DOES THE 2HDM VIOLATE CP?

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CP
BREAKING
NEWS

The 2HDM potential

$$
\begin{aligned}
V(\Phi_1,\Phi_2) &= -\frac{1}{2}\left\{m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + \left[m_{12}^2\Phi_1^\dagger\Phi_2 + \mathrm{h.c.}\right]\right\} \\
&\quad + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) \\
&\quad + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{1}{2}\left[\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \mathrm{h.c.}\right] \\
&\quad + \left\{\left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right](\Phi_1^\dagger\Phi_2) + \mathrm{h.c.}\right\} \\
&\equiv Y_{a\bar{b}}\Phi_{\bar{a}}^\dagger\Phi_b + \frac{1}{2}Z_{a\bar{b}c\bar{d}}(\Phi_{\bar{a}}^\dagger\Phi_b)(\Phi_{\bar{c}}^\dagger\Phi_d)
\end{aligned}
$$

EWSB

- Vacuum expectation values – most general form that preserves $U(1)_{em}$:

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 e^{i\xi_1} \end{pmatrix}, \quad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\xi_2} \end{pmatrix}$$

Phases ξ_1 and ξ_2 are not ”physical”, and do not themselves appear in physical observables.

$\xi \equiv \xi_2 - \xi_1$ does!

$$v_1^2 + v_2^2 = v^2 = (246\,\mathrm{GeV})^2$$

Stationary point equations

- VEVs should be at minima of the potential
- Demand that derivatives of the fields should be zero here → Stationary point equations
- Use stationary point equations to eliminate the parameters m_{11}^2, m_{22}^2 and $\mathrm{Im}\, m_{12}^2$.
- Left with the following set of parameters:

$$\{\lambda_1, \lambda_2, \lambda_3, \lambda_4, \mathrm{Re}\lambda_5, \mathrm{Im}\lambda_5, \mathrm{Re}\lambda_6, \mathrm{Im}\lambda_6, \mathrm{Re}\lambda_7, \mathrm{Im}\lambda_7, \mu^2, v_1, v_2, \xi, \}$$

- Here, $\mu^2 = \frac{v^2}{2v_1 v_2}\mathrm{Re}\, m_{12}^2$

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Extracting the physical fields

$$\Phi_j = e^{i\xi_j} \begin{pmatrix} \varphi_j^+ \\ (v_j + \eta_j + i\chi_j)/\sqrt{2} \end{pmatrix}, \quad j = 1, 2.$$

Introduce orthogonal states:

$$\begin{pmatrix} G_0 \\ \eta_3 \end{pmatrix} = \begin{pmatrix} v_1/v & v_2/v \\ -v_2/v & v_1/v \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}$$

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} v_1/v & v_2/v \\ -v_2/v & v_1/v \end{pmatrix} \begin{pmatrix} \varphi_1^\pm \\ \varphi_2^\pm \end{pmatrix}.$$

Neutral sector mass matrix

Introduce neutral sector rotation matrix:

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix},$$

satisfying $$R\mathcal{M}^2 R^{\mathrm{T}} = \mathrm{diag}(M_1^2, M_2^2, M_3^2),$$

$$R = \begin{pmatrix} c_1\, c_2 & s_1\, c_2 & s_2 \\ -(c_1\, s_2\, s_3 + s_1\, c_3) & c_1\, c_3 - s_1\, s_2\, s_3 & c_2\, s_3 \\ -c_1\, s_2\, c_3 + s_1\, s_3 & -(c_1\, s_3 + s_1\, s_2\, c_3) & c_2\, c_3 \end{pmatrix}$$

$c_i \equiv \cos\alpha_i,\ s_i \equiv \sin\alpha_i$

Change of parameters

$$RM^2R^{\mathrm{T}} = \mathrm{diag}(M_1^2, M_2^2, M_3^2),$$

constitutes six equations.

- Along with expression for mass of charged scalar we have a total of seven equations.
- Solve this set for seven parameters of the potential: $\{\lambda_1, \lambda_2, \lambda_3, \lambda_4, \mathrm{Re}\lambda_5, \mathrm{Im}\lambda_6, \mathrm{Im}\lambda_7\}$ and replace.
- New set of "physical" parameters

$$\{M_{H^\pm}^2, \mu^2, M_1^2, M_2^2, M_3^2, \mathrm{Im}\lambda_5, \mathrm{Re}\lambda_6, \mathrm{Re}\lambda_7, v_1, v_2, \xi, \alpha_1, \alpha_2, \alpha_3\}$$

Change of basis

- Initial expression of potential is defined with respect to fields Φ_1 and Φ_2 - this defines our initial basis.
- We can change to a new basis by the following transformation

$$\bar{\Phi}_i = U_{ij}\Phi_j$$

where *U* is a U(2) matrix.

- Observables must not depend on choice of basis – they should be basis-independent.

CP violation

- If a basis exists in which all the parameters of the potential and the VEVs are all real. Then the 2HDM is CP conserving.
- If not, the 2HDM breaks CP.
- One can further distinguish between explicit and spontaneous CP violation.

CP violation (Lavoura&Silva)

- Identified three CP-odd invariants. Expressed these in the Higgs-basis;

$$
\begin{aligned}
J_1^{LS} &= (M_1^2 - M_2^2)(M_1^2 - M_3^2)(M_2^2 - M_3^2) R_{11} R_{21} R_{31} \\
J_2^{LS} &= M_k^2 q_k q_i (R_{k2} R_{i3} - R_{k3} R_{i2}) \\
&\quad - M_k^2 R_{k1} q_i q_j (R_{k2} R_{i3} - R_{k3} R_{i2}) \\
J_3^{LS} &= M_k^2 R_{k1} q_i (R_{k2} R_{i3} - R_{k3} R_{i2})
\end{aligned}
$$

$$q_i \equiv \mathrm{Coefficient}(V, H_i H^- H^+)$$

CP violation (Gunion & Haber)

- Identified three CP-breaking invariants. Not limited to Higgs-basis, but basis independent.
- If J_1, J_2, J_3 are all real, CP is conserved.
- If at least one of J_1, J_2, J_3 is complex, CP is broken.

$$\mathrm{Im}\, J_1 = -\frac{2}{v^2}\mathrm{Im}\left[\hat{v}^*_{\bar{a}} Y_{a\bar{b}} Z^{(1)}_{b\bar{d}} \hat{v}_d\right],$$

$$\mathrm{Im}\, J_2 = \frac{4}{v^4}\mathrm{Im}\left[\hat{v}^*_{\bar{b}} \hat{v}^*_{\bar{c}} Y_{b\bar{e}} Y_{c\bar{f}} Z_{e\bar{a}f\bar{d}} \hat{v}_a \hat{v}_d\right],$$

$$\mathrm{Im}\, J_3 = \mathrm{Im}\left[\hat{v}^*_{\bar{b}} \hat{v}^*_{\bar{c}} Z^{(1)}_{b\bar{e}} Z^{(1)}_{c\bar{f}} Z_{e\bar{a}f\bar{d}} \hat{v}_a \hat{v}_d\right].$$

- Distinguishes between sponaneous and explicit CP violation.

CP violation (Ivanov)

- Geometric approach. Minkowski-space formalism.
- Basis independent.
- Introduces vector M^μ and matrix $\Lambda^{\mu\nu}$.
- CP is broken if at most one of the eigenvectors of Λ_{ij} is orthogonal to M_i.
- Distinguishes between sponaneous and explicit CP violation.

Desire

- Find conditions for CP conservation/violation expressed in terms of <u>observable quantities.</u>
- Observable quantities: Masses and couplings.
- Require basis-independent formalism.
- Then find which processes can be used to determine the CP properties of 2HDM.
- Formalism of Gunion&Haber seems well suited for this.

Approach

- Write out Im J_1, Im J_2, Im J_3 in general basis.
- Write out all the couplings from Higgs-sector and weak-sector. No Yukawas.
- Introduce ”physical parameter set” in the expressions above

$$\{M_{H^\pm}^2, \mu^2, M_1^2, M_2^2, M_3^2, \mathrm{Im}\lambda_5, \mathrm{Re}\lambda_6, \mathrm{Re}\lambda_7, v_1, v_2, \xi, \alpha_1, \alpha_2, \alpha_3\}$$

- Compare and try to find connections.
- MATHEMATICA efficient tool for this.
- Success!

Findings for Im J_2

$$\begin{aligned}\operatorname{Im} J_2 &= 2\frac{e_1 e_2 e_3}{v^9}(M_1^2 - M_2^2)(M_2^2 - M_3^2)(M_3^2 - M_1^2) \\ &= \frac{2}{v^9}\sum_{i,j,k}\epsilon_{ijk} e_i e_j e_k M_i^4 M_k^2 = \frac{2 e_1 e_2 e_3}{v^9}\sum_{i,j,k}\epsilon_{ijk} M_i^4 M_k^2\end{aligned}$$

where

$e_i \equiv v_1 R_{i1} + v_2 R_{i2}$

is a basis-invariant expression representing several couplings.

The coupling e_i:

$$H_i H_j Z_\mu : \quad \frac{g}{2v\cos\theta_W}\epsilon_{ijk}e_k(p_i - p_j)_\mu, \qquad H_i H_j G_0 : \quad i\frac{M_i^2 - M_j^2}{v^2}\epsilon_{ijk}e_k,$$

and

$$H_i Z_\mu Z_\nu : \quad \frac{ig^2}{2\cos^2\theta_W}e_i\, g_{\mu\nu}, \qquad H_i W_\mu^+ W_\nu^- : \quad \frac{ig^2}{2}e_i\, g_{\mu\nu},$$

$$H_i G_0 G_0 : \quad \frac{-iM_i^2 e_i}{v^2}, \qquad H_i G^+ G^- : \quad \frac{-iM_i^2 e_i}{v^2},$$

$$H_i G^+ A_\mu W_\nu^- : \quad \frac{ig^2 \sin\theta_W}{2v}e_i\, g_{\mu\nu}, \qquad H_i G^- A_\mu W_\nu^+ : \quad \frac{ig^2 \sin\theta_W}{2v}e_i\, g_{\mu\nu},$$

$$H_i G^+ Z_\mu W_\nu^- : \quad -\frac{ig^2}{2v}\frac{\sin^2\theta_W}{\cos\theta_W}e_i\, g_{\mu\nu} \qquad H_i G^- Z_\mu W_\nu^+ : \quad -\frac{ig^2}{2v}\frac{\sin\theta_W^2}{\cos\theta_W}e_i\, g_{\mu\nu},$$

$$H_i G_0 Z_\mu : \quad \frac{g}{2v\cos\theta_W}e_i(p_i - p_0)_\mu,$$

$$H_i G^+ W_\mu^- : \quad i\frac{g}{2v}e_i(p_i - p^+)_\mu, \qquad H_i G^- W_\mu^+ : \quad -i\frac{g}{2v}e_i(p_i - p^-)_\mu.$$

The coupling e_i:

$$H_iH_jZ_\mu : \quad \frac{g}{2v\cos\theta_W}\epsilon_{ijk}e_k(p_i - p_j)_\mu,$$

and

$$H_iZ_\mu Z_\nu : \quad \frac{ig^2}{2\cos^2\theta_W}e_i\, g_{\mu\nu}, \qquad H_iW_\mu^+W_\nu^- : \quad \frac{ig^2}{2}e_i\, g_{\mu\nu},$$

$$e_i \equiv v_1 R_{i1} + v_2 R_{i2}$$

$$e_1^2 + e_2^2 + e_3^3 = v^2 = (246\,\mathrm{GeV})^2$$

Findings for Im J_1

$$
\begin{aligned}
\operatorname{Im} J_1 &= \frac{1}{v^5} \sum_{i,j,k} \epsilon_{ijk} M_i^2 e_i e_k q_j \\
&= \frac{1}{v^5} [M_1^2 e_1 (e_3 q_2 - e_2 q_3) + M_2^2 e_2 (e_1 q_3 - e_3 q_1) \\
&\qquad + M_3^2 e_3 (e_2 q_1 - e_1 q_2)].
\end{aligned}
$$

Findings for Im J_3

$$\mathrm{Im}\, J_3 = K\,\mathrm{Im}\, J_1 + \mathrm{Im}\, J_2 + \frac{2}{v^7}\sum_{i,j,k}\epsilon_{ijk}(v^2 q_i + 2e_i M_i^2)M_i^2 e_j q_k$$

If we put Im J_1 = Im J_2 = 0, we find the only term in this expression that survives is the second last term.

$$\mathrm{Im}\, J_{30} = \frac{1}{v^5}\sum_{i,j,k}\epsilon_{ijk} q_i M_i^2 e_j q_k$$

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Reformulated conditions for CP violation

- If Im J_1 = Im J_2 = Im J_{30} = 0, CP is conserved.
- If at least one of Im J_1, Im J_2, Im J_{30} is nonzero, CP is broken.

Expressed as determinants

$$\operatorname{Im} J_1 = \frac{1}{v^5} \begin{vmatrix} q_1 & q_2 & q_3 \\ e_1 & e_2 & e_3 \\ e_1 M_1^2 & e_2 M_2^2 & e_3 M_3^2 \end{vmatrix},$$

$$\operatorname{Im} J_2 = \frac{2}{v^9} \begin{vmatrix} e_1 & e_2 & e_3 \\ e_1 M_1^2 & e_2 M_2^2 & e_3 M_3^2 \\ e_1 M_1^4 & e_2 M_2^4 & e_3 M_3^4 \end{vmatrix},$$

$$\operatorname{Im} J_{30} = \frac{1}{v^5} \begin{vmatrix} e_1 & e_2 & e_3 \\ q_1 & q_2 & q_3 \\ q_1 M_1^2 & q_2 M_2^2 & q_3 M_3^2 \end{vmatrix}.$$

Comparing to Lavoura&Silva:

$$
\begin{aligned}
[\mathrm{Im}\, J_2]_{\text{Higgs-Basis}} &= -2v^6 J_1^{\text{LS}}, \\
[\mathrm{Im}\, J_1]_{\text{Higgs-Basis}} &= v^3 J_3^{\text{LS}}, \\
[\mathrm{Im}\, J_{30}]_{\text{Higgs-Basis}} &= -v^4 J_2^{\text{LS}}.
\end{aligned}
$$

When is CP conserved?

- Put Im J_1 = Im J_2 = Im J_{30} = 0 and solve

6 distinct cases:

- Case 1: $M_1=M_2=M_3$. Full mass degeneracy.
- Case 2: $M_1=M_2$ and $e_1q_2 = e_2q_1$
- Case 3: $M_2=M_3$ and $e_2q_3 = e_3q_2$
- Case 4: $e_1=0$ and $q_1=0$
- Case 5: $e_2=0$ and $q_2=0$
- Case 6: $e_3=0$ and $q_3=0$

If none of the above occur, then CP is broken!

When is CP conserved?

- Put Im J_1 = Im J_2 = Im J_{30} = 0 and solve

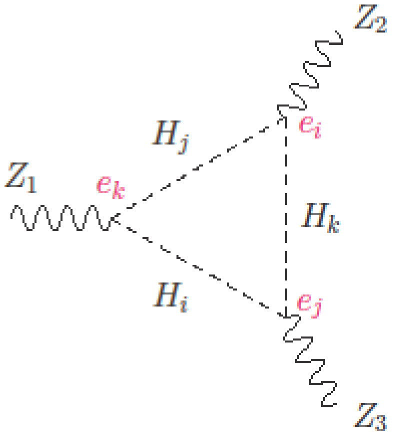
6 distinct cases:

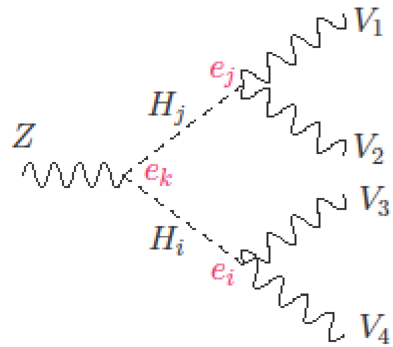
- Case 1: $M_1=M_2=M_3$. Full mass degeneracy.
- Case 2: $M_1=M_2$ and $(H_1ZZ)(H_2H^+H^-) = (H_2ZZ)(H_1H^+H^-)$
- Case 3: $M_2=M_3$ and $(H_2ZZ)(H_3H^+H^-) = (H_3ZZ)(H_2H^+H^-)$
- Case 4: $(H_1ZZ) = 0$ and $(H_1H^+H^-) = 0$
- Case 5: $(H_2ZZ) = 0$ and $(H_2H^+H^-) = 0$
- Case 6: $(H_3ZZ) = 0$ and $(H_3H^+H^-) = 0$

If none of the above occur, then CP is broken!

Processes containing Im J_2:

- ZZZ vertex or $Z \to VVVV$



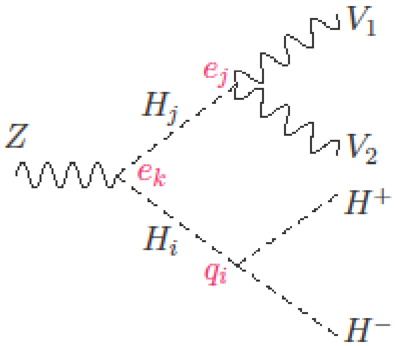


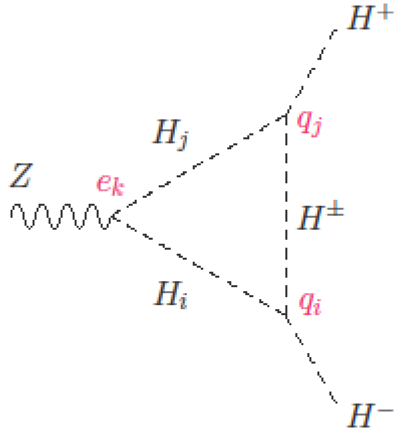
- Summing over all possible combinations of *i,j,k*, we find

$$\mathcal{M} \propto \mathrm{Im} J_2$$

Processes containing Im J_1 and Im J_3 :

- $Z \to VVH^+H^-$ or $Z \to H^+H^-$



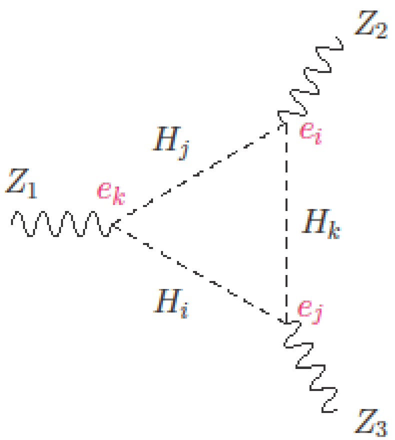


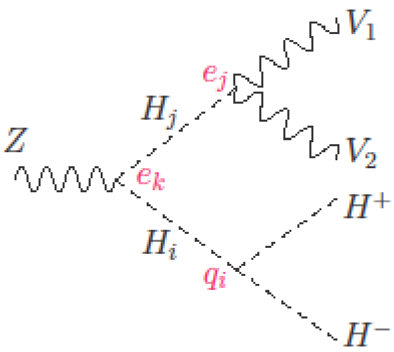
- Summing over all possible combinations of *i,j,k*, we find

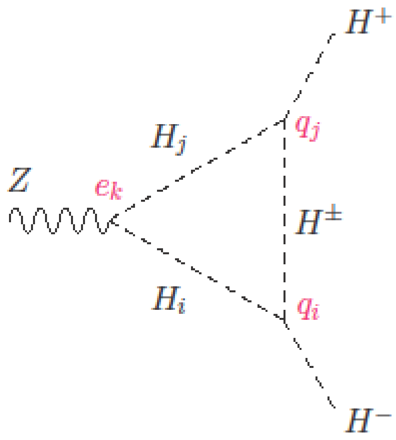
$\mathcal{M}$ contains $\mathrm{Im} J_1$ $\mathcal{M}$ contains $\mathrm{Im} J_3$

Summary

- Three processes identified in order to determine if Higgs-sector of 2HDM is CP violating or not!







- Experimental challenge!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AYE.. THE MORE I DRINK THE LESS THINGS MATTER
VIEWER
R. LINGLEY 4.89