The approach to CP conservation in the 2HDM

MHD 2014
Lisboa September 2014

Per Osland
University of Bergen

actually, two talks

(for the price of one)

CP violation in the 2HDM

(generalities on explicit and spontaneous CP violation)

CP violating ZZZ vertex

(“Trailer” for OMO’s talk)

CP violation in the 2HDM

(generalities on explicit and spontaneous CP violation)

Work with:

Bohdan Grzadkowski, Odd Magne Øgreid

JHEP 2014, see also talk by OMØ, Scalars 2011

2HDM notation 1

$$
\begin{aligned}
V(\Phi_1, \Phi_2) &= -\frac{1}{2}\left\{m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}\right]\right\} \\
&+ \frac{\lambda_1}{2}(\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\
&+ \frac{1}{2}\left[\lambda_5(\Phi_1^\dagger \Phi_2)^2 + \text{h.c.}\right] + \left\{\left[\lambda_6(\Phi_1^\dagger \Phi_1) + \lambda_7(\Phi_2^\dagger \Phi_2)\right](\Phi_1^\dagger \Phi_2) + \text{h.c.}\right\} \\
&\equiv Y_{a\bar{b}} \Phi_{\bar{a}}^\dagger \Phi_b + \frac{1}{2} Z_{a\bar{b}c\bar{d}}(\Phi_{\bar{a}}^\dagger \Phi_b)(\Phi_{\bar{c}}^\dagger \Phi_d)
\end{aligned}
$$

No FCNC: $\lambda_6 = 0; \quad \lambda_7 = 0$ (first part)

Allow CPV: $M_{12}^2, \lambda_5, \lambda_6, \lambda_7$ complex

2HDM notation 2

$$\Phi_j = e^{i\xi_j} \left(\begin{array}{c} \varphi_j^+ \\ (v_j + \eta_j + i\chi_j)/\sqrt{2} \end{array} \right), \quad j = 1, 2$$

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}$$

$$\eta_3 = -\sin\beta\chi_1 + \cos\beta\chi_2$$

$$R\mathcal{M}^2 R^{\mathrm{T}} = \mathcal{M}^2_{\mathrm{diag}} = \mathrm{diag}(M_1^2, M_2^2, M_3^2)$$

The rotation matrix R

$$R = R_3\, R_2\, R_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_3 & \sin\alpha_3 \\ 0 & -\sin\alpha_3 & \cos\alpha_3 \end{pmatrix} \begin{pmatrix} \cos\alpha_2 & 0 & \sin\alpha_2 \\ 0 & 1 & 0 \\ -\sin\alpha_2 & 0 & \cos\alpha_2 \end{pmatrix} \begin{pmatrix} \cos\alpha_1 & \sin\alpha_1 & 0 \\ -\sin\alpha_1 & \cos\alpha_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_1\, c_2 & s_1\, c_2 & s_2 \\ -(c_1\, s_2\, s_3 + s_1\, c_3) & c_1\, c_3 - s_1\, s_2\, s_3 & c_2\, s_3 \\ -c_1\, s_2\, c_3 + s_1\, s_3 & -(c_1\, s_3 + s_1\, s_2\, c_3) & c_2\, c_3 \end{pmatrix}$$

$$c_i = \cos\alpha_i,\; s_i = \sin\alpha_i$$

3-dimensional parameter space

CP will be conserved on certain manifolds embedded in this space.

In the absence of a Yukawa sector
(which might distinguish the two doublets),
there is a reparametrization invariance

If CP is conserved, or spontaneously violated, then a basis exists in which all the parameters of the potential are real

We follow approach by Gunion & Haber, study invariants:

CP conservation:

Define:

$$
\begin{aligned}
\operatorname{Im} J_1 &= -\frac{2}{v^2}\operatorname{Im}\left[\hat{v}^*_{\bar{a}} Y_{a\bar{b}} Z^{(1)}_{b\bar{d}}\,\hat{v}_d\right] \\
&= -\frac{v_1^2 v_2^2}{v^4}(\lambda_1-\lambda_2)\operatorname{Im}\lambda_5 \\
\operatorname{Im} J_2 &= \frac{4}{v^4}\operatorname{Im}\left[\hat{v}^*_{\bar{b}}\hat{v}^*_{\bar{c}} Y_{b\bar{e}} Y_{c\bar{f}} Z_{e\bar{a}f\bar{d}}\hat{v}_a\hat{v}_d\right] \\
&= -\frac{v_1^2 v_2^2}{v^8}\left[\left((\lambda_1-\lambda_3-\lambda_4)^2-|\lambda_5|^2\right)v_1^4 + 2(\lambda_1-\lambda_2)\operatorname{Re}\lambda_5 v_1^2 v_2^2 \right. \\
&\qquad\qquad \left. -\left((\lambda_2-\lambda_3-\lambda_4)^2-|\lambda_5|^2\right)v_2^4\right]\operatorname{Im}\lambda_5 \\
\operatorname{Im} J_3 &= \operatorname{Im}\left[\hat{v}^*_{\bar{b}}\hat{v}^*_{\bar{c}} Z^{(1)}_{b\bar{e}} Z^{(1)}_{c\bar{f}} Z_{e\bar{a}f\bar{d}}\hat{v}_a\hat{v}_d\right] \\
&= \frac{v_1^2 v_2^2}{v^4}(\lambda_1-\lambda_2)(\lambda_1+\lambda_2+2\lambda_4)\operatorname{Im}\lambda_5
\end{aligned}
$$

CPC: $\boxed{\operatorname{Im} J_1 = \operatorname{Im} J_2 = \operatorname{Im} J_3 = 0}$

CP conservation:

Define:

Second lines valid in “2HDM5”

$$\mathrm{Im}\,J_1 = -\frac{2}{v^2}\mathrm{Im}\left[\hat{v}^*_{\bar{a}} Y_{a\bar{b}} Z^{(1)}_{b\bar{d}} \hat{v}_d\right]$$

$$\longrightarrow \quad = -\frac{v_1^2 v_2^2}{v^4}(\lambda_1 - \lambda_2)\mathrm{Im}\,\lambda_5$$

$$\mathrm{Im}\,J_2 = \frac{4}{v^4}\mathrm{Im}\left[\hat{v}^*_{\bar{b}}\hat{v}^*_{\bar{c}} Y_{b\bar{e}} Y_{c\bar{f}} Z_{e\bar{a}f\bar{d}} \hat{v}_a \hat{v}_d\right]$$

$$\longrightarrow \quad = -\frac{v_1^2 v_2^2}{v^8}\left[\left((\lambda_1 - \lambda_3 - \lambda_4)^2 - |\lambda_5|^2\right) v_1^4 + 2(\lambda_1 - \lambda_2)\mathrm{Re}\,\lambda_5 v_1^2 v_2^2 - \left((\lambda_2 - \lambda_3 - \lambda_4)^2 - |\lambda_5|^2\right) v_2^4\right]\mathrm{Im}\,\lambda_5$$

$$\mathrm{Im}\,J_3 = \mathrm{Im}\left[\hat{v}^*_{\bar{b}}\hat{v}^*_{\bar{c}} Z^{(1)}_{b\bar{e}} Z^{(1)}_{c\bar{f}} Z_{e\bar{a}f\bar{d}} \hat{v}_a \hat{v}_d\right]$$

$$\longrightarrow \quad = \frac{v_1^2 v_2^2}{v^4}(\lambda_1 - \lambda_2)(\lambda_1 + \lambda_2 + 2\lambda_4)\mathrm{Im}\,\lambda_5$$

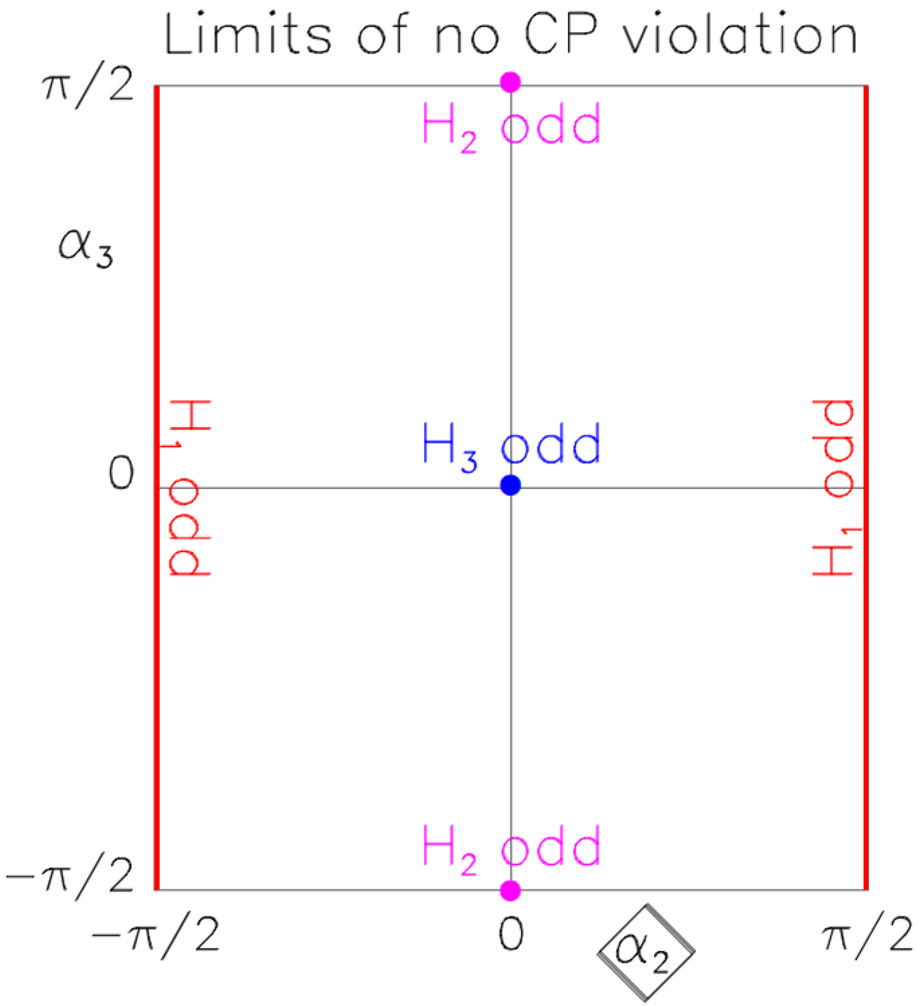
$$\boxed{\text{CPC:} \qquad \mathrm{Im}\,J_1 = \mathrm{Im}\,J_2 = \mathrm{Im}\,J_3 = 0}$$

CP conservation:

(1) Where in the α space do we have CP conservation?

$$\alpha = \{\alpha_1, \alpha_2, \alpha_3\}$$

Answered in part in hep-ph/0702097



Explicit or Spontaneous CP conservation?

more invariants!

$$\begin{aligned}
I_{Y3Z} &= \mathrm{Im}\left[Z^{(1)}_{a\bar{c}} Z^{(1)}_{e\bar{b}} Z_{b\bar{e}c\bar{d}} Y_{d\bar{a}}\right] \\
&= 0 \\
I_{2Y2Z} &= \mathrm{Im}\left[Y_{a\bar{b}} Y_{c\bar{d}} Z_{b\bar{a}d\bar{f}} Z^{(1)}_{f\bar{c}}\right] \\
&= \frac{1}{4}(\lambda_1 - \lambda_2)\mathrm{Im}\ \left[(m_{12}^2)^2 \lambda_5^*\right] \\
&= \frac{v_1^2 v_2^2}{4v^4}(\lambda_1 - \lambda_2)\left[4v^2\mu^2 \mathrm{Re}\,\lambda_5 - 4\mu^4 + v^4(\mathrm{Im}\,\lambda_5)^2\right]\mathrm{Im}\,\lambda_5 \\
I_{3Y3Z} &= \mathrm{Im}\left[Z_{a\bar{c}b\bar{d}} Z_{c\bar{e}d\bar{g}} Z_{e\bar{h}f\bar{q}} Y_{g\bar{a}} Y_{h\bar{b}} Y_{q\bar{f}}\right] \\
&= -\frac{1}{8}(m_{11}^2 - m_{22}^2)\left[(\lambda_1 - \lambda_3 - \lambda_4)(\lambda_2 - \lambda_3 - \lambda_4) - |\lambda_5|^2\right]\mathrm{Im}\ \left[(m_{12}^2)^2\lambda_5^*\right] \\
&= -\frac{v_1^2 v_2^2}{8v^6}\left[(\lambda_1 - \lambda_3 - \lambda_4)(\lambda_2 - \lambda_3 - \lambda_4) - |\lambda_5|^2\right] \\
&\times \left[(v_1^2 - v_2^2)(2\mu^2 - v^2(\lambda_3 + \lambda_4 + \mathrm{Re}\,\lambda_5)) + v^2(v_1^2\lambda_1 - v_2^2\lambda_2)\right] \\
&\times \left[4v^2\mu^2\mathrm{Re}\,\lambda_5 - 4\mu^4 + v^4(\mathrm{Im}\,\lambda_5)^2\right]\mathrm{Im}\,\lambda_5 \\
I_{6Z} &= \mathrm{Im}\left[Z_{a\bar{b}c\bar{d}} Z^{(1)}_{b\bar{f}} Z^{(1)}_{d\bar{h}} Z_{f\bar{a}j\bar{k}} Z_{k\bar{j}m\bar{n}} Z_{n\bar{m}h\bar{c}}\right] \\
&= 0
\end{aligned}$$

In general the CP violation is *explicit* if

$$I_{Y3Z} \neq 0 \quad and/or \quad I_{2Y2Z} \neq 0 \quad and/or \quad I_{3Y3Z} \neq 0 \quad and/or \quad I_{6Z} \neq 0$$

(2) Where in the α space is CP violation spontaneous?

Plots for

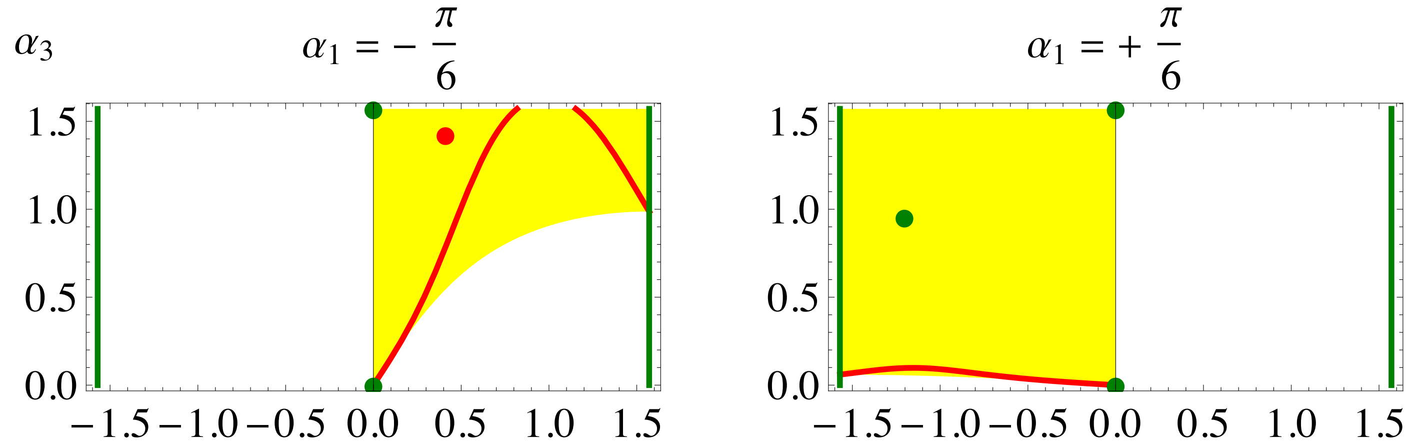
$M_1 = 125\ \mathrm{GeV}, \quad M_2 = 200\ \mathrm{GeV}, \quad M_{H^\pm} = 350\ \mathrm{GeV}, \quad \mu = 250\ \mathrm{GeV}$

$\tan\beta = 2$

Green (lines, dots): CPC

Red (lines, dots): SCPV

Yellow region: ECPV



White regions:

No solution to

$$M_3^2 > 0 \quad \text{and} \quad M_3 > M_2$$

Plots for

$M_1 = 125$ GeV, $\quad M_2 = 200$ GeV, $\quad M_{H^\pm} = 350$ GeV, $\quad \mu = 250$ GeV

$\tan\beta = 2$

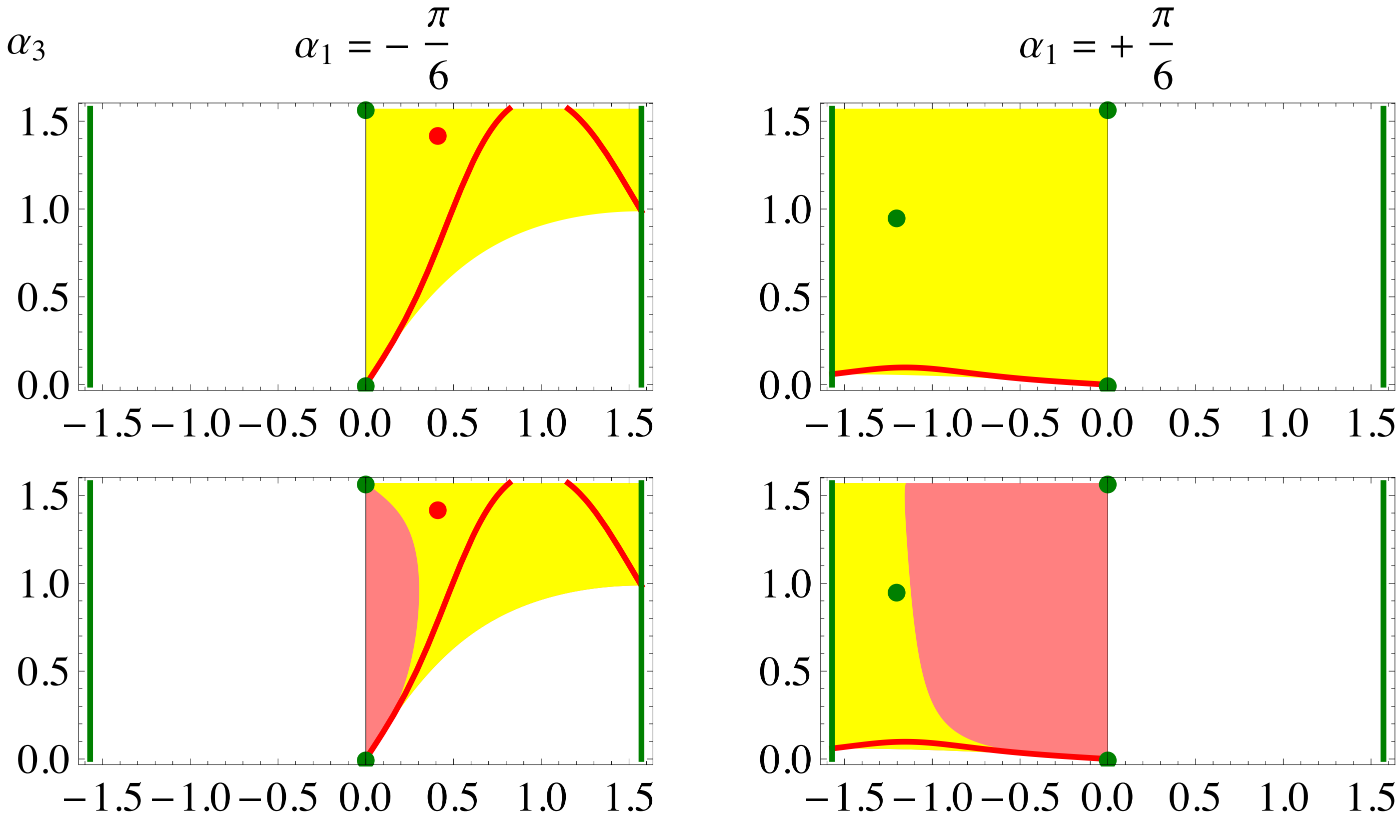
Green (lines, dots): CPC

Red (lines, dots): SCPV

Yellow region: ECPV

Violations:

Pink region: positivity



Plots for

$M_1 = 125$ GeV, $\quad M_2 = 200$ GeV, $\quad M_{H^\pm} = 350$ GeV, $\quad \mu = 250$ GeV

$\tan\beta = 2$

Green (lines, dots): CPC

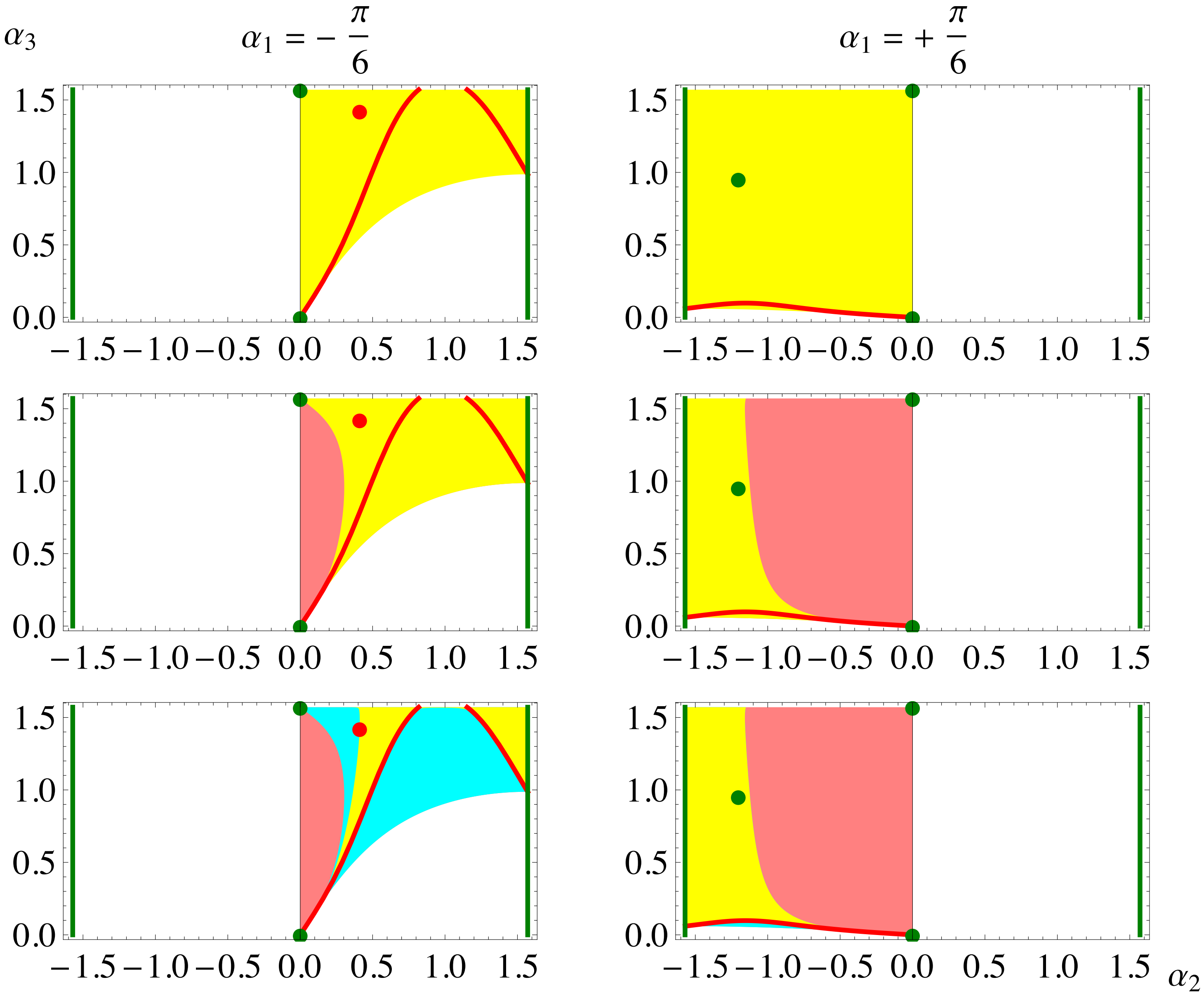
Red (lines, dots): SCPV

Yellow region: ECPV

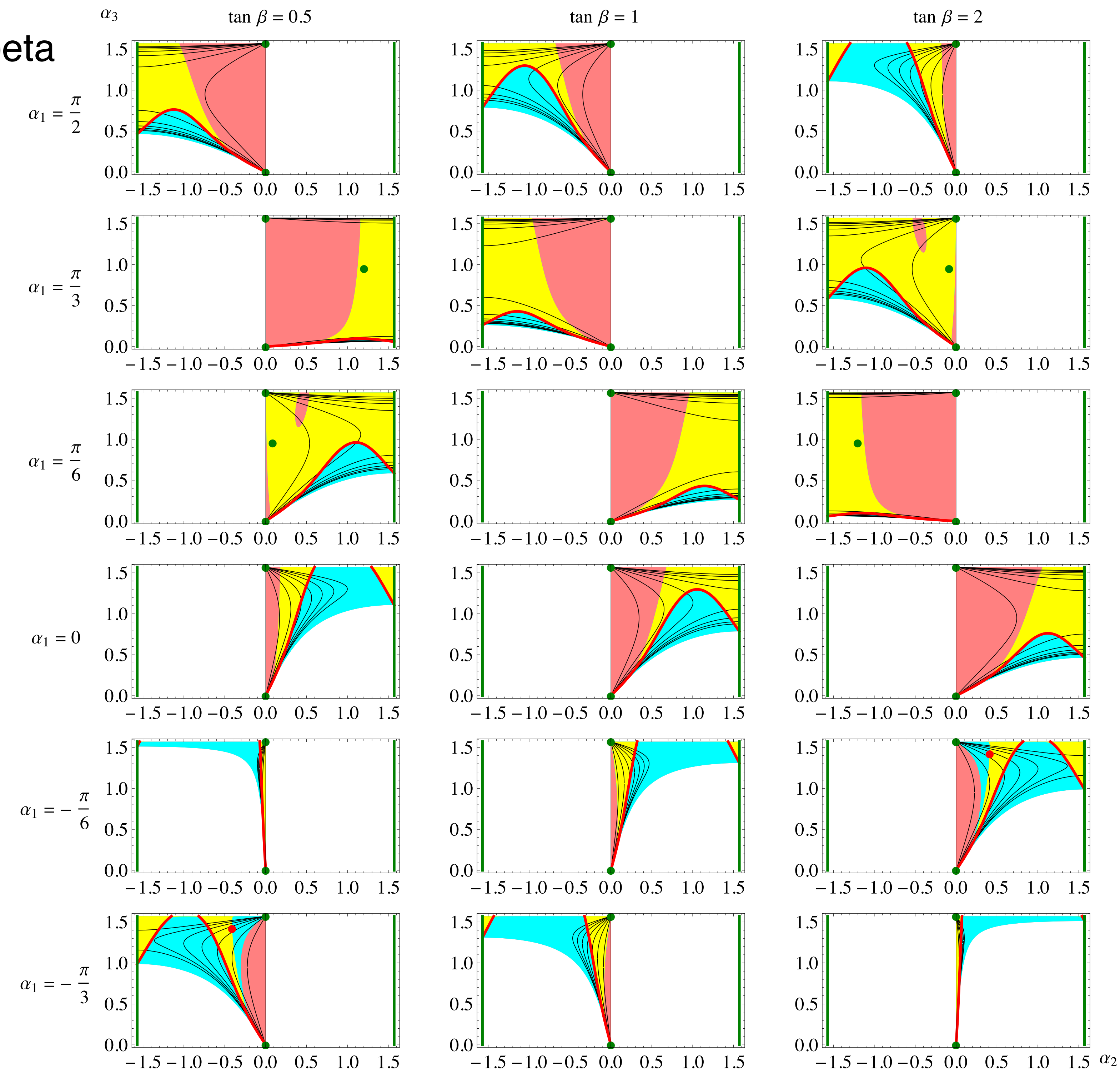
Excluded:

Pink region: positivity

Cyan region: global min

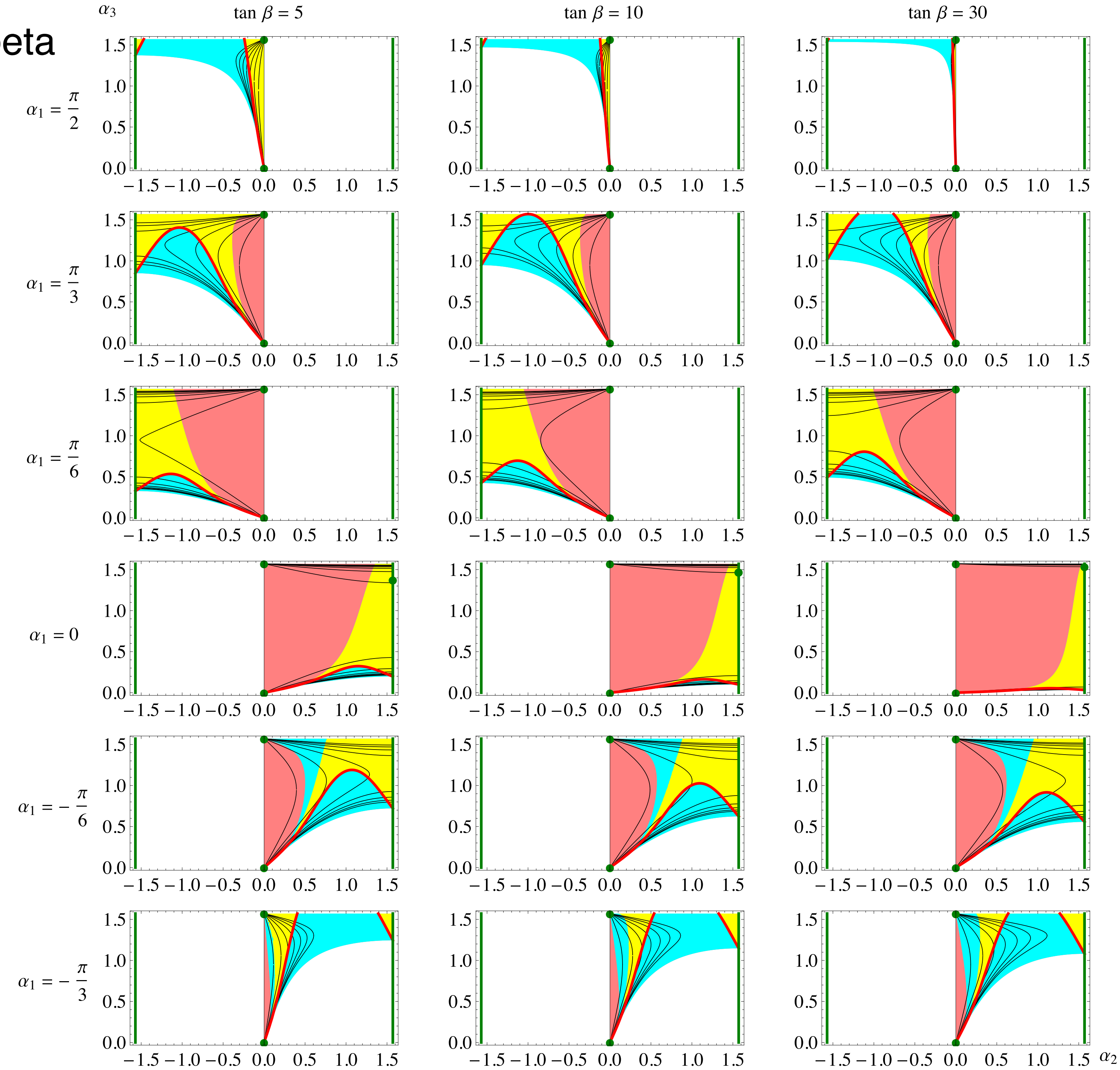


Low tanbeta



α_3
$\tan\beta = 0.5$
$\tan\beta = 1$
$\tan\beta = 2$
$\alpha_1 = \frac{\pi}{2}$
$\alpha_1 = \frac{\pi}{3}$
$\alpha_1 = \frac{\pi}{6}$
$\alpha_1 = 0$
$\alpha_1 = -\frac{\pi}{6}$
$\alpha_1 = -\frac{\pi}{3}$
0.0 0.5 1.0 1.5
−1.5 −1.0 −0.5 0.0 0.5 1.0 1.5
α_2

High tanbeta



INTERMISSION

CP violating ZZZ vertex

("Trailer" for OMO's talk)

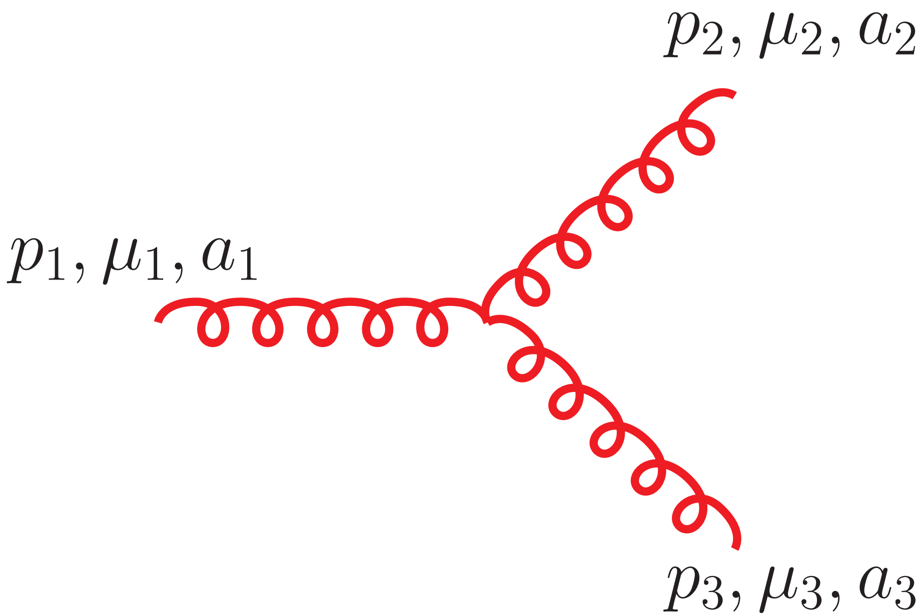
Work with:

Bohdan Grzadkowski, Odd Magne Øgreid

in progress…

Preamble

ggg vertex in QCD



SU(3)

Structure:

$$-\mathrm{i}\; g\; f^{a_1 a_2 a_3}\left[g_{\mu_1\mu_2}(p_1 - p_2)_{\mu_3} + g_{\mu_2\mu_3}(p_2 - p_3)_{\mu_1} + g_{\mu_3\mu_1}(p_3 - p_1)_{\mu_2}\right]$$

Origin of ggg coupling, SU(3):

$$\mathcal{L} \ni \mathrm{Tr}\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu}$$

$$\mathcal{F}^{\mu\nu} = \partial^\nu A^\mu - \partial^\mu A^\nu - ig[A^\mu, A^\nu]$$

$$F_a^{\mu\nu} = \partial^\nu A_a^\mu - \partial^\mu A_a^\nu + g f_{abc} A_b^\mu A_c^\nu$$

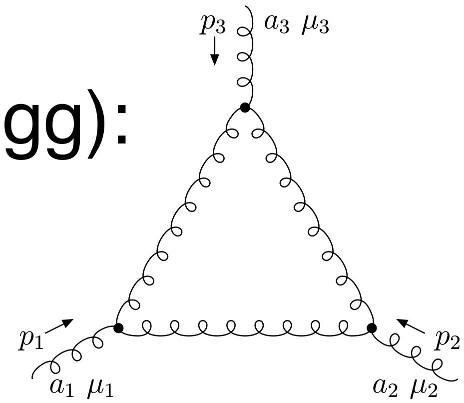
structure constant

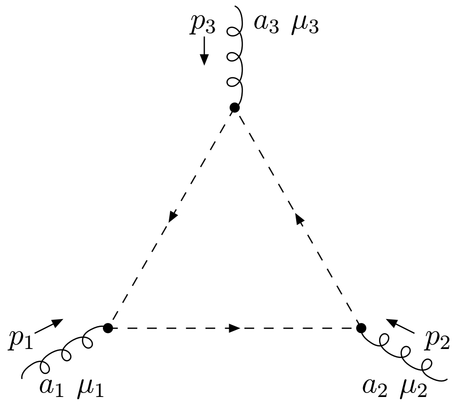
SU(2): Completely analogous.

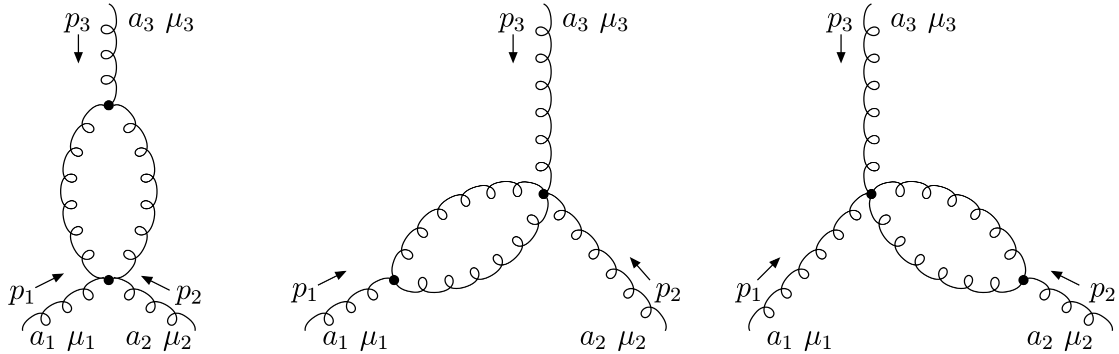
But structure constant is zero unless
all 3 gauge fields are present.
Have W^+W^-Z vertex, but
No ZZZ vertex (at tree level).

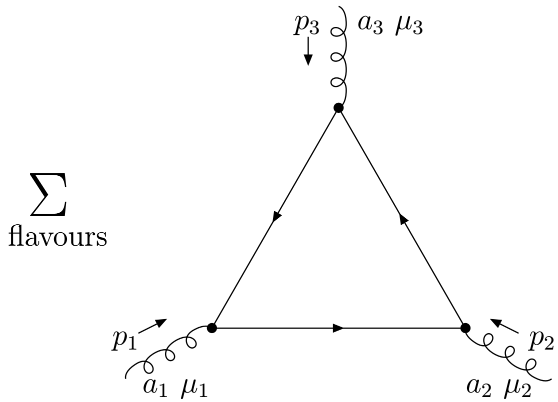
Back to QCD

At one loop (ggg):









More Lorentz structures:

tree level structure

$$\Gamma_{\mu_1\mu_2\mu_3}(p_1,p_2,p_3) = A(p_1^2,p_2^2;p_3^2)\; g_{\mu_1\mu_2}(p_1-p_2)_{\mu_3} + B(p_1^2,p_2^2;p_3^2)\; g_{\mu_1\mu_2}(p_1+p_2)_{\mu_3}$$
$$-C(p_1^2,p_2^2;p_3^2)\; \Big((p_1p_2)g_{\mu_1\mu_2} - p_{1\mu_2}p_{2\mu_1}\Big)\; (p_1-p_2)_{\mu_3}$$
$$+\tfrac{1}{3}\; S(p_1^2,p_2^2,p_3^2)\; \Big(p_{1\mu_3}p_{2\mu_1}p_{3\mu_2} + p_{1\mu_2}p_{2\mu_3}p_{3\mu_1}\Big)$$
$$+F(p_1^2,p_2^2;p_3^2)\; \Big((p_1p_2)g_{\mu_1\mu_2} - p_{1\mu_2}p_{2\mu_1}\Big)\; \Big(p_{1\mu_3}(p_2p_3) - p_{2\mu_3}(p_1p_3)\Big)$$
$$+H(p_1^2,p_2^2,p_3^2)\Big[-g_{\mu_1\mu_2}\Big(p_{1\mu_3}(p_2p_3)-p_{2\mu_3}(p_1p_3)\Big) + \tfrac{1}{3}\Big(p_{1\mu_3}p_{2\mu_1}p_{3\mu_2}-p_{1\mu_2}p_{2\mu_3}p_{3\mu_1}\Big)\Big]$$
$$+\{\text{ cyclic permutations of } (p_1,\mu_1),(p_2,\mu_2),(p_3,\mu_3)\ \}\,.$$

(in QCD) all conserve CP

ZZZ vertex at one loop

General structure

Gaemers & Gounaris (1979);
Hagiwara, Peccei, Zeppenfeld (1986);
Baur & Rainwater (2000)

Bose symmetry and Lorentz invariance: Coupling vanishes if all are on-shell

For two Zs (Z_2, Z_3) being on-shell:

$$\Gamma^{\mu_1\mu_2\mu_3}_{VVV} = \frac{p_1^2 - M_Z^2}{M_Z^2}\left[i f_4 (p_1^{\mu_2} g^{\mu_1\mu_3} + p_1^{\mu_3} g^{\mu_1\mu_2}) + i f_5 \epsilon^{\mu_1\mu_2\mu_3\rho}(p_2 - p_3)_\rho - f_6^V \epsilon^{\mu_1\mu_2\mu_3\rho} p_{1\rho}\right]$$

Violates CP Preserve CP

plus "scalar" terms $\propto p_1^{\mu_1}$

SM: quark loop contributes to f_5, f_6

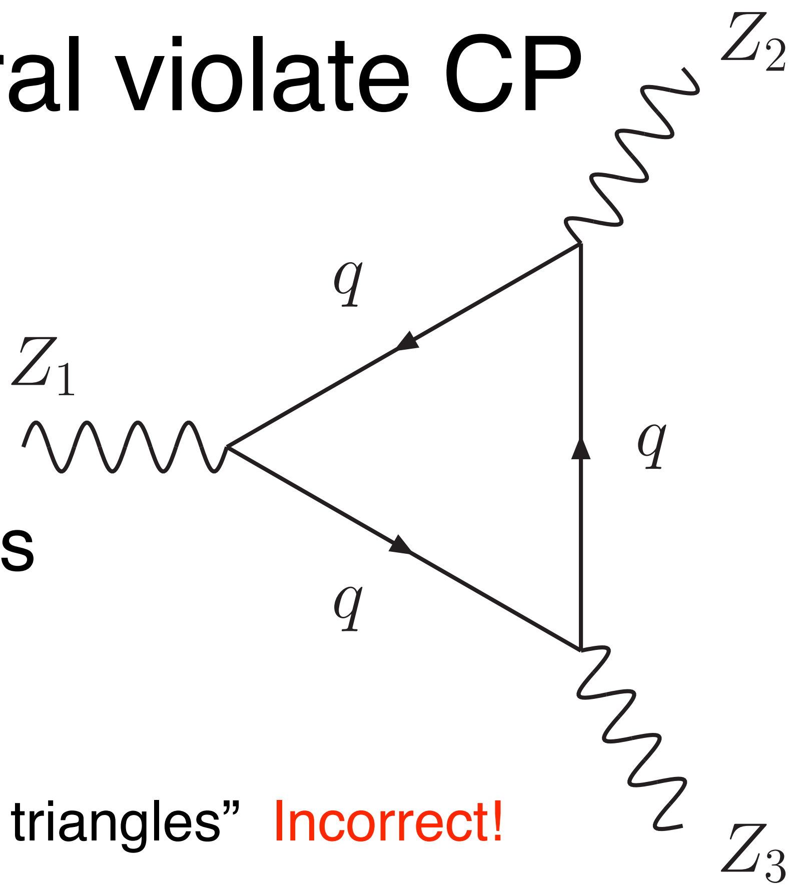
LHC: $|f_4| \lesssim 0.01$

ZZZ vertex at one loop

Z: odd under C

ZZ: even under C

ZZZ vertex will in general violate CP



SM contribution via fermion loops
preserves CP

Gounaris, Layssac, Renard (2000):
"Vertex can only be generated by fermionic triangles" Incorrect!

ZZZ vertex at one loop, in 2HDM

non-zero contribution from bosonic sector

Allow for CP violation in Higgs potential

2HDM notation 1

$$
\begin{aligned}
V(\Phi_1,\Phi_2) &= -\frac{1}{2}\left\{m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + \left[m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right]\right\}\\
&+ \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)\\
&+ \frac{1}{2}\left[\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right] + \left\{\left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right](\Phi_1^\dagger\Phi_2) + \text{h.c.}\right\}\\
&\equiv Y_{a\bar{b}}\Phi_{\bar{a}}^\dagger\Phi_b + \frac{1}{2}Z_{a\bar{b}c\bar{d}}(\Phi_{\bar{a}}^\dagger\Phi_b)(\Phi_{\bar{c}}^\dagger\Phi_d)
\end{aligned}
$$

No FCNC: $\lambda_6 = 0; \quad \lambda_7 = 0$ (relax this)

Allow CPV: $M_{12}^2, \lambda_5, \lambda_6, \lambda_7$ complex

2HDM notation 2

$$\Phi_j = e^{i\xi_j} \begin{pmatrix} \varphi_j^+ \\ (v_j + \eta_j + i\chi_j)/\sqrt{2} \end{pmatrix}, \quad j = 1, 2$$

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}$$

$$\eta_3 = -\sin\beta\chi_1 + \cos\beta\chi_2$$

$$R\mathcal{M}^2 R^{\mathrm{T}} = \mathcal{M}^2_{\mathrm{diag}} = \mathrm{diag}(M_1^2, M_2^2, M_3^2)$$

CP violation in the 2HDM

- Lavoura and Silva, 1994
- Botella and Silva, 1995
- Gunion and Haber, 2005
- Ivanov, Nishi, Maniatis…, 2006-2007

Gunion and Haber expressed invariants

vev

$$\mathrm{Im}\, J_1 = -\frac{2}{v^2}\mathrm{Im}\left[\hat{v}^*_{\bar{a}} Y_{a\bar{b}} Z^{(1)}_{b\bar{d}}\,\hat{v}_d\right]$$

$$\mathrm{Im}\, J_2 = \frac{4}{v^4}\mathrm{Im}\left[\hat{v}^*_{\bar{b}}\hat{v}^*_{\bar{c}} Y_{b\bar{e}} Y_{c\bar{f}} Z_{e\bar{a}f\bar{d}}\hat{v}_a\hat{v}_d\right]$$

$$\mathrm{Im}\, J_3 = \mathrm{Im}\left[\hat{v}^*_{\bar{b}}\hat{v}^*_{\bar{c}} Z^{(1)}_{b\bar{e}}\, Z^{(1)}_{c\bar{f}}\, Z_{e\bar{a}f\bar{d}}\hat{v}_a\hat{v}_d\right]$$

CP conserved iff $\boxed{\mathrm{Im}\, J_1 = \mathrm{Im}\, J_2 = \mathrm{Im}\, J_3 = 0}$

What is the physical content?

Can

$$\operatorname{Im} J_1, \ \operatorname{Im} J_2, \ \operatorname{Im} J_3$$

be rephrased in terms of "physical" quantities?

The physical content

$$\mathrm{Im}\, J_1 = \frac{1}{v^5}\begin{vmatrix} q_1 & q_2 & q_3 \\ e_1 & e_2 & e_3 \\ e_1 M_1^2 & e_2 M_2^2 & e_3 M_3^2 \end{vmatrix}$$

$$\mathrm{Im}\, J_2 = \frac{2}{v^9}\begin{vmatrix} e_1 & e_2 & e_3 \\ e_1 M_1^2 & e_2 M_2^2 & e_3 M_3^2 \\ e_1 M_1^4 & e_2 M_2^4 & e_3 M_3^4 \end{vmatrix}$$

$$\mathrm{Im}\, J_{30} = \frac{1}{v^5}\begin{vmatrix} e_1 & e_2 & e_3 \\ q_1 & q_2 & q_3 \\ q_1 M_1^2 & q_2 M_2^2 & q_3 M_3^2 \end{vmatrix}$$

Footnote: $\mathrm{Im}\, J_3 = \mathrm{Im}\, J_{30} + \text{ terms } \propto \mathrm{Im}\, J_1, \mathrm{Im}\, J_2$

Three determinants:

$$\mathrm{Im}\, J_1 = \frac{1}{v^5} \begin{vmatrix} q_1 & q_2 & q_3 \\ e_1 & e_2 & e_3 \\ e_1 M_1^2 & e_2 M_2^2 & e_3 M_3^2 \end{vmatrix}$$

$$\mathrm{Im}\, J_2 = \frac{2}{v^9} \begin{vmatrix} e_1 & e_2 & e_3 \\ e_1 M_1^2 & e_2 M_2^2 & e_3 M_3^2 \\ e_1 M_1^4 & e_2 M_2^4 & e_3 M_3^4 \end{vmatrix}$$

$$\mathrm{Im}\, J_{30} = \frac{1}{v^5} \begin{vmatrix} e_1 & e_2 & e_3 \\ q_1 & q_2 & q_3 \\ q_1 M_1^2 & q_2 M_2^2 & q_3 M_3^2 \end{vmatrix}$$

Footnote: $\mathrm{Im}\, J_3 = \mathrm{Im}\, J_{30} + \text{ terms } \propto \mathrm{Im}\, J_1, \mathrm{Im}\, J_2$

Couplings:

antisymmetric

$i, j, k : 1, 2, 3$

$$Z^\mu H_i H_j : \quad \frac{g}{2v\cos\theta_\mathrm{W}} \epsilon_{ijk} e_k (p_i - p_j)^\mu$$

$$H^+ H^- H_i : \quad -iq_i$$

$$e_i = v_1 R_{i1} + v_2 R_{i2}$$

$q_i = \cdots$ more complicated

Recall CP-conserving 2HDM

Let
$$H_1 = h$$
$$H_2 = H$$
$$H_3 = A$$

Then

(ZHA)	$e_1 \neq 0$	(hH^+H^-)	$q_1 \neq 0$
(ZhA)	$e_2 \neq 0$	(HH^+H^-)	$q_2 \neq 0$
(ZhH)	$e_3 = 0$	(AH^+H^-)	$q_3 = 0$

The invariants $\mathrm{Im}\, J_1,\ \mathrm{Im}\, J_2,\ \mathrm{Im}\, J_3$ **vanish**

This result was actually published by Lavoura and Silva in 1994

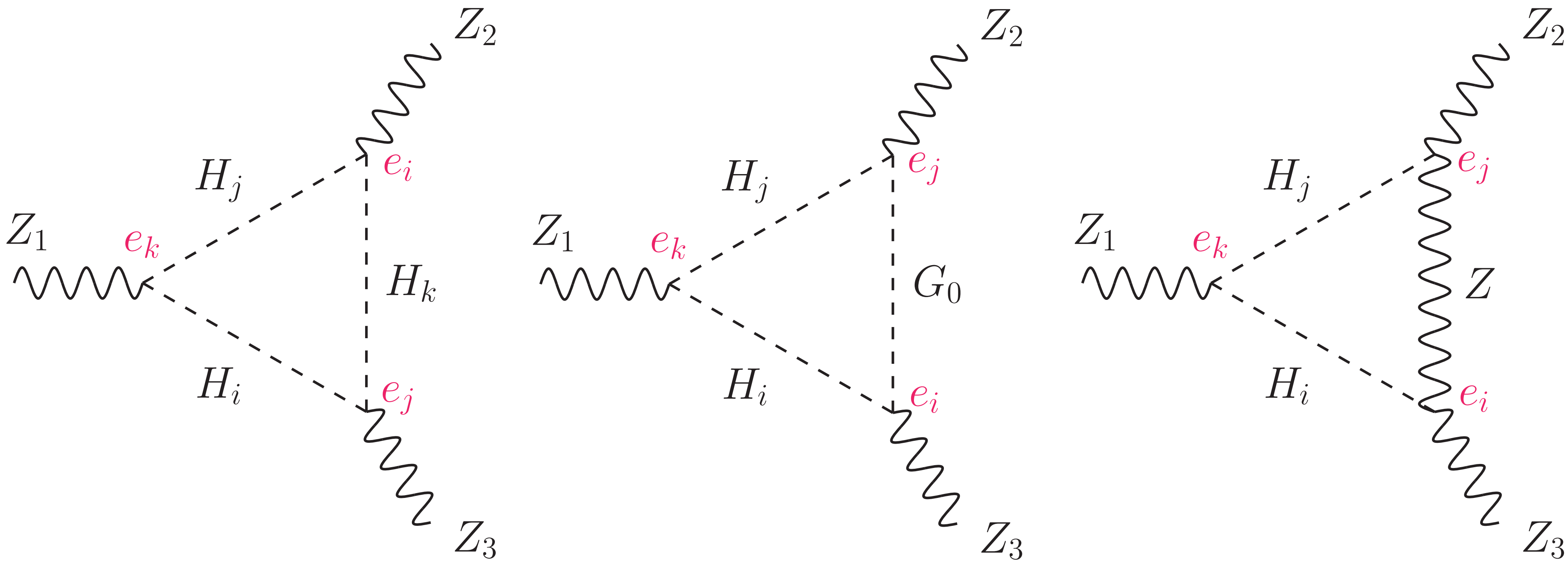
$$[\mathrm{Im}\, J_2]_{\text{Higgs Basis}} = -v^6\, J_1^{\mathrm{LS}}$$

$$[\mathrm{Im}\, J_1]_{\text{Higgs Basis}} = v^3\, J_3^{\mathrm{LS}}$$

$$[\mathrm{Im}\, J_{30}]_{\text{Higgs Basis}} = -v^4\, J_2^{\mathrm{LS}}$$

and is reminiscent of the Jarlskog determinant

ZZZ vertex at one loop



Note sum over i, j, k all different.
All three neutral Higgs bosons involved.

ZZZ vertex at one loop

All three neutral Higgs bosons involved.

$\boxed{\text{Proportional to Im } J_2}$

$$f_4^Z(p_1^2) = \frac{2\alpha}{\pi \sin^3(2\theta_{\mathrm{W}})} \frac{M_Z^2}{p_1^2 - M_Z^2} \frac{e_1 e_2 e_3}{v^3}$$

$$\times \sum_{i,j,k} \epsilon_{ijk} \big[C_{001}(p_1^2, M_Z^2, M_Z^2, M_i^2, M_j^2, M_Z^2) + C_{001}(p_1^2, M_Z^2, M_Z^2, M_Z^2, M_j^2, M_k^2)$$

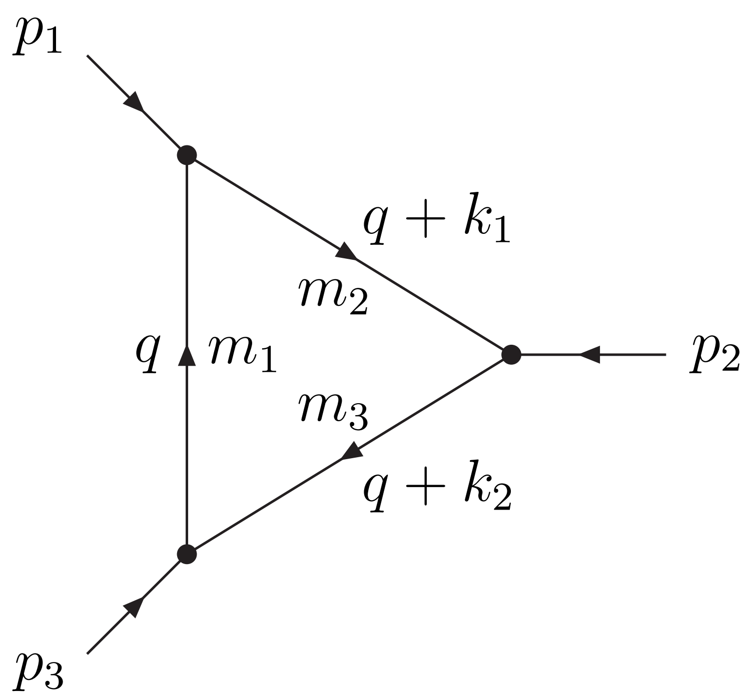
$$+ C_{001}(p_1^2, M_Z^2, M_Z^2, M_i^2, M_Z^2, M_k^2) - C_{001}(p_1^2, M_Z^2, M_Z^2, M_i^2, M_j^2, M_k^2)$$

$$+ M_Z^2 C_1(p_1^2, M_Z^2, M_Z^2, M_i^2, M_Z^2, M_k^2) \big]$$

Z_2, Z_3 assumed on shell

non-symmetric expression

Three-point functions

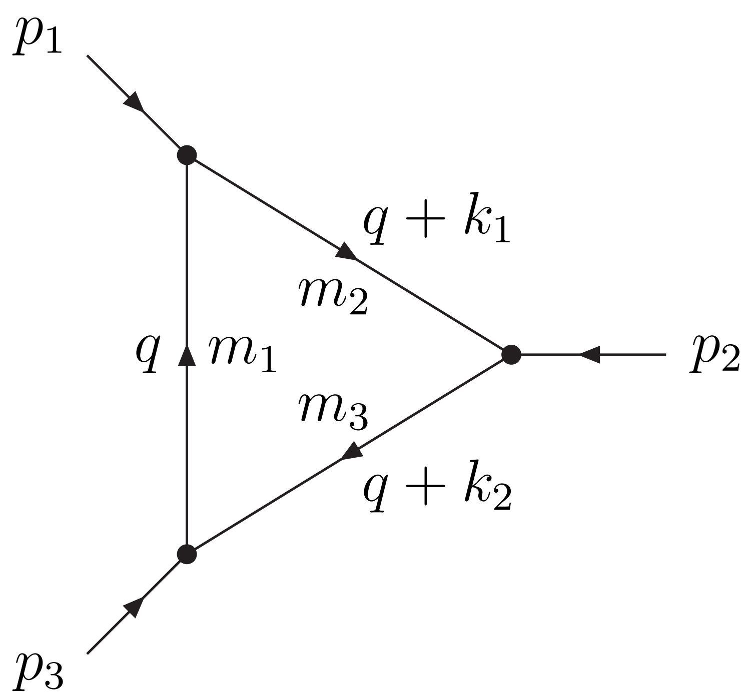


May have up to three Lorentz indices (from Lorentz vectors in integrand)

Expand in “tensor coefficients”

LoopTools (Thomas Hahn)

Three-point functions



Tensor coefficients:

$$C_\mu = k_{1\mu} C_1 + k_{2\mu} C_2 = \sum_{i=1}^{2} k_{i\mu} C_i \ ,$$

$$C_{\mu\nu} = g_{\mu\nu} C_{00} + \sum_{i,j=1}^{2} k_{i\mu} k_{j\nu} C_{ij} \ ,$$

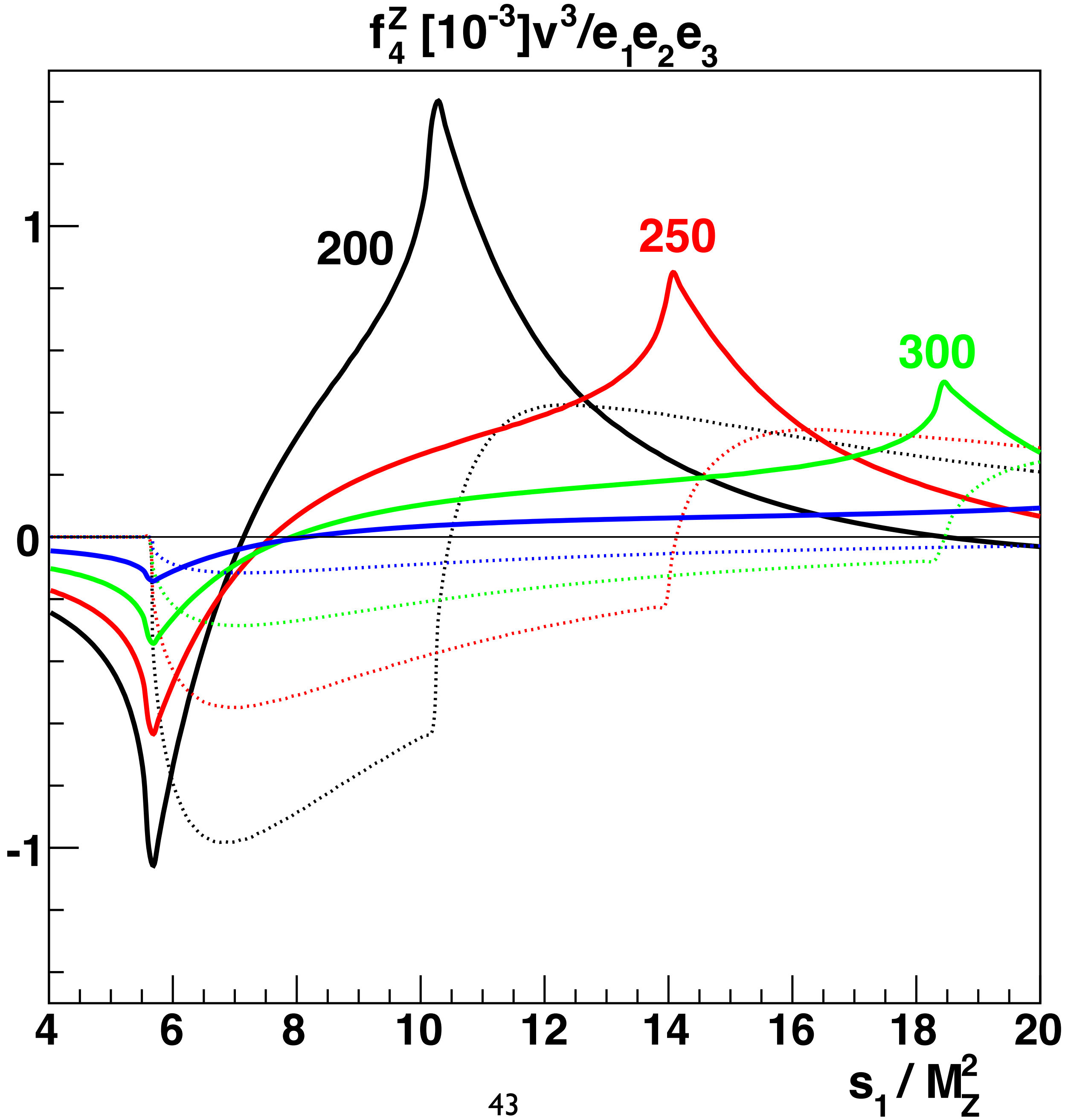
$$C_{\mu\nu\rho} = \sum_{i=1}^{2} \big(g_{\mu\nu} k_{i\rho} + g_{\nu\rho} k_{i\mu} + g_{\mu\rho} k_{i\nu} \big) C_{00i} + \sum_{i,j,\ell=1}^{2} k_{i\mu} k_{j\nu} k_{\ell\rho} C_{ij\ell} \ ,$$

Numerical example

$$M_1 = 125 \text{ GeV},$$
$$M_2 = (200, 250, 300, 350) \text{ GeV},$$
$$M_3 = 400 \text{ GeV}$$

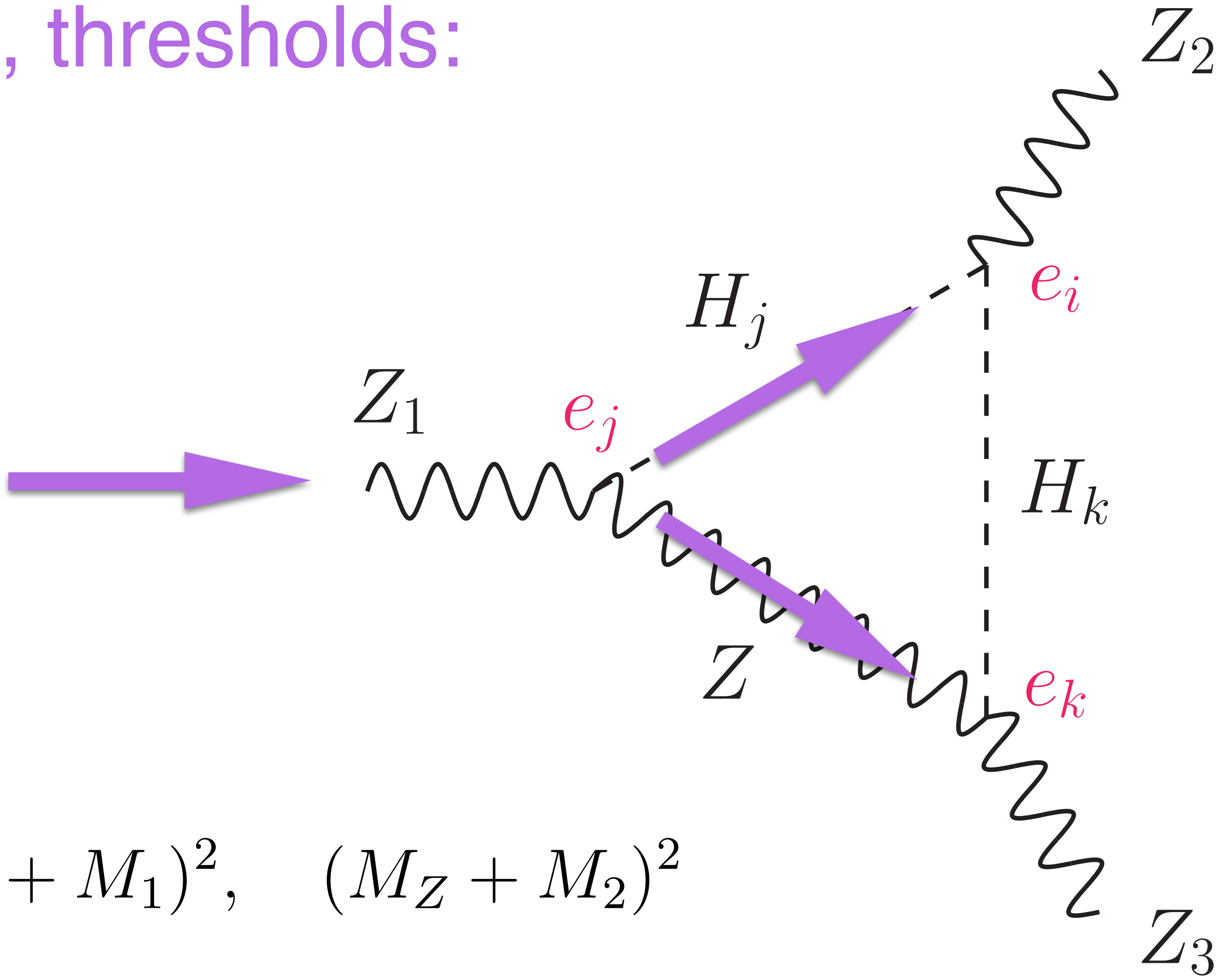
$$f_4^Z(s_1 = p_1^2) \text{ vs } s_1/M_Z^2$$

ZZZ vertex at one loop



Qualitative features

Resonances, thresholds:



at:

$$s_1 = (M_Z + M_1)^2, \quad (M_Z + M_2)^2$$

Product of couplings:

$$e_1^2 + e_2^2 + e_3^2 = v^2$$

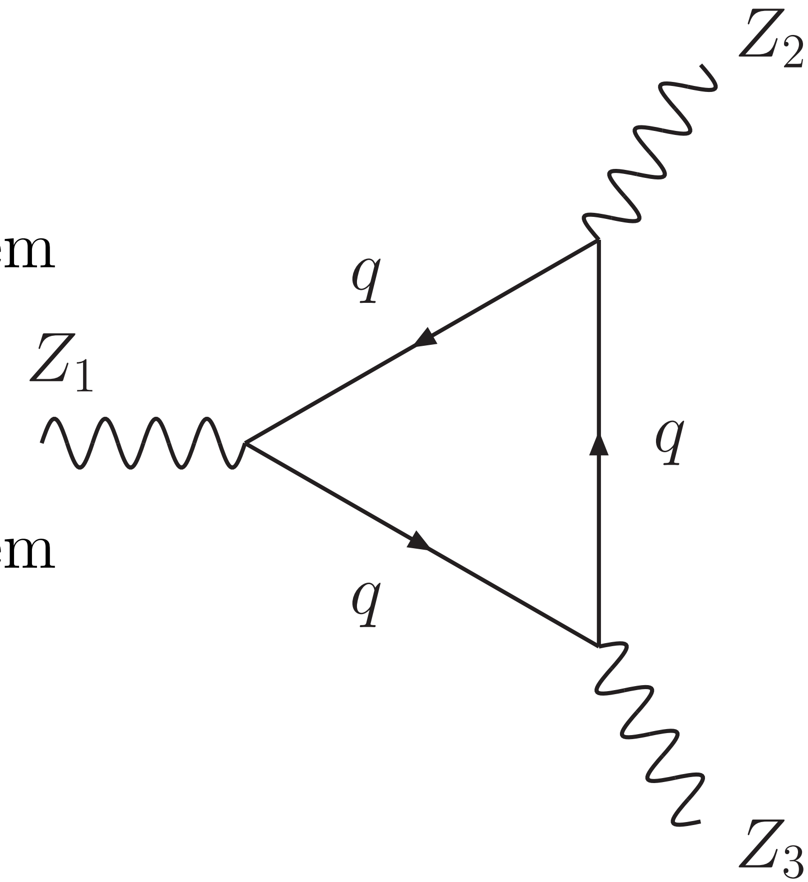
$$\max \frac{e_1 e_2 e_3}{v^3} = \frac{1}{3\sqrt{3}} \simeq 0.1925$$

For benchmarks studied in Basso et al, 1205.6569:

$$\frac{e_1 e_2 e_3}{v^3} = \mathcal{O}(0.01)$$

SM background

- $VVV :\to 0$ Furry's Theorem
- $VVA :\sim \epsilon_{\alpha\beta\gamma\delta}$ Violates P
- $VAA :\to 0$ Furry's Theorem
- $AAA :\sim \epsilon_{\alpha\beta\gamma\delta}$ Violates P



Contributions to f_5

No contribution to f_6 when Z_2, Z_3 are on-shell

Summary

- Invariants Im J_1, Im J_2, Im J_3 can be expressed in terms of physical couplings and masses
- All three neutral Higgses are "involved"
- Couplings lead to a calculable CP-violating ZZZ coupling
- It will be a challenge to measure it

Thanks for local support!