Higgs Bosons and Grand Unification

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Based on work by DJM, António Morais JHEP 1310 (2013) 226
[arXiv:1307.1373] and arXiv:1408.3013.

Workshop on Multi-Higgs Models
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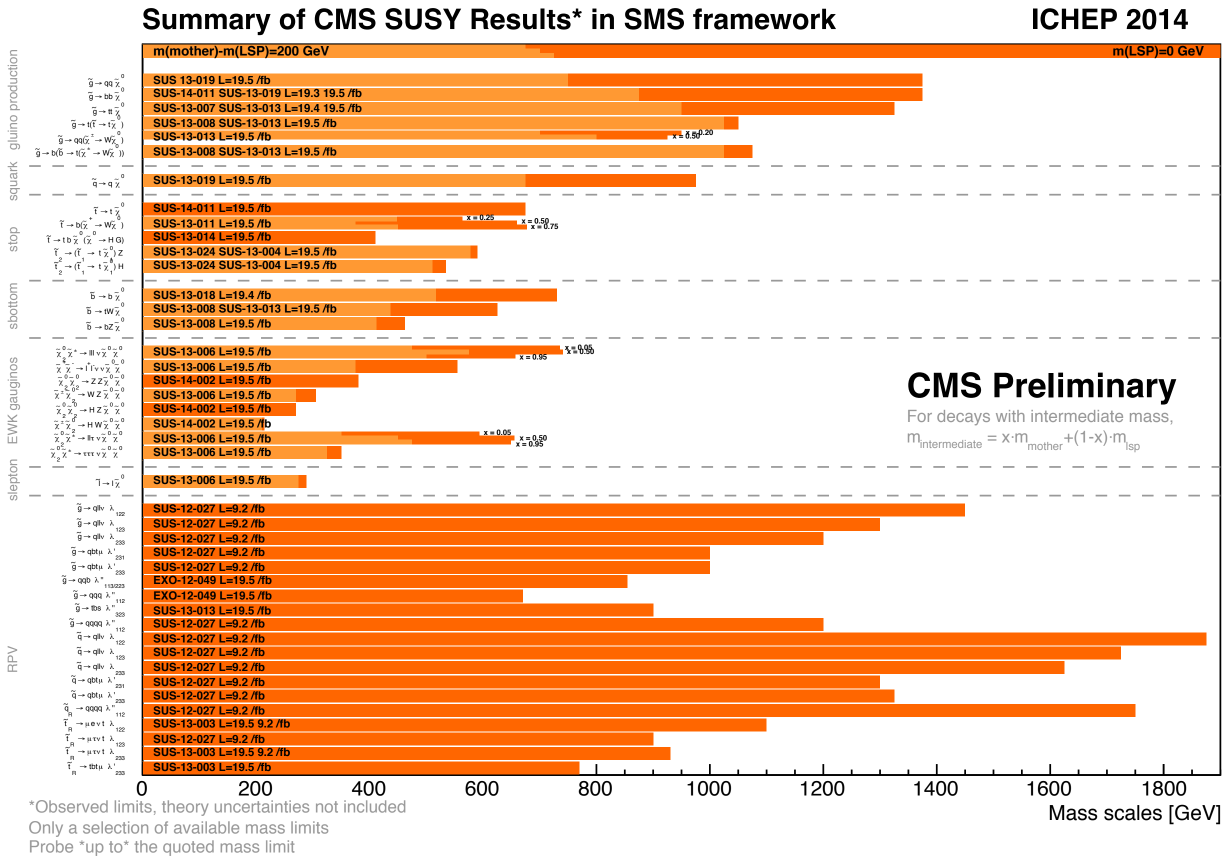
Introduction

One of the community’s favourite **multi-Higgs** models is supersymmetry.

Lately superymmetry has been looking like it is in trouble; LHC exclusions are pushing SUSY to higher energy.



There is still room for a lightish stop, but this is shrinking fast. What happens when it is gone?

Heavy mass spectrum → fine-tuning

What does this mean for GUT theories?

Fine-tuning in supersymmetry

As usual, it is the Higgs boson that causes the problem.

At tree-level the Z-boson mass is given by

$$M_Z^2 = -2\left(m_{H_u}^2 + |\mu|^2\right) + \frac{2}{\tan^2\beta}\left(m_{H_d}^2 - m_{H_u}^2\right) + \mathcal{O}\left(1/\tan^4\beta\right)$$

If m_{H_u} or μ are **large**, natural fluctuations will give large fluctuations in M_Z.

Measure fine-tuning by $\Delta = \max\{\Delta_{\mathcal{P}_i}\}$ with $\Delta_{\mathcal{P}_i} = \left|\frac{\mathcal{P}_i}{M_Z^2}\frac{\partial M_Z^2}{\partial \mathcal{P}_i}\right|$

[Barbieri and Guidice, 1988]

Then $\Delta_\mu \approx \frac{4|\mu|^2}{M_Z^2}$

For $\Delta_\mu \lesssim 10$ we need to have $\mu \lesssim \sqrt{5/2}M_Z \approx 150\,\mathrm{GeV}$

Partial fine-tuning

But μ is an peculiar parameter anyway. It suffers from the **μ-problem**.

It is not a supersymmmetry breaking parameter like the other mass scales but sits in the superpotential $W \supset \mu H_u H_d$.

Could the susy fine-tuning problems be originating from μ alone?

Note:

- I am not saying fine-tuning in μ is not a problem. It is. But maybe this problem is tied up with the μ-problem?
- I have no fix for this problem [neither Guidice-Masiero nor NMSSM help].
- This wouldn't work for the unconstrained MSSM since one would also have fluctuations in m_{H_u}. However, in **GUT models**, m_{H_u} is not a fundamental parameter either.

SU(5) & SO(10) GUTs

We examined SU(5), SO(10) and an orbifold model to confront them with:

- New Higgs mass bounds
- LHC SUSY searches
- Electroweak precision
- Dark Matter
- Fine-tuning

However, in this talk I will not discuss proton decay bounds and the doublet-triplet splitting problem.

SU(5) breaks to the SM trivially, $SU(5) \to SU(3) \times SU(2) \times U(1)$

$$\begin{aligned} \mathbf{1} &\to (\mathbf{1},\mathbf{1})_0\,, \\ \mathbf{5} &\to (\mathbf{1},\mathbf{2})_3 \oplus (\mathbf{3},\mathbf{1})_{-2}\,, \\ \overline{\mathbf{5}} &\to (\mathbf{1},\mathbf{2})_{-3} \oplus \left(\overline{\mathbf{3}},\mathbf{1}\right)_2\,, \\ \mathbf{10} &\to (\mathbf{1},\mathbf{1})_6 \oplus \left(\overline{\mathbf{3}},\mathbf{1}\right)_{-4} \oplus (\mathbf{3},\mathbf{2})_1\,, \end{aligned}$$

SO(10) may break via SU(5)…

$$SO(10) \to SU(5) \times U(1)_X \to SU(3) \times SU(2) \times U(1)_Z \times U(1)_X \to G_{SM}$$

$$\mathbf{16} \to \mathbf{1}_{-5} \oplus \overline{\mathbf{5}}_3 \oplus \mathbf{10}_{-1},$$

$$\mathbf{10} \to \mathbf{5}_2 \oplus \overline{\mathbf{5}}_{-2},$$

$$\mathbf{1} \to (\mathbf{1},\mathbf{1})_0\,,$$

$$\mathbf{5} \to (\mathbf{1},\mathbf{2})_3 \oplus (\mathbf{3},\mathbf{1})_{-2}\,,$$

$$\overline{\mathbf{5}} \to (\mathbf{1},\mathbf{2})_{-3} \oplus \left(\overline{\mathbf{3}},\mathbf{1}\right)_2\,,$$

$$\mathbf{10} \to (\mathbf{1},\mathbf{1})_6 \oplus \left(\overline{\mathbf{3}},\mathbf{1}\right)_{-4} \oplus (\mathbf{3},\mathbf{2})_1\,,$$

…either "normal" or "flipped" ($e_R \leftrightarrow N_R$ and $u_R \leftrightarrow d_R$)

or via Pati-Salam…

$$SO(10) \to SU(4) \times SU(2)_L \times SU(2)_R \to SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_W \to G_{SM},$$

$$\mathbf{16} \to (\mathbf{4},\mathbf{2},\mathbf{1}) \oplus \left(\overline{\mathbf{4}},\mathbf{1},\mathbf{2}\right),$$

$$\mathbf{10} \to (\mathbf{1},\mathbf{2},\mathbf{2}) \oplus (\mathbf{6},\mathbf{1},\mathbf{1})\,,$$

$$\left(\overline{\mathbf{4}},\mathbf{1},\mathbf{2}\right) = \begin{pmatrix} \hat{u}_x^\dagger & \hat{N}^\dagger \\ \hat{d}_x^\dagger & \hat{e}^\dagger \end{pmatrix}_R$$

$$(\mathbf{4},\mathbf{2},\mathbf{1}) = \begin{pmatrix} \hat{u}^x & \hat{\nu} \\ \hat{d}^x & \hat{e} \end{pmatrix}_L$$

$$(\mathbf{1},\mathbf{2},\mathbf{2}) = \begin{pmatrix} \hat{h}_u^+ & \hat{h}_d^0 \\ \hat{h}_u^0 & \hat{h}_d^- \end{pmatrix}$$

$$(\mathbf{4},\mathbf{2},\mathbf{1}) \to (\mathbf{1},\mathbf{2},\mathbf{1})_3 \oplus (\mathbf{3},\mathbf{2},\mathbf{1})_{-1}\,,$$

$$\left(\overline{\mathbf{4}},\mathbf{1},\mathbf{2}\right) \to (\mathbf{1},\mathbf{1},\mathbf{2})_{-3} \oplus (\mathbf{3},\mathbf{1},\mathbf{2})_1\,,$$

$$(\mathbf{1},\mathbf{2},\mathbf{2}) \to (\mathbf{1},\mathbf{2},\mathbf{2})_0\,,$$

…again either "normal" or "flipped"

Boundary Conditions

Scalar masses:

SO(10)

SU(5)

$$m^2_{Q_{ij}}(0) = m^2_{u_{ij}}(0) = m^2_{e_{ij}}(0) = \begin{pmatrix} K_{\mathbf{16}} & 0 & 0 \\ 0 & K_{\mathbf{16}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \left(m^2_{\mathbf{16}} + g^2_{10} D\right),$$

$$m^2_{\mathbf{10}}$$

$$m^2_{L_{ij}}(0) = m^2_{d_{ij}}(0) = \begin{pmatrix} K_{\mathbf{16}} & 0 & 0 \\ 0 & K_{\mathbf{16}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \left(m^2_{\mathbf{16}} - 3g^2_{10} D\right),$$

$$m^2_{\overline{\mathbf{5}}}$$

$$m^2_{N_{ij}}(0) = \begin{pmatrix} K_{\mathbf{16}} & 0 & 0 \\ 0 & K_{\mathbf{16}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \left(m^2_{\mathbf{16}} + 5g^2_{10} D\right),$$

$$\left.\begin{aligned} m^2_{H_u}(0) &= m^2_{\mathbf{10+126}} - 2g^2_{10} D, \\ m^2_{H_d}(0) &= m^2_{\mathbf{10+126}} + 2g^2_{10} D, \end{aligned}\right\} m^2_{\mathbf{5}'}$$

Trilinear couplings: $a_t(0) = a_b(0) = a_\tau(0) = a_{\mathbf{10}}$

$$a_t(0) = a_{\mathbf{5}'},$$
$$a_b(0) = a_\tau(0) = a_{\overline{\mathbf{5}}'}.$$

The four different SO(10) embeddings give the same scalar masses and D-terms but give different gaugino masses, depending on how they are broken.
To quantify our non-universal Gaugino masses we set: $M_1/\rho_1 = M_2/\rho_2 = M_3 \equiv M_{1/2}$

Theoretical and Experimental Constraints

LHC susy constraints: $m_{\tilde{q}} > 1.7\,\mathrm{TeV}$, $m_{\tilde{g}} > 1.2\,\mathrm{TeV}$

$m_{\tilde{q}} > 1.4\,\mathrm{TeV}$

$m_{\tilde{g}} > 0.8\,\mathrm{TeV}$

for SU(5)

LHC Higgs mass constraint $m_H = 125.7 \pm 2.1\mathrm{GeV}$

Direct Dark Matter constraint from **LUX** or **XENON100** (for SU(5))

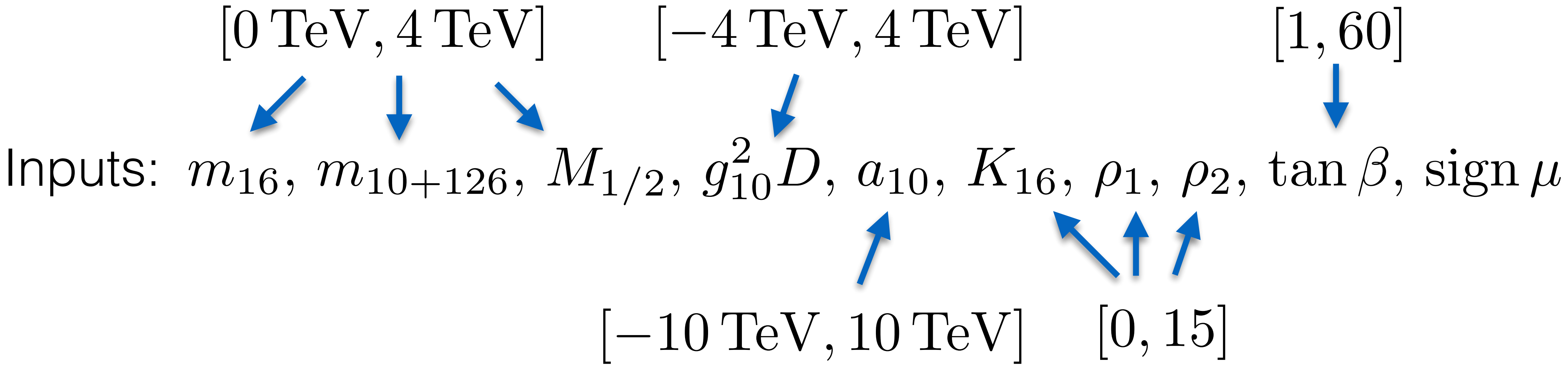
Relic Abundance $\Omega_c h^2 = 0.1157 \pm 0.0023$ (WMAP)

Other low energy constraints from $b \to s\gamma,\ B_s \to \mu^+\mu^-,\ B \to \tau\nu_\tau, a_\mu$

$$P_{\mathrm{tot}} = P_{m_h} \cdot P_{\Omega_c h} \cdot P_{b\to s\gamma} \cdot P_{\mathcal{R}_{\tau\nu_\tau}} \cdot P_{B_s\to\mu\mu} \cdot P_{a_\mu} > 10^{-3}$$

Only if deviation greater than in SM

We also implement **vacuum stability** as described by Casas, Lleyda, Munoz (1996)



$[0\,\mathrm{TeV}, 4\,\mathrm{TeV}]$ $[-4\,\mathrm{TeV}, 4\,\mathrm{TeV}]$ $[1, 60]$

Inputs: $m_{16},\ m_{10+126},\ M_{1/2},\ g_{10}^2 D,\ a_{10},\ K_{16},\ \rho_1,\ \rho_2,\ \tan\beta,\ \mathrm{sign}\,\mu$

$[-10\,\mathrm{TeV}, 10\,\mathrm{TeV}]$ $[0, 15]$

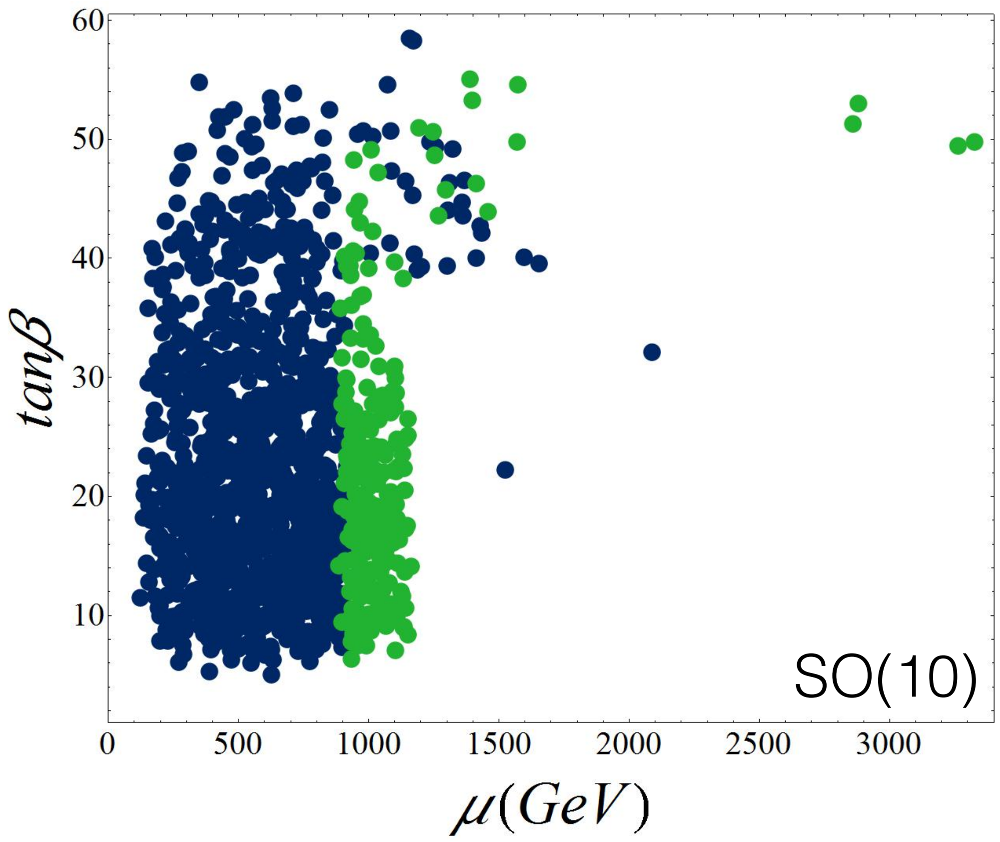
We used:

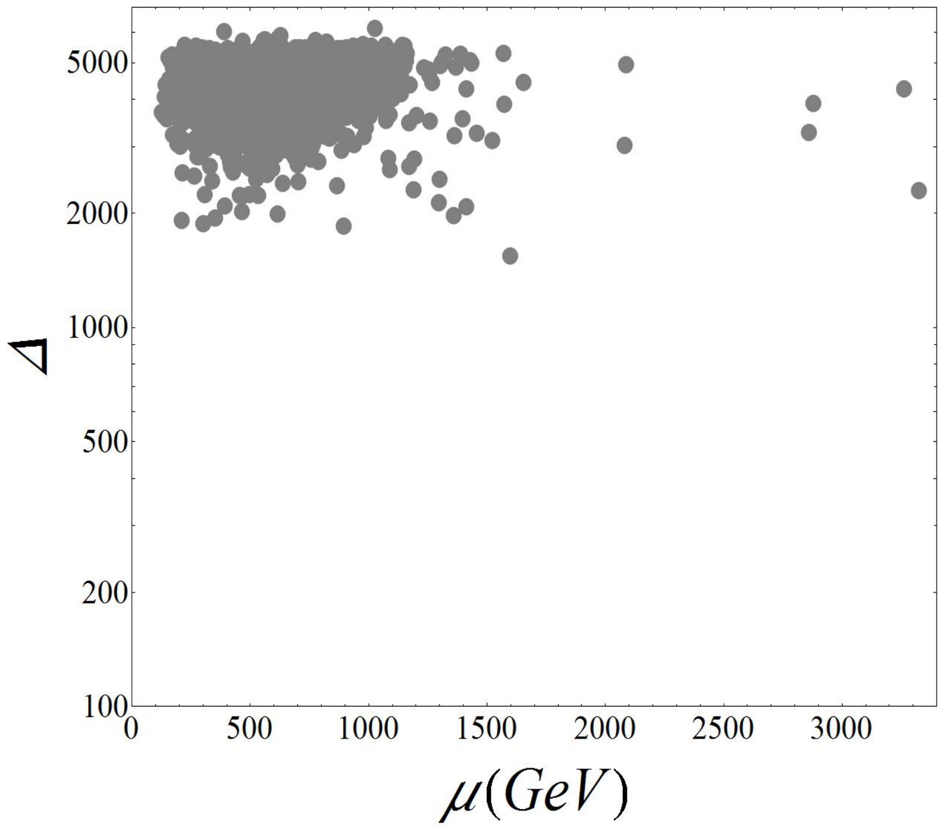
SOFTSUSY 3.3.0 (Allananch 2002) for the RGE running and fine-tuning measure.

micoOMEGAs 2.4.5 (Belanger et al 2006) for Relic density, Dark Matter nucleon cross-section and other low energy constraints.

Universal Gaugino Masses

First we looked at scenarios with universal gauging masses $\rho_1 = \rho_2 = 1$

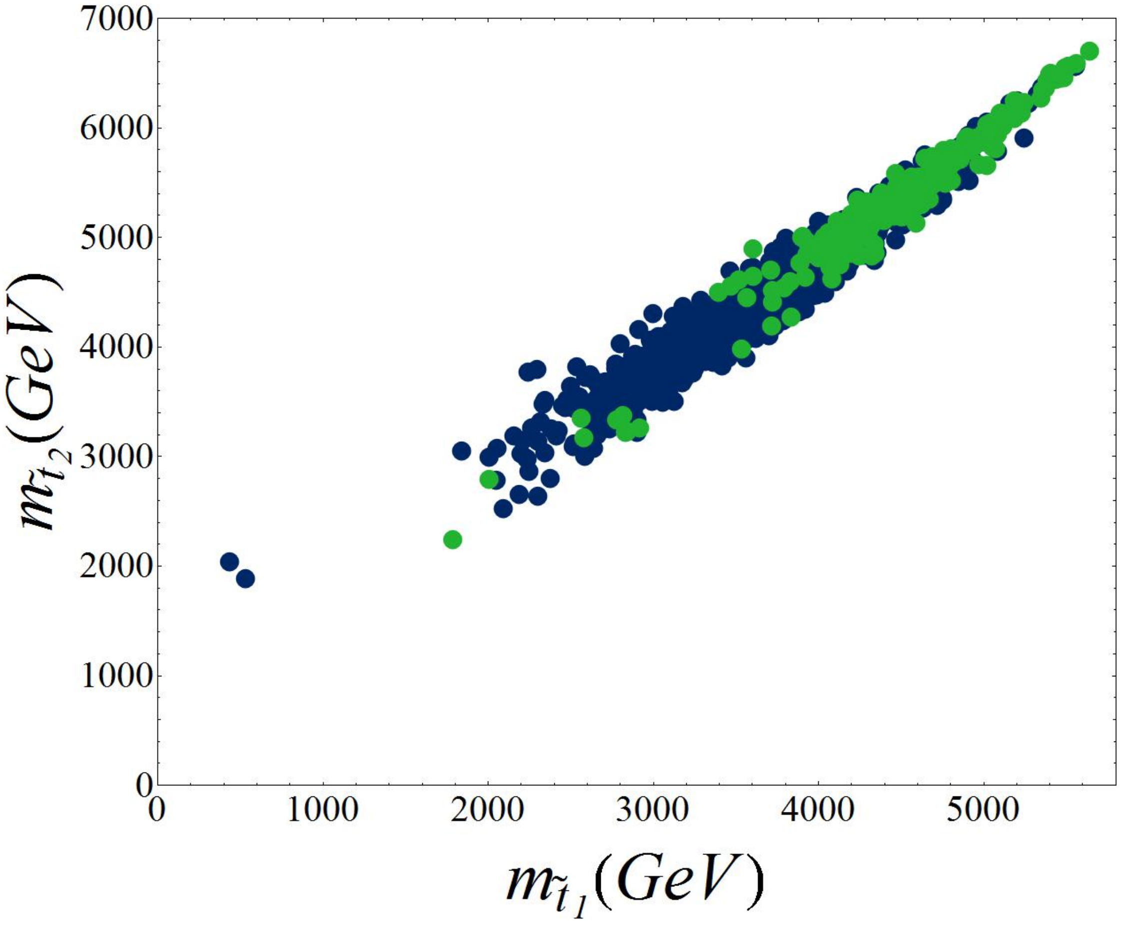


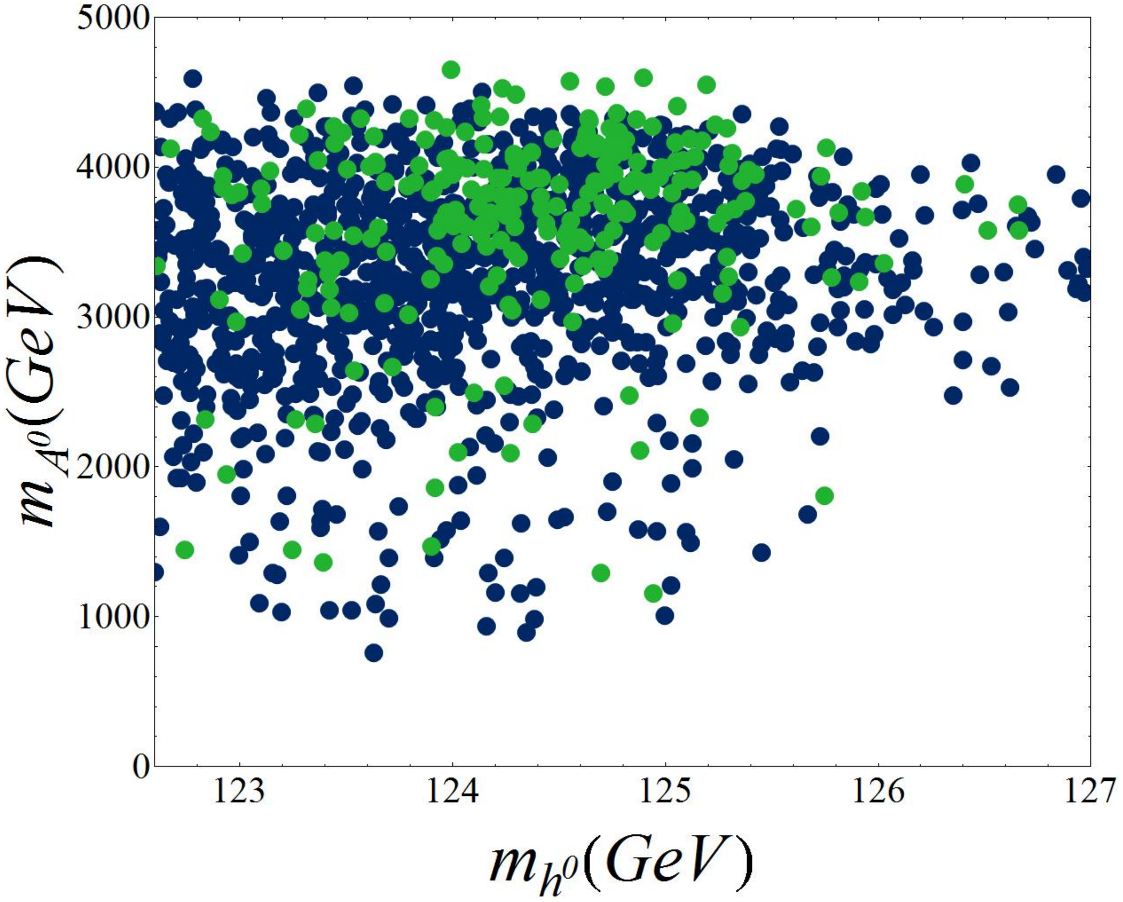


Green points have the correct relic density, while **blue** points have too little.

Although there are plenty of viable points, we could only find ones that are fine-tuned, even neglecting fine-tuning from μ.

These scenarios have heavy spectra.





The Higgs boson is in the decoupling regime, so the light Higgs would look exactly like the SM Higgs.

Non-Universal Gauginos

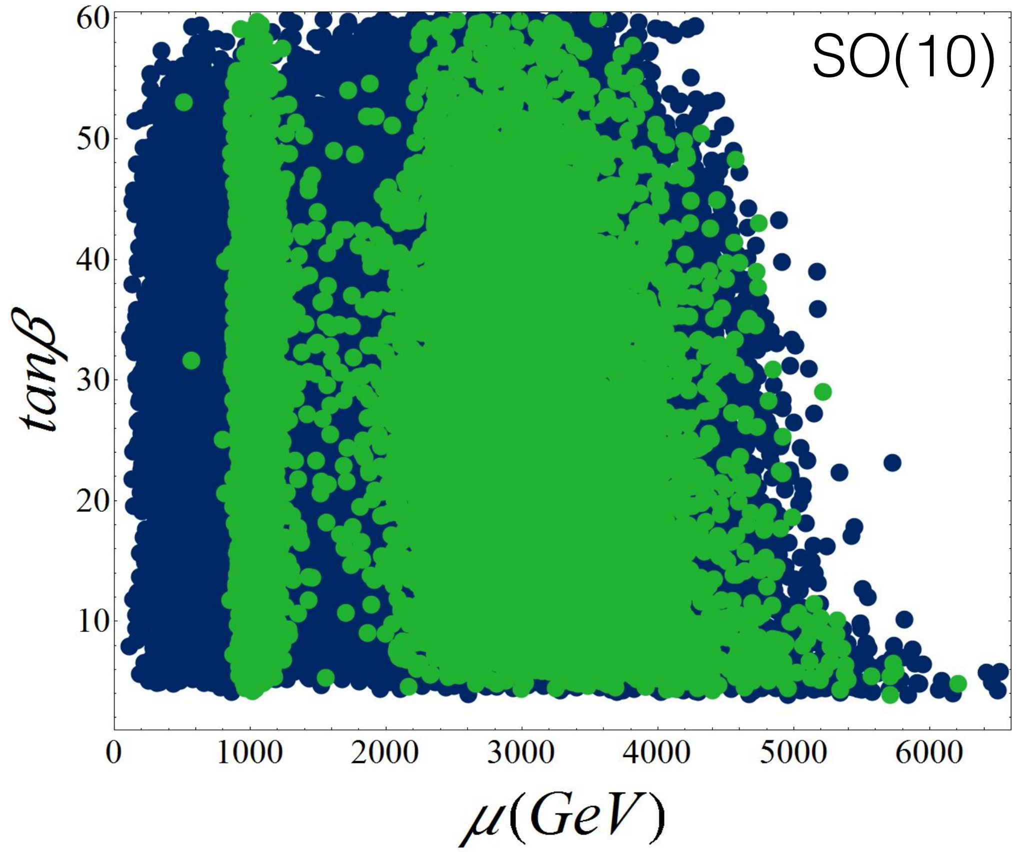
Generally one might expect the gauginos to have non-universal masses at the high scale. For example, if the symmetry is broken by some hidden sector field $\hat{X}$ with an F-term F_X then we generate masses of the form

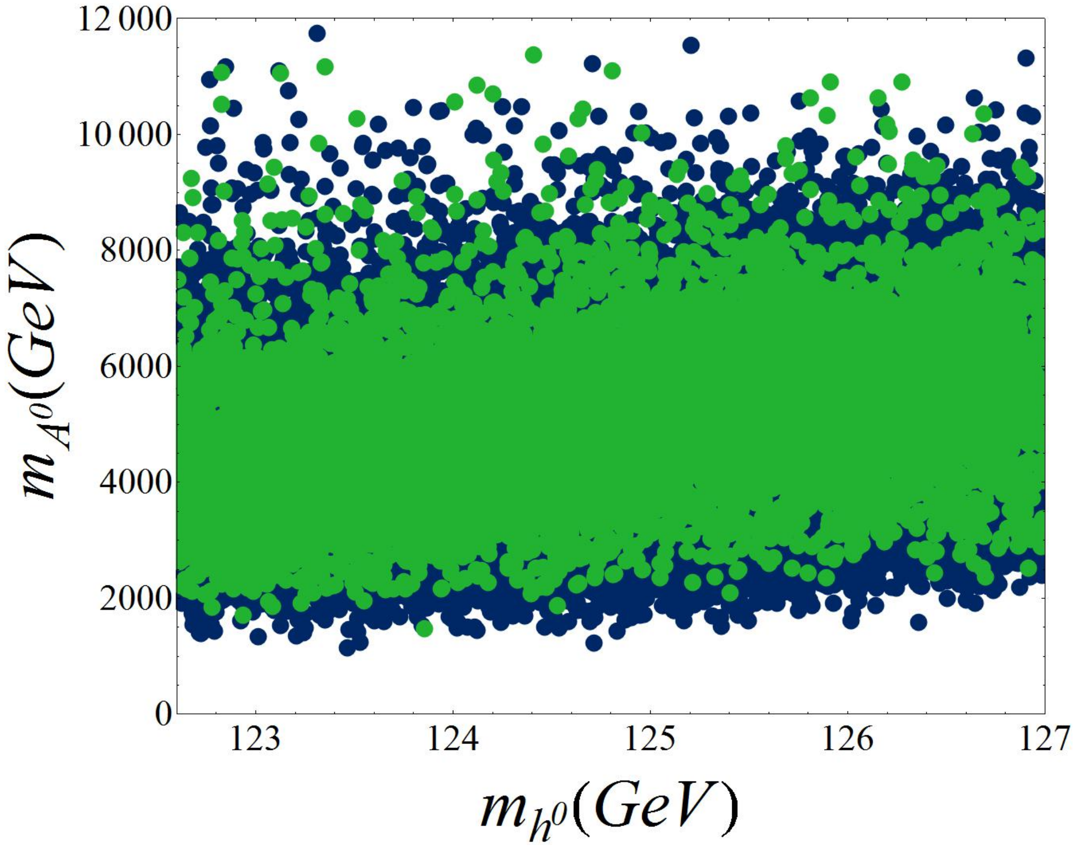
$$\frac{1}{2}\frac{\langle F_X^j \rangle}{\langle Re f_{\alpha\beta} \rangle}\left\langle \frac{\partial f_{\alpha\beta}^*}{\partial \varphi^{j*}} \right\rangle \tilde{\lambda}^\alpha \tilde{\lambda}^\beta$$

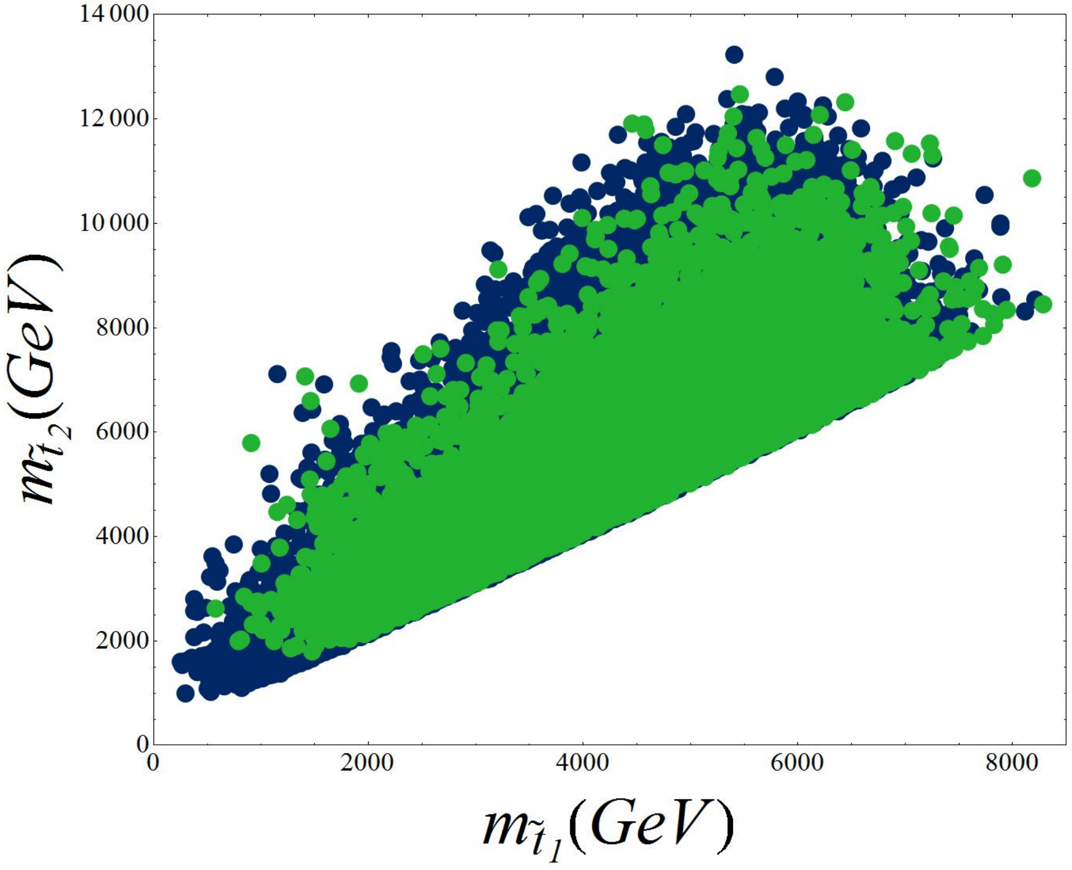
If $\hat{X}$ is a singlet, this gives **universal** gauginos, but if it is not we will find **non-universal** gaugino masses.

At the GUT scale we set

$$M_1/\rho_1 = M_2/\rho_2 = M_3 \equiv M_{1/2}$$



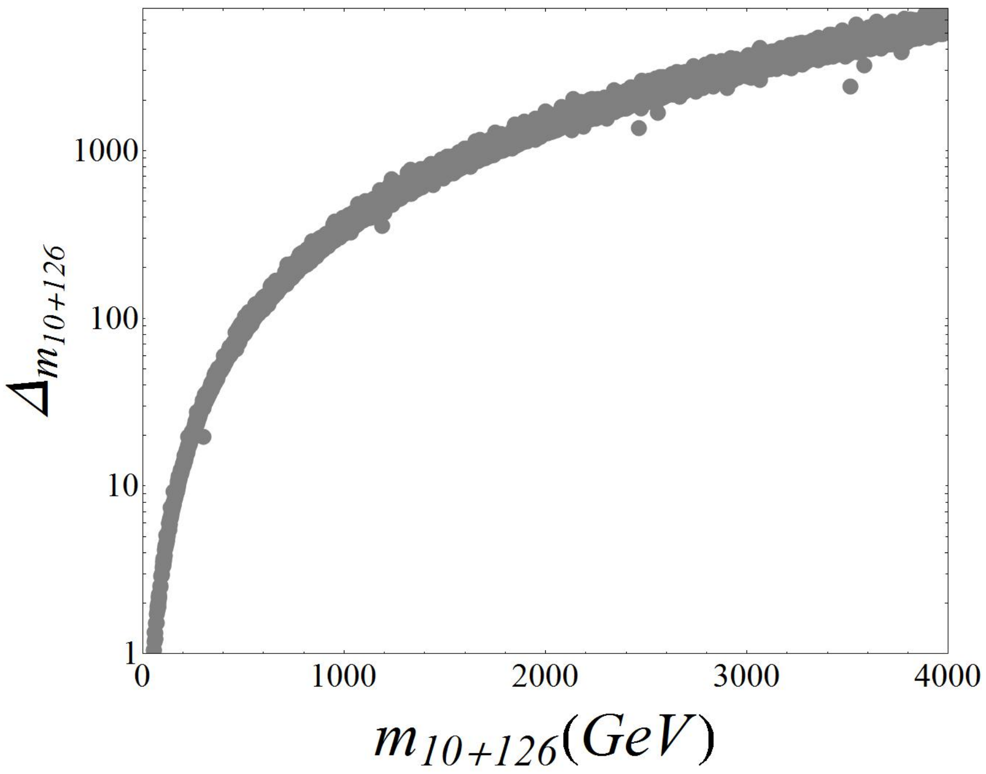


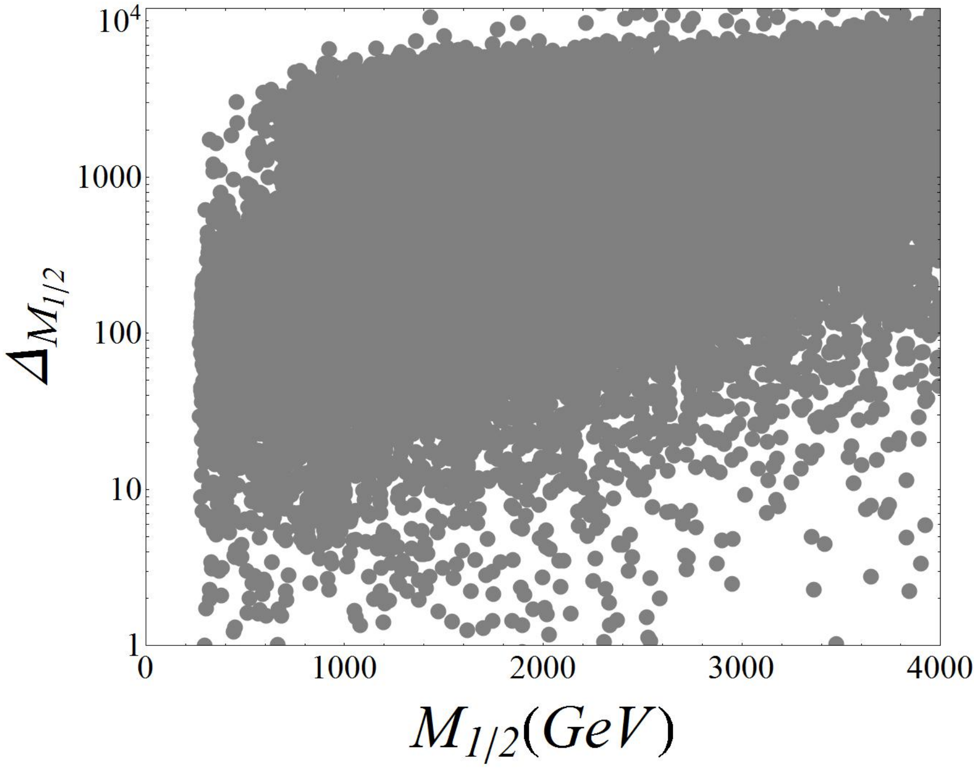


Lots of scenarios open up, some with quite light stops.

But it is very difficult to get a small μ and the correct relic density.

Fine-tuning



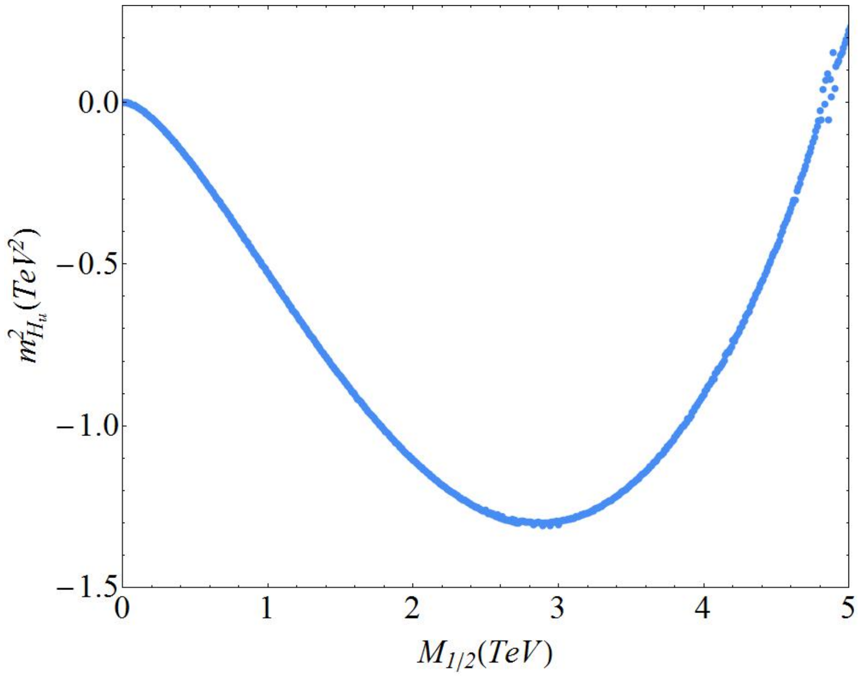


Fine-tuning arising from scalar masses (and D-terms, trilinears) grows with the mass but $M_{1/2}$ seems to allow low fine-tuning even for large values.

$$M_Z^2 = -2\left(m_{H_u}^2 + |\mu|^2\right) + \frac{2}{\tan^2\beta}\left(m_{H_d}^2 - m_{H_u}^2\right) + \mathcal{O}\left(1/\tan^4\beta\right)$$

m_{H_u} is not an input parameter. It is a complicated function of the other inputs.

If we set all the masses other than $M_{1/2}$ to zero then one expects $m_{H_u}^2 = aM_{1/2}^2$

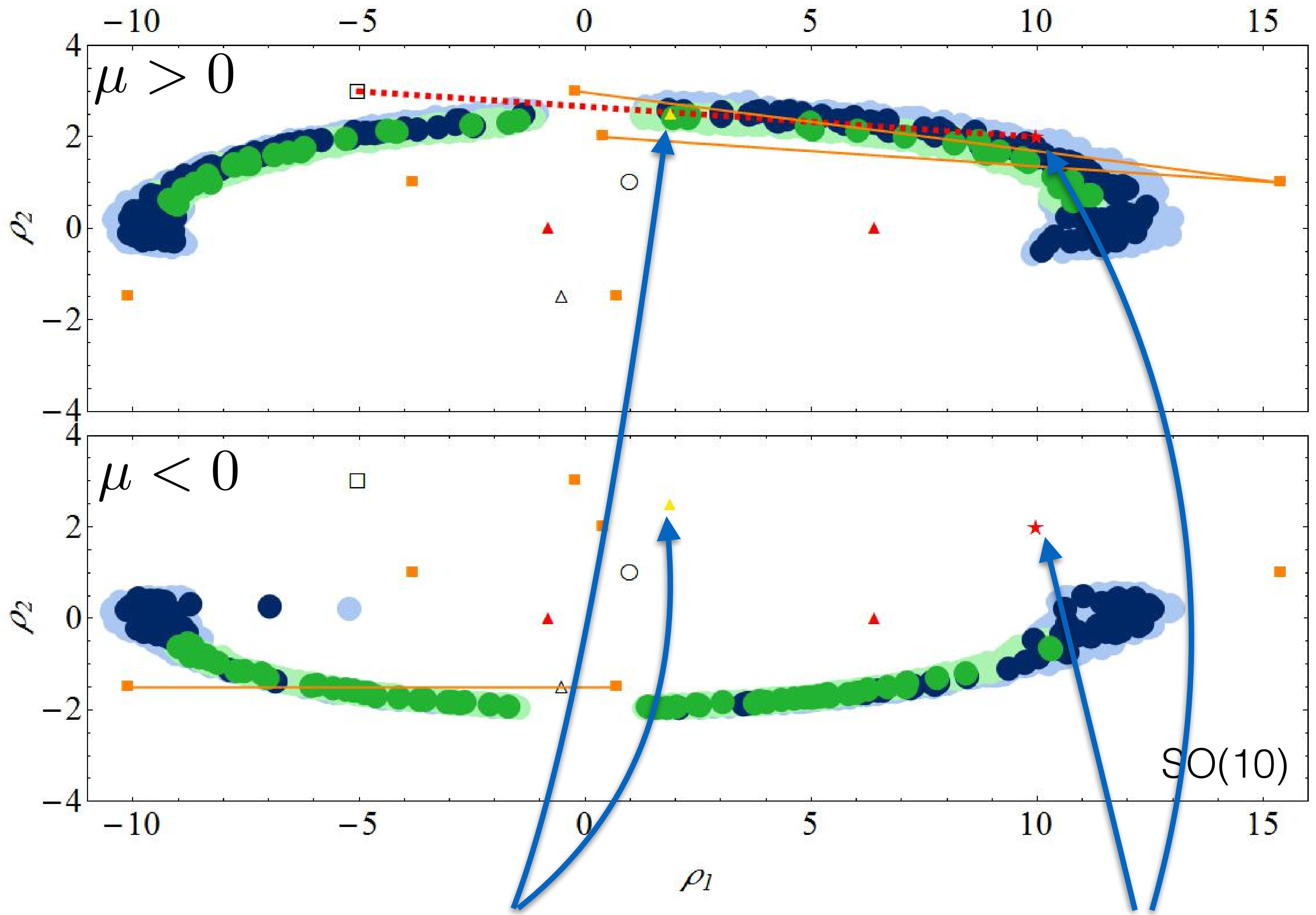


However, adding radiative corrections at the low scale, makes this more complicated and a also becomes $M_{1/2}$ dependent.

The dependence of m_{H_u} on $M_{1/2}$ gains a minimum.

This plot was made with SOFTSUSY. This behaviour persists also with Spheno, but the position of the minima moves.

Set the scalar masses and trilinear $< 150\,\mathrm{GeV}$ (they will fed by $M_{1/2}$ during running) and see what happens:



Light:

$10 < \Delta < 100$

Dark:

$\Delta < 10$

The symbols are each a different embedding at the GUT scale.

The yellow triangle is a Pati-Salam embedding in SO(10).

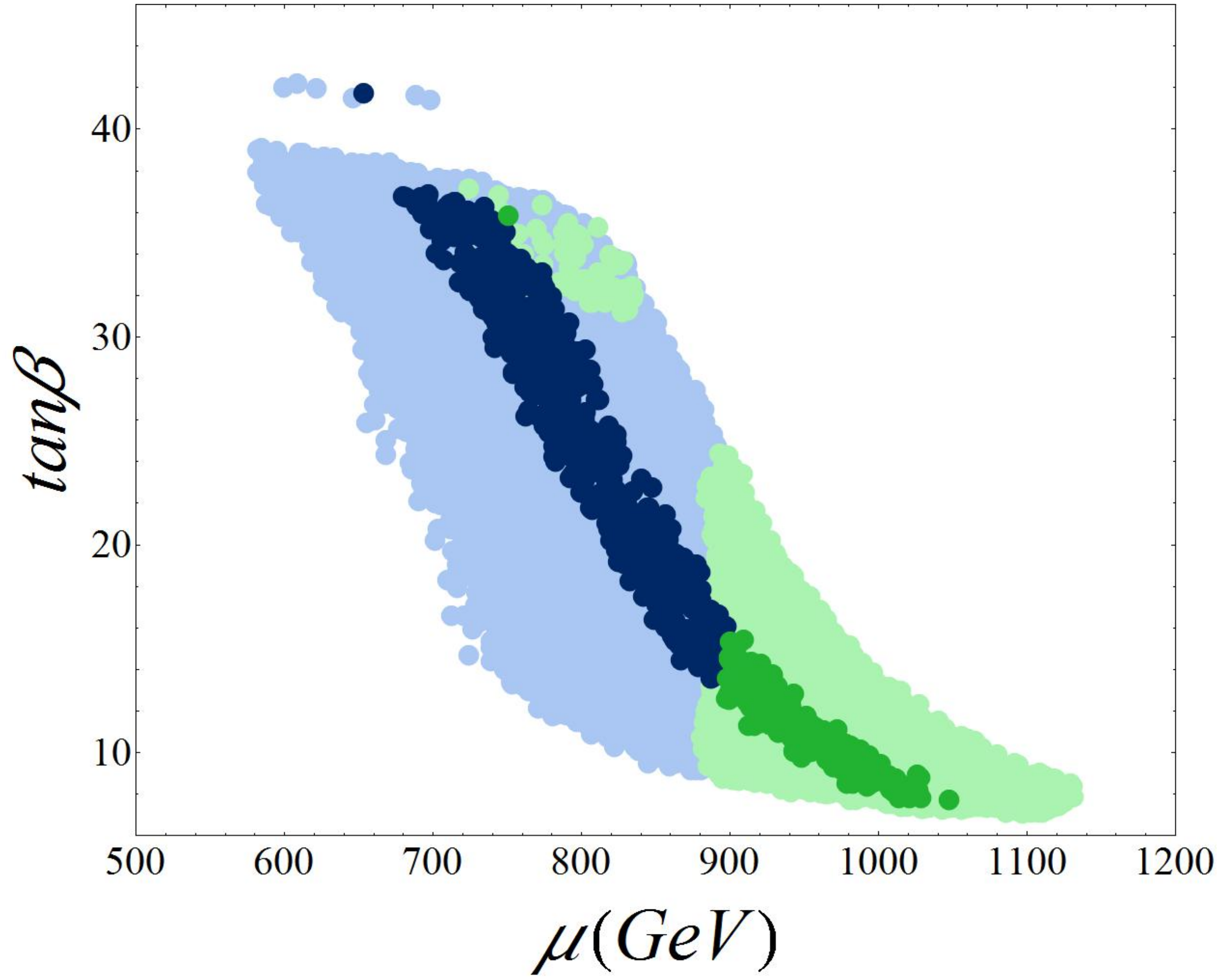
200 of SU(5)

An Example: Pati-Salam Embedding

PS breaking (the yellow triangle)

$$\begin{cases} SO(10) \to SU(4) \times SU(2)_R \\ \mathbf{770} \to (\mathbf{1},\mathbf{1}) \end{cases}$$

$$\rho_1 = \frac{19}{10},\ \rho_2 = \frac{5}{2}$$



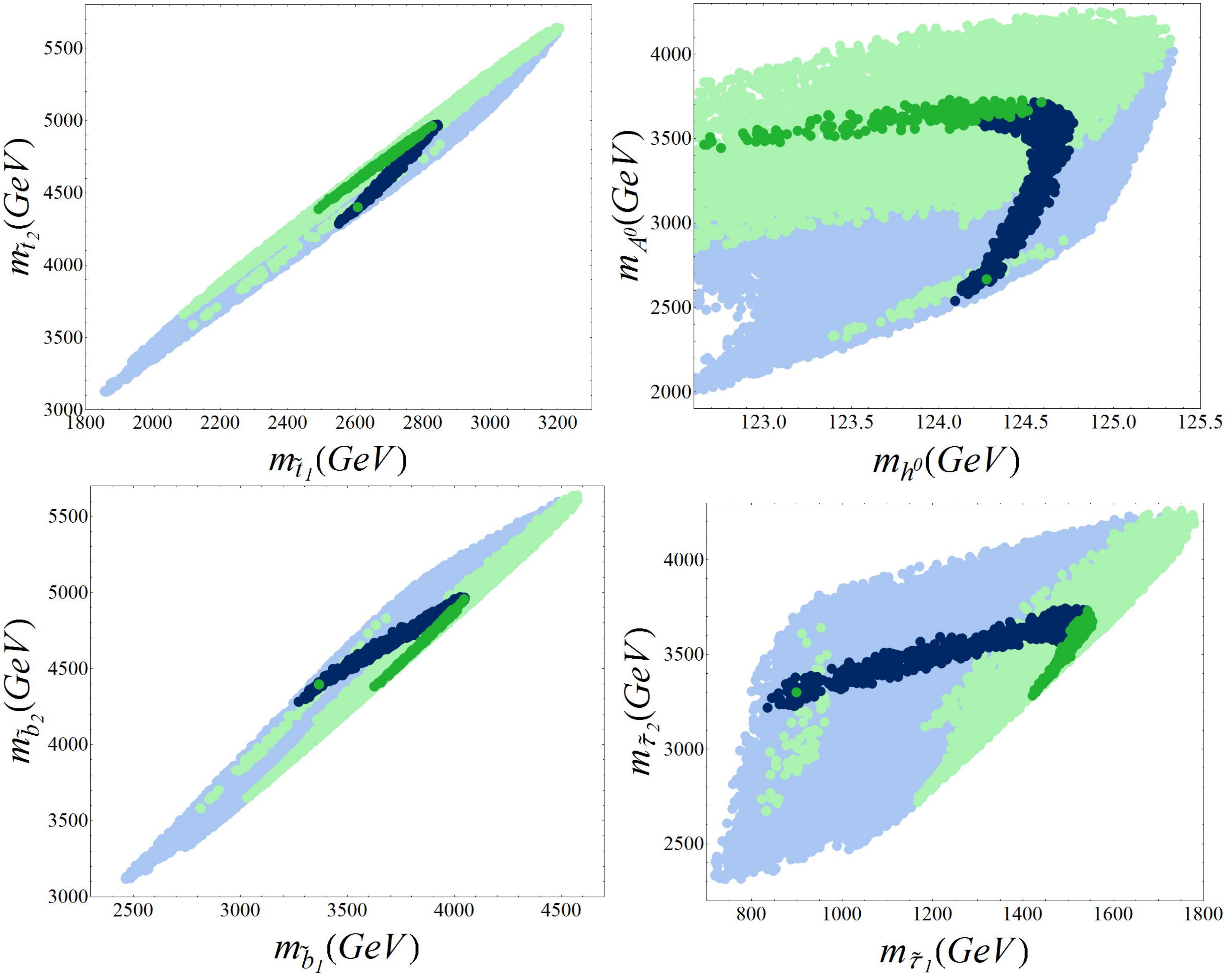
All scenarios with the correct relic density have higgsino LSP and charging NLSP.

Light:

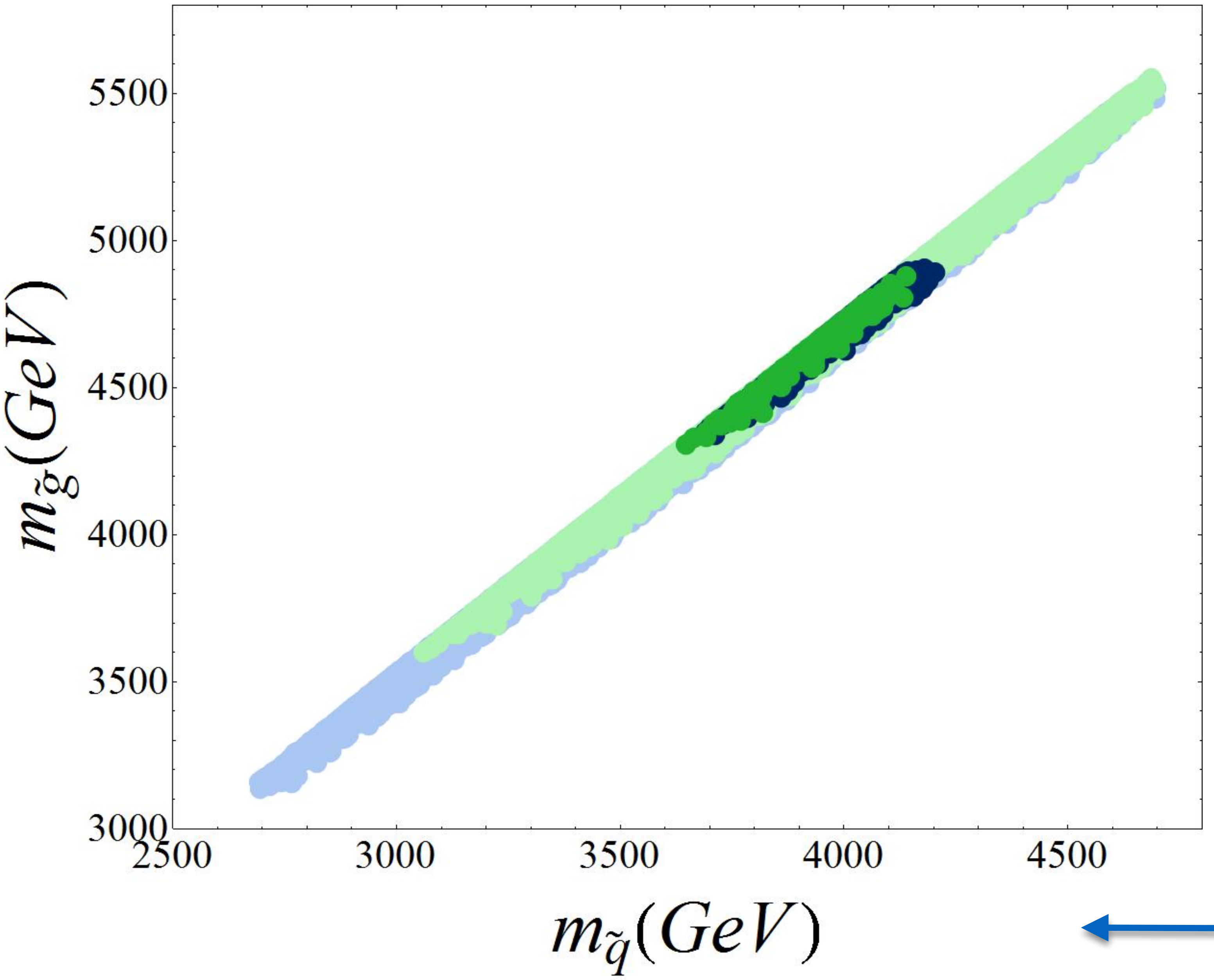
$$10 < \Delta < 100$$

Dark:

$$\Delta < 10$$



Unfortunately the mass spectrum is very heavy, so this is very challenging to see.



first/second gen. squarks

Since the scalar masses are generated by $M_{1/2}$ these models predict $m_{\tilde{d}_R} \approx 0.9 m_{\tilde{g}}$

Orbifolds

We have also examined Orbifold models by Brignole, Ibáñez and Muñoz (1994).

In this model supersymmetry is broken by compactification of e.g. String Theory in higher dimensions, via F-terms of **dilaton** and **moduli** fields in a hidden sector. This gives rise to a **goldstino**

$$\tilde{\eta} = \tilde{S} \sin\theta + \tilde{T} \cos\theta$$

goldstino angle

dilaton

moduli

Transformation properties of moduli → modular weights

BIM O-I:

$$\begin{aligned} n_{Q_L} = n_{d_R} &= -1, \\ n_{u_R} &= -2, \\ n_{L_L} = n_{e_R} &= -3, \\ n_H + n_{\overline{H}} &= -5, -4. \end{aligned}$$

BIM O-II:

$$n_i = -1$$

$$\sin\theta \to 0$$

BIM O-I

Gaugino masses at the “GUT” scale:

$$
\begin{aligned}
M_1 &= 1.18\sqrt{3}m_{3/2}\left[\sin\theta - \left(\frac{51}{5} + \delta_{GS}\right) 2.9 \times 10^{-2}\cos\theta\right],\\
M_2 &= 1.06\sqrt{3}m_{3/2}\left[\sin\theta - (7 + \delta_{GS})\, 2.9 \times 10^{-2}\cos\theta\right],\\
M_3 &= \sqrt{3}m_{3/2}\left[\sin\theta - (6 + \delta_{GS})\, 2.9 \times 10^{-2}\cos\theta\right],
\end{aligned}
$$

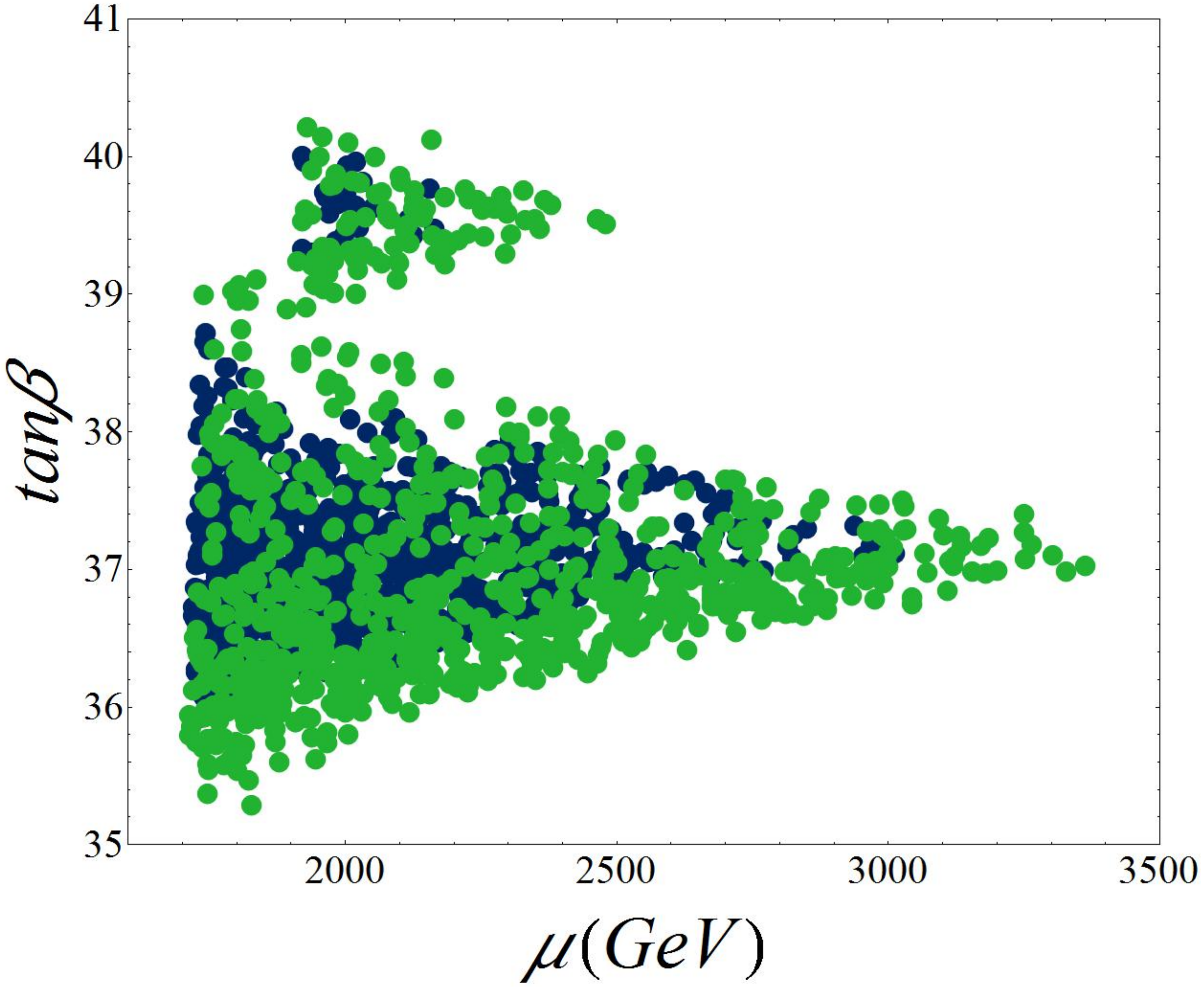
Green-Schwarz counterterm for anomaly cancellation: choose $\delta_{GS} = -5$

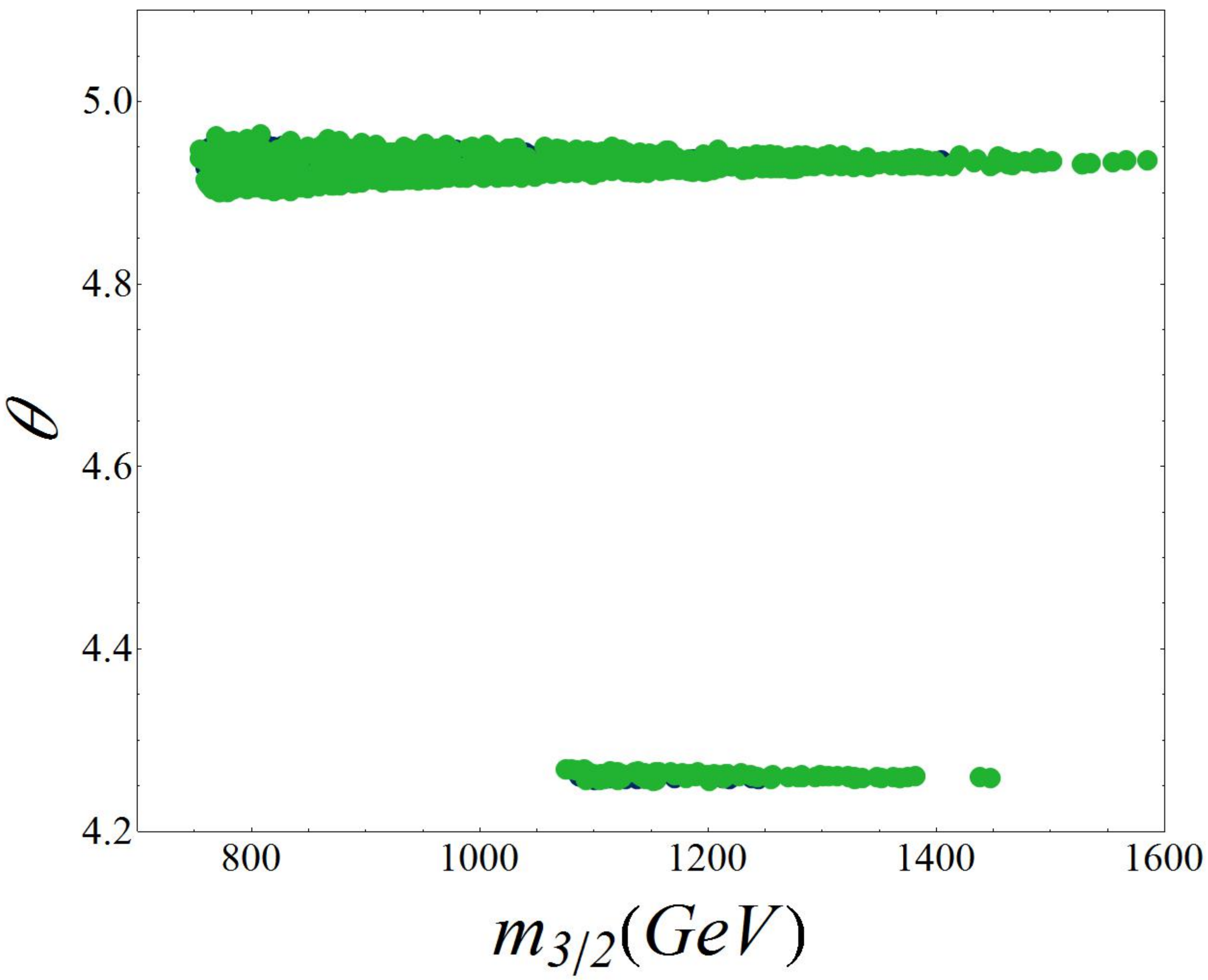
Scalar masses: $m_i^2 = m_{3/2}^2(1 - n_i\cos^2\theta)$

$$
\begin{aligned}
m^2_{\tilde{Q}_L} = m^2_{\tilde{d}_R} &= m^2_{-1} = m^2_{3/2}\sin^2\theta,\\
m^2_{\tilde{u}_R} &= m^2_{-2} = m^2_{3/2}\left(1 - 2\cos^2\theta\right)\\
m^2_{\tilde{L}_L} = m^2_{\tilde{e}_R} &= m^2_{-3} = m^2_{3/2}\left(1 - 3\cos^2\theta\right) \qquad \longleftarrow \qquad \sin^2\theta \geq 2/3
\end{aligned}
$$

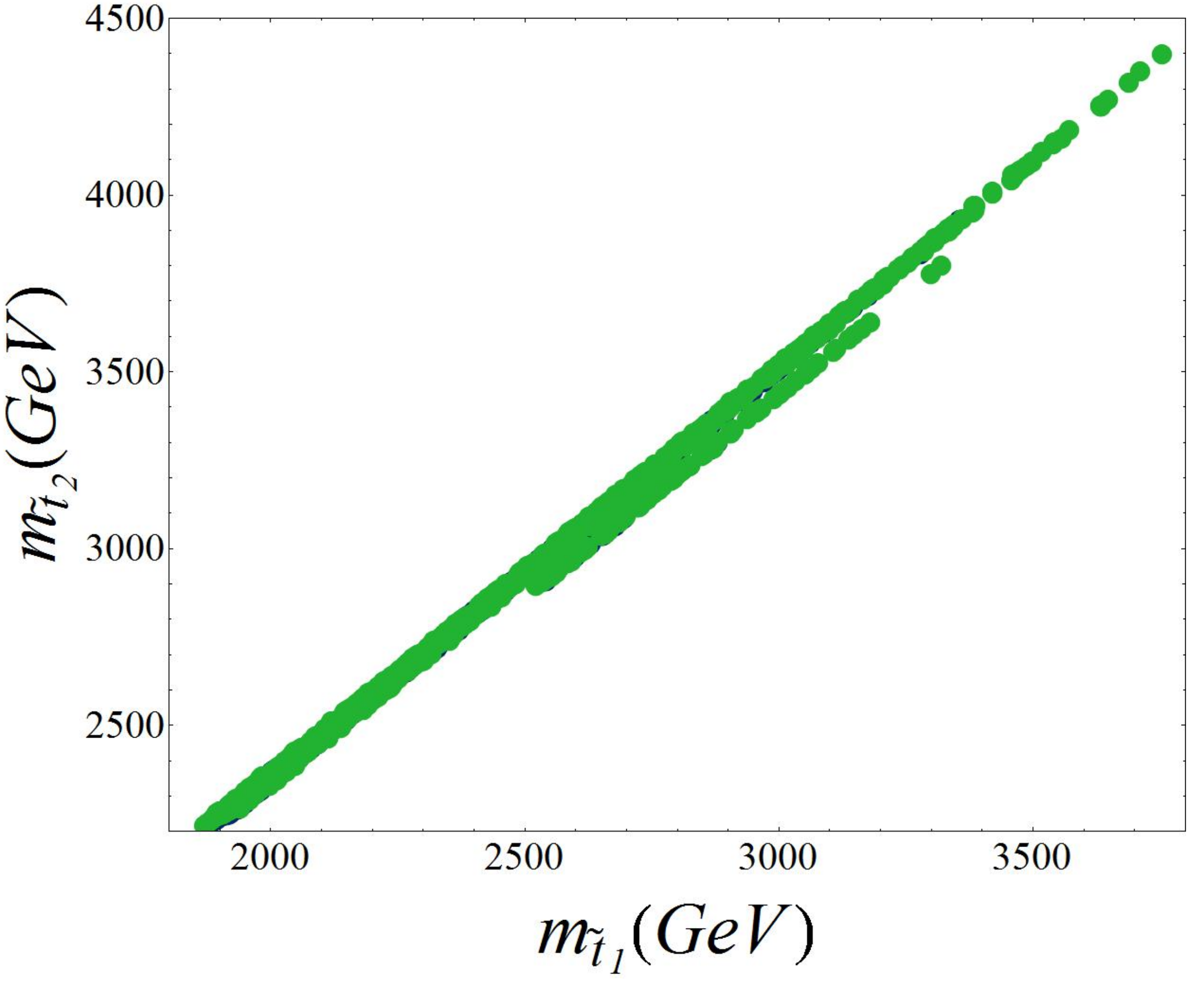
$$m^2_{H_u} = m^2_{H_d} = m^2_{3/2}\left(1 - 2\cos^2\theta\right)$$

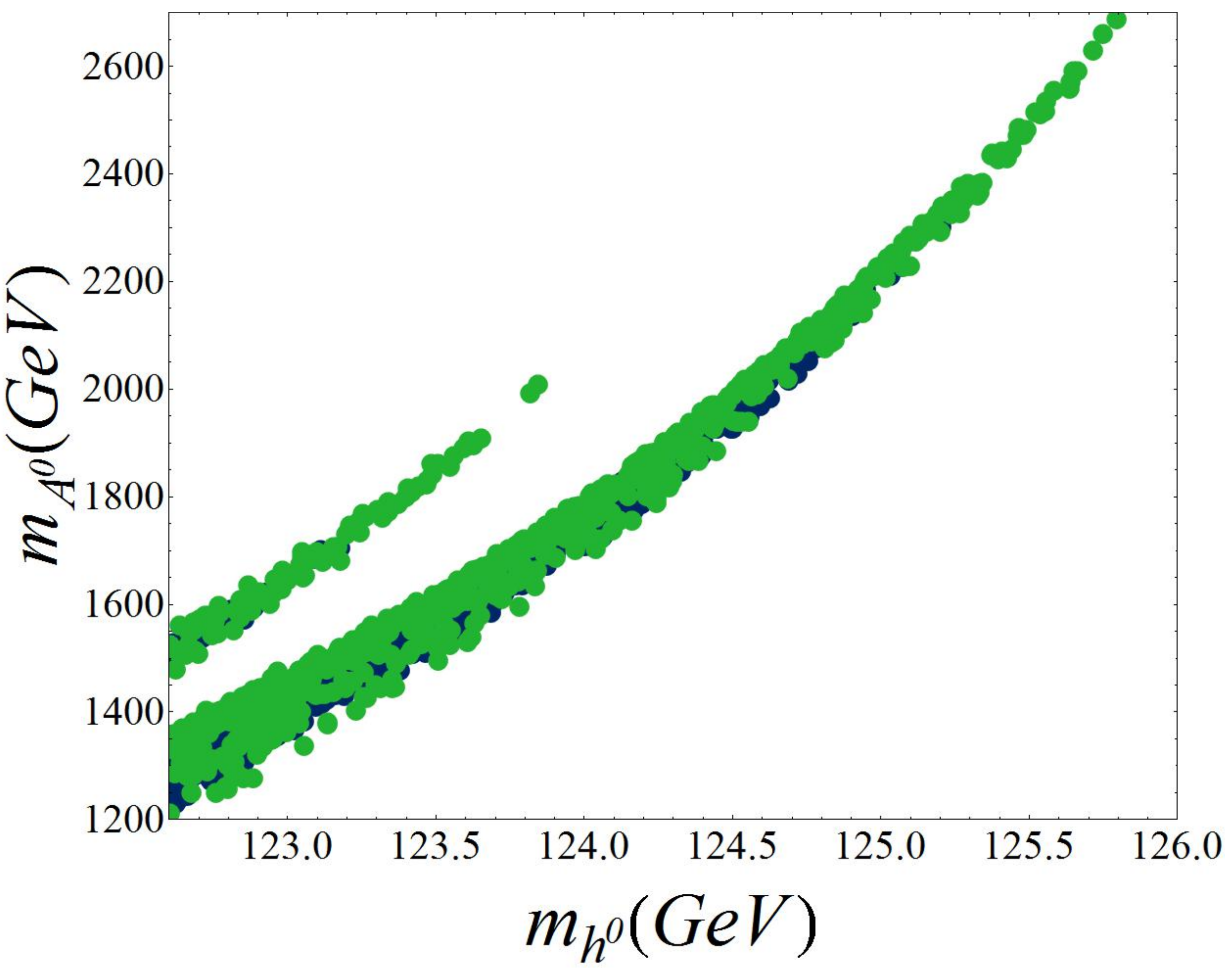
Trilinear: $a_0 = -m_{3/2}\left(\sqrt{3}\sin\theta + n_{H_d}\cos\theta\right)$



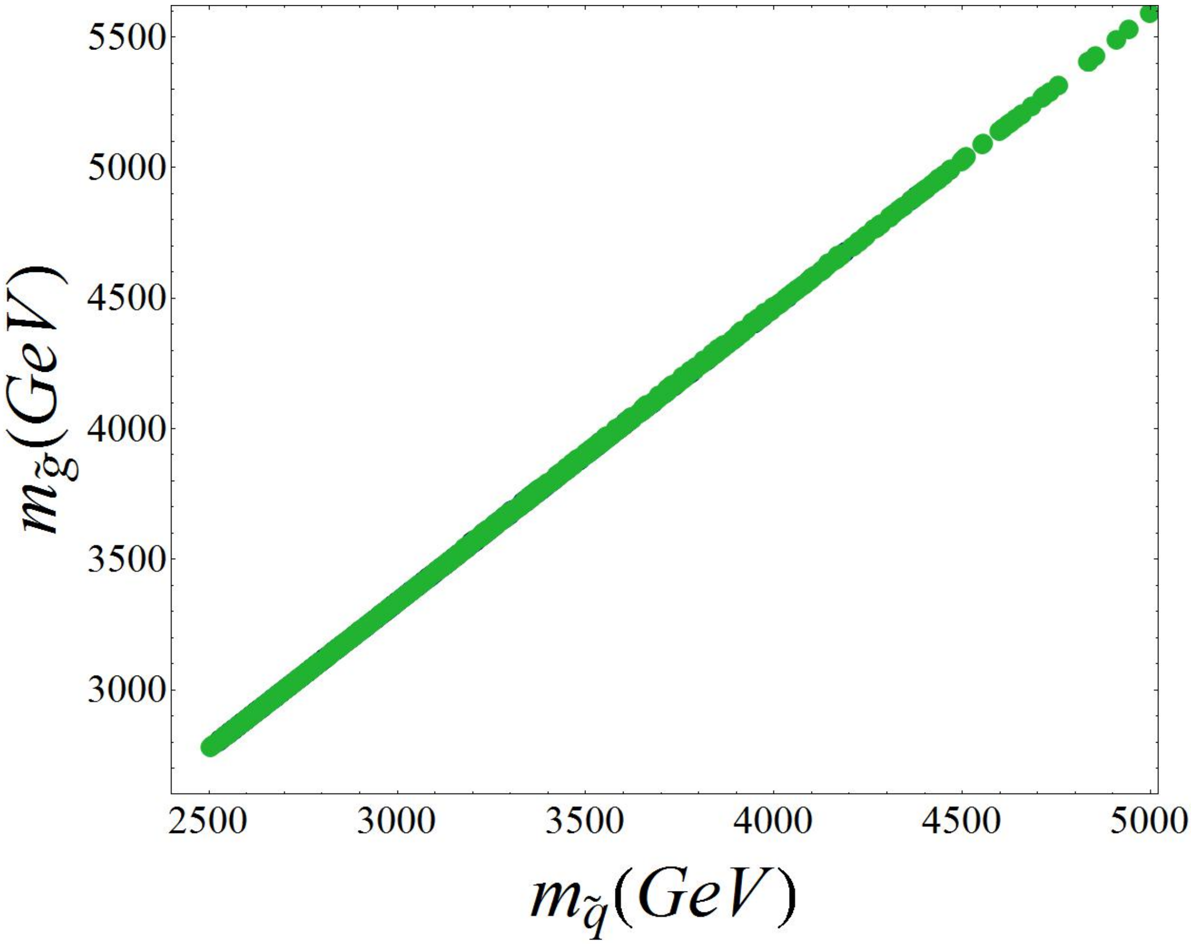


Note that these scenarios all suffer from fine-tuning $\Delta \sim 1000$.


4500
4000
3500
3000
2500
2000
2500
3000
3500
$m_{\tilde{t}_2}(GeV)$
$m_{\tilde{t}_1}(GeV)$


2600
2400
2200
2000
1800
1600
1400
1200
123.0
123.5
124.0
124.5
125.0
125.5
126.0
$m_{A^0}(GeV)$
$m_{h^0}(GeV)$

Interestingly, this model also predicts squarks and gluinos close in mass, even though the scalar GUT scale masses are not small.



$$m_{\tilde{d}_R}(t_{EW}) \approx \sqrt{0.78} m_{\tilde{g}} \sqrt{1 + 0.78 \frac{m_{3/2}^2}{m_{\tilde{g}}^2}}$$

BIM O-II

Gaugino masses at the “GUT” scale:

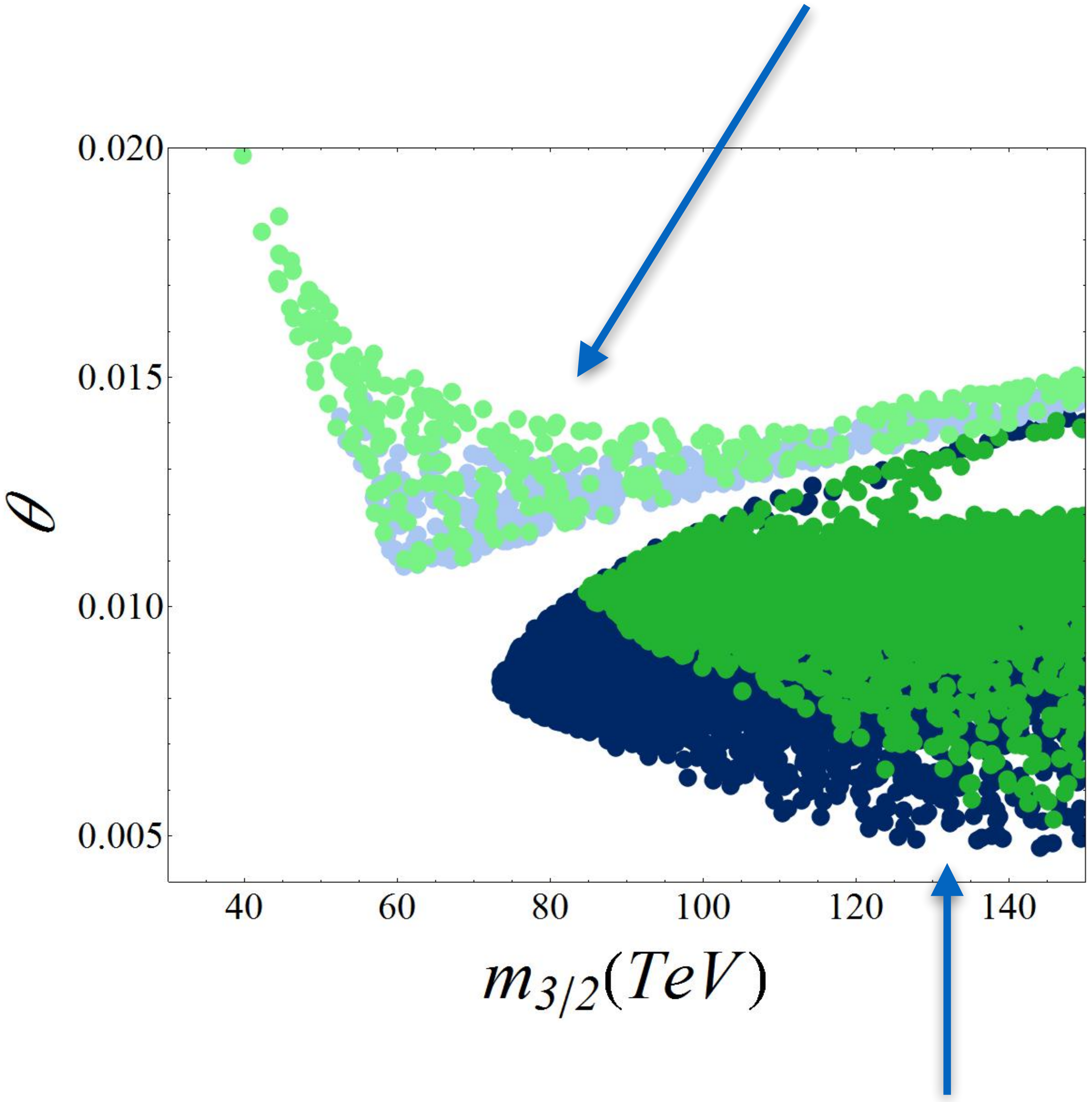
$$
\begin{aligned}
M_1 &= 1.18\sqrt{3}m_{3/2}\left[\sin\theta - \left(\frac{-33}{5}+\delta_{GS}\right)4.6\times 10^{-4}\cos\theta\right],\\
M_2 &= 1.06\sqrt{3}m_{3/2}\left[\sin\theta - \left(-1+\delta_{GS}\right)4.6\times 10^{-4}\cos\theta\right],\\
M_3 &= \sqrt{3}m_{3/2}\left[\sin\theta - \left(3+\delta_{GS}\right)4.6\times 10^{-4}\cos\theta\right].
\end{aligned}
$$

Since $\sin\theta \to 0$, LHC limits imply $m_{3/2} \gtrsim 126\ \mathrm{TeV}$

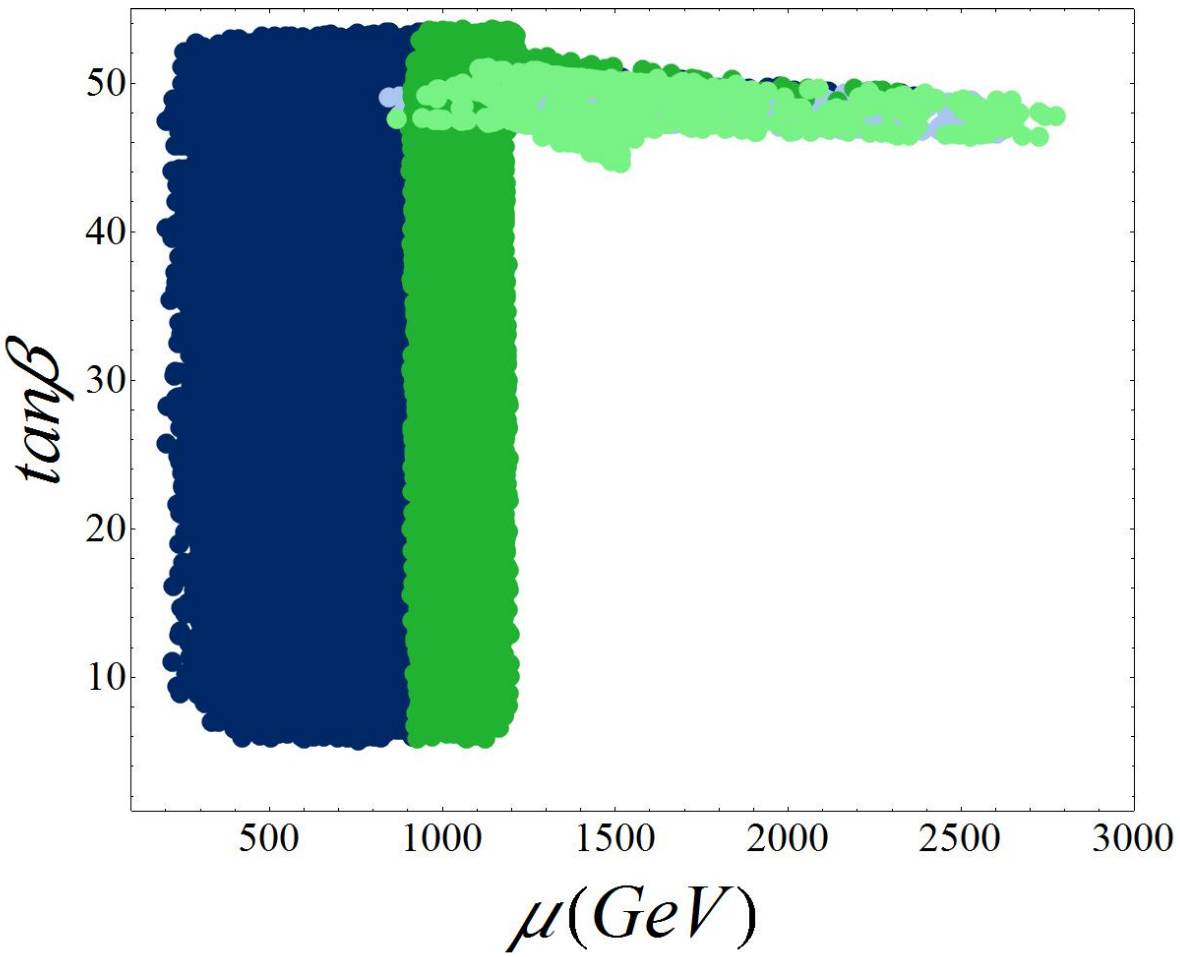
$$m_0^2 \approx m_{3/2}^2\,(-\delta_{GS})\times 10^{-3} \gtrsim (10\,\mathrm{TeV})^2$$

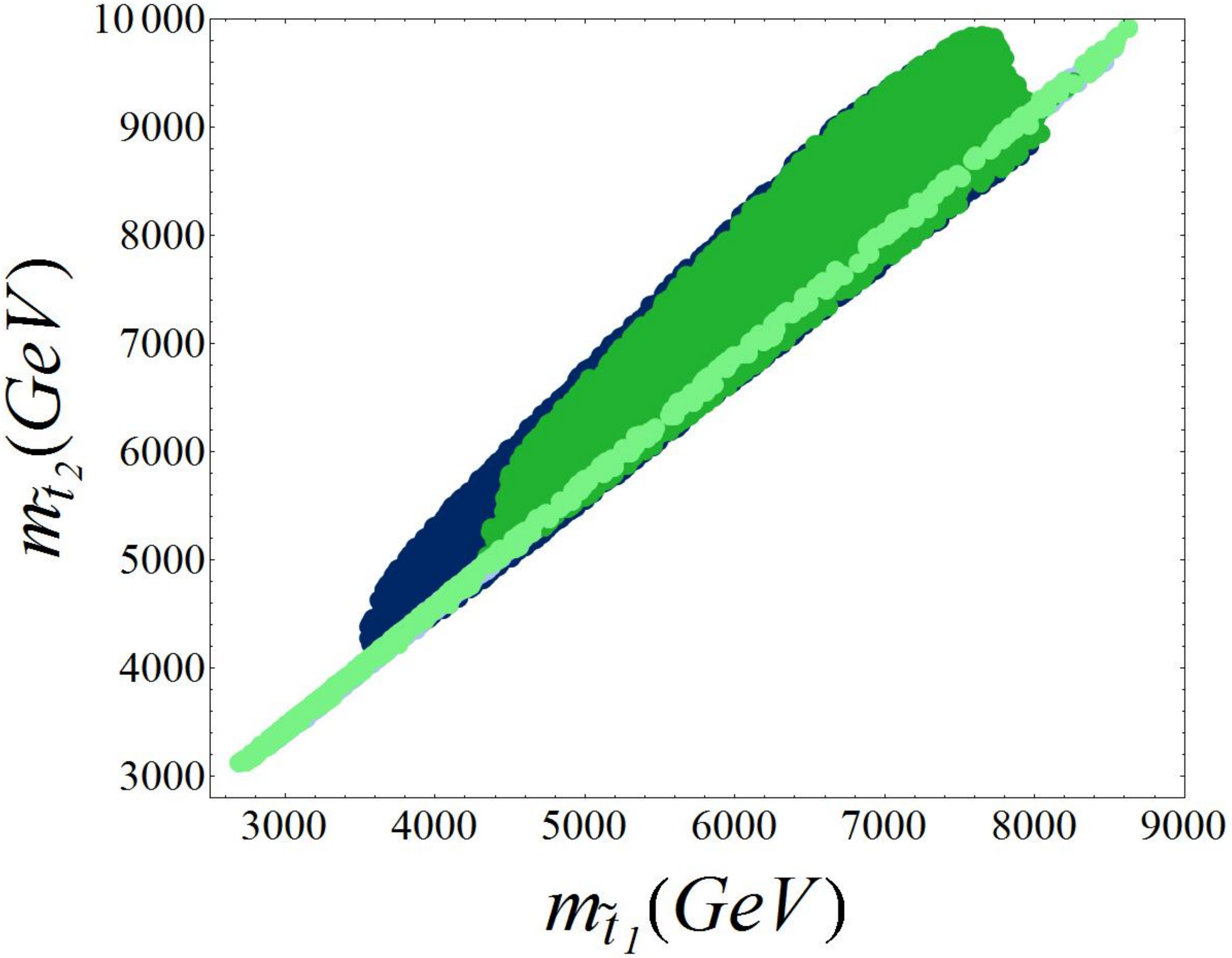
$$a_0 = -\sqrt{3}m_{3/2}\sin\theta$$

Bino dominated DM



Higgsino dominated DM




10 000
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$m_{\tilde{t}_2}(GeV)$
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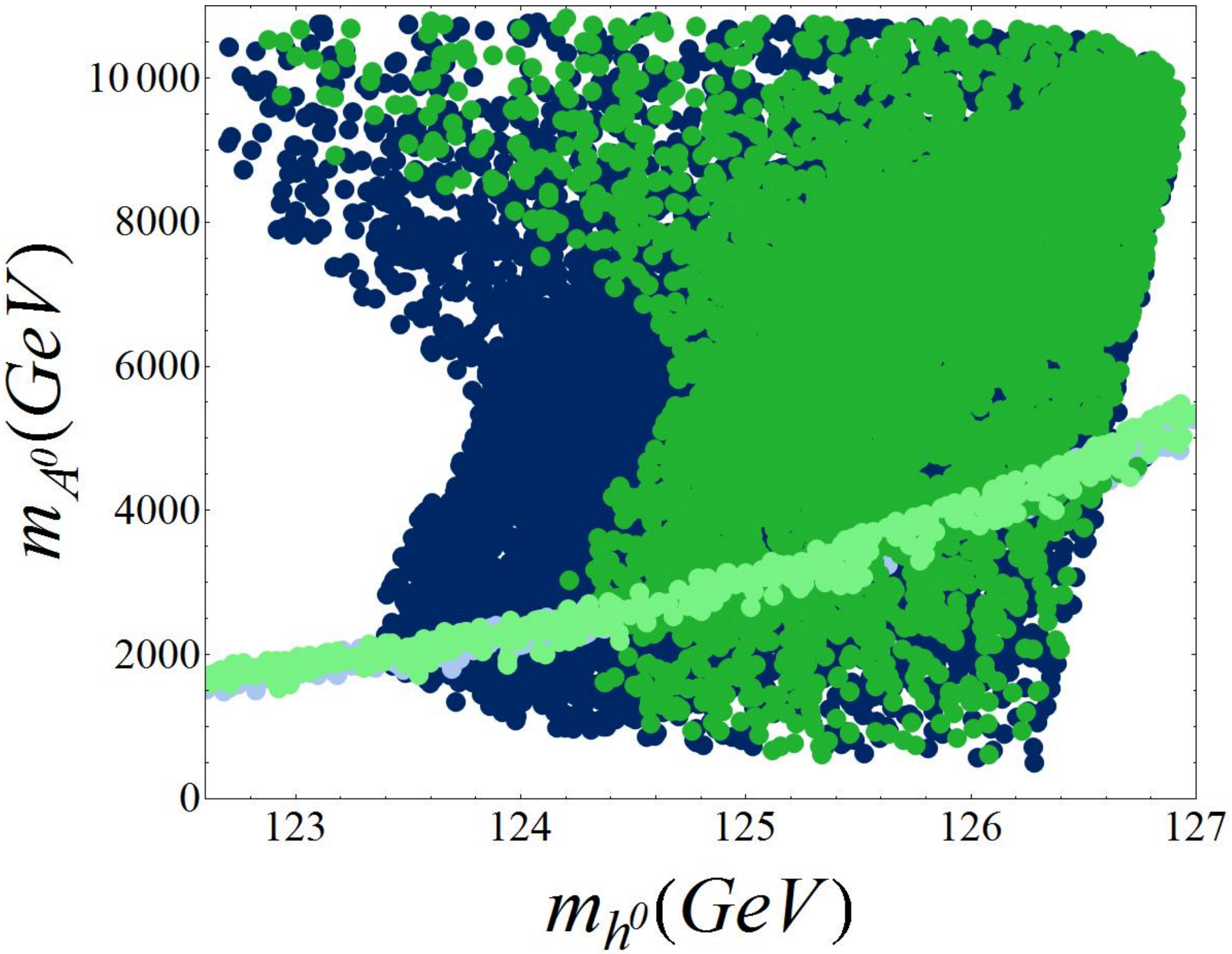
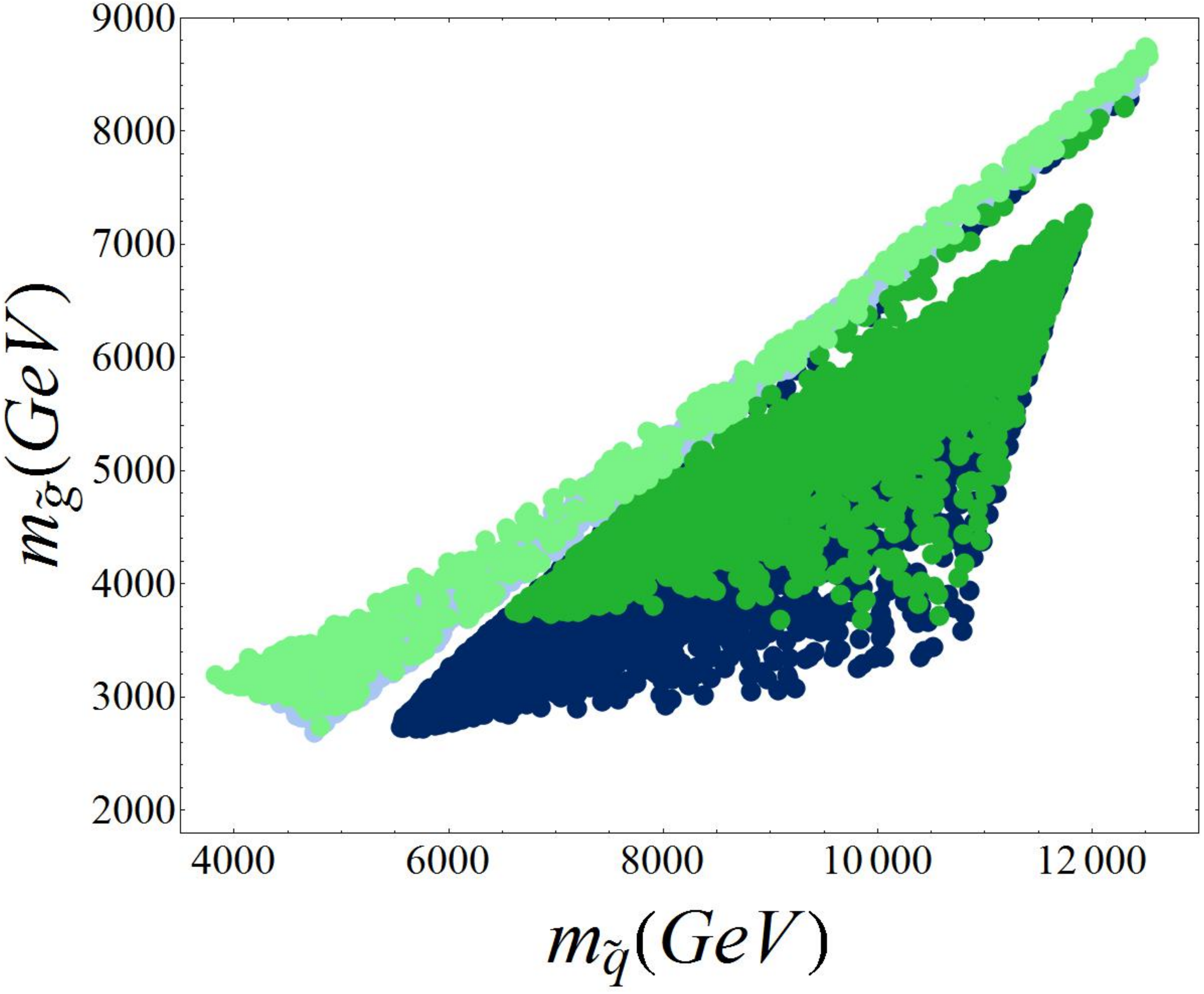
9000
$m_{\tilde{t}_1}(GeV)$


10 000
8000
6000
4000
2000
0
$m_{A^0}(GeV)$
123
124
125
126
127
$m_{h^0}(GeV)$



Lose the correlation between squark and gluino masses due to the really large GUT scale scalar mass.

Summary of Key Points

Saw SUSY scenarios with non-universal gaugino masses where the only fine-tuning arises from μ.

Challenge the community to think up theories where μ is fixed by the UV completion.

In our SO(10) & SU(5) GUTs, constraints from the Higgs discovery and Dark Matter constrain the parameter space much more than direct SUSY searches.

These models can have a very heavy spectrum that will be difficult to see.

We see similar effects in models motivated by orbifolds.

Backup Slides

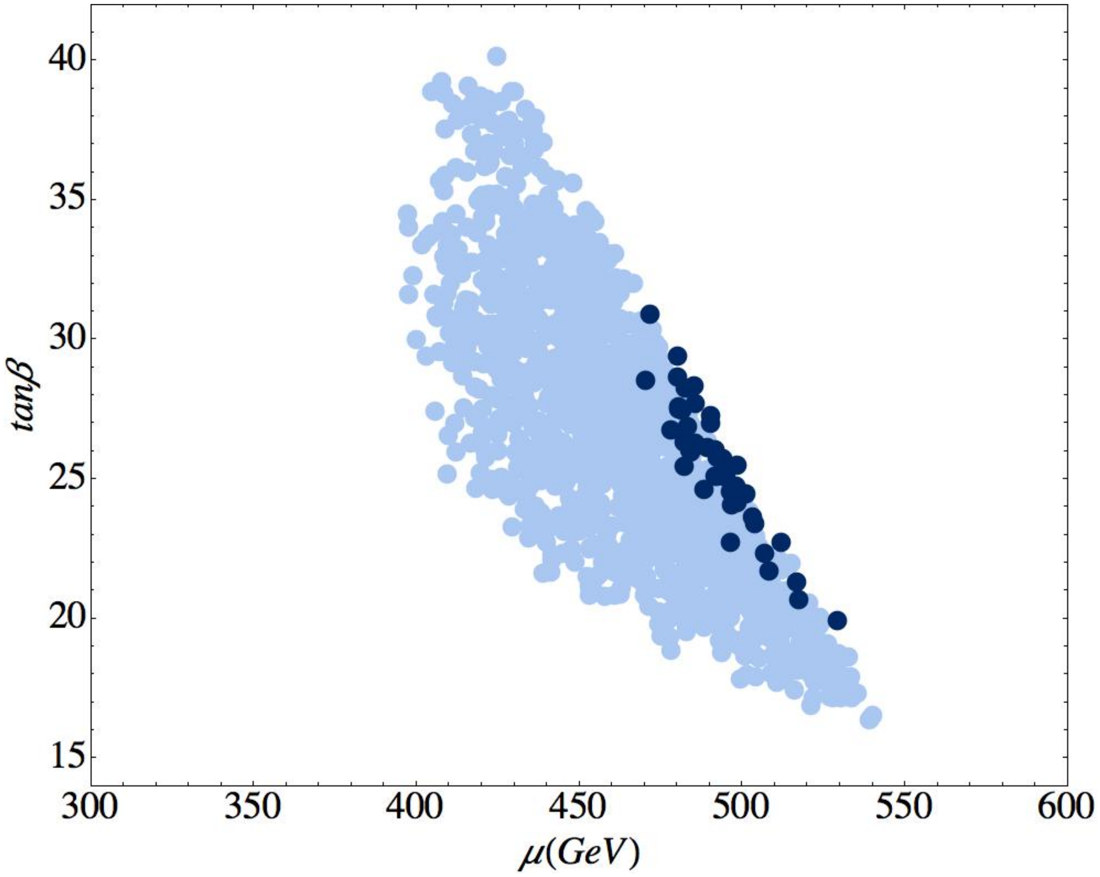
200 of SU(5)

Consider a **200** of SU(5)

$$\rho_1 = 10, \ \rho_2 = 2$$

or $\begin{cases} SO(10) \to SU(5) \times U(1) \\ \mathbf{770} \to \mathbf{220} \end{cases}$

Although this is close to the ellipse is it very difficult to get low fine-tuning and the correct relic density.



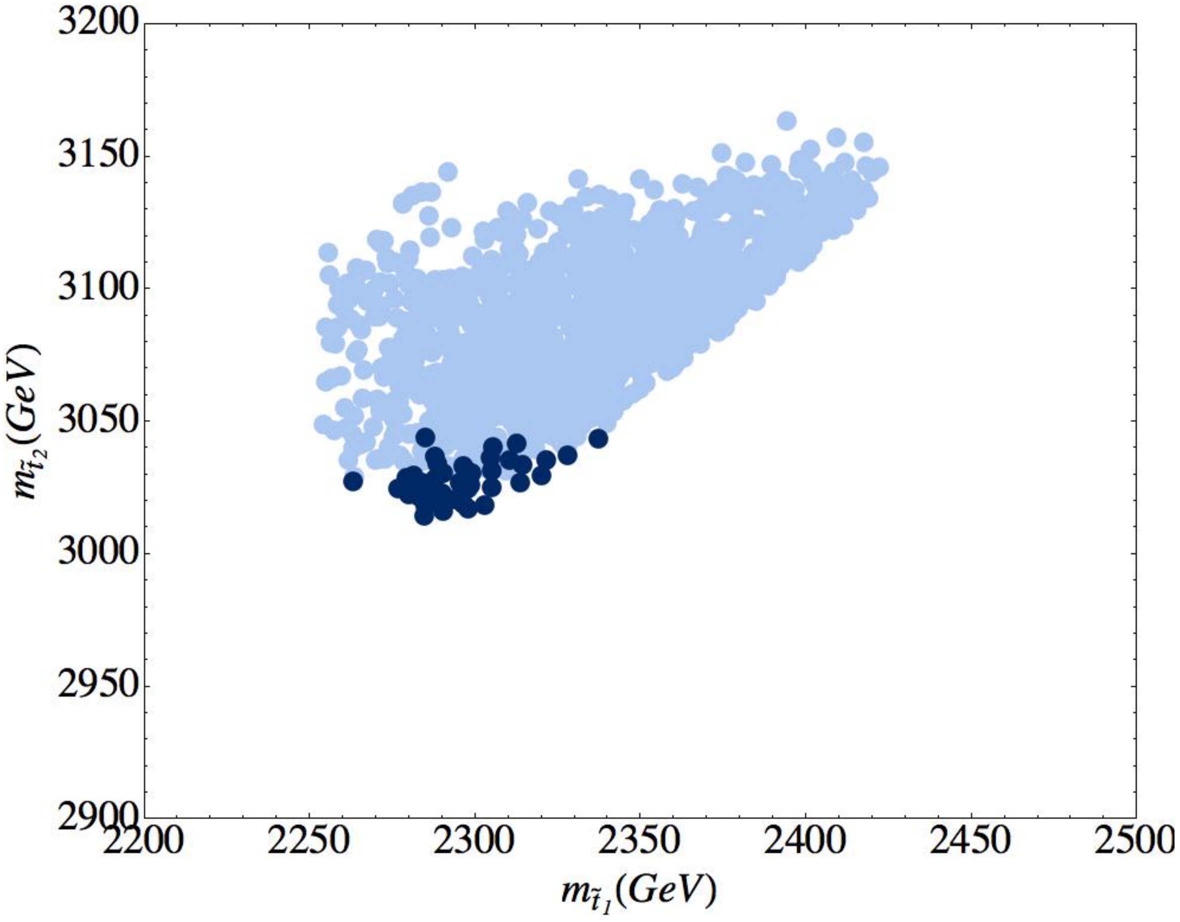
In this plot,

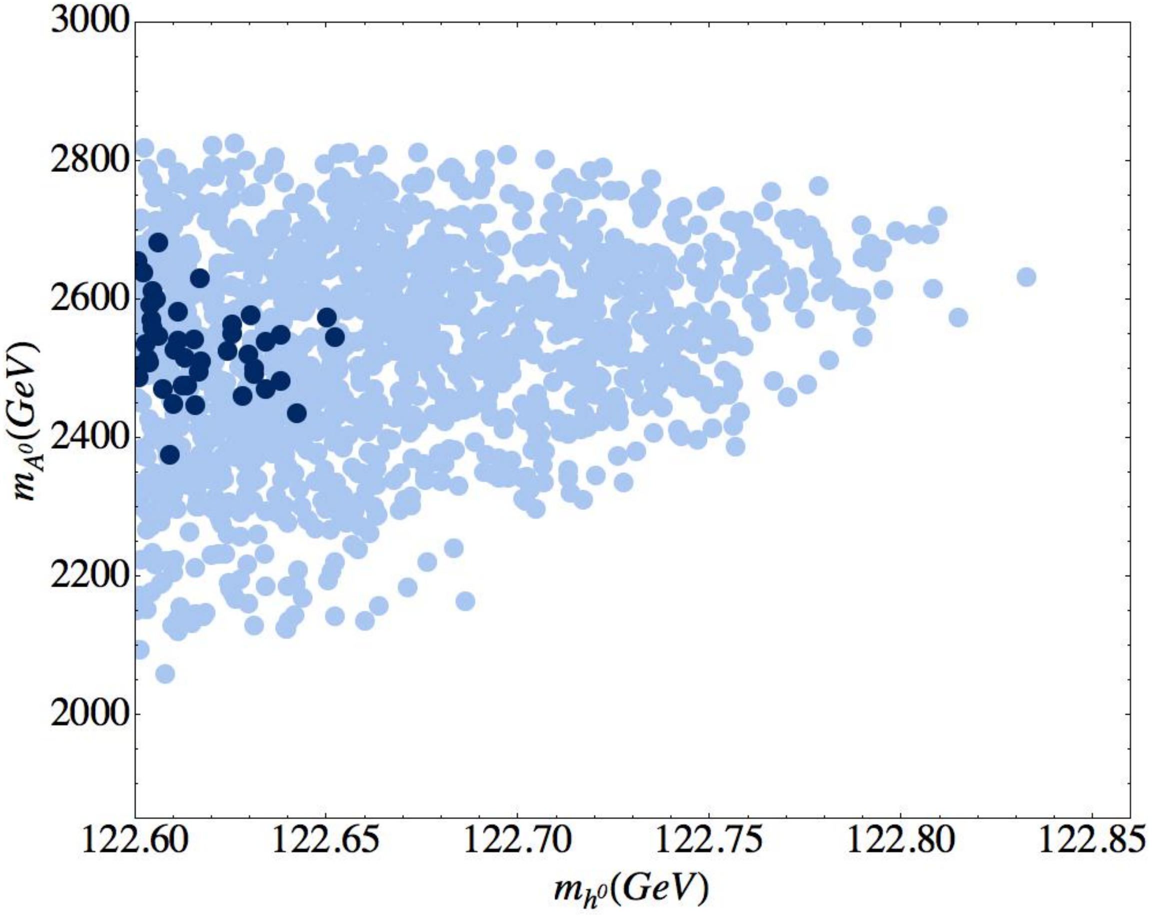
Light:

$$80 < \Delta < 100$$

Dark:

$$\Delta < 80$$





All of these scenarios have too little Dark Matter.

An example SO(10) Pati-Salam scenario:

$m_{\mathbf{16}}$	113.8
$K_{\mathbf{16}}$	12.3
$m_{\mathbf{10+126}}$	132.5
$g_{10}^2 D$	-6674
$a_{\mathbf{10}}$	-116.7
$M_{1/2}$	2471
ρ_1	1.90
ρ_2	2.50

$m_{\tilde{u}_L}$	5785
$m_{\tilde{u}_R}$	4481
$m_{\tilde{d}_L}$	5786
$m_{\tilde{d}_R}$	4417
$m_{\tilde{e}_L}$	4036
$m_{\tilde{e}_R}$	1765
$m_{\tilde{\nu}1}$	4035

$m_{\tilde{t}_1}$	2987
$m_{\tilde{t}_2}$	5243
$m_{\tilde{b}_1}$	4240
$m_{\tilde{b}_2}$	5239
$m_{\tilde{\tau}_1}$	1577
$m_{\tilde{\tau}_2}$	3955
$m_{\tilde{\nu}3}$	3954

$M_{\tilde{g}}$	5175	
$M_{\tilde{\chi}_1^0}$	949.4	(LSP)
$M_{\tilde{\chi}_2^0}$	952.2	
$M_{\tilde{\chi}_3^0}$	2050	
$M_{\tilde{\chi}_4^0}$	5040	
$M_{\tilde{\chi}_1^\pm}$	951.3	(NLSP)
$M_{\tilde{\chi}_2^\pm}$	5040	

m_{h^0}	125.0
m_{A^0}	3842
m_{H^0}	3842
$m_{H^\pm}$	3843
μ	907.5
$\tan\beta$	19.13

$R_{tb\tau}$	4.76
$R_{b\tau}$	1.32
Δ	33.62
Δ_μ	453.5
$\Omega_c h^2$	0.0934