Two Higgs doublet models with an S_3 symmetry

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- Some attempts have been done to extend this analysis into the Yukawa sector (P.M. Ferreira and J. P. Silva Phys. Rev. D **83**, 065026; Eur. Phys. J. C **69**, 45)
- But there was not classification of all possible implementation on non-Abelian symmetries in both sectors

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\begin{aligned}
e &: (x_1, x_2, x_3) \rightarrow (x_1, x_2, x_3),\\
a_1 &: (x_1, x_2, x_3) \rightarrow (x_2, x_1, x_3),\\
a_2 &: (x_1, x_2, x_3) \rightarrow (x_3, x_2, x_1),\\
a_3 &: (x_1, x_2, x_3) \rightarrow (x_1, x_3, x_2),\\
a_4 &: (x_1, x_2, x_3) \rightarrow (x_3, x_1, x_2),\\
a_5 &: (x_1, x_2, x_3) \rightarrow (x_2, x_3, x_1).
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- By defining $a_1 = a, a_2 = b$, all of elements are written as $\{e, a, b, ab, ba, bab\}$

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- These elements are classified to three conjugacy classes,

$$C_1 : \{e\}, \quad C_2 : \{ab, ba\}, \quad C_3 : \{a, b, bab\}. \tag{2}$$

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- And the matrix form of the elements a and b in the real representation are:

$$a = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad b = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}. \tag{3}$$

S_3 symmetry

- And in the complex representation:

$$a_C = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad b_C = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}, \tag{4}$$

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$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix} \tag{6}$$

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- The multiplication rules for S_3 are:

$$\begin{aligned} \mathbf{1} \otimes \text{any} &= \text{any}, \\ \mathbf{1}' \otimes \mathbf{1}' &= \mathbf{1}, \\ \mathbf{1}' \otimes \mathbf{2} &= \mathbf{2}, \\ \mathbf{2} \otimes \mathbf{2} &= \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{2}. \end{aligned} \tag{7}$$

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- In the real representation, the product of two doublets $x = (x_1, x_2)^\intercal$ and $y = (y_1, y_2)^\intercal$, gives

$$\begin{aligned} (x \otimes y)_{\mathbf{1}} &= x_1 y_1 + x_2 y_2, \\ (x \otimes y)_{\mathbf{1}'} &= x_1 y_2 - x_2 y_1, \\ (x \otimes y)_{\mathbf{2}} &= (x_2 y_2 - x_1 y_1, x_1 y_2 + x_2 y_1)^\intercal. \end{aligned} \tag{8}$$

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- We will denote $\Phi \sim (\mathbf{1}, \mathbf{1})$ when both scalars are in the singlet representation of S_3
- In this case we obtain the generic scalar potential of the 2HDM, which may be written as:

$$\begin{aligned} V_H &= m_{11}^2|\Phi_1|^2 + m_{22}^2|\Phi_2|^2 - m_{12}^2\, \Phi_1^\dagger \Phi_2 - (m_{12}^2)^*\, \Phi_2^\dagger \Phi_1 \\ &+ \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4\,(\Phi_1^\dagger\Phi_2)\,(\Phi_2^\dagger\Phi_1) \\ &+ \left[\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 + \left(\lambda_6|\Phi_1|^2 + \lambda_7|\Phi_2|^2\right)(\Phi_1^\dagger\Phi_2) + \mathrm{h.c.}\right] \end{aligned} \quad (12)$$

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- In this case, if $\arg(\lambda_5) = 2\arg(m_{12}^2)$, the phases can be removed (real 2HDM and the scalar sector preserves CP), otherwise the phases cannot be removed (complex 2HDM (C2HDM) and the scalar sector violates CP)

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- In both models with softly broken Z_2 symmetry, the conditions for a bounded from below potential are (N. G. Deshpande and E. Ma, Phys. Rev. D. **18**, 2574)

$$\lambda_1 > 0, \quad \lambda_2 > 0, \quad \sqrt{\lambda_1\lambda_2} > -\lambda_3, \quad \sqrt{\lambda_1\lambda_2} > |\lambda_5| - \lambda_3 - \lambda_4. \quad (13)$$

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- Transforming, respectively, as **1**, **1**$'$, and **2**

- Thus, the most general potential of a doublet of S_3, consistent with the real representation is:

$$V_R = \mu \left(|\varphi_2|^2 + |\varphi_1|^2\right) + d_1 \left(|\varphi_2|^2 + |\varphi_1|^2\right)^2 + d_2 \left(\varphi_1^\dagger \varphi_2 - \varphi_2^\dagger \varphi_1\right)^2 + d_3 \left[\left(|\varphi_2|^2 - |\varphi_1|^2\right)^2 + \left(\varphi_1^\dagger \varphi_2 + \varphi_2^\dagger \varphi_1\right)^2\right]. \quad (15)$$

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- This coincides with the generic potential in Eq. (12), subject to the conditions

$$m_{11}^2 = m_{22}^2, \quad m_{12}^2 = 0, \quad \lambda_1 = \lambda_2, \quad \lambda_5 = \lambda_1 - \lambda_3 - \lambda_4, \quad (16)$$

identified in Table I of (P. M. Ferreira, H. E. Habber and J. P. Silva, Phys. Rev. D **79**, 116004) as the CP3 model

$\Phi = (\varphi_1, \varphi_2)^\intercal \sim 2$, complex representation

- Remark! If $(\phi_1, \phi_2)^\intercal \sim \mathbf{2}$, in the complex representation one has $(\phi_2^\dagger, \phi_1^\dagger)^\intercal \sim \mathbf{2}$ (E. Ma, arXiv:hep-ph/0409075)

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  transforming, respectively, as $\mathbf{1}$, $\mathbf{1}'$, and $\mathbf{2}$
- Thus, the most general potential of a doublet of S_3, consistent with the complex representation of Eq. (4) is (E. Ma, B. Melic, Phys. Lett. B **725**,402)

$$\begin{aligned} V_C \; = \; & \mu_1^2 \left(|\phi_2|^2 + |\phi_1|^2\right) + \tfrac{1}{2}\ell_1 \left(|\phi_2|^2 + |\phi_1|^2\right)^2 + \tfrac{1}{2}\ell_2 \left(|\phi_2|^2 - |\phi_1|^2\right)^2 \\ & + \ell_3 (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1). \end{aligned} \tag{18}$$

- This is the same as Eq. (15), through the transformation $\Phi' = U\Phi$, with $\mu_1^2 = \mu$, $\ell_1 = 2d_1$, $\ell_2 = -2d_2$, and $\ell_3 = 4d_3$

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- The potential obtained coincides with the generic potential in Eq. (12), subject to the conditions

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- However, they are the same conditions, but seen in different basis
- (E. Ma, B. Melic, Phys. Lett. B **725**,402) also include in the potential a term which breaks S_3 softly, while preserving the $\phi_1 \leftrightarrow \phi_2$ symmetry

$$V_{soft} = -\mu_2^2(\phi_1^\dagger \phi_2 + \phi_2^\dagger \phi_1). \tag{20}$$

  This term is needed since otherwise there would be a massless pseudoscalar

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- Considering the vev (v, v), we obtain the scalar masses:

$$\begin{aligned} m_{H^\pm}^2 &= 2\mu_2^2 - \tfrac{1}{2}\ell_3 v^2, \\ m_A^2 &= 2\mu_2^2, \\ m_h^2 &= \tfrac{1}{2}(2\ell_1 + \ell_3)v^2, \\ m_H^2 &= 2\mu_2^2 + \tfrac{1}{2}(2\ell_2 - \ell_3)v^2, \end{aligned} \tag{22}$$

for the charged scalars ($H^\pm$), the pseudoscalar (A), the light (h) and the heavy (H) CP even scalars, respectively

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- The charged scalar and pseudoscalar masses become

$$m_{H^\pm}^2 = -\ell_2 v^2, \tag{23}$$

$$m_A^2 = -\frac{1}{2}(2\ell_2 - \ell_3) v^2, \tag{24}$$

while the CP even scalar mass matrix is

$$M_n = \begin{pmatrix} \ell_1 v_1^2 + \frac{1}{2}\ell_3 v_2^2 + \ell_2 (v_1^2 - v_2^2) & \frac{1}{2}(2\ell_1 + \ell_3) v_1 v_2 \\ \frac{1}{2}(2\ell_1 + \ell_3) v_1 v_2 & \ell_1 v_2^2 + \frac{1}{2}\ell_3 v_1^2 - \ell_2 (v_1^2 - v_2^2) \end{pmatrix}. \tag{25}$$

- Its trace and determinant are

$$m_h^2 + m_H^2 = \mathrm{Tr}\,(M_n) \;=\; \frac{1}{2}(2\ell_1 + \ell_3)v^2 \tag{26}$$

$$m_h^2\, m_H^2 = \mathrm{Det}\,(M_n) \;=\; -\frac{1}{2}(\ell_1 + \ell_2)(2\ell_2 - \ell_3)(v_1^2 - v_2^2)^2. \tag{27}$$

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- We find:

$$\tan(2\alpha) = \frac{2\ell_1 + \ell_3}{2\ell_1 + 4\ell_2 - \ell_3}\,\frac{2v_1 v_2}{v_1^2 - v_2^2} = \frac{m_h^2 + m_H^2}{m_h^2 + m_H^2 - 2m_A^2}\tan(2\beta). \tag{29}$$

S_3 potential with the most general real soft violations of S_3

- $$\begin{aligned} V &= \mu_1^2 \left(|\phi_2|^2 + |\phi_1|^2\right) - \mu_2^2(\phi_1^\dagger \phi_2 + \phi_2^\dagger \phi_1) - \mu_3^2 \left(|\phi_2|^2 - |\phi_1|^2\right) + \\ & \tfrac{1}{2}\ell_1 \left(|\phi_2|^2 + |\phi_1|^2\right)^2 + \tfrac{1}{2}\ell_2 \left(|\phi_2|^2 - |\phi_1|^2\right)^2 + \ell_3(\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1). \end{aligned} \tag{30}$$

S_3 potential with the most general real soft violations of S_3

- $$V = \mu_1^2 \left(|\phi_2|^2 + |\phi_1|^2\right) - \mu_2^2(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1) - \mu_3^2 \left(|\phi_2|^2 - |\phi_1|^2\right) + \frac{1}{2}\ell_1 \left(|\phi_2|^2 + |\phi_1|^2\right)^2 + \frac{1}{2}\ell_2 \left(|\phi_2|^2 - |\phi_1|^2\right)^2 + \ell_3(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1). \tag{30}$$

- Repeating the previous steps, we find

$$m_{H^\pm}^2 = -\ell_2 v^2 - 2\mu_3^2 \sec(2\beta), \tag{31}$$

$$m_A^2 = -\frac{1}{2}\left[(2\ell_2 - \ell_3)v^2 + 4\mu_3^2 \sec(2\beta)\right], \tag{32}$$

$$T \equiv m_h^2 + m_H^2 = \frac{1}{2}\left[(2\ell_1 + \ell_3)v^2 - 4\mu_3^2 \sec(2\beta)\right], \tag{33}$$

$$D \equiv m_h^2\, m_H^2 = -\frac{v^2}{2}[(2\ell_2 - \ell_3)\cos(2\beta)((\ell_1 + \ell_2)v^2 \cos(2\beta) + 2\mu_3^2) + 2(2\ell_1 + \ell_3)\mu_3^2 \sec(2\beta)] \tag{34}$$

- Finally we obtain the following relation among $\mu_3, \beta, and \alpha$

$$\frac{D}{c_{2\beta}^2} = m_A^2(T - m_A^2) + \frac{T^2}{4} t_{2\beta}^2 - \left(\frac{T}{2} - m_A^2\right)^2 t_{2\alpha}^2. \tag{35}$$

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Yukawa couplings, complex representation

- General form:

$$-\mathcal{L}_Y = \bar{q}_L(\Gamma_1\Phi_1 + \Gamma_2\Phi_2)n_R + \bar{q}_L(\Delta_1\tilde{\Phi}_1 + \Delta_2\tilde{\Phi}_2)p_R + \mathrm{h.c.}, \quad (36)$$

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- The mass matrices are diagonalized through the transformations:

$$\begin{aligned} \text{diag}(m_d, m_s, m_b) = D_d &= \frac{1}{\sqrt{2}} U_{d_L}^\dagger \left[v_1\,\Gamma_1 + v_2\Gamma_2\right] U_{d_R}, \\ \text{diag}(m_u, m_c, m_t) = D_u &= \frac{1}{\sqrt{2}} U_{u_L}^\dagger \left[v_1\,\Delta_1 + v_2\Delta_2\right] U_{u_R}, \quad (37) \end{aligned}$$

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- Where $V = U_{u_L}^\dagger U_{d_L}$ is the CKM matrix

- Now we define:

$$Y_d = v_1 \Gamma_1 + v_2 \Gamma_2, \qquad Y_u = v_1 \Delta_1 + v_2 \Delta_2, \tag{38}$$

and the hermitian matrices

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- For CP violation, we used

$$J = \mathrm{Det}(H_d H_u - H_u H_d) \tag{40}$$

Example 1: Φ in singlet; fermions in doublets

- Let us consider the possibility

$$\Phi \sim (\mathbf{1}, \mathbf{1}'), \quad \bar{q}_L \sim (\mathbf{2}, \mathbf{1}), \quad n_R \sim (\mathbf{2}, \mathbf{1}), \quad p_R \sim (\mathbf{2}, \mathbf{1}). \tag{41}$$

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- Thus, the products with the scalars into a singlet are

$$\Phi_1 \left(\bar{q}_{L1} n_{R2} + \bar{q}_{L2} n_{R1} \right), \tag{43}$$
$$\Phi_2 \left(\bar{q}_{L1} n_{R2} - \bar{q}_{L2} n_{R1} \right). \tag{44}$$

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$$Y_d = \begin{bmatrix} 0 & av_1 + bv_2 & 0 \\ av_1 - bv_2 & 0 & 0 \\ 0 & 0 & cv_1 \end{bmatrix} \tag{46}$$

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- Thus, these S_3 assignments cannot be used for the quarks

Example 2: doublets in all sectors

- We now turn to

$$\Phi \sim \mathbf{2}, \quad \bar{q}_L \sim (\mathbf{2}, \mathbf{1}), \quad n_R \sim (\mathbf{2}, \mathbf{1}), \quad p_R \sim (\mathbf{2}, \mathbf{1}). \tag{47}$$

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- For a n_{R3} in a singlet, we find

$$\left(\begin{array}{c} \Phi_1 \\ \Phi_2 \end{array} \right) \otimes \left(\begin{array}{c} \bar{q}_{L1} \\ \bar{q}_{L2} \end{array} \right) \Bigg|_{\mathbf{1}} \otimes n_{R3} = \Phi_1 \bar{q}_{L2} n_{R3} + \Phi_2 \bar{q}_{L1} n_{R3}. \tag{50}$$

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- Multiplying Eqs. (49), (50), and (51) by complex coefficients a, b, and c, respectively, we find

$$Y_d = \begin{bmatrix} a v_1 & 0 & b v_2 \\ 0 & a v_2 & b v_1 \\ c v_2 & c v_1 & 0 \end{bmatrix}. \qquad (52)$$

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- And Eq. (40) to show that we can generate a nonzero CP violating phase
- We note that $J \neq 0$ even if one takes the vevs to be real
- Implying that this model does not coincide with the CP3 model with quarks presented in Ref (P.M. Ferreira and J. P. Silva, Eur. Phys. J. C **69**, 45) where a complex vev was needed in order to get a non-vanishing J.

Example 3: singlet only on right-handed sectors

- Let us consider

$$\Phi \sim \mathbf{2}, \quad \bar{q}_L \sim (\mathbf{2}, \mathbf{1}), \quad n_R \sim \mathbf{s}, \quad p_R \sim (\mathbf{2}, \mathbf{1}). \tag{54}$$

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- A combination of both problems occurs in

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Example 4: singlet only on left-handed sector

- The last case to be considered is

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- Thus, examples 3 and 4 are ruled out

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- All fields are in singlets or, else, all fields sectors have a doublet representation
- Even in the most general real soft-breaking term, there is a relation between α and β, shown in Eq. (35). As far as we know, this is a new result.