# Higgs Physics in 3HDMs and 6HDMs with Inert Scalar Doublets

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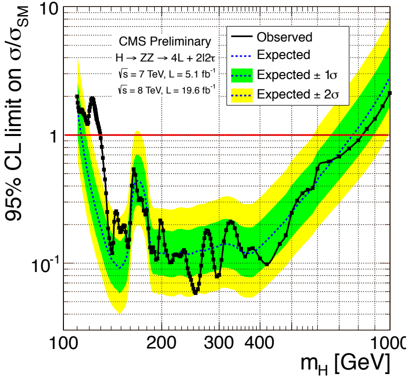
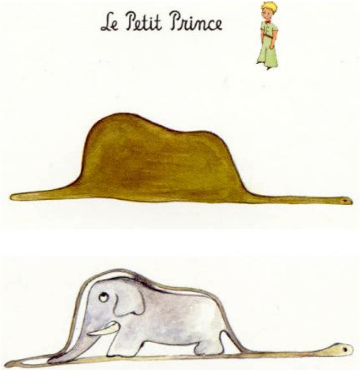
# Experimental evidence and possible New Physics

Evidence for a Higgs-like boson with mass 125 GeV:




'On ne voit bien qu'avec le coeur. L'essentiel est invisible pour les yeux.'

# Motivations for introducing more than one doublet

- LHC: Higgs (doublet) seen (very SM-like), no BSM
- No fundamental reason for only one doublet (ignore singlet, not seen)
- Hierarchy of the Yukawa couplings
- Sources of *CP* violation
- Source of FCNCs
- Axion models with Peccei-Quinn symmetry
- Dark matter candidates (IHDMs)

# N-Higgs Doublet Models (NHDMs)

N copies of the Higgs doublet with identical quantum numbers:

$$\Phi_\alpha = \begin{pmatrix} \phi_\alpha^+ \\ \frac{1}{\sqrt{2}}(\rho_\alpha + i\eta_\alpha) \end{pmatrix}, \qquad \alpha = 1, 2, \cdots N$$

The most general potential

$$V = Y_{ab}(\Phi_a^\dagger \Phi_b) + Z_{abcd}(\Phi_a^\dagger \Phi_b)(\Phi_c^\dagger \Phi_d)$$

contains $N^2(N^2+3)/2$ free parameters.

All Abelian symmetries realisable in NHDM have been found.
[Ivanov, et al., J.Phys.A 45,215201 (2012)]

- 3HDM and symmetries In 3HDMs, all finite symmetries are known

$$Z_2, \quad Z_3, \quad Z_4, \quad Z_2 \times Z_2, \quad D_6, \quad D_8, \quad A_4, \quad S_4, \quad \Delta(54)/Z_3, \quad \Sigma(36)$$

[Ivanov, et al., Eur.Phys.J.C 73,2309 (2013)]

- Some credentials

Popular 3HDMs, Private Higgs model:
[Weinberg,et al. Phys.Rev.D15,1958 (1977)], [Paschos, Phys.Rev.D15,1966 (1977)]
[Adler, Phys.Rev.D60,015002 (1999)], [Zee,et al., Phys.Lett.B666,491 (2008)]

Groups with triplet representations $A_4, S_4$:
[Ma, et al., Phys.Lett.B 552, (2003)], [Altarelli, et al., Nucl.Phys.B 720, (2005)]
[Lam, Phys.Rev.Lett. 101, (2008)], [Morisi, et al., Phys.Rev.D 80, (2009)]
[King, et al., Phys.Lett.B 687, (2010)]

# 3HDM $\longrightarrow$ 6HDM

- Experiment: search for NP through the Higgs sector high on agenda.
- Guided by existence of 3 generations of fermions we pick the 3HDM, assigning one doublet to be the SM one, we are left with two inert doublets plus one Higgs doublet (I(2+1)HDM).
- Inspired by Supersymmetry, which requires even number of doublets, we double-up the above into one with four inert doublets plus two Higgs doublets (I(4+2)HDM), e.g., $E_6$SSM.

We previously studied full list of symmetries in 3HDMs:
[V. Keus, S.F. King and SM, JHEP 1401, 052 (2014)
and extended them to 6HDMs by

$$\Phi_\alpha = \begin{pmatrix} \Phi_{\alpha u} \\ \Phi_{\alpha d} \end{pmatrix} \qquad i = 1, 2, 3$$

while keeping potential symmetric under the desired symmetry group.

# The I(2+1)HDM and I(4+2)HDM

We study the 3HDMs with $(0, 0, v)$ and 6HDMs with $(0, 0, 0, 0, v_u, v_d)$ v.e.v. alignment symmetric under:

- continuous Abelian groups

$$U(1), \quad U(1) \times U(1), \quad U(1) \times Z_2,$$

- finite Abelian groups

$$Z_2 \text{ (2HDM standard)}, \quad Z_3, \quad Z_4, \quad Z_2 \times Z_2,$$

- finite non-Abelian groups

$$D_6, \quad D_8, \quad A_4, \quad S_4, \quad \Delta(54)/Z_3, \quad \Sigma(36).$$

The DM in each case is protected by either the original symmetry of the potential or the remnant of the symmetry after EWSB.

[Keus, et al., JHEP 1401 (2014) 052]

# Constructing the ($Z_2$ symmetric) I(2+1)HDM

Start with the phase invariant potential:

$$V_0 = -|\mu_i^2|(\Phi_i^\dagger\Phi_i) + \lambda_{ii}(\Phi_i^\dagger\Phi_i)^2 + \lambda_{ij}(\Phi_i^\dagger\Phi_i)(\Phi_j^\dagger\Phi_j) + \lambda'_{ij}(\Phi_i^\dagger\Phi_j)(\Phi_j^\dagger\Phi_i)$$

and add the terms

$$V_{Z_2} = -\mu_{12}^2(\Phi_1^\dagger\Phi_2) + \lambda_1(\Phi_1^\dagger\Phi_2)^2 + \lambda_2(\Phi_2^\dagger\Phi_3)^2 + \lambda_3(\Phi_3^\dagger\Phi_1)^2 + h.c.$$

that ensure the $Z_2$ symmetry generated by

$$g^{Z_2} = (-,-,+)$$

where

$$\Phi_\alpha = \begin{pmatrix} H_\alpha^\pm \\ \frac{1}{\sqrt{2}}\left(H_\alpha^0 + iA_\alpha^0\right) \end{pmatrix}, \quad \alpha = 1,2,3$$

where 1,2 are inert, 3 is active.

# Dark matter in the I(2+1)HDM

The VEV alignment $\langle \Phi_i \rangle = (0, 0, v)$ respects the $Z_2$ symmetry: lightest neutral fields from the inert doublets, $H_{1,2}, A_{1,2}$ are viable DM candidates.

$$\Phi_\alpha = \begin{pmatrix} H^\pm_\alpha \\ \frac{1}{\sqrt{2}}\left(H^0_\alpha + iA^0_\alpha\right) \end{pmatrix}, \quad \alpha = 1, 2$$

$\longrightarrow$ See D. Sokolowska's talk on this !

Notes:

- To make sure whole Lagrangian is $Z_2$ symmetric, assign even $Z_2$ parity to all SM particles, identical to $Z_2$ parity of only doublet coupling to them, i.e., active $\phi_3$.
- With this parity assignment FCNCs are avoided as extra doublets are forbidden to decay to fermions by $Z_2$ conservation.

# Higgs sector of the I(2+1)HDM

Free parameters

- $\mu_3, \lambda_{33}$: Higgs field parameters, given by Higgs mass,

$$m_h^2 = 2\mu_3^2 = 2\lambda_{33} v^2 \quad (v \equiv v_{\rm SM}).$$

- $\mu_1, \mu_2, \mu_{12}, \lambda_{31}, \lambda_{23}, \lambda'_{31}, \lambda'_{23}, \lambda_2, \lambda_3$: mass parameters and couplings of inert scalars to visible sector, 9 parameters (can be determined by 6 masses and 3 mixing angles):

$$-10 \ \ \text{TeV}^2 < \mu_1^2, \mu_2^2, \mu_{12}^2 < 10 \ \text{TeV}^2,$$
$$-0.5 < \lambda_{31}, \lambda_{23}, \lambda'_{31}, \lambda'_{23}, \lambda_2, \lambda_3 < 0.5.$$

- $\lambda_{11}, \lambda_{22}, \lambda_{12}, \lambda'_{12}$: inert self-interactions (NB: relic density calculations do not depend on these, bounds would come from collider limits)

$$0 < \lambda_{11}, \lambda_{22}, \lambda_{12}, \lambda'_{12} < 0.5.$$

# Higgs sector of the I(2+1)HDM

Physical Higgs states

- One active one: $h_{\rm SM}$ + $G^0(G^\pm)$ Goldstones to make $Z(W^\pm)$ massive.
- Two generations of inert ones: $(H_1, A_1, H_1^\pm)$ chosen lighter than $(H_2, A_2, H_2^\pm)$ $\rightarrow$ $H_1$ being the lightest, i.e., the DM candidate:

$$m_{H_1} < m_{H_2}, m_{A_{1,2}}, m_{H^\pm_{1,2}} \quad (\text{implies } 2\lambda_2, 2\lambda_3 < \lambda'_{23}, \lambda'_{31} < 0).$$

1. Introduce matrix

$$R_{\theta_i} = \begin{pmatrix} \cos\theta_i & \sin\theta_i \\ -\sin\theta_i & \cos\theta_i \end{pmatrix}, \qquad \theta_i = \theta_h, \theta_a, \theta_c,$$

   $\theta_{h(a)[c]}$ rotation angles of scalar(pseudo-scalar)[charged] inert sector.

2. Can express mass spectrum in terms of
   $\Sigma = 4\mu_{12}^4 + (\mu_1^2 - \Lambda_{\phi_1} - \mu_2^2 + \Lambda_{\phi_2})^2$ (same for $\Sigma'^{('')}$ vs $\Lambda'^{('')}_{\phi_i}$, $i = 1, 2$)

# Higgs sector of the I(2+1)HDM

with

$$\Lambda_{\phi_1} = \frac{1}{2}(\lambda_{31} + \lambda'_{31} + 2\lambda_3)v^2 \qquad \Lambda_{\phi_2} = \frac{1}{2}(\lambda_{23} + \lambda'_{23} + 2\lambda_2)v^2$$

$$\tan 2\theta_h = \frac{2\mu_{12}^2}{\mu_1^2 - \Lambda_{\phi_1} - \mu_2^2 + \Lambda_{\phi_2}}$$

$$\Lambda'_{\phi_1} = \frac{1}{2}(\lambda_{31})v^2 \qquad \Lambda'_{\phi_2} = \frac{1}{2}(\lambda_{23})v^2$$

$$\tan 2\theta_c = \frac{2\mu_{12}^2}{\mu_1^2 - \Lambda'_{\phi_1} - \mu_2^2 + \Lambda'_{\phi_2}}$$

$$\Lambda''_{\phi_1} = \frac{1}{2}(\lambda_{31} + \lambda'_{31} - 2\lambda_3)v^2 \qquad \Lambda''_{\phi_2} = \frac{1}{2}(\lambda_{23} + \lambda'_{23} - 2\lambda_2)v^2$$

$$\tan 2\theta_a = \frac{2\mu_{12}^2}{\mu_1^2 - \Lambda''_{\phi_1} - \mu_2^2 + \Lambda''_{\phi_2}}$$

# Higgs sector of the I(2+1)HDM

- Possible mass spectrum is

| | |
|---|---|
| $m_h^2$ | $2\mu_3^2$ |
| $m_{H_2^\pm}^2$ | $(-\mu_1^2 + \Lambda'_{\phi_1} - \mu_2^2 + \Lambda'_{\phi_2} + \sqrt{\Sigma'})/2$ |
| $m_{H_1^\pm}^2$ | $(-\mu_1^2 + \Lambda'_{\phi_1} - \mu_2^2 + \Lambda'_{\phi_2} - \sqrt{\Sigma'})/2$ |
| $m_{A_2}^2$ | $(-\mu_1^2 + \Lambda''_{\phi_1} - \mu_2^2 + \Lambda''_{\phi_2} + \sqrt{\Sigma''})/2$ |
| $m_{A_1}^2$ | $(-\mu_1^2 + \Lambda''_{\phi_1} - \mu_2^2 + \Lambda''_{\phi_2} - \sqrt{\Sigma''})/2$ |
| $m_{H_2}^2$ | $(-\mu_1^2 + \Lambda_{\phi_1} - \mu_2^2 + \Lambda_{\phi_2} + \sqrt{\Sigma})/2$ |
| $m_{H_1}^2$ | $(-\mu_1^2 + \Lambda_{\phi_1} - \mu_2^2 + \Lambda_{\phi_2} - \sqrt{\Sigma})/2$ |
| $m_{G^0}^2, m_{G^\pm}^2$ | $0$ |

# Higgs sector of the I(2+1)HDM

Theoretical constraints

- Positivity of mass eigenstates
  - $\mu_3^2 > 0$
  - $-2\mu_1^2 + \lambda_{31} v^2 > 0$
  - $-2\mu_1^2 + (\lambda_{31} + \lambda'_{31}) v^2 > 0$
  - $-2\mu_1^2 + (\lambda_{31} + \lambda'_{31} - 2\lambda_3) v^2 > 0$
  - $-2\mu_2^2 + \lambda_{23} v^2 > 0$
  - $-2\mu_2^2 + (\lambda_{23} + \lambda'_{23}) v^2 > 0$
  - $-2\mu_2^2 + (\lambda_{23} + \lambda'_{23} - 2\lambda_2) v^2 > 0$
  - $-2\mu_1^2 - 2\mu_2^2 + (\lambda_{31} + \lambda_{23}) v^2 > 4|\mu_{12}^2|$
  - $-2\mu_1^2 - 2\mu_2^2 + (\lambda_{31} + \lambda_{23} + \lambda'_{31} + \lambda'_{23}) v^2 > 4|\mu_{12}^2|$
  - $-2\mu_1^2 - 2\mu_2^2 + (\lambda_{31} + \lambda_{23} + \lambda'_{31} + \lambda'_{23} - 2\lambda_3 - 2\lambda_2) v^2 > 4|\mu_{12}^2|$

# Higgs sector of the I(2+1)HDM

- Bounded-ness of potential

  - $\lambda_{11}, \lambda_{22}, \lambda_{33} > 0$
  - $\lambda_{12} + \lambda'_{12} > -2\sqrt{\lambda_{11}\lambda_{22}}$
  - $\lambda_{23} + \lambda'_{23} > -2\sqrt{\lambda_{22}\lambda_{33}}$
  - $\lambda_{31} + \lambda'_{31} > -2\sqrt{\lambda_{33}\lambda_{11}}$

  Also require parameters $V_{Z_2}$ be smaller than $V_0$ ones:

  - $$|\lambda_1|, |\lambda_2|, |\lambda_3| < |\lambda_{ii}|, |\lambda_{ij}|, |\lambda'_{ij}|, \quad i \neq j : 1, 2, 3 \qquad (1)$$

- Positive-definite-ness of Hessian

  - $\mu_3^2 > 0$
  - $-2\mu_2^2 + (\lambda_{23} + \lambda'_{23})v^2 > 0$
  - $-2\mu_1^2 + (\lambda_{31} + \lambda'_{31})v^2 > 0$
  - $\left(-2\mu_1^2 + (\lambda_{31} + \lambda'_{31})v^2\right)\left(-2\mu_2^2 + (\lambda_{23} + \lambda'_{23})v^2\right) > 4\mu_{12}^4$

# Higgs sector of the I(2+1)HDM

Experimental constraints (LEP)
Searches for Higgs states give

$$\bullet \quad m_{H_i^\pm} + m_{H_i,A_i} > m_{W^\pm} \qquad (2)$$
$$\bullet \quad m_{H_i} + m_{A_i} > m_Z$$
$$\bullet \quad 2m_{H_i^\pm} > m_Z$$

$$\bullet \quad m_{H_i^\pm} > 70 \quad \text{GeV.} \qquad (3)$$

Searches for charginos and neutralinos translate to

$$\bullet \quad m_H < 80 \quad \text{GeV} \quad \text{and} \quad m_A < 100 \quad \text{GeV}$$

and

$$\bullet \quad m_A - m_H > 8 \quad \text{GeV.} \qquad (4)$$

(Limit enforced for any pair CP-even/odd pair.)

# Higgs sector of the I(2+1)HDM

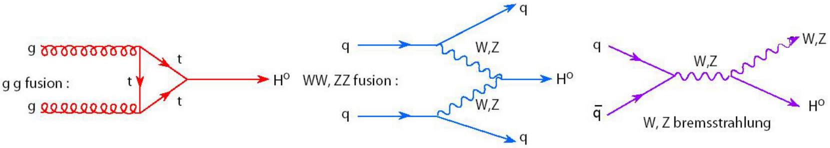
Invisible Higgs decay limits (LHC)

1. Direct detection limits:

- ATLAS limits in Zh channel: BR($h \to$ invisible) $<$ 65% at 95% CL
- CMS limits in Zh channel: BR($h \to$ invisible) $<$ 75% at 95% CL
- CMS limits in VBF channel: BR($h \to$ invisible) $<$ 69% at 95% CL

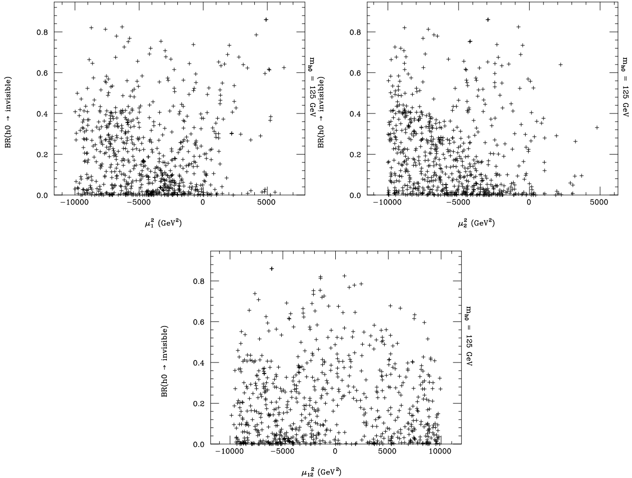
2. Global fits on Higgs signal strengths:

- Higgs boson with SM couplings but additional invisible decay modes: BR($h \to$ invisible) $<$ 20% (or so) at 95% CL

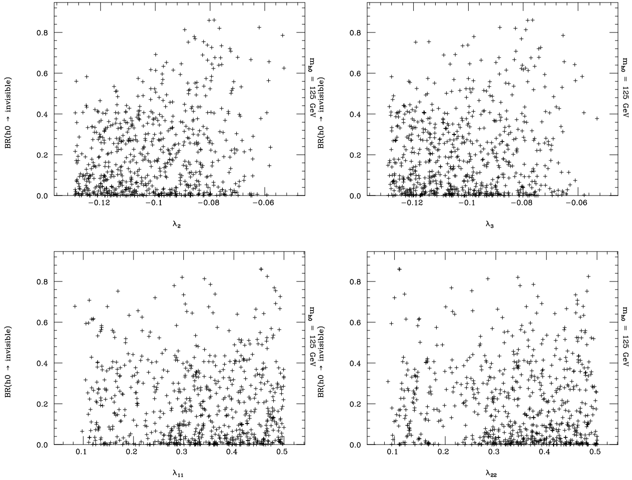


# Results for the I(2+1)HDM

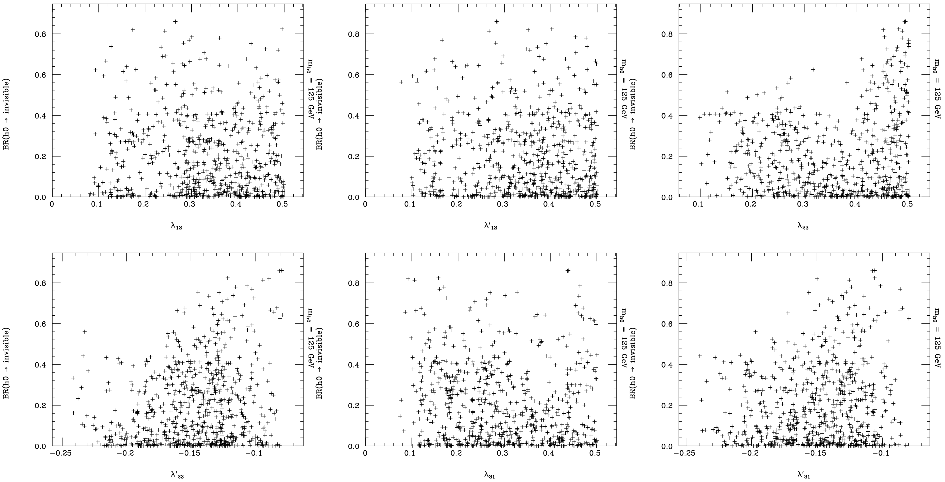
- $h(\equiv h_{\rm SM}) \to$ invisible ($m_h = 125$ GeV).



# Results for the I(2+1)HDM



# Results for the I(2+1)HDM



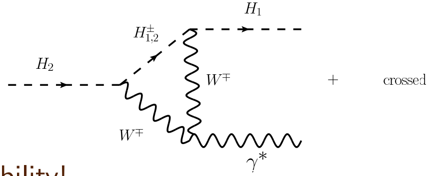
# Results for the I(2+1)HDM

- However, most striking I(2+1)HDM signal is radiative decays of heavy inert Higgs states into DM candidate:

$$h \to H_1 H_2, \qquad h \to H_2 H_2,$$

wherein

$$H_2 \to H_1(\equiv \mathrm{DM})\ \gamma^*(\to e^+e^-)$$

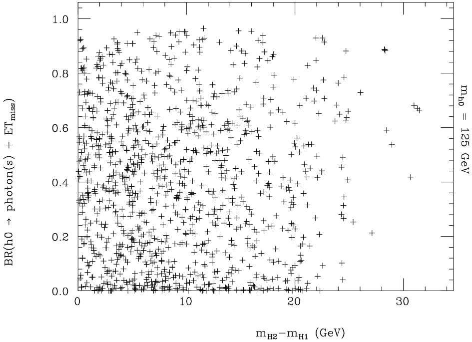



with 100% probability!

- $A_1$, $H_1^{\pm}$, $A_2$ and $H_2^{\pm}$ never involved ($H_1$ and $H_2$ always lightest inerts).

# Results for the I(2+1)HDM

- Search for EM showers, one or two at a time, alongside significant missing (transverse) energy, $E^T_{\mathrm{miss}}$ from DM pair.




- Can enable I(2+1)HDM to be distinguished from I(1+1)HDM, as here CP-conservation prevents such radiative decays.

# Constructing the I(4+2)HDM

Start from the $Z_2$-symmetric 3HDM potential:

$$
\begin{aligned}
V \;=\; & -|\mu_i^2|(\Phi_i^\dagger \Phi_i) + \lambda_{ii}(\Phi_i^\dagger \Phi_i)^2 + \lambda_{ij}(\Phi_i^\dagger \Phi_i)(\Phi_j^\dagger \Phi_j) + \lambda'_{ij}(\Phi_i^\dagger \Phi_j)(\Phi_j^\dagger \Phi_i) \\
& -\mu_{12}^2(\Phi_1^\dagger \Phi_2) + \lambda_1(\Phi_1^\dagger \Phi_2)^2 + \lambda_2(\Phi_2^\dagger \Phi_3)^2 + \lambda_3(\Phi_3^\dagger \Phi_1)^2 + h.c.
\end{aligned}
$$

and extend the potential to 6HDM (acquires $S_3$ symmetry)

$$
\Phi_\alpha = \begin{pmatrix} H_{\alpha u} \\ H_{\alpha d} \end{pmatrix} \qquad i = 1, 2, 3
$$

The potential is symmetric under:

$$
g_{Z_2^H} = \mathrm{diag}\,(-,-,-,-,+,+)
$$

The DM candidate could also be protected by:

$$
g_{Z_2^{DM}} = \mathrm{diag}\,(-,+,+,+,+,+)
$$

respected by the vacuum alignment:

$$
\langle H_{1u}\rangle = \langle H_{1d}\rangle = \langle H_{2u}\rangle = \langle H_{2d}\rangle = 0, \qquad \langle H_{3u}\rangle = v_u, \quad \langle H_{3d}\rangle = v_d
$$

# $\tan\beta = 1$ case $(v_u = v_d)$

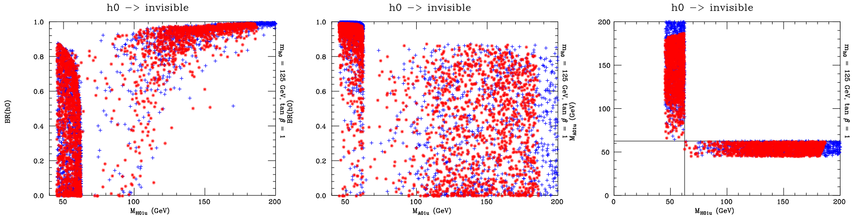
- Physical states from active doublets:

$$h = \frac{H^0_{3u} + H^0_{3d}}{\sqrt{2}}, \qquad H = \frac{H^0_{3u} - H^0_{3d}}{\sqrt{2}}$$

- Parameter constraint: $v^2 = \frac{m_h^2}{4\lambda_a + 4\lambda_{aa}}$.
- Coupling of $h$ to all SM matter: identical to those of $h_{\mathrm{SM}}$.
- Coupling of $H$ to gauge bosons: $HVV = 0$.
- Dominant decay channel of $H$: $H \to t\bar{t}$.
- Enlarged Higgs sector: new interactions between $h$ and its inert partners.

# h $\rightarrow$ invisible in $\tan\beta = 1$ case

- A significant contribution from $h \rightarrow H^0_{1u} H^0_{1u}$ or $h \rightarrow A^0_{1u} A^0_{1u}$.



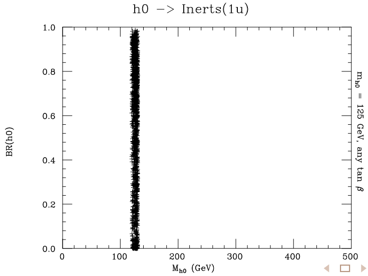

- Include theoretical and experimental constraints (also LHC on heavy Higgses).
- Red-star(Blue-cross) symbol is for $\lambda_{aa} = 0 (\neq 0)$ mixing in active sector.

# $\tan\beta \neq 1$ case ($v_u \neq v_d$)

- Physical states from active doublets:

$$h_{\rm SM} = c_\beta H^0_{3u} + s_\beta H^0_{3d}, \qquad H = -s_\beta H^0_{3u} + c_\beta H^0_{3d}$$

- $HVV = 0 \quad \Longrightarrow \quad$ dominant $H \to t\bar{t}$.
- Now add $\chi^2$ fit to ATLAS and CMS signal strenghts.
- Invisible decays: $h_{\rm SM} \to H^0_{1u} H^0_{1u} + A^0_{1u} A^0_{1u}$ (same as for $\tan\beta = 1$).



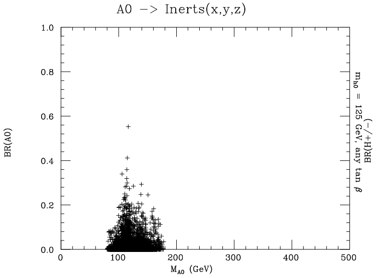
# $\tan\beta \neq 1$ case ($v_u \neq v_d$)

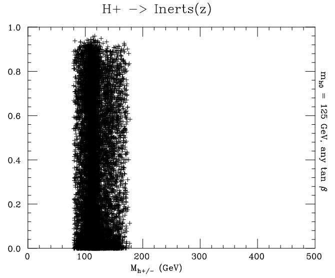
- Heavy Higgs decay channels:

$$A \to A_x H_x + A_y H_y + A_x H_y + A_y H_x + A_z H_z,$$

$$H^\pm \to H_z^\pm H_z + H_z^\pm A_z,$$

i.e., cascades (absent in I(1+1)HDM): heavy inert $\to$ light inert (DM).




- $g_{Z_2^{DM}}$ governs pattern.

# Summary

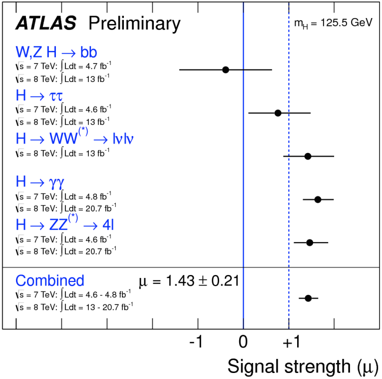
- NHDMs are good for you!
- Our 3HDM→6HDM studies are inspired by $E_6$SSM but can be well motivated on their own.
- The $Z_2$ symmetric 3HDMs/6HDMs contain non-SM features in the Higgs sector which are testable at the LHC.
- These models contain viable DM candidates leading to a relic abundance in agreement with the observed data (Dorota's talk).
- Essentially same parameter space used in above two analyses (3HDM).

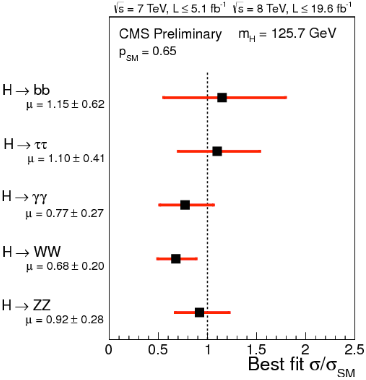
# Backup slides

Backup slides

# Are there any BSM hints?

Deviations from the SM hint at a non-minimal Higgs sectors.




Many non-minimal Higgs sectors have been studied:

[Accomando et al., arXiv:hep-ph/0608079]

# The I(4+2)HDM potential

$$
\begin{aligned}
V \;=\;& \mu_5^2|H_{3u}|^2 + \mu_6^2|H_{3d}|^2 + \mu_i^2\left(|H_{1u}|^2 + |H_{1d}|^2 + |H_{2u}|^2 + |H_{2d}|^2\right) \\
&+\mu_a'^2\left(H_{3u}^\dagger H_{3d}\right) + \mu_i'^2\left(H_{1d}^\dagger H_{2u} + H_{1d}^\dagger H_{2d} + H_{2u}^\dagger H_{2d}\right) + h.c. \\
&+\lambda_a\left(|H_{3u}|^2 + |H_{3d}|^2\right)^2 + \lambda_i\left(|H_{1u}|^2 + |H_{1d}|^2 + |H_{2u}|^2 + |H_{2d}|^2\right)^2 \\
&+\lambda_{ai}\left[(|H_{3u}|^2 + |H_{3d}|^2)(|H_{1u}|^2 + |H_{1d}|^2 + |H_{2u}|^2 + |H_{2d}|^2)\right] \\
&+\lambda_{ai}'\Big[|H_{3u}^\dagger H_{1u}|^2 + |H_{3u}^\dagger H_{1d}|^2 + |H_{3u}^\dagger H_{2u}|^2 + |H_{3u}^\dagger H_{2d}|^2 \\
&\qquad +|H_{3d}^\dagger H_{1u}|^2 + |H_{3d}^\dagger H_{1d}|^2 + |H_{3d}^\dagger H_{2u}|^2 + |H_{3d}^\dagger H_{2d}|^2\Big] + \lambda_{aa}\left[H_{3u}^\dagger H_{3d} + H_{3d}^\dagger H_{3u}\right]^2 \\
&+\lambda_1\left[H_{1d}^\dagger H_{3u} + H_{1d}^\dagger H_{3d} + H_{2u}^\dagger H_{3u} + H_{2u}^\dagger H_{3d} + H_{2d}^\dagger H_{3u} + H_{2d}^\dagger H_{3d}\right]^2 + h.c. \\
&+\lambda_1'\left[H_{1u}^\dagger H_{3u} + H_{1u}^\dagger H_{3d}\right]^2 + h.c.
\end{aligned}
$$

# The I(4+2)HDM spectrum

- Minimising the potential for $v_u = v' \cos\beta$ and $v_d = v' \sin\beta$ requires

$$v'^2 = \frac{-\mu_5^2 - \tan\beta \mu_a'^2}{\lambda_a + 2\sin^2\beta\lambda_{aa}} = \frac{-\tan\beta\mu_6^2 - \mu_a'^2}{\tan\beta\lambda_a + 2\cos\beta\sin\beta\lambda_{aa}} \quad (5)$$

with $\tan\beta$ solution of

$$\begin{aligned} &\tan^4\beta\,(\mu_a'^2\lambda_a) + \tan^3\beta\,((\mu_5^2 - \mu_6^2)\lambda_a - 2\mu_6^2\lambda_{aa}) \\ &+ \tan\beta\,((\mu_5^2 - \mu_6^2)\lambda_a + 2\mu_5^2\lambda_{aa}) + (-\mu_a'^2\lambda_a) = 0. \end{aligned} \quad (6)$$

- Physical mass eigenstates are found upon EWSB driven by

$$\langle H_{1u}\rangle = \langle H_{1d}\rangle = \langle H_{2u}\rangle = \langle H_{2d}\rangle = 0, \qquad \langle H_{3u}\rangle = \frac{v_u}{\sqrt{2}}, \quad \langle H_{3d}\rangle = \frac{v_d}{\sqrt{2}}. \quad (7)$$

By construction, this pattern of minimum respects $Z_2^H \times S_3 \times Z_2^{DM}$.

# The I(4+2)HDM spectrum

- Use Higgs basis by rotating the doublets $H_{3u}$ and $H_{3d}$ and defining the new doublets $\widehat{H_{3u}}$ and $\widehat{H_{3d}}$ as

$$\widehat{H_{3u}} = \cos\beta H_{3u} + \sin\beta H_{3d}, \qquad \widehat{H_{3d}} = -\sin\beta H_{3u} + \cos\beta H_{3d}. \tag{8}$$

Rotation changes VEV alignment to

$$\langle\widehat{H_{3u}}\rangle = \cos\beta v_u + \sin\beta v_d = \frac{v'}{\sqrt{2}}, \tag{9}$$
$$\langle\widehat{H_{3d}}\rangle = -\sin\beta v_u + \cos\beta v_d = 0.$$

- Expand potential around $(0,0,0,0,\frac{v'}{\sqrt{2}},0)$ to obtain

$$v'^2 = -\frac{\mu_5^2 + \mu_a'^2 + \tan\beta(\mu_6^2 - \mu_5^2 + 2\mu_a'^2) + \tan_\beta^2(\mu_6^2 - \mu_a'^2)}{\lambda_a(\tan_\beta^2 + 1) + 2\sin\beta\lambda_{aa}(2\sin\beta + \cos\beta - \tan_\beta \sin\beta)} \tag{10}$$

# The I(4+2)HDM spectrum

- Independent input parameters are:

1. $\mu_5^2$, $\mu_6^2$, $\mu'^2_a$, $\mu_i^2$, $\mu'^2_i$ (masses)
2. $\lambda_{ai}$, $\lambda'_{ai}$, $\lambda_{aa}$, $\lambda_i$, $\lambda_1$ and $\lambda'_1$ (self-couplings)

- Active mass spectrum is

$$\mathbf{h} \equiv \widehat{\mathbf{H}^0_{3u}} = c_\beta H^0_{3u} + s_\beta H^0_{3d} : \quad m^2 = 2(\lambda_a + \lambda_{aa})v'^2$$
$$\mathbf{H} \equiv \widehat{\mathbf{H}^0_{3d}} = -s_\beta H^0_{3u} + c_\beta H^0_{3d} : \quad m^2 = \mu_5^2 + \mu_6^2 + 2(\lambda_a + \lambda_{aa})v'^2$$
$$\mathbf{G}^\pm \equiv \widehat{\mathbf{H}^\pm_{3u}} = c_\beta H^\pm_{3u} + s_\beta H^\pm_{3d} : \quad m^2 = 0$$
$$\mathbf{H}^\pm \equiv \widehat{\mathbf{H}^\pm_{3d}} = -s_\beta H^\pm_{3u} + c_\beta H^\pm_{3d} : \quad m^2 = \mu_5^2 + \mu_6^2 + 2\lambda_a v'^2$$
$$\mathbf{G}^0 \equiv \widehat{\mathbf{A}^0_{3u}} = c_\beta A^0_{3u} + s_\beta A^0_{3d} : \quad m^2 = 0$$
$$\mathbf{A} \equiv \widehat{\mathbf{A}^0_{3d}} = -s_\beta A^0_{3u} + c_\beta A^0_{3d} : \quad m^2 = \mu_5^2 + \mu_6^2 + 2\lambda_a v'^2$$

# The I(4+2)HDM spectrum

- Inert mass spectrum is

$$
\begin{aligned}
&\mathbf{H}^{\pm}_{1u}: \quad m^2 = {\mu_i}^2 + \frac{\lambda_{ai}}{2} v'^2 \\
&\mathbf{H}^{0}_{1u}: \quad m^2 = \mu_i^2 + (\frac{\lambda_{ai} + \lambda'_{ai}}{2} + (1 + 2c_\beta s_\beta)\lambda'_1) v'^2 \\
&\mathbf{A}^{0}_{1u}: \quad m^2 = \mu_i^2 + (\frac{\lambda_{ai} + \lambda'_{ai}}{2} - (1 + 2c_\beta s_\beta)\lambda'_1) v'^2 \\
&\mathbf{H}_x = \frac{H^0_{1d} - H^0_{2d}}{\sqrt{2}}: \quad m^2 = \mu_i^2 - \mu_i'^2 + (\frac{\lambda_{ai} + \lambda'_{ai}}{2}) v'^2 \\
&\mathbf{H}_y = \frac{H^0_{1d} - H^0_{2u}}{\sqrt{2}}: \quad m^2 = \mu_i^2 - \mu_i'^2 + (\frac{\lambda_{ai} + \lambda'_{ai}}{2}) v'^2 \\
&\mathbf{H}_z = \frac{H^0_{1d} + H^0_{2u} + H^0_{2d}}{\sqrt{3}}: \quad m^2 = \mu_i^2 + 2\mu_i'^2 + (\frac{\lambda_{ai} + \lambda'_{ai}}{2} + 3(1 + 2c_\beta s_\beta)\lambda_1) v'^2
\end{aligned}
\tag{11}
$$

# The I(4+2)HDM spectrum

$$\mathbf{A}_x = \frac{A^0_{1d} - A^0_{2d}}{\sqrt{2}} : \quad m^2 = \mu_i^2 - \mu_i'^2 + \left(\frac{\lambda_{ai} + \lambda'_{ai}}{2}\right) v'^2$$

$$\mathbf{A}_y = \frac{A^0_{1d} - A^0_{2u}}{\sqrt{2}} : \quad m^2 = \mu_i^2 - \mu_i'^2 + \left(\frac{\lambda_{ai} + \lambda'_{ai}}{2}\right) v'^2$$

$$\mathbf{A}_z = \frac{A^0_{1d} + A^0_{2u} + A^0_{2d}}{\sqrt{3}} : \quad m^2 = \mu_i^2 + 2\mu_i'^2 + \left(\frac{\lambda_{ai} + \lambda'_{ai}}{2} - 3(1 + 2c_\beta s_\beta)\lambda_1\right) v'^2$$

$$\mathbf{H}^\pm_x = \frac{H^\pm_{1d} - H^\pm_{2d}}{\sqrt{2}} : \quad m^2 = \mu_i^2 - \mu_i'^2 + \frac{\lambda_{ai}}{2} v'^2$$

$$\mathbf{H}^\pm_y = \frac{H^\pm_{1d} - H^\pm_{2u}}{\sqrt{2}} : \quad m^2 = \mu_i^2 - \mu_i'^2 + \frac{\lambda_{ai}}{2} v'^2$$

$$\mathbf{H}^\pm_z = \frac{H^\pm_{1d} + H^\pm_{2u} + H^\pm_{2d}}{\sqrt{3}} : \quad m^2 = \mu_i^2 + 2\mu_i'^2 + \frac{\lambda_{ai}}{2} v'^2$$

where $c_\beta(s_\beta)$ is $\cos\beta(\sin\beta)$.