned} A_{11} &= \lambda_4 \\ A_{22} &= \lambda_1 + \lambda_3 \\ A_{33} &= A_{22} \\ A_{12} &= A_{21} = \frac{1}{2}\lambda_5 \\ A_{13} &= A_{31} = \frac{1}{2}(\lambda_5 + \lambda_6 - 2\lambda_7) \\ A_{23} &= A_{32} = \lambda_1 - \lambda_3 - 2\lambda_2, \end{aligned} \tag{20}$$

A new proposal

- $a_{ii} \geqslant 0$ and $v_{ij} = a_{ij} + \sqrt{a_{ii}a_{jj}} \geqslant 0$ and $\sqrt{a_{11}a_{22}a_{33}} + a_{12}\sqrt{a_{33}} + a_{13}\sqrt{a_{22}} + a_{23}\sqrt{a_{11}} + \sqrt{v_{12}v_{13}v_{23}} \geqslant 0$

$$\begin{aligned} &\lambda_4 \geqslant 0 \\ &\lambda_1 + \lambda_3 \geqslant 0 \\ &\lambda_5 + \sqrt{\lambda_4(\lambda_1 + \lambda_3)} \geqslant 0 \\ &\lambda_5 + \lambda_6 - 2\lambda_7 + \sqrt{\lambda_4(\lambda_1 + \lambda_3)} \geqslant 0 \\ &\lambda_1 - \lambda_2 \geqslant 0 \end{aligned} \tag{21}$$

From a more complicated analysis we also have one more condition.

$$-2\lambda_3 < \lambda_2 \leq \frac{9}{2}\lambda_3. \tag{22}$$

V. Keus at all in arXiv:1407.7859

CP violation

If the VEVs are complex and imposing

$$v_1 e^{i\theta_1} = v_2 e^{i\theta_2} = v_3 e^{i\theta_3} = Ve^{i\Theta}.$$

Θ can be transformed away with a global $U(1)$ transformation as it happens in the standard model.

If $\theta_1 \neq \theta_2 \neq \theta_3 \rightarrow$ there is no inert feature of the two doublets.

There is no spontaneous *CP* violation through the VEVs.

CP violation

Hard CP violation through complex coupling constants λ_6 and λ_8.

Defining $\lambda_6 = |\lambda_6| e^{i\alpha_6}$ and $\lambda_8 = |\lambda_8| e^{i\alpha_8}$.

It is possible to absorve the λ_6

$S \rightarrow S e^{i a_S}$ and $D \rightarrow D e^{i a_D}$

Choosing $a_D - a_S = \alpha_6/2$, so the λ_6 phase can be absorved.

A similar analysis follows for λ_8, with $\alpha_8 = \alpha_6/2$.

Degeneracy Break

Adding $\mu^2_{nm} H^\dagger_n H_m$, $n, m = 2, 3$ to the scalar potential.
The masses matrices are now of the form:

$$M_n^2 = \begin{pmatrix} a_n & b_n & b_n \\ b_n & a_n + \mu^2_{22} & b_n + \mu^2_{23} \\ b_n & b_n + \mu^2_{23} & a_n + \mu^2_{33} \end{pmatrix}, \tag{23}$$

where μ^2_{nm} are naturally small and real for the sake of simplicity. To maintain the inert (be diagonalized by tribimaximal matrix) feature and obtain the correct number of Goldstones bosons we have to impose that $\mu^2_{22} = \mu^2_{33} = -\mu^2_{23} \equiv \mu^2$. The eigenvalues are now $(2a_n + b_n, a_n - b_n, a_n - b_n + \mu^2)$

Dark Matter

Relic Density : Freeze-out mechanism $\Omega h^2 \propto \frac{1}{<\sigma v_e ff>}$
Including $h_i^0 h_i^0 \to XX$ and $H_i^0 A_i^0$, $H_i^0 H_i^\pm \to XX$.
Where $i = 2, 3$ and X represents the other particles of the Model.
In order to calculate the DM abundance we have used the MicrOMEGAs package to numerically solve the Boltzmann equation after implementing all the interactions of the model in the CalcHEP package.

Dark Matter

	scenario 1a	scenario 1b	scenario 2a	scenario 2b	scenario 2c
$m_{H_2^0}$	54.1	79.9	63.4	59.1	168
$m_{H_3^0}$	54.1	79.9	86.59	83.47	178.04
$m_{A_2^0}$	112.44	127.95	117.19	117.25	196.16
$m_{A_3^0}$	112.44	127.95	131.19	131.24	204.83
$m_{h_2^+}$	85.02	95.36	83.09	83.13	84.70
$m_{h_3^+}$	85.02	95.36	101.89	101.92	103.21
μ_d	48.53	78.1	72	72.1	173
v	–	–	41.7	41.7	41.7
λ'	0.019	0.009	0.019	0.001	0.001
Ω	0.11	0.11	0.108	0.11	0.11
σv	0.0832	0.003	6.17	0.0013	0.74
σ^{SI}_{proton}	7.33×10^{-46}	7.44×10^{-47}	5.31×10^{-46}	1.7×10^{-48}	2.019×10^{-49}
$\sigma^{SI}_{neutron}$	8.38×10^{-46}	8.52×10^{-47}	6.08×10^{-46}	1.9×10^{-48}	2.32×10^{-49}

Figure : Parameters choice for Scenario 1 and 2 with $m_h = 125$ GeV. The other masses units are in GeV, σv is in units of $10^{-26}\ cm^3/s$ and the units for σ^{SI} are in cm^2. The parameters $\lambda'' = 0.34$ and $\lambda_5 = 0.4$ for scenarios 1a,1b, 2a, 2b and $\lambda_5 = -0.4$ for scenario 2c.

Dark Matter

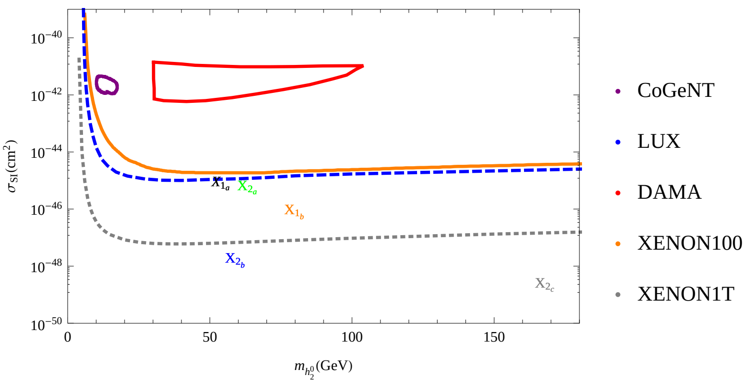


Figure : Limits for σ_{SI} according to the experiments CoGent, DAMA, XENON100, XENON1T and LUX. The points X_{1a}, X_{1b}, X_{2a}, X_{2b} and X_{2c} are the ones refer to scenarios 1a, 1b, 2a, 2b and 2c given in Table above.

Final Thoughts

Here we have proved that the $IDMS_3$ also has DM candidates at least in the second region.

The model has, besides the SM particles, six scalar bosons which are inert.

In the case of degenerated masses (Scenario 1), two neutral scalars plays the role of DM and in the case of non-degenerated masses (Scenario 2), one of the neutral scalars is the DM candidate.

The model can also accommodate pseudoscalars as DM candidates.

Final Thoughts

Processes induced at loop level have always been important to seek for existence of new physics.

This is the case of the decays $h \to \gamma\gamma$ and $h \to Z\gamma$ because they have contributions from the new charged particles. These studies will be shown in the talk:

Two inert scalars doublet model: Status in $h \to \gamma\gamma$ and $h \to Z\gamma$
Javier Domínguez