Scalar symmetries and their breaking patterns in NHDM

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based on:

Ivanov, Vdovin, EPJC 73, 2309 (2013);
Degee, Ivanov, Keus, JHEP 1302, 125 (2013);
Ivanov, Nishi, *in preparation*.

Outline

Symmetries in multi-Higgs models

- $\mathcal{L}_{\rm bSM}$ → impose symmetries → get structures → study phenomenological consequences (CP, FCNC, astroparticle, flavour, etc).
- The more fields with the same quantum numbers, the larger is the symmetry space. NHDM is a conservative bSM model with numerous phenomenological consequences, often linked to the symmetries imposed.
- Classic papers on 3HDM with symmetries: *CP*-violating $\mathbb{Z}_2 \times \mathbb{Z}_2$ *[Weinberg, 1979]*, *CP*-conserving $\mathbb{Z}_2 \times \mathbb{Z}_2$ *[Branco, 1980]*, S_3 *[Pakvasa, Sugawara, 1978]*, A_4 *[Ma, Rajasekaran, 2001]*, $\Delta(27)$ *[Branco, Gerard, Grimus, 1984]*, + many dozens of more recent works.
- Systematic exploration of all symmetry-related issues in a model with N doublets is a challenging but rewarding undertaking. General understanding of how symmetries work in bSM models will support the mainstream model-building activity.

Symmetries in multi-Higgs models

I will focus on discrete symmetries in the scalar sector of 3HDM, with some discussion of NHDM.

Both Higgs-family transformations $\phi_i \mapsto U_{ij}\phi_j$ and generalized-CP (GCP) transformations $\phi_i \mapsto U_{ij}\phi_j^*$ transformations will be considered.

Main questions:

- Which Higgs-family symmetry groups G_{HF} can the 3HDM scalar sector have?
- What are the CP-consequences of each G_{HF}?
- How do these symmetries break upon minimization of the potential?

Notation: $\mathbb{Z}_2^*$ generated by a GCP transformation, $\mathbb{Z}_2$ generated by a Higgs-family transformation. Symmetry group of CP conserving model $G \simeq G_{HF} \rtimes \mathbb{Z}_2^*$.

Technical remarks

- Since we work only with the scalar sector, unitary transformations are considered up to overall rephasing:
  $U_{ij} \in U(N)/U(1) \simeq SU(N)/\mathbb{Z}_N = PSU(N)$.

  The most important example is $\Delta(27) \in U(3)$ in 3HDM,

$$\Delta(27) = \langle a_3,\, b \rangle\,, \quad a_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}\,, \quad b = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}\,,$$

  becomes $\Delta(27)/\mathbb{Z}_3 \simeq \mathbb{Z}_3 \times \mathbb{Z}_3 \in PSU(3)$.

- Important difference between imposing and deriving symmetry group: below, G-symmetric 3HDM means that V is not only G-invariant, but also does not have any other symmetry beyond G.

  G represents the full symmetry content of the model.

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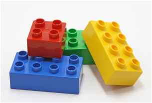
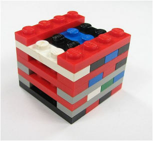
Finding discrete symmetries in the 3HDM scalar sector

“Abelian LEGO” strategy

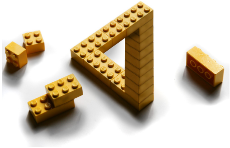
Step 1: find all possible discrete abelian groups A_i; any allowed G can have only those abelian subgroups. These are “LEGO bricks” with which we will build a non-abelian model.



Step 2: build G by combining various A_i but avoid producing abelian groups not in the list!



Step 3: for each G built, check that it fits $PSU(3)$ and that it does not automatically produce any higher symmetry.



3HDM scalar symmetries

Deriving discrete symmetries in 3HDM *[Ivanov, Vdovin, 2013]*:

- Allowed abelian Higgs-family groups:

$$\mathbb{Z}_2\,,\quad \mathbb{Z}_3\,,\quad \mathbb{Z}_4\,,\quad \mathbb{Z}_2\times\mathbb{Z}_2\,,\quad \mathbb{Z}_3\times\mathbb{Z}_3\,.$$

- G_{HF} can contain only these abelian subgroups $\to$ $|G_{HF}| = 2^p 3^q$ $\to$ using Burnside's theorem and applying some finite group theory results, we obtain the key result:

$$G_{HF} = A \rtimes K\,,\quad K \subseteq Aut(A)\,.$$

- Checking all As one by one, we establish all possible G_{HF}.

3HDM scalar symmetries

Discrete non-abelian G_{HF}'s allowed in the 3HDM scalar sector:

$$G_{HF} = S_3\,,\quad D_4\,,\quad A_4\,,\quad S_4\,,\quad \Delta(54)/\mathbb{Z}_3\,,\quad \Sigma(36)\,.$$

This list is complete: trying to impose any other finite Higgs-family symmetry group on the 3HDM potential will unavoidably lead to a continuous symmetry.

Checking CP properties

Explicitly CP-violating 3HDM:

$$G = \mathbb{Z}_2, \quad \mathbb{Z}_3, \quad \mathbb{Z}_2 \times \mathbb{Z}_2\,, \quad S_3\,, \quad \Delta(54)/\mathbb{Z}_3\,.$$

Explicitly CP-conserving 3HDM:

$$\begin{aligned} G \;=\; & \mathbb{Z}_2^*, \quad \mathbb{Z}_2 \times \mathbb{Z}_2^*, \quad \mathbb{Z}_4^*, \quad \mathbb{Z}_3 \rtimes \mathbb{Z}_2^*, \quad \mathbb{Z}_4 \rtimes \mathbb{Z}_2^*, \quad \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2^*\,, \\ & S_3 \times \mathbb{Z}_2^*, \quad D_4 \times \mathbb{Z}_2^*, \quad A_4 \rtimes \mathbb{Z}_2^*, \quad S_4 \times \mathbb{Z}_2^*, \\ & (\Delta(54)/\mathbb{Z}_3) \rtimes \mathbb{Z}_2^*, \quad \Sigma(36) \times \mathbb{Z}_2^*\,. \end{aligned}$$

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Breaking discrete symmetries

Symmetry breaking patterns in NHDM

Consider NHDM with scalar symmetry group G. After EWSB, we get a neutral vacuum with a certain vev alignment $\langle\phi_i^0\rangle = v_i e^{i\xi_i}/\sqrt{2}$ invariant under a residual symmetry group $G_v \subseteq G$.

There situations are possible:

- symmetry is conserved: $G_v = G$;
- symmetry is partially broken: $\{e\} \subset G_v \subset G$;
- symmetry is completely broken: $G_v = \{e\}$.

The goal: for each G, establish its symmetry breaking patterns.

Phenomenology depends a lot on how much of the original symmetry is broken!
Quark sector: NHDM with symmetry group G can lead to viable quark masses and CKM only if G is broken completely *[Leurer, Nir, Seiberg, 1993; Gonzalez Felipe, Ivanov, Nishi, Serodio, Silva, 2014]*.

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Symmetry breaking patterns in NHDM

If we could explicitly find the global minimum of any G-symmetric potential, we could easily find the symmetry breaking patterns. Straightforward algebra does not help $\rightarrow$ general group-theoretical or algebraic considerations are needed.

There exist algebraic methods for minimization of the Higgs potential such as Gröbner basis *[Maniatis, von Manteuffel, Nachtmann, 2007]* and NPHC *[Maniatis, Mehta, 2012]*, which resort to numerical calculations only at the final stage. Probably they could help with this task, but here, I stick to purely analytical calculations.

Symmetry breaking patterns in NHDM

One easy result: if Higgses are not in irrep and contain a singlet, say ϕ_1, then it is possible to conserve the symmetry, $G_v = G$, by choosing free parameters leading to $(v_1, 0, \ldots, 0)$. Vice versa, if Higgses are in the irrep, the symmetry must always break: $G_v \subset G$.

A much harder question: which groups G can be broken completely? There should exist general criteria which answer it without explicit minimization of the NHDM potential. We have not yet found them. As a step towards this goal, we decided to work out the 3HDM case explicitly, hoping that it will provide further insights for general N.

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Minimization of 3HDM potentials

Some remarks:

- For small groups G lots of free parameters make any symmetry breaking pattern possible.
- Groups such as $Z_3 \rtimes \mathbb{Z}_2^*$ and $Z_4 \rtimes \mathbb{Z}_2^*$ have very few phase-sensitive terms, and it helps to derive conclusions on symmetry breaking.

  For example, $Z_4 \rtimes \mathbb{Z}_2^*$-symmetric 3HDM is $V = V_0 + V_{ph}$, where V_0 depends only on $|\phi_i|^2$ and

$$V_{ph} = \lambda(\phi_2^\dagger \phi_1)(\phi_3^\dagger \phi_1) + \lambda'(\phi_3^\dagger \phi_2)^2 + h.c.$$

  with real λ, λ'. It is symmetric under $a_4 = \mathrm{diag}(1, i, -i)$ and CP. Parametrizing vev alignment as $(v_1,\ v_2 e^{i\xi_2},\ v_3 e^{i\xi_3})$ and differentiating V, one gets rigid phases such as $\xi_2 = -\xi_3 = \pi/4$. As a result, the full breaking of $Z_4 \rtimes \mathbb{Z}_2^*$ is impossible.

Minimization of 3HDM potentials

For large symmetry groups, the potential has trivial quadratic part and very few terms in the quartic part. The global minimum can be found much more efficiently with a geometric method rather than the traditional sequence *[Degee, Ivanov, Keus, 2013]*.

The main idea is to rewrite the G-symmetric potential as a linear function of certain real variables:

$$V = -\frac{1}{2}m^2 v^2 + \frac{1}{4}v^4 \left(\Lambda_0 + \Lambda_1 x_1 + \Lambda_2 x_2 + \cdots + \Lambda_k x_k\right) .$$

Variables x_i do not depend on v; they reflect the relative structures in the vev alignment, and satisfy certain inequalities. Finding these inequalities defines the orbit space in the space of all x_i.

Once the shape of this orbit space is constructed, the minimization of V becomes a trivial geometric exercise and can be done for all possible values of free parameters.

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Minimization of the "$\Delta(27)$" 3HDM

To illustrate this method, consider "CP-violating $\Delta(27)$" 3HDM (with the true symmetry group $\Delta(54)/\mathbb{Z}_3$):

$$\begin{aligned} V_1 &= -m^2\left[\phi_1^\dagger\phi_1+\phi_2^\dagger\phi_2+\phi_3^\dagger\phi_3\right]+\lambda_0\left[\phi_1^\dagger\phi_1+\phi_2^\dagger\phi_2+\phi_3^\dagger\phi_3\right]^2\\ &+\frac{\lambda_1}{3}\left[(\phi_1^\dagger\phi_1)^2+(\phi_2^\dagger\phi_2)^2+(\phi_3^\dagger\phi_3)^2-(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2)-(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3)-(\phi_3^\dagger\phi_3)(\phi_1^\dagger\phi_1)\right]\\ &+\lambda_2\left[|\phi_1^\dagger\phi_2|^2+|\phi_2^\dagger\phi_3|^2+|\phi_3^\dagger\phi_1|^2\right]\\ &+\left(\lambda_3\left[(\phi_1^\dagger\phi_2)(\phi_1^\dagger\phi_3)+(\phi_2^\dagger\phi_3)(\phi_2^\dagger\phi_1)+(\phi_3^\dagger\phi_1)(\phi_3^\dagger\phi_2)\right]+h.c.\right), \end{aligned}$$

which can be cast in the form

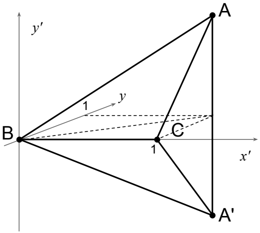
$$V_1=-\frac{1}{2}m^2v^2+\frac{1}{4}v^4\left(\Lambda_0+\Lambda_1x+\Lambda_1'x'+\Lambda_2y+\Lambda_2'y'\right),$$

with appropriately defined x, y, x', y'. With some algebra, one finds that

$$x=1\,,\quad 0\le y\le x'\le 1\,,\quad |y'|\le y\,.$$

Minimization of the "Δ(27)" 3HDM

These conditions define a tetrahedron in the (x', y, y')-space. We need to minimize a linear function on the orbit space which lies inside this tetrahedron (but does not fill it completely).



The key observation: the four vertices belong to the orbit space → the global minimum can only be at those points.

Up to cyclic permutations,

$$A:\ (\omega,\, 1,\, 1)\,, \quad A':\ (\omega^2,\, 1,\, 1)\,, \quad B:\ (1,\, 0,\, 0)\,,$$

and

$$C:\ (1,\, 1,\, 1)\,, \quad (1,\, \omega,\, \omega^2)\,, \quad (1,\, \omega^2,\, \omega)\,.$$

Finding explicit conditions on parameters Λ is straightforward. No other vev can be the global minimum of this potential for any values of Λ's.

Minimization of the "$\Delta(27)$" 3HDM

Passing to larger groups is straightforward: just collapse the orbit space on the plane $y' = 0$, for "CP-conserving $\Delta(27)$", and further on the axis y, for $\Sigma(36)$.

In all three cases, the global minimum can only reside at these points. In the CP-conserving case, points A and A' realize the geometric-CP violation *[Branco, Gerard, Grimus, 1984]*. Remarkably, the same geometric phase persists even for explicitly CP-violating!

Since we now have all possible vev alignments, we can proceed with symmetry breaking patterns at each minimum.

Symmetry breaking in 3HDM

Results on strongest and weakest breaking of discrete symmetries in 3HDM, as well as on spontaneous *CP*-violation.

group	$\|G\|$	$\|G_v\|_{min}$	$\|G_v\|_{max}$	sCPv possible?
abelian	2, 3, 4, 8	1	$\|G\|$	yes
$\mathbb{Z}_3 \rtimes \mathbb{Z}_2^*$	6	1	6	yes
S_3	6	1	6	—
$\mathbb{Z}_4 \rtimes \mathbb{Z}_2^*$	8	2	8	no
$S_3 \times \mathbb{Z}_2^*$	12	2	12	yes
$D_4 \times \mathbb{Z}_2^*$	16	2	16	no
$A_4 \rtimes \mathbb{Z}_2^*$	24	4	8	no
$S_4 \times \mathbb{Z}_2^*$	48	6	16	no
CP-violating $\Delta(27)$	18	6	6	—
CP-conserving $\Delta(27)$	36	6	12	yes
$\Sigma(36)$	72	12	12	no

Symmetry breaking in 3HDM

- Spontaneous *CP*-violation is possible only for those Higgs-family groups G_{HF}, for which there exists an explicitly *CP*-violating model. If G_{HF} forbids explicit *CP* violation, it also forbids spontaneous *CP*-violation. Explicit and spontaneous *CP* violations come in pairs.
- When we break a discrete symmetry group, we have several degenerate minima. Usual expectation: all minima lie on a single *G*-orbit: one can link any pair of minima by a broken symmetry $\in G$. Then, the number of degenerate minima is equal to the length of the orbit

$$\ell = |G|/|G_v|\,.$$

In one case, this expectation breaks: $G = A_4 \rtimes \mathbb{Z}_2^*$, $|G| = 24$, breaks at $(\pm 1, \omega, \omega^2)$ to $G_v = \mathbb{Z}_3 \rtimes \mathbb{Z}_2^*$, $|G_v| = 6$, producing eight minima lying on two disjoint orbits.
- Unlike 2HDM, minima with different symmetry breaking can coexist in 3HDM and even be degenerate.

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Towards NHDM result: a possible line of attack

What prevents sufficiently large discrete groups from complete breaking? There must exist an upper bound n_{max} on the number of minima of the NHDM potential. Explicit calculations show that $n_{max} = 2$ for 2HDM and $n_{max} = 8$ for 3HDM. Therefore, groups with $|G| > n_{max}$ cannot break completely. The difficult question is to actually find n_{max} for general N.

When working in the space on bilinears $r_a = \phi_i^\dagger \lambda^a_{ij} \phi_j$, minimization of the NHDM potential can be cast in purely geometric terms. Search for the global minimum = search for contact points between two algebraic manifolds, the potential V and the orbit space.

If two algebraic manifolds in $\mathbb{R}^k$ of degrees m_1 and m_2 intersect, there must exist an upper bound on the number of connected components. For planar curves, it is $m_1 m_2$ (Bezout's theorem); we just need its analog for higher k.

Note that n_{max} depends on the algebraic order of the potential → G-symmetric higher-order terms might lead to stronger symmetry breaking than quadratic+quartic.

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Conclusions

- We investigated in full detail discrete symmetries and their breaking in 3HDM. All allowed symmetry groups are found, and for each group all symmetry breaking patterns are established. Interplay between Higgs-family symmetries and *CP*-violation is investigated. Some peculiar regularities were observed.
- This study serves both as an input to specific 3HDM models and as a step towards general understanding of discrete symmetry breaking in the scalar sector of NHDM. We hope that, with further efforts based on algebraic-geometric and group-theoretic properties of the problem, will lead us to the answer.