Higgs potentials with two and more Higgs-boson doublets

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Bilinears

O. Nachtmann, A. Manteuffel, M.M. **EPJC 48** (2006)

C. Nishi **PRD 74** (2006)

- In the nHDM the Higgs sector is extended

$$\varphi_1 = \begin{pmatrix} \varphi_1^+ \\ \varphi_1^0 \end{pmatrix}, \quad \varphi_2 = \begin{pmatrix} \varphi_2^+ \\ \varphi_2^0 \end{pmatrix}, \quad \ldots, \quad \varphi_n = \begin{pmatrix} \varphi_n^+ \\ \varphi_n^0 \end{pmatrix},$$

- Gauge invariance requires to restrict Higgs potential to terms

$$\varphi_i^\dagger \varphi_j, \qquad i,j \in \{1,\ldots,n\}.$$

- Focus on 3 Higgs doublets here. MM, O. Nachtmann arxiv 1408.0833

- Collect all Higgs doublets in one 3×2 matrix

$$\phi = \begin{pmatrix} \varphi_1^+ & \varphi_1^0 \\ \varphi_2^+ & \varphi_2^0 \\ \varphi_3^+ & \varphi_3^0 \end{pmatrix}$$

- Arrange all $SU(2)_L \times U(1)_Y$ invariants into hermitian 3×3 matrix

$$\underline{K} = \phi\phi^\dagger = \begin{pmatrix} \varphi_1^\dagger\varphi_1 & \varphi_2^\dagger\varphi_1 & \varphi_3^\dagger\varphi_1 \\ \varphi_1^\dagger\varphi_2 & \varphi_2^\dagger\varphi_2 & \varphi_3^\dagger\varphi_2 \\ \varphi_1^\dagger\varphi_3 & \varphi_2^\dagger\varphi_3 & \varphi_3^\dagger\varphi_3 \end{pmatrix}.$$

- Basis for $\underline{K}$ are Gell-Mann matrices λ_α, $\lambda_0 = \sqrt{\frac{2}{3}}\mathbb{1}_3$,

$$\underline{K} = \frac{1}{2} K_\alpha \lambda_\alpha, \qquad \alpha = 0, 1, \ldots, 8$$

- The real coefficients K_α are given by traces

$$K_\alpha = \mathrm{tr}(\underline{K}\lambda_\alpha).$$

- Coefficients K_α called bilinears in the following,

$$\begin{aligned}
K_0 &= \sqrt{\frac{2}{3}}\left(\varphi_1^\dagger\varphi_1 + \varphi_2^\dagger\varphi_2 + \varphi_3^\dagger\varphi_3\right),\\
K_1 &= \varphi_1^\dagger\varphi_2 + \varphi_2^\dagger\varphi_1,\\
K_2 &= -i\varphi_1^\dagger\varphi_2 + i\varphi_2^\dagger\varphi_1,\\
K_3 &= \varphi_1^\dagger\varphi_1 - \varphi_2^\dagger\varphi_2,\\
K_4 &= \varphi_1^\dagger\varphi_3 + \varphi_3^\dagger\varphi_1,\\
K_5 &= -i\varphi_1^\dagger\varphi_3 + i\varphi_3^\dagger\varphi_1,\\
K_6 &= \varphi_2^\dagger\varphi_3 + \varphi_3^\dagger\varphi_2,\\
K_7 &= -i\varphi_2^\dagger\varphi_3 + i\varphi_3^\dagger\varphi_2,\\
K_8 &= \frac{1}{\sqrt{3}}\left(\varphi_1^\dagger\varphi_1 + \varphi_2^\dagger\varphi_3 - 2\varphi_1^\dagger\varphi_3\right).
\end{aligned}$$

- Properties of matrix $\underline{K} = \phi\phi^\dagger$:
  - By construction hermitian.
  - By construction positive semidefinte.
  - By construction $\operatorname{rank}\underline{K} \leq 2$
  - Matrix $\underline{K}$ is hermitian, positive semidefinite, diagonalization,

$$U\underline{K}U^\dagger = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}, \qquad \text{with all } \lambda_i \geq 0.$$

- Suppose $\underline{K}$ rank 2, then

$$\begin{aligned} \det(\underline{K}) &= \lambda_1\lambda_2\lambda_3 &= 0, \\ \mathrm{tr}(\underline{K}))^2 - \mathrm{tr}(\underline{K}^2) &= 2\lambda_1\lambda_2 + 2\lambda_2\lambda_3 + 2\lambda_1\lambda_3 &> 0, \\ \mathrm{tr}(\underline{K}) &= \lambda_1 + \lambda_2 + \lambda_3 &> 0. \end{aligned}$$

- Suppose, $\underline{K}$ rank 1, then

$$\begin{aligned} \det(\underline{K}) &= \lambda_1\lambda_2\lambda_3 &= 0, \\ \mathrm{tr}(\underline{K}))^2 - \mathrm{tr}(\underline{K}^2) &= 2\lambda_1\lambda_2 + 2\lambda_2\lambda_3 + 2\lambda_1\lambda_3 &= 0, \\ \mathrm{tr}(\underline{K}) &= \lambda_1 + \lambda_2 + \lambda_3 &> 0. \end{aligned}$$

- Suppose $\underline{K}$ rank 0, then $K_\alpha = 0$ and also $V = 0$.
- One-to-one correspondence

- In terms of

$$K_0, \qquad \boldsymbol{K} = \begin{pmatrix} K_1 \\ \vdots \\ K_8 \end{pmatrix}$$

the most general potential can be written

$$V = \xi_0 K_0 + \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{K} + \eta_{00} {K_0}^2 + 2 K_0 \, \boldsymbol{\eta}^{\mathrm{T}} \boldsymbol{K} + \boldsymbol{K}^{\mathrm{T}} E \boldsymbol{K},$$

- with real parameters

$$\xi_0, \; \eta_{00}, \; \boldsymbol{\xi}, \; \boldsymbol{\eta}, \; E = E^{\mathrm{T}}$$

Change of basis

- Consider the following unitary mixing of the doublets

$$\begin{pmatrix} \varphi_1' \\ \varphi_2' \\ \varphi_3' \end{pmatrix} = U \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix}.$$

- Bilinears transform as

$$K_0' = K_0, \qquad \boldsymbol{K}' = R(U)\boldsymbol{K},$$

with $R \in SO(8)$, proper rotations in $\boldsymbol{K}$-space.

- Under a change of basis $K_0' = K_0$, $\boldsymbol{K}' = R(U)\boldsymbol{K}$ potential remains invariant if

$$\xi_0' = \xi_0, \quad \eta_{00}' = \eta_{00},$$
$$\boldsymbol{\xi}' = R\,\boldsymbol{\xi}, \quad \boldsymbol{\eta}' = R\,\boldsymbol{\eta}, \quad E' = R\,E\,R^{\mathrm{T}}.$$

- Total number of (real) parameters 54.

- Let us consider a simple example:

$$V_{\text{expl}} = -\mu^2 \varphi_1^\dagger \varphi_1 + \lambda(\varphi_1^\dagger \varphi_1 + \varphi_2^\dagger \varphi_2 + \varphi_3^\dagger \varphi_3)^2.$$

- In terms of bilinears

$$V_{\text{expl}} = -\frac{\mu^2}{\sqrt{6}} K_0 - \frac{\mu^2}{2} K_3 - \frac{\mu^2}{2\sqrt{3}} K_8 + \frac{3\lambda}{2} {K_0}^2.$$

Stability

- Stability: potential to be bounded from below.
- Formulate stability in terms of the biliners.
- For $K_0 = \sqrt{\frac{2}{3}}(\varphi_1^\dagger \varphi_1 + \varphi_2^\dagger \varphi_2 + \varphi_3^\dagger \varphi_3) = 0 \Rightarrow \varphi_i = 0 \Rightarrow V = 0$.
- For $K_0 > 0$ we define

$$\boldsymbol{k} = \frac{\boldsymbol{K}}{K_0}$$

- $\boldsymbol{k}$ is defined on the domain

$$2 - \boldsymbol{k}^2 \geq 0, \qquad \det(\sqrt{2/3}\mathbb{1}_3 + k_a \lambda_a) = 0.$$

- Potential V reads

$$V = K_0 \underbrace{\left(\xi_0 + \boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{k}\right)}_{J_2(\boldsymbol{k})} + {K_0}^2 \underbrace{\left(\eta_{00} + 2\boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{k} + \boldsymbol{k}^{\mathrm{T}}E\boldsymbol{k}\right)}_{J_4(\boldsymbol{k})}$$

- Stability determined by behavior of V in the limit $K_0 \to \infty$.
- Stability in the strong sense requires $J_4(\boldsymbol{k}) > 0$ or for all stationary points on the domain.
- Stability in the weak sense requires $J_4(\boldsymbol{k}) \geq 0$ and $J_2(\boldsymbol{k}) \geq 0$ where $J_4(\boldsymbol{k}) = 0$.

- The stationary points in the interior of domain of $J_4(\boldsymbol{k})$

$$\begin{cases} \nabla_{k_1,\ldots,k_8,u}\left[J_4(\boldsymbol{k}) - u\Big(\det(\sqrt{2/3}\mathbb{1}_3 + k_a\lambda_a)\Big)\right] = 0, \\ 2 - \boldsymbol{k}^2 > 0 \end{cases}$$

- The stationary points on domain of $J_4(\boldsymbol{k})$

$$\nabla_{k_1,\ldots,k_8,u_1,u_2}\left[J_4(\boldsymbol{k}) - u_1\Big(\det(\sqrt{2/3}\mathbb{1}_3 + k_a\lambda_a)\Big) - u_2\Big(2 - \boldsymbol{k}^2\Big)\right] = 0,$$

- In our example we have

$$J_4(\boldsymbol{k}) = \frac{3}{2}\lambda.$$

$J_4(\boldsymbol{k})$ positive in any direction $\boldsymbol{k}$, stability guaranteed.

Electroweak symmetry breaking

- Suppose the potential is stable.
- At the global minimum

$$\langle\phi\rangle = \left\langle \begin{pmatrix} \varphi_1^+ & \varphi_1^0 \\ \varphi_2^+ & \varphi_2^0 \\ \varphi_3^+ & \varphi_3^0 \end{pmatrix} \right\rangle = \begin{pmatrix} v_1^+ & v_1^0 \\ v_2^+ & v_2^0 \\ v_3^+ & v_3^0 \end{pmatrix}.$$

- $\langle\phi\rangle$ rank 2, **no** EW gauge transformation with $v_i^+ = 0$, ~~EW~~.
  rank 2 of $\underline{K}$ corresponds to
  $$\mathrm{tr}(\underline{K}) > 0, \qquad (\mathrm{tr}(\underline{K}))^2 - \mathrm{tr}(\underline{K}^2) > 0, \qquad \det(\underline{K}) = 0.$$

- $\langle \phi \rangle$ rank 1, EW gauge transformation with $v_i^+ = 0$,

$$\langle \phi \rangle = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & v_0/\sqrt{2} \end{pmatrix}, \quad v_0 > 0, \quad SU(2)_L \times U(1)_Y \ \rightarrow U(1)_{em} \, .$$

  rank 1 of $\underline{K}$ corresponds to

$$\mathrm{tr}(\underline{K}) > 0, \qquad (\mathrm{tr}(\underline{K}))^2 - \mathrm{tr}(\underline{K}^2) = 0, \qquad \det(\underline{K}) = 0.$$

- $\langle \phi \rangle$ rank 0, $\underline{K}$=0, $\varphi_i = 0$, $V = 0$, unbroken $SU(2)_L \times U(1)_Y$.

Stationarity equations

- Stationarity equations for rank 2

$$\begin{cases} \nabla_{K_0,\ldots,K_8,u}\Big[V(K_0,\ldots,K_8) - u\det(\underline{K})\Big] = 0, \\ 2{K_0}^2 - K_a K_a > 0, \\ K_0 > 0 \end{cases}$$

- Stationarity equations for rank 1

$$\begin{cases} \nabla_{K_0,\ldots,K_8,u_1,u_2}\Big[V(K_0,\ldots,K_8) - u_1(2{K_0}^2 - K_a K_a) - u_2\det(\underline{K})\Big] = 0, \\ K_0 > 0 \end{cases}$$

- Stationary solutions of rank 0 correspond to $V = 0$.
- Polynomial systems of equations.

- Groebner basis approach, homotopy continuation.
- Deepest solution is global minimum.
- Example: rank 1 global minimum at

$$\frac{\sqrt{6}}{2}K_0 = \sqrt{3}K_8 = K_3 = \frac{\mu^2}{(2\lambda)},$$

$$K_{1/2/4/5/6/7} = 0$$

corresponding to $V = -\frac{1}{4}\frac{(\mu^2)^2}{\lambda}$.

Mass matrices

- Suppose stable potential, breaking $SU(2)_L \times U(1)_Y \ \to U(1)_{em}$
- In the unitary gauge, we have

$$\varphi_1(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_0 + h_0(x) \end{pmatrix}, \quad \varphi_{2/3}(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} H^+_{2/3}(x) \\ H^0_{2/3}(x) + iA^0_{2/3}(x) \end{pmatrix}.$$

- Collecting the quadratic terms of the potential,

$$V_{\text{quad.}} = \frac{1}{2} \left(h^0 \, H^0_2 \, H^0_3 \, A^0_2 \, A^0_3 \right) \mathcal{M}^2_{\text{neutral}} \begin{pmatrix} h^0 \\ H^0_2 \\ H^0_3 \\ A^0_2 \\ A^0_3 \end{pmatrix} + \left(H^+_2 \, H^+_3 \right) \mathcal{M}^2_{\text{charged}} \begin{pmatrix} H^-_2 \\ H^-_3 \end{pmatrix}.$$

- Example potential yields

$$\mathcal{M}^2_{\text{neutral}} = \text{diag}(3\lambda v_0^2 - \mu^2, \lambda v_0^2, \lambda v_0^2, \lambda v_0^2, \lambda v_0^2), \quad \mathcal{M}^2_{\text{charged}} = \lambda v_0^2 \mathbb{1}_2 \ .$$

Symmetries

- Symmetry desirable in order to comply with experiment, for instance restricting large FCNC.
- Symmetries easily formulated in terms of bilinears.
- Transformation $K_0 \to K_0$, $\boldsymbol{K} \to R\boldsymbol{K}$, R rotation, is symmetry of potential iff

$$\boldsymbol{\xi} = R\,\boldsymbol{\xi}, \qquad \boldsymbol{\eta} = R\,\boldsymbol{\eta}, \qquad E = R\,E\,R^{\mathrm{T}}.$$

see for instance
I. P. Ivanov and C. C. Nishi, Phys. Rev D82, 2010
V. Keus, S.F. King, S. Moretti, JHEP 1401
B. Grzadkowski, MM, J. Wudka, JHEP 1111
P. M. Ferreira, H. E. Haber, MM, O. Nachtmann, J. P. Silva, Int.J.Mod.Phys. A26, 2011

Conclusion

- Bilinears are powerful tool in nHDM.
- Basis-transformations, stability, EWSB, symmetries easily formulated.
- Polynomial systems of equations for stability and stationarity.
- Open for many studies in the nHDM.
- Thank you for your attention!