Higgs Spectra from Maximally Symmetric Two Higgs Doublet Model Potential

P. S. Bhupal Dev

Consortium for Fundamental Physics,
The University of Manchester, United Kingdom

PSBD and A. Pilaftsis, arXiv:1408.3405 [hep-ph].

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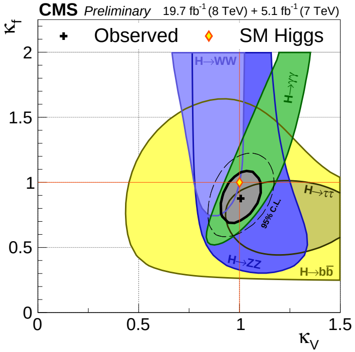
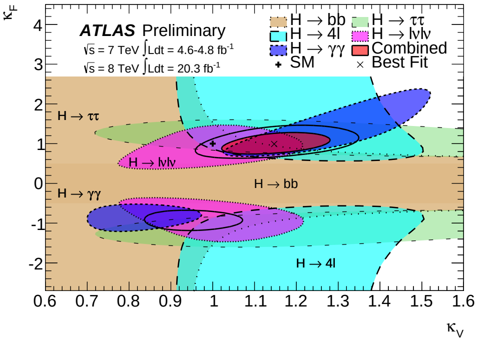
Outline

- Introduction
- Symmetry classifications of the 2HDM Potential
- Spectrum Analysis for the Maximally Symmetric Potential
- Some Collider Phenomenology
- Conclusion

Introduction





- Couplings of the discovered Higgs boson at $M_H = 125 \pm 2$ GeV are within $\mathcal{O}(10\%)$ of the SM predictions.
- Opportunity in the search of (or constraining) BSM physics through Higgs portal.
  - Precision Higgs Study (Higgcision).
  - Search for additional Higgses.

Two Higgs Doublet Model

- Several theoretical motivations to go beyond the SM Higgs sector.
- In a modest bottom-up approach, consider the simplest Higgs-sector extension of the SM, i.e. two $SU(2)_L$ doublets $\Phi_i = \begin{pmatrix} \phi_i^+ \\ \phi_i^0 \end{pmatrix}$ (with $i = 1, 2$).
- Most general 2HDM potential in doublet field space $\Phi_{1,2}$:

$$
\begin{aligned}
V = & -\mu_1^2(\Phi_1^\dagger\Phi_1) - \mu_2^2(\Phi_2^\dagger\Phi_2) - \left[m_{12}^2(\Phi_1^\dagger\Phi_2) + \mathrm{H.c.}\right] \\
& + \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\
& + \left[\frac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \lambda_7(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_2) + \mathrm{H.c.}\right].
\end{aligned}
$$

- Four real mass parameters $\mu_{1,2}^2$, Re(m_{12}^2), Im(m_{12}^2), and 10 real quartic couplings $\lambda_{1,2,3,4}$, Re($\lambda_{5,6,7}$), Im($\lambda_{5,6,7}$).
- Motivated by the LHC Higgs data, scrutinize the SM alignment limit of the 2HDM potential.
- Explore possible symmetries of the 2HDM potential to *naturally* justify the alignment limit.

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An Alternative Formulation of the 2HDM Potential

- Gauge-invariant bilinear scalar-field formalism. [Nishi '06; Ivanov '06; Maniatis *et al* '06]
- Extend the SL(2,$\mathbb{C}$) group to the maximal reparametrization group of the 2HDM potential, namely, the complex linear group GL(8,$\mathbb{C}$), by introducing an 8-dimensional complex multiplet: [Battye, Brawn, Pilaftsis '11; Nishi '11; Pilaftsis '12]

$$\mathbf{\Phi} = \begin{pmatrix} \Phi_1 \\ \Phi_2 \\ i\sigma^2\Phi_1^* \\ i\sigma^2\Phi_2^* \end{pmatrix} .$$

- $\mathbf{\Phi}$ satisfies the Majorana property: $\mathbf{\Phi} = C\mathbf{\Phi}^*$, where $C = \sigma^2 \otimes \sigma^0 \otimes \sigma^2$.
- Define a *null* 6-dimensional Lorentz vector: $R^A = \Phi^\dagger \Sigma^A \Phi$ (with $A = 0, 1, 2, 3, 4, 5$), where

$$\Sigma^0 = \frac{1}{2}\sigma^0 \otimes \sigma^0 \otimes \sigma^0 \equiv \frac{1}{2}\mathbf{1}_8, \quad \Sigma^1 = \frac{1}{2}\sigma^0 \otimes \sigma^1 \otimes \sigma^0, \quad \Sigma^2 = \frac{1}{2}\sigma^3 \otimes \sigma^2 \otimes \sigma^0,$$
$$\Sigma^3 = \frac{1}{2}\sigma^0 \otimes \sigma^3 \otimes \sigma^0, \quad \Sigma^4 = -\frac{1}{2}\sigma^2 \otimes \sigma^2 \otimes \sigma^0, \quad \Sigma^5 = -\frac{1}{2}\sigma^1 \otimes \sigma^2 \otimes \sigma^0.$$

- The general 2HDM potential takes a simple form:

$$V = -\frac{1}{2}M_A R^A + \frac{1}{4}L_{AB}R^A R^B,$$

- The bilinear field space spanned by the 6-vector R^A realizes an *orthochronous* $SO(1,5)$ symmetry.

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Symmetry Classifications of the 2HDM Potential

- Three classes of accidental symmetries of the 2HDM potential:
  - Higgs Family (HF) Symmetries involving transformations of $\Phi_{1,2}$ only (but not $\Phi^*_{1,2}$), e.g. Z_2 [Glashow, Weinberg '58], $\mathrm{U(1)_{PQ}}$ [Peccei, Quinn '77], $\mathrm{SO(3)_{HF}}$ [Deshpande, Ma '78; Ivanov '07; Ma, Maniatis '09; Ferreira, Haber, Maniatis, Nachtmann, Silva '10].
  - CP Symmetries relating $\Phi_{1,2}$ to $\Phi^*_{1,2}$, e.g. $\Phi_{1(2)} \to \Phi^*_{1(2)}$ (CP1) [Lee '73; Branco '80], $\Phi_{1(2)} \to (-)\Phi^*_{2(1)}$ (CP2) [Davidson, Haber '05], CP1 combined with $\mathrm{SO(2)_{HF}}/Z_2$ (CP3) [Ivanov '07; Ferreira, Haber, Silva '09; Ma, Maniatis '09; Ferreira, Haber, Maniatis, Nachtmann, Silva '10].
  - Additional mixed HF and CP symmetries larger than O(3) that leave the gauge-kinetic terms of $\Phi_{1,2}$ invariant [Battye, Brawn, Pilaftsis '11].
- The Majorana condition, together with $\mathrm{SU(2)}_L \times \mathrm{U(1)}_Y$ gauge invariance, reduces GL(8,$\mathbb{C}$) to two subgroups isomorphic to GL(4,$\mathbb{R}$): one related to HF and another to generalized CP.
- Maximum of 13 *distinct* accidental symmetries of the general 2HDM potential. [Battye, Brawn, Pilaftsis '11; Pilaftsis '12]
- Each of them imposes specific relations among the scalar parameters.

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Symmetry Classifications of the 2HDM Potential

[Pilaftsis '12]

Table 1
Parameter relations for the 13 accidental symmetries [1] related to the $U(1)_Y$-invariant 2HDM potential in the diagonally reduced basis, where $\mathrm{Im}\,\lambda_5 = 0$ and $\lambda_6 = \lambda_7$. A dash signifies the absence of a constraint.

No.	Symmetry	μ_1^2	μ_2^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	$\mathrm{Re}\,\lambda_5$	$\lambda_6 = \lambda_7$
1	$Z_2 \times O(2)$	–	–	Real	–	–	–	–	–	Real
2	$(Z_2)^2 \times SO(2)$	–	–	0	–	–	–	–	–	0
3	$(Z_2)^3 \times O(2)$	–	μ_1^2	0	–	λ_1	–	–	–	0
4	$O(2) \times O(2)$	–	–	0	–	–	–	–	0	0
5	$Z_2 \times [O(2)]^2$	–	μ_1^2	0	–	λ_1	–	–	$2\lambda_1 - \lambda_{34}$	0
6	$O(3) \times O(2)$	–	μ_1^2	0	–	λ_1	–	$2\lambda_1 - \lambda_3$	0	0
7	$SO(3)$	–	–	Real	–	–	–	–	λ_4	Real
8	$Z_2 \times O(3)$	–	μ_1^2	Real	–	λ_1	–	–	λ_4	Real
9	$(Z_2)^2 \times SO(3)$	–	μ_1^2	0	–	λ_1	–	–	$\pm\lambda_4$	0
10	$O(2) \times O(3)$	–	μ_1^2	0	–	λ_1	$2\lambda_1$	–	0	0
11	$SO(4)$	–	–	0	–	–	–	0	0	0
12	$Z_2 \times O(4)$	–	μ_1^2	0	–	λ_1	–	0	0	0
13	$SO(5)$	–	μ_1^2	0	–	λ_1	$2\lambda_1$	0	0	0

- In the diagonally reduced basis [Gunion, Haber '05; Maniatis, Nachtmann '11], $\mathrm{Im}(\lambda_5) = 0$ and $\lambda_6 = \lambda_7$.
- 7 independent quartic couplings for the $U(1)_Y$-invariant 2HDM potential.

- *Maximal* symmetry group in the bilinear field space: $G^R_{\mathrm{2HDM}} = SO(5)$.
- *Maximal* symmetry group in the original Φ-field space: $\mathrm{G}^{\Phi}_{\mathrm{2HDM}} = (\mathrm{Sp}(4)/\mathrm{Z}_2) \otimes \mathrm{SU}(2)_L$.
- Conjecture: In a general nHDM, $\mathrm{G}^{\Phi}_{n\mathrm{HDM}} = (\mathrm{Sp}(2n)/\mathrm{Z}_2) \otimes \mathrm{SU}(2)_L$.
- For the SM (with $n = 1$), reproduce the well-known result $\mathrm{G}^{\Phi}_{\mathrm{SM}} = (\mathrm{SU}(2)_C/\mathrm{Z}_2) \otimes \mathrm{SU}(2)_L$, using group isomorphy: $\mathrm{Sp}(2) \sim \mathrm{SU}(2)_C$ (custodial). [Sikivie, Susskind, Voloshin, Zakharov '80]

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Maximally Symmetric 2HDM

- Following relations between the scalar potential parameters in the SO(5) limit:

$$\mu_1^2 \;=\; \mu_2^2\,, \quad m_{12}^2 \;=\; 0\,, \quad \lambda_2 \;=\; \lambda_1\,, \quad \lambda_3 \;=\; 2\lambda_1\,, \quad \lambda_4 \;=\; \mathrm{Re}(\lambda_5) \;=\; \lambda_6 \;=\; \lambda_7 \;=\; 0\,.$$

- 2HDM potential parametrized by *single* mass parameter μ^2 and *single* quartic coupling λ:

$$V \;=\; -\,\mu^2\left(|\Phi_1|^2+|\Phi_2|^2\right) \;+\; \lambda\left(|\Phi_1|^2+|\Phi_2|^2\right)^2 \;=\; -\,\frac{\mu^2}{2}\,\mathbf{\Phi}^\dagger\,\mathbf{\Phi} \;+\; \frac{\lambda}{4}\,(\mathbf{\Phi}^\dagger\,\mathbf{\Phi})^2\,.$$

- More minimal than the MSSM scalar potential, even in the custodial limit $g' \to 0$, which possesses a smaller symmetry: $\mathrm{O}(2)\otimes\mathrm{O}(3)\subset\mathrm{SO}(5)$.
- After EWSB in the MS-2HDM, one massive Higgs boson H with mass $M_H^2 = 2\lambda_2 v^2$, whilst remaining four scalar fields (h, a and $h^\pm$) are massless (pseudo)-Goldstone bosons.
- **Natural SM alignment limit with $\alpha = \beta$.** [Recall $H_{\mathrm{SM}} \;=\; H\cos(\beta-\alpha)+h\sin(\beta-\alpha)$]
- (Pseudo)-Goldstones can naturally pick up mass due to g' and Yukawa coupling effects.

Custodial Symmetries in the MS-2HDM

- Quark-sector Yukawa Lagrangian

$$
\begin{aligned}
-\mathcal{L}_Y^q &= \bar{Q}_L(h_1^u\Phi_1 + h_2^u\Phi_2)u_R + \bar{Q}_L(h_1^d\widetilde{\Phi}_1 + h_2^d\widetilde{\Phi}_2)d_R \\
&= (\bar{u}_L\,,\,\bar{d}_L)\,\left(\Phi_1\,,\,\Phi_2\,,\,\widetilde{\Phi}_1\,,\,\widetilde{\Phi}_2\right)\underbrace{\begin{pmatrix} h_1^u & \mathbf{0} \\ h_2^u & \mathbf{0} \\ \mathbf{0} & h_1^d \\ \mathbf{0} & h_2^d \end{pmatrix}}_{\mathcal{H}}\begin{pmatrix} u_R \\ d_R \end{pmatrix}.
\end{aligned}
$$

- To find *all* the custodial symmetries of this Lagrangian, consider the 10 Lie generators of Sp(4) group: $K^a = \kappa^a \otimes \sigma^0$, where with the normalization: $\mathrm{Tr}(\kappa^a\,\kappa^b) = \delta^{ab}$,

$$
\kappa^{0,1,3} = \frac{1}{2}\sigma^3 \otimes \sigma^{0,1,3}\,, \quad \kappa^2 = \frac{1}{2}\sigma^0 \otimes \sigma^2\,, \quad \kappa^4 = \frac{1}{2}\sigma^1 \otimes \sigma^0\,, \quad \kappa^5 = \frac{1}{2}\sigma^1 \otimes \sigma^3\,,
$$

$$
\kappa^6 = \frac{1}{2}\sigma^2 \otimes \sigma^0\,, \quad \kappa^7 = \frac{1}{2}\sigma^2 \otimes \sigma^3\,, \quad \kappa^8 = \frac{1}{2}\sigma^1 \otimes \sigma^1\,, \quad \kappa^9 = \frac{1}{2}\sigma^2 \otimes \sigma^1\,.
$$

- Candidate Sp(4) generators of the custodial symmetry $\mathrm{SU}(2)_C$ are those which do *not* commute with the hypercharge generator K^0, i.e. K^a with $a = 4, 5, 6, 7, 8, 9$.
- 3 inequivalent realizations: (i) $K^{0,4,6}$, (ii) $K^{0,5,7}$, (iii) $K^{0,8,9}$ [equivalent in SO(5)).
- Satisfy the symmetry commutation relation $\kappa^a\,\mathcal{H} - \mathcal{H}\,t^b = \mathbf{0}_{4\times 2}$ (where $t^b = \sigma^b/2$).

$$
\text{(i) } h_{1,2}^u = e^{i\theta}\,h_{1,2}^d\,, \quad \text{(ii) } h_1^u = e^{i\theta}\,h_1^d,\ \ h_2^u = -e^{i\theta}\,h_2^d\,, \quad \text{(iii) } h_1^u = e^{i\theta}\,h_2^d,\ \ h_2^u = e^{-i\theta}\,h_1^d\,.
$$

Custodial Symmetries in the MS-2HDM

- Quark-sector Yukawa Lagrangian

$$-\mathcal{L}_Y^q \;=\; \bar{Q}_L(h_1^u\Phi_1 + h_2^u\Phi_2)u_R \;+\; \bar{Q}_L(h_1^d\widetilde{\Phi}_1 + h_2^d\widetilde{\Phi}_2)d_R$$

$$= \;(\bar{u}_L\,,\,\bar{d}_L)\;\left(\Phi_1\,,\,\Phi_2\,,\,\widetilde{\Phi}_1\,,\,\widetilde{\Phi}_2\right)\underbrace{\begin{pmatrix} h_1^u & \mathbf{0} \\ h_2^u & \mathbf{0} \\ \mathbf{0} & h_1^d \\ \mathbf{0} & h_2^d \end{pmatrix}}_{\mathcal{H}}\begin{pmatrix} u_R \\ d_R \end{pmatrix}\,.$$

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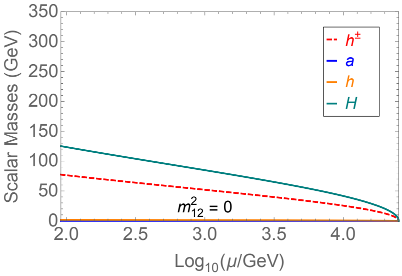
g' and Yukawa Coupling Effects

- Custodial symmetry broken by non-zero g' and Yukawa couplings.

$$\begin{aligned}
\mathrm{SO}(5) \otimes \mathrm{SU}(2)_L \quad &\xrightarrow{g' \neq 0} \quad \mathrm{O}(3) \otimes \mathrm{O}(2) \otimes \mathrm{SU}(2)_L \sim \mathrm{O}(3) \otimes \mathrm{U}(1)_Y \otimes \mathrm{SU}(2)_L \\
&\xrightarrow{\text{Yukawa}} \quad \mathrm{O}(2) \otimes \mathrm{U}(1)_Y \otimes \mathrm{SU}(2)_L \sim \mathrm{U}(1)_{\mathrm{PQ}} \otimes \mathrm{U}(1)_Y \otimes \mathrm{SU}(2)_L \\
&\xrightarrow{\langle \Phi_{1,2} \rangle \neq 0} \quad \mathrm{U}(1)_{\mathrm{em}} \, .
\end{aligned}$$

- To study their effects on the Higgs spectrum in a technically natural manner, assume SO(5)-symmetry scale $\mu_X \gg v$, and use RG running down to the weak scale.
- Does NOT yield a viable Higgs spectrum.

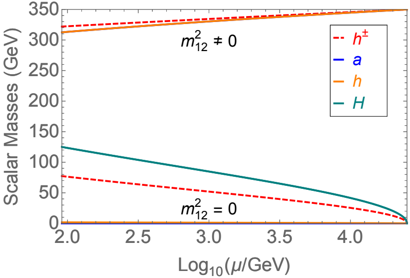


g' and Yukawa Coupling Effects

- Custodial symmetry broken by non-zero g' and Yukawa couplings.

$$\begin{aligned} \mathrm{SO}(5)\otimes\mathrm{SU}(2)_L \quad &\xrightarrow{g'\neq 0} \quad \mathrm{O}(3)\otimes\mathrm{O}(2)\otimes\mathrm{SU}(2)_L \;\sim\; \mathrm{O}(3)\otimes\mathrm{U}(1)_Y\otimes\mathrm{SU}(2)_L \\ &\xrightarrow{\text{Yukawa}} \quad \mathrm{O}(2)\otimes\mathrm{U}(1)_Y\otimes\mathrm{SU}(2)_L \;\sim\; \mathrm{U}(1)_{\mathrm{PQ}}\otimes\mathrm{U}(1)_Y\otimes\mathrm{SU}(2)_L \\ &\xrightarrow{\langle\Phi_{1,2}\rangle\neq 0} \quad \mathrm{U}(1)_{\mathrm{em}}\ . \end{aligned}$$

- Include soft SO(5)-breaking effects by $\mathrm{Re}(m_{12}^2)\neq 0$.
- Does yield a viable Higgs spectrum.



Soft Breaking Effects

- In the $SO(5)$ limit for quartic couplings, but with $\mathrm{Re}(m_{12}^2) \neq 0$,

$$M_S^2 = M_a^2 \begin{pmatrix} s_\beta^2 & -s_\beta c_\beta \\ -s_\beta c_\beta & c_\beta^2 \end{pmatrix} + 2\lambda_2 v^2 \begin{pmatrix} c_\beta^2 & s_\beta c_\beta \\ s_\beta c_\beta & s_\beta^2 \end{pmatrix}$$

$$= \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} 2\lambda_2 v^2 & 0 \\ 0 & M_a^2 \end{pmatrix} \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \equiv O \widehat{M}_S^2 O^{\mathsf{T}} .$$

- $M_H^2 = 2\lambda_2 v^2 \,, \quad \text{and} \quad M_h^2 = M_a^2 = M_{h^\pm}^2 = \dfrac{\mathrm{Re}(m_{12}^2)}{s_\beta c_\beta}.$
- Natural alignment, irrespective of other 2HDM parameters.
- In the general 2HDM, $\widehat{M}_S^2 = \begin{pmatrix} \widehat{A} & \widehat{C} \\ \widehat{C} & \widehat{B} \end{pmatrix}$ with

$$\widehat{A} = 2v^2 \Big[c_\beta^4 \lambda_1 + s_\beta^2 c_\beta^2 \lambda_{345} + s_\beta^4 \lambda_2 + 2 s_\beta c_\beta \big(c_\beta^2 \lambda_6 + s_\beta^2 \lambda_7 \big) \Big] ,$$

$$\widehat{B} = M_a^2 + \lambda_5 v^2 + 2v^2 \Big[s_\beta^2 c_\beta^2 \big(\lambda_1 + \lambda_2 - \lambda_{345} \big) - s_\beta c_\beta \big(c_\beta^2 - s_\beta^2 \big) \big(\lambda_6 - \lambda_7 \big) \Big] ,$$

$$\widehat{C} = v^2 \Big[s_\beta^3 c_\beta \big(2\lambda_2 - \lambda_{345} \big) - c_\beta^3 s_\beta \big(2\lambda_1 - \lambda_{345} \big) + c_\beta^2 \big(1 - 4 s_\beta^2 \big) \lambda_6 + s_\beta^2 \big(4 c_\beta^2 - 1 \big) \lambda_7 \Big]$$

- Alignment iff $\widehat{C} = 0$.

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- $M_H^2 = 2\lambda_2 v^2$, and $M_h^2 = M_a^2 = M_{h^\pm}^2 = \frac{\mathrm{Re}(m_{12}^2)}{s_\beta c_\beta}$.
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- Alignment iff $\widehat{C} = 0$.

Natural Alignment Condition

$$t_\beta^4 \lambda_7 - t_\beta^3 (2\lambda_2 - \lambda_{345}) + 3 t_\beta^2 (\lambda_6 - \lambda_7) + t_\beta (2\lambda_1 - \lambda_{345}) - \lambda_6 \; = \; 0 \, .$$

- Requires $\lambda_6 = \lambda_7 = 0$, and $\tan^2\beta \; = \; \frac{2\lambda_1 - \lambda_{345}}{2\lambda_2 - \lambda_{345}} \; > \; 0$.
- Alignment without decoupling, i.e. independent of M_a.
  (similar to [Gunion, Haber '03; Carena, Low, Shah, Wagner '13])
- CP-even Higgs masses are given by

$$\begin{aligned} M_H^2 &= 2v^2(\lambda_1 c_\beta^4 + \lambda_{345} s_\beta^2 c_\beta^2 + \lambda_2 s_\beta^4) \; \equiv \; \lambda_{\rm SM} v^2 \, , \\ M_h^2 &= M_a^2 \; + \; \lambda_5 v^2 \; + \; 2v^2 s_\beta^2 c_\beta^2 (\lambda_1 + \lambda_2 - \lambda_{345}) \, . \end{aligned}$$

- In the decoupling limit $M_a \gg v$, can use a seesaw-like approximation to obtain

$$\begin{aligned} M_H^2 &\simeq \lambda_{\rm SM} v^2 - \frac{v^4 s_\beta^2 c_\beta^2}{M_a^2 + \lambda_5 v^2} \left[s_\beta^2 \left(2\lambda_2 - \lambda_{345} \right) - c_\beta^2 \left(2\lambda_1 - \lambda_{345} \right) \right]^2 , \\ M_h^2 &\simeq M_a^2 + \lambda_5 v^2 \; \gg \; v^2 \, . \end{aligned}$$

- Includes the possibility of decoupling via a large λ_5. [Ginzburg, Krawczyk '04]
- Apart from SO(5), only two other symmetries can lead to natural alignment:

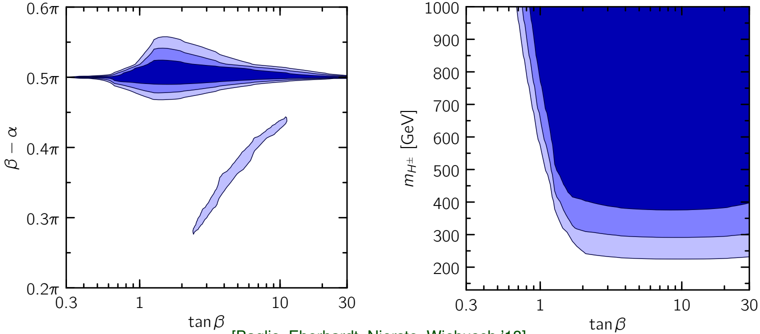
$$\begin{aligned} &\text{(i)} \quad \mathrm{O}(3) \otimes \mathrm{O}(2): \qquad \mu_1^2 = \mu_2^2 \, , \quad \lambda_1 = \lambda_2 = \lambda_{34}/2 \, , \\ &\text{(ii)} \quad \mathrm{Z}_2 \otimes [\mathrm{O}(2)]^2: \qquad \mu_1^2 = \mu_2^2 \, , \quad \lambda_1 = \lambda_2 = \lambda_{345}/2 \, . \end{aligned}$$

Theoretical and Experimental Constraints

- Stability of the potential: [Branco *et al* '12]

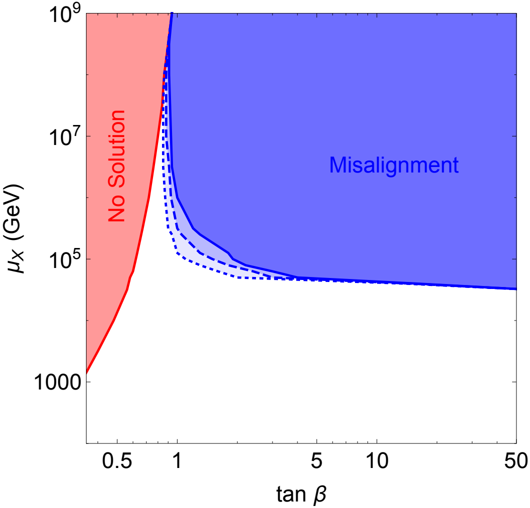
$$\lambda_{1,2} > 0, \quad \lambda_3 + \sqrt{\lambda_1 \lambda_2} > 0, \quad \lambda_3 + \lambda_4 + \sqrt{\lambda_1 \lambda_2} - \mathrm{Re}(\lambda_5) > 0.$$

- Perturbativity of the Higgs self-couplings: $\|S_{\phi\phi\to\phi\phi}\| < \frac{1}{8}$.
- Electroweak precision observables.
- LHC signal strengths of the light *CP*-even Higgs boson.
- Limits on heavy *CP*-even scalar from $H \to WW, ZZ, \tau\tau$ searches.
- Flavor observables such as B_s mixing and $B \to X_s\gamma$.



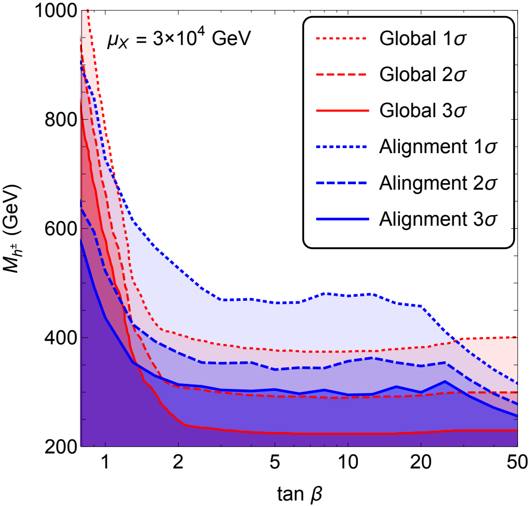
[Baglio, Eberhardt, Nierste, Wiebusch '13]

Misalignment Predictions



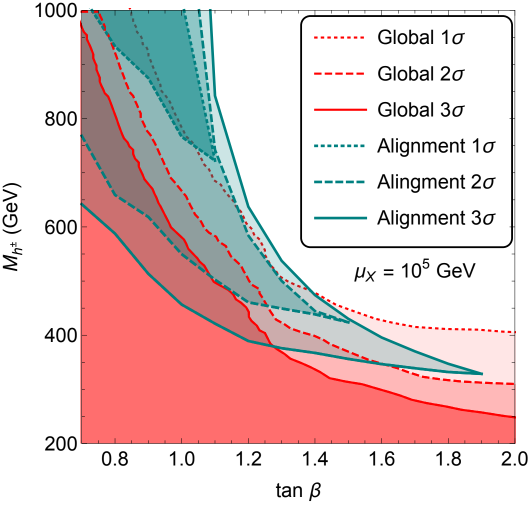
[PSBD, Pilaftsis '14]

Lower Limit on Charged Higgs Mass



[PSBD, Pilaftsis '14]

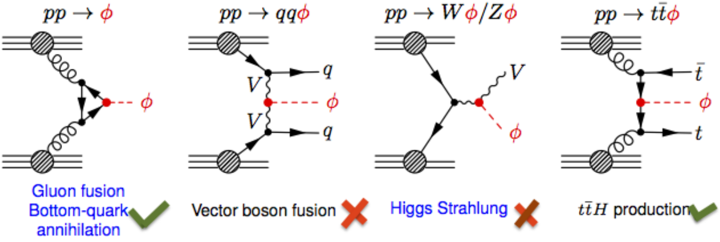
Lower *and* Upper Limits on Charged Higgs Mass

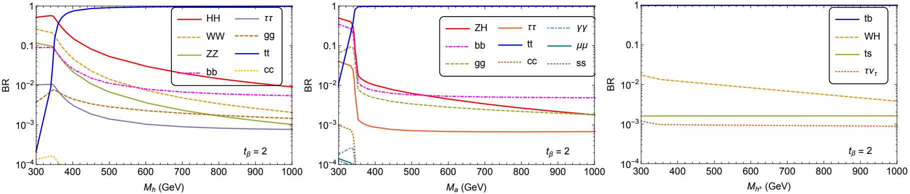


[PSBD, Pilaftsis '14]

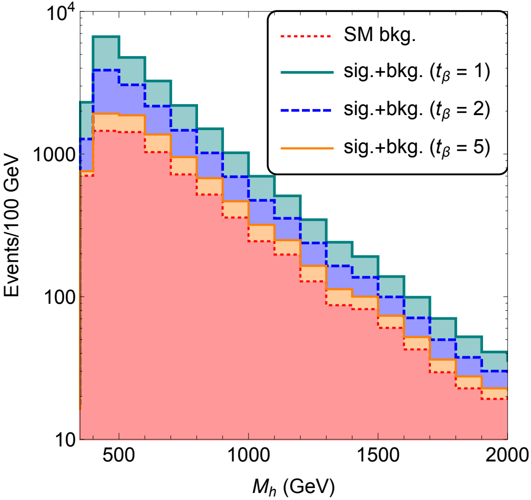
Implications for LHC

Higgs production processes:



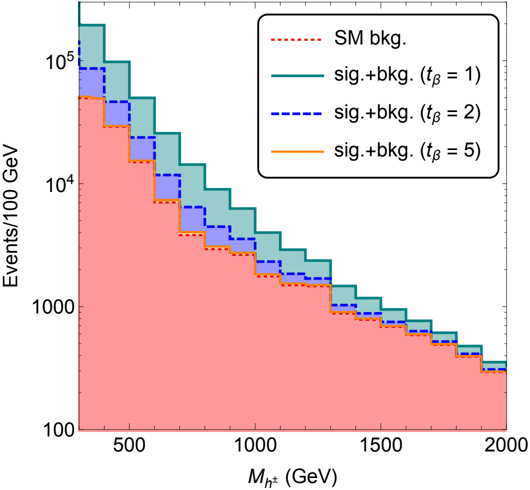


$pp \to t\bar{t}h \to t\bar{t}t\bar{t}$ Signal



[PSBD, Pilaftsis ’14]

$pp \to \bar{t}bh^{+} \to t\bar{t}b\bar{b}$ Signal



[PSBD, Pilaftsis '14]

Conclusions

- We analyzed the scalar potential of the Maximally Symmetric 2HDM.
- SM Alignment limit can be realized naturally, *independently* of the heavy Higgs spectrum and the value of tan β.
- Deviations from the alignment limit are induced by RGE effects due to the hypercharge gauge coupling g' and third generation Yukawa couplings, which also break the custodial symmetry of the theory.
- In addition, non-zero soft SO(5)-breaking mass parameter is required to yield a viable Higgs spectrum consistent with the existing experimental constraints.
- Using the current Higgs signal strength data from the LHC, which disfavour large deviations from the alignment limit, we derive important constraints on the MS-2HDM parameter space.
- Predict lower limits on the heavy Higgs spectrum, which prevail the present limits in a wide range of parameter space.
- Depending on the scale where the maximal symmetry could be realized in nature, we also obtain an upper limit on the heavy Higgs masses in certain cases, which could be completely probed during the run-II phase of the LHC.
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THANK YOU.

2HDM Potential in Bilinear Space

$$V = -\frac{1}{2} M_A R^A + \frac{1}{4} L_{AB} R^A R^B,$$

where

$$M_A = \left(\mu_1^2 + \mu_2^2, \quad 2\mathrm{Re}(m_{12}^2), \quad -2\mathrm{Im}(m_{12}^2), \quad \mu_1^2 - \mu_2^2, \quad 0, \quad 0\right),$$

$$R^A = \begin{pmatrix} \Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 \\ \Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1 \\ -i(\Phi_1^\dagger \Phi_2 - \Phi_2^\dagger \Phi_1) \\ \Phi_1^\dagger \Phi_1 - \Phi_2^\dagger \Phi_2 \\ \Phi_1^\mathsf{T} i\sigma^2 \Phi_2 - \Phi_2^\dagger i\sigma^2 \Phi_1^* \\ -i(\Phi_1^\mathsf{T} i\sigma^2 \Phi_2 + \Phi_2^\dagger i\sigma^2 \Phi_1^*) \end{pmatrix},$$

$$L^A_B = \begin{pmatrix} \lambda_1 + \lambda_2 + \lambda_3 & \mathrm{Re}(\lambda_6 + \lambda_7) & -\mathrm{Im}(\lambda_6 + \lambda_7) & \lambda_1 - \lambda_2 & 0 & 0 \\ \mathrm{Re}(\lambda_6 + \lambda_7) & \lambda_4 + \mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_5) & \mathrm{Re}(\lambda_6 - \lambda_7) & 0 & 0 \\ -\mathrm{Im}(\lambda_6 + \lambda_7) & -\mathrm{Im}(\lambda_5) & \lambda_4 - \mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_6 - \lambda_7) & 0 & 0 \\ \lambda_1 - \lambda_2 & \mathrm{Re}(\lambda_6 - \lambda_7) & -\mathrm{Im}(\lambda_6 - \lambda_7) & \lambda_1 + \lambda_2 - \lambda_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Symmetry Generators

Table 2
Symmetry generators [cf. (10), (14)] and discrete group elements [cf. (17)] for the 13 accidental symmetries of the $U(1)_Y$-invariant 2HDM potential. For each symmetry, the maximally broken SO(5) generators compatible with a neutral vacuum are displayed, along with the pseudo-Goldstone bosons (given in parentheses) that result from the Goldstone theorem.

No.	Symmetry	Generators $T^a \leftrightarrow K^a$	Discrete group elements	Maximally broken SO(5) generators	Number of pseudo-Goldstone bosons
1	$Z_2 \times O(2)$	T^0	D_{CP1}	–	0
2	$(Z_2)^2 \times SO(2)$	T^0	D_{Z_2}	–	0
3	$(Z_2)^3 \times O(2)$	T^0	D_{CP2}	–	0
4	$O(2) \times O(2)$	T^3, T^0	–	T^3	1 (a)
5	$Z_2 \times [O(2)]^2$	T^2, T^0	D_{CP1}	T^2	1 (h)
6	$O(3) \times O(2)$	$T^{1,2,3}, T^0$	–	$T^{1,2}$	2 (h, a)
7	$SO(3)$	$T^{0,4,6}$	–	$T^{4,6}$	2 $(h^\pm)$
8	$Z_2 \times O(3)$	$T^{0,4,6}$	$D_{Z_2} \cdot D_{CP2}$	$T^{4,6}$	2 $(h^\pm)$
9	$(Z_2)^2 \times SO(3)$	$T^{0,5,7}$	$D_{CP1} \cdot D_{CP2}$	$T^{5,7}$	2 $(h^\pm)$
10	$O(2) \times O(3)$	$T^3, T^{0,8,9}$	–	T^3	1 (a)
11	$SO(4)$	$T^{0,3,4,5,6,7}$	–	$T^{3,5,7}$	3 $(a, h^\pm)$
12	$Z_2 \times O(4)$	$T^{0,3,4,5,6,7}$	$D_{Z_2} \cdot D_{CP2}$	$T^{3,5,7}$	3 $(a, h^\pm)$
13	$SO(5)$	$T^{0,1,2,\ldots,9}$	–	$T^{1,2,8,9}$	4 $(h, a, h^\pm)$

[Pilaftsis '12]

- T^a and K^a are the generators of $SO(5)$ and $Sp(4)$ respectively ($a = 0, \ldots, 9$).
- T^0 is the hypercharge generator in R-space, which is equivalent to the electromagnetic generator $Q_{\rm em} = \frac{1}{2}\sigma^0 \otimes \sigma^0 \otimes \sigma^3 + K^0$ in Φ-space.
- $Sp(4)$ contains the custodial symmetry group $SU(2)_C$.
- Three *independent* realizations of custodial symmetry induced by (i) $K^{0,4,6}$, (ii) $K^{0,5,7}$, (iii) $K^{0,8,9}$.

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Higgs Masses and Couplings in a General 2HDM

- In the CP-even sector,

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} .$$

$$\left(M_S^2\right)_{ij} \equiv \begin{pmatrix} A & C \\ C & B \end{pmatrix} = M_A^2 \begin{pmatrix} s_\beta^2 & -s_\beta c_\beta \\ -s_\beta c_\beta & c_\beta^2 \end{pmatrix}$$
$$+ v^2 \begin{pmatrix} 2\lambda_1 c_\beta^2 + \mathrm{Re}(\lambda_5) s_\beta^2 + 2\mathrm{Re}(\lambda_6) s_\beta c_\beta & \lambda_{34} s_\beta c_\beta + \mathrm{Re}(\lambda_6) c_\beta^2 + \mathrm{Re}(\lambda_7) s_\beta^2 \\ \lambda_{34} s_\beta c_\beta + \mathrm{Re}(\lambda_6) c_\beta^2 + \mathrm{Re}(\lambda_7) s_\beta^2 & 2\lambda_2 s_\beta^2 + \mathrm{Re}(\lambda_5) c_\beta^2 + 2\mathrm{Re}(\lambda_7) s_\beta c_\beta \end{pmatrix}$$

with $\lambda_{34} = \lambda_3 + \lambda_4$ and $\tan 2\alpha = \frac{2C}{A-B}$. [Pilaftsis, Wagner '99]

- The SM Higgs boson is given by

$$H_{\mathrm{SM}} = \rho_1 \cos\beta + \rho_2 \sin\beta = H\cos(\beta-\alpha) + h\sin(\beta-\alpha) .$$

- With respect to the SM Higgs couplings $H_{\mathrm{SM}}VV$ ($V = W^\pm, Z$),

$$g_{hVV} = \sin(\beta-\alpha) , \qquad g_{HVV} = \cos(\beta-\alpha) .$$

Unitarity constraints uniquely fix other V-Higgs-Higgs couplings [Gunion, Haber, Kane, Dawson '90]

$$g_{haZ} = \frac{g}{2\cos\theta_w}\cos(\beta-\alpha) , \qquad g_{HaZ} = \frac{g}{2\cos\theta_w}\sin(\beta-\alpha) ,$$
$$g_{h^+hW^-} = \frac{g}{2}\cos(\beta-\alpha) , \qquad g_{h^+HW^-} = \frac{g}{2}\sin(\beta-\alpha) .$$

Higgs Masses and Couplings in a General 2HDM

- Consider normal vacua with real vevs $v_{1,2}$, where $\sqrt{v_1^2 + v_2^2} = v_{\rm SM}$ and $\tan\beta = v_2/v_1$.
- Eight real scalar fields: $\phi_j = \begin{pmatrix} \phi_j^+ \\ \frac{1}{\sqrt{2}}(v_j + \rho_j + i\eta_j) \end{pmatrix}$ (with $j = 1, 2$).
- After EWSB, three Goldstone bosons ($G^\pm$, G^0), which are eaten by $W^\pm$ and Z, and five physical scalar fields: two CP-even (h, H), one CP-odd (a) and two charged ($h^\pm$).
- In the charged sector,

$$\begin{pmatrix} G^\pm \\ h^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix} .$$

with $$M_{h^\pm}^2 = \frac{1}{s_\beta c_\beta}\left[\mathrm{Re}(m_{12}^2) - \frac{1}{2}\left(\{\lambda_4 + \mathrm{Re}(\lambda_5)\}\, s_\beta c_\beta + \mathrm{Re}(\lambda_6) c_\beta^2 + \mathrm{Re}(\lambda_7) s_\beta^2\right)\right].$$

- In the CP-odd sector,

$$\begin{pmatrix} G^0 \\ a \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} .$$

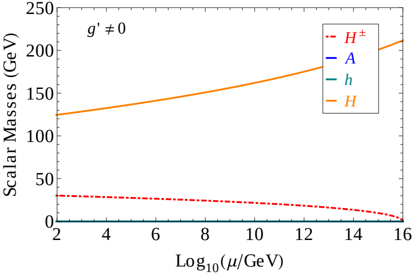
with $$\begin{aligned} M_a^2 &= \frac{1}{s_\beta c_\beta}\left[\mathrm{Re}(m_{12}^2) - v^2\left(\mathrm{Re}(\lambda_5) s_\beta c_\beta + \frac{1}{2}\left\{\mathrm{Re}(\lambda_6) c_\beta^2 + \mathrm{Re}(\lambda_7) s_\beta^2\right\}\right)\right] \\ &= M_{h^\pm}^2 + \frac{1}{2}\left[\lambda_4 - \mathrm{Re}(\lambda_5)\right] v^2. \end{aligned}$$

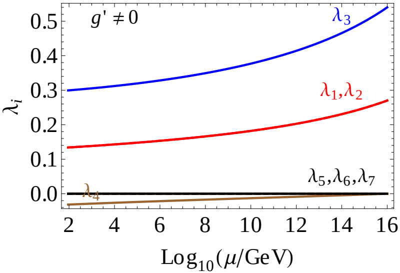
Quark Yukawa Couplings

- By convention, choose $h_1^u = 0$. For Type-I (Type-II) 2HDM, $h_1^d (h_2^d) = 0$.
- Quark yukawa couplings w.r.t. the SM are given by

Coupling	Type-I	Type-II
$g_{ht\bar{t}}$	$\cos\alpha/\sin\beta$	$\cos\alpha/\sin\beta$
$g_{hb\bar{b}}$	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$
$g_{Ht\bar{t}}$	$\sin\alpha/\sin\beta$	$\sin\alpha/\sin\beta$
$g_{Hb\bar{b}}$	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$
$g_{at\bar{t}}$	$\cot\beta$	$\cot\beta$
$g_{ab\bar{b}}$	$-\cot\beta$	$\tan\beta$

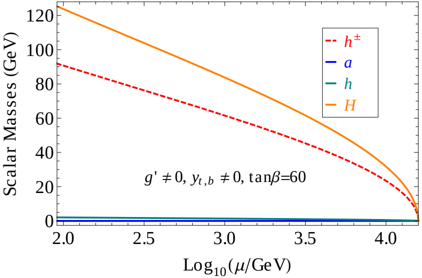
g' Effect

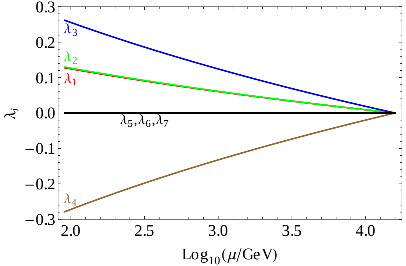




No.	Symmetry	Generators $T^a \leftrightarrow K^a$	Discrete group elements	Maximally broken SO(5) generators	Number of pseudo-Goldstone bosons
1	$Z_2 \times O(2)$	T^0	D_{CP1}	–	0
2	$(Z_2)^2 \times SO(2)$	T^0	D_{Z_2}	–	0
3	$(Z_2)^3 \times O(2)$	T^0	D_{CP2}	–	0
4	$O(2) \times O(2)$	T^3, T^0	–	T^3	1 (a)
5	$Z_2 \times [O(2)]^2$	T^2, T^0	D_{CP1}	T^2	1 (h)
6	$O(3) \times O(2)$	$T^{1,2,3}, T^0$	–	$T^{1,2}$	2 (h, a)
7	$SO(3)$	$T^{0,4,6}$	–	$T^{4,6}$	2 $(h^{\pm})$
8	$Z_2 \times O(3)$	$T^{0,4,6}$	$D_{Z_2} \cdot D_{CP2}$	$T^{4,6}$	2 $(h^{\pm})$
9	$(Z_2)^2 \times SO(3)$	$T^{0,5,7}$	$D_{CP1} \cdot D_{CP2}$	$T^{5,7}$	2 $(h^{\pm})$
10	$O(2) \times O(3)$	$T^3, T^{0,8,9}$	–	T^3	1 (a)
11	$SO(4)$	$T^{0,3,4,5,6,7}$	–	$T^{3,5,7}$	3 $(a, h^{\pm})$
12	$Z_2 \times O(4)$	$T^{0,3,4,5,6,7}$	$D_{Z_2} \cdot D_{CP2}$	$T^{3,5,7}$	3 $(a, h^{\pm})$
13	$SO(5)$	$T^{0,1,2,\ldots,9}$	–	$T^{1,2,8,9}$	4 $(h, a, h^{\pm})$

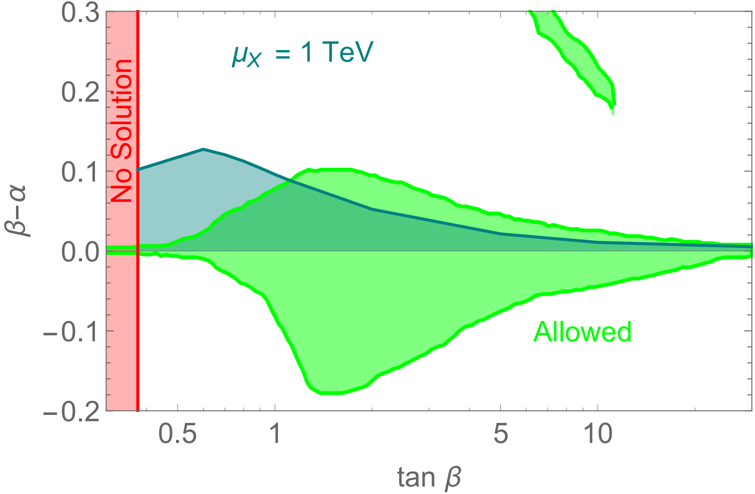
Yukawa Coupling Effects





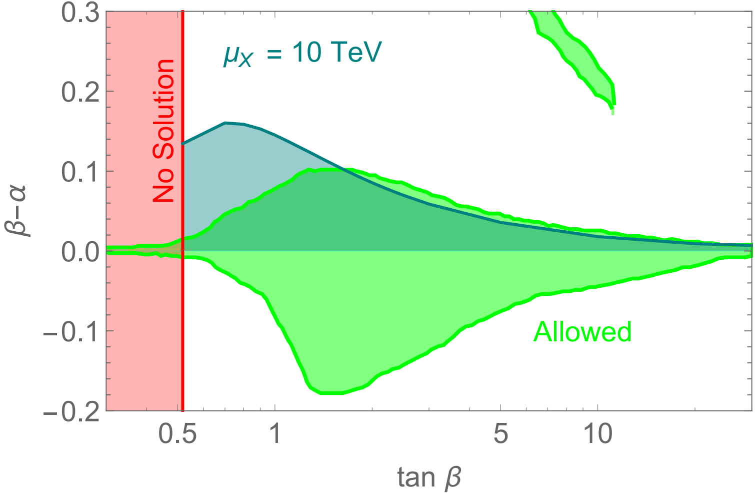
No.	Symmetry	Generators $T^a \leftrightarrow K^a$	Discrete group elements	Maximally broken SO(5) generators	Number of pseudo-Goldstone bosons
1	$Z_2 \times O(2)$	T^0	D_{CP1}	–	0
2	$(Z_2)^2 \times SO(2)$	T^0	D_{Z_2}	–	0
3	$(Z_2)^3 \times O(2)$	T^0	D_{CP2}	–	0
4	$O(2) \times O(2)$	T^3, T^0	–	T^3	1 (a)
5	$Z_2 \times [O(2)]^2$	T^2, T^0	D_{CP1}	T^2	1 (h)
6	$O(3) \times O(2)$	$T^{1,2,3}, T^0$	–	$T^{1,2}$	2 (h, a)
7	$SO(3)$	$T^{0,4,6}$	–	$T^{4,6}$	2 $(h^\pm)$
8	$Z_2 \times O(3)$	$T^{0,4,6}$	$D_{Z_2} \cdot D_{CP2}$	$T^{4,6}$	2 $(h^\pm)$
9	$(Z_2)^2 \times SO(3)$	$T^{0,5,7}$	$D_{CP1} \cdot D_{CP2}$	$T^{5,7}$	2 $(h^\pm)$
10	$O(2) \times O(3)$	$T^3, T^{0,8,9}$	–	T^3	1 (a)
11	$SO(4)$	$T^{0,3,4,5,6,7}$	–	$T^{3,5,7}$	3 $(a, h^\pm)$
12	$Z_2 \times O(4)$	$T^{0,3,4,5,6,7}$	$D_{Z_2} \cdot D_{CP2}$	$T^{3,5,7}$	3 $(a, h^\pm)$
13	$SO(5)$	$T^{0,1,2,\ldots,9}$	–	$T^{1,2,8,9}$	4 $(h, a, h^\pm)$

With $SO(5)$ Boundary Conditions at μ_X



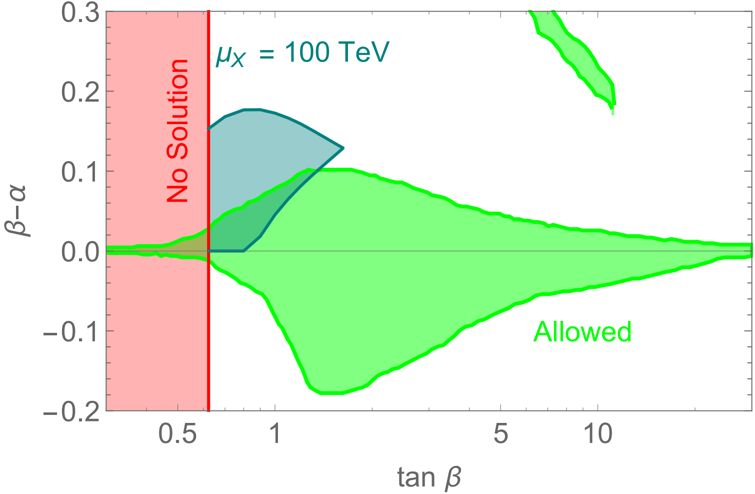
[PSBD, Pilaftsis (preliminary)]

With $SO(5)$ Boundary Conditions at μ_X



[PSBD, Pilaftsis (preliminary)]

With $SO(5)$ Boundary Conditions at μ_X



[PSBD, Pilaftsis (preliminary)]