A quick review of $D = 6$ 1-loop effective action

Application to the 2HDM (Work In Progress!)

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Outline

1. Introduction
2. $D = 6$ effective action
   - Simplified formalism
   - Evaluate $\Delta S_{\text{eff, 1-loop}}$
3. Application to the 2HDM
4. Conclusions

Outline

Introduction (1/2)

Disclaimer

It's a quick review of existing results ($\to$ nothing new for the technique) that I want to use for the 2HDM.

- Schwinger, Goldstone, Salam, Weinberg, Jona-Lasinio (1960's) ...
- Cheyette & Gaillard (1980's): Covariant Derivative Expansion
- Henning, Lu, Murayama (2014): 1-loop formula using CDE with degenerated masses.
- Drozd, Ellis, Quevillon, You (2014-2015): Same with non-degenerated masses.

Introduction (2/2)

Why do I present it?

- Write an EFT with explicit $SU(3) \times SU(2) \times U(1)$ gauge-invariance.
- Some operators can be generated at tree-level (e.g. exchange of a heavy field), some others are at loop-level only.
- I don't want to find & compute all the possible 1-loop diagrams (using fields in physical basis) and then guess from which gauge-invariant operators (written with fields in gauge basis) they came from.
- Tree-level EFT $\rightarrow$ talk by Duarte.

Outline

A Quantum Field Theory

A quantum field theory, with light and heavy fields: φ and Φ, can be described by its partition function (and its action):

$$\mathcal{Z} = \int \mathcal{D}\varphi\mathcal{D}\Phi \ \mathrm{e}^{iS[\varphi,\Phi]} , \quad S[\varphi,\Phi] = \int \mathrm{d}^4 x \, \mathcal{L}[\varphi(x),\Phi(x)] . \tag{2.1}$$

The "physical" field configurations extremize $\mathcal{Z}$, i.e. when the exponential is stationary, corresponding to an extremum of $S \Rightarrow$ EOMs for φ, Φ.

Classical field configurations φ_c, Φ_c are solutions of the EOMs; quantum fluctuations around φ_c, Φ_c.

Obtaining the EFT (1/2)

$$\mathcal{Z} = \int \mathcal{D}\varphi\mathcal{D}\Phi \ \mathrm{e}^{iS[\varphi,\Phi]}\,, \quad S[\varphi,\Phi] = \int \mathrm{d}^4 x \, \mathcal{L}[\varphi(x),\Phi(x)]\,.$$

The heavy fields Φ have a mass $m_\Phi \gtrsim \Lambda \gg m_\varphi$ (with Λ: matching scale). At this scale, Φ can be integrated out:

$$\mathcal{Z} = \int \mathcal{D}\varphi \ \mathrm{e}^{iS_{\mathrm{eff}}[\varphi]}\,, \quad \mathrm{e}^{iS_{\mathrm{eff}}[\varphi]} = \int \mathcal{D}\Phi \ \mathrm{e}^{iS[\varphi,\Phi]}\,, \tag{2.2}$$

where $S_{\mathrm{eff}}[\varphi]$ is the effective action where the light fields φ are kept fixed. At fixed φ the EOM for Φ writes:

$$\frac{\delta S}{\delta \Phi}[\varphi, \Phi = \Phi_c] = 0\,, \tag{2.3}$$

$\Phi_c \equiv \Phi_c[\varphi]$: "classical" value for Φ.

Obtaining the EFT (2/2)

Now expand the action around Φ_c. Writing $\Phi = \Phi_c + \eta$, we get:

$$S[\varphi, \Phi_c + \eta] = S[\varphi, \Phi_c] + \underbrace{\frac{\delta S}{\delta \Phi}[\varphi, \Phi_c]}_{=0 \text{ by definition}} \eta + \frac{1}{2}\frac{\delta^2 S}{\delta \Phi^2}[\varphi, \Phi_c]\eta^2 + \mathcal{O}(\eta^3) \; . \quad (2.4)$$

The effective action is then defined and computed by evaluating the path integral:

$$\begin{aligned} e^{iS_{\text{eff}}[\varphi]} &= \int \mathcal{D}\eta \;\; e^{iS[\varphi, \Phi_c + \eta]} \\ &\approx e^{iS[\varphi, \Phi_c]} \int \mathcal{D}\eta \;\; e^{\frac{1}{2}\frac{\delta^2 S}{\delta \Phi^2}[\varphi, \Phi_c]\eta^2} = e^{iS[\varphi, \Phi_c]} \left[\det \left(-\frac{\delta^2 S}{\delta \Phi^2}[\varphi, \Phi_c] \right) \right]^{-1/2} \end{aligned}$$

so that S_{eff} is given by:

$$S_{\text{eff}}[\varphi] \approx S[\varphi, \Phi_c] + \frac{i}{2} \operatorname{Tr} \ln \left(-\frac{\delta^2 S}{\delta \Phi^2}[\varphi, \Phi_c] \right) \; . \quad (2.5)$$

Recap.: Effective action

$$S_{\text{eff}}[\varphi] \approx \underbrace{S[\varphi, \Phi_c]}_{\text{Tree-level effective action}} + \underbrace{\frac{i}{2} \operatorname{Tr} \ln \left(-\frac{\delta^2 S}{\delta \Phi^2}[\varphi, \Phi_c] \right)}_{\text{1-loop effective action: } \Delta S_{\text{eff, 1-loop}}} .$$

- How to evaluate the $\operatorname{Tr} \ln \left(-\frac{\delta^2 S}{\delta \Phi^2}[\varphi, \Phi_c] \right) \propto \Delta S_{\text{eff, 1-loop}}$?
- Keep the computations explicitely gauge-invariant

For a general 2HDM: Lagrangian of the theory (in Higgs basis):

$$\mathcal{L} = \mathcal{L}_{\text{SM}}^{\text{no Higgs}} + |D_\mu H_1|^2 + |D_\mu H_2|^2 + \mathcal{L}_Y - V_H , \quad -\mathcal{L}_Y = Y_f \overline{f_R} H_1^\dagger f_L + \frac{\eta_f}{t_\beta} Y_f \overline{f_R} H_2^\dagger f_L + \text{h.c.} ,$$

$$V_H = Y_1 |H_1|^2 + Y_2 |H_2|^2 + (Y_3 H_1^\dagger H_2 + \text{h.c.}) + \frac{Z_1}{2}|H_1|^4 + \frac{Z_2}{2}|H_2|^4 + Z_3 |H_1|^2 |H_2|^2$$
$$+ Z_4 (H_1^\dagger H_2)(H_2^\dagger H_1) + \left\{ \frac{Z_5}{2}(H_1^\dagger H_2)^2 + (Z_6 |H_1|^2 + Z_7 |H_2|^2)(H_1^\dagger H_2) + \text{h.c.} \right\}$$

Suppose H_2 is the heavy doublet (plays the role of Φ), and its "mass" matrix m^2 is $\equiv Y_2$.

Cast $\mathcal{L}$ into:

$$\mathcal{L} = \underbrace{\mathcal{L}[\varphi]}_{\text{typically:}\mathcal{L}_{\text{SM}}^{\text{no Higgs}}} + \underbrace{\left(\Phi^\dagger F[\varphi] + \text{h.c.}\right)}_{\text{generates tree-level EFT}} + \underbrace{\Phi^\dagger \left[P^2 - m^2 - U[\varphi]\right] \Phi}_{\text{generates 1-loop contrib.}} + \mathcal{O}(\Phi^3)$$

(Notation: $P_\mu \equiv iD_\mu$; $F[\varphi]$, $U[\varphi]$: coupling matrices; m^2 : diagonalized mass matrix)

$$\Delta S_{\text{eff, 1-loop}} = \int \mathrm{d}^4 x \, \Delta\mathcal{L}_{\text{eff, 1-loop}} = i c_s \,\mathrm{Tr} \ln \left(-P^2 + m^2 + U[\varphi(x)]\right) \,,$$

with c_s depending on the species integrated out (e.g. $c_s = 1/2$ for real scalars, 1 for complex scalars, 1/2 for gauge bosons, -1/2 for fermions, ...). (An equivalent form exists when integrating out fermions.)

For a general 2HDM: The Lagrangian can contain non-holomorphic couplings, for example $\supset ((H_1^\dagger H_2)^2 + \text{h.c.}) \Rightarrow$ use a multiplet $\Sigma = \begin{pmatrix} \delta H_2 \\ \delta H_2^* \end{pmatrix}$ and use the trick $\mathcal{L} = \frac{1}{2}\mathcal{L} + \frac{1}{2}\mathcal{L}^T$, to write the quadratic term $\mathcal{L} \supset \frac{1}{2}\Sigma^\dagger \cdot \left[P^2 - Y_2^2 - U\right] \cdot \Sigma$.

Evaluate $\Delta S_{\text{eff, 1-loop}}$? (1/2) The CDE (Cheyette, Gaillard)

Usual technique (Peskin, ...): $\text{Tr} \ln \mathcal{O}$ equals sum over eigenvalues of $\ln \mathcal{O}$:

$$\text{Tr} \ln \left(-P^2 + m^2 + U[\varphi(x)] \right) = \int d^4 x \int \frac{d^4 k}{(2\pi)^4} \, \text{tr} \left(e^{ik\cdot x} \ln \left(-P^2 + m^2 + U[\varphi(x)] \right) e^{-ik\cdot x} \right)$$

(tr: trace on internal indices only).

Use Backer-Campbell-Hausdorff formula to re-express:

$$\text{tr} \left(e^{ik\cdot x} \ln \left(-P^2 + m^2 + U[\varphi] \right) e^{-ik\cdot x} \right) \Rightarrow \text{tr} \ln \left(-(P_\mu - k_\mu)^2 + m^2 + U[\varphi] \right) .$$

- Easy when e.g. $\mathcal{O} \sim (\partial^2 + m^2) \Rightarrow -k^2 + m^2$,
- What about $\mathcal{O} \supset D^2 = (\partial_\mu - igA_\mu)^2$?

We want to keep explicit gauge covariance $\Rightarrow$ (do NOT split D and) write gauge-invariant objects. $\Rightarrow$ Use a "Covariant Derivative Expansion" [Cheyette, Gaillard (1985-1987)]: insertion of $e^{\pm P_\mu \frac{\partial}{\partial k_\mu}}$ operators:

$$\text{tr} \ln \left[\overrightarrow{e^{P_\mu \frac{\partial}{\partial k_\mu}}} \left(-(P_\mu - k_\mu)^2 + m^2 + U[\varphi] \right) \overleftarrow{e^{-P_\mu \frac{\partial}{\partial k_\mu}}} \right]$$

allows to rewrite the operator inside "tr ln" as being an expansion in commutators of P_μ with $G_{\mu\nu}$ and U.

Evaluate $\Delta S_{\text{eff, 1-loop}}$? (2/2) (Henning, Lu, Murayama)

When supposing the mass matrix being degenerate [Henning, Lu, Murayama (2014)] (in dimensional regularization and $\overline{\text{MS}}$ renormalization scheme, μ is the renormalization scale. Notation: $(AB) \equiv [A, B]$, $G'_{\mu\nu} = [D_\mu, D_\nu]$):

$$\begin{aligned}\Delta\mathcal{L}_{\text{eff, 1-loop}} = \frac{c_s}{(4\pi)^2} \operatorname{tr} \Bigg\{ & m^4 \left[-\frac{1}{2}\left(\ln\frac{m^2}{\mu^2} - \frac{3}{2} \right) \right] + m^2 \left[-\left(\ln\frac{m^2}{\mu^2} - 1 \right) U \right] \\ & + m^0 \left[-\frac{1}{12}\left(\ln\frac{m^2}{\mu^2} - 1 \right) {G'_{\mu\nu}}^2 - \frac{1}{2}\ln\frac{m^2}{\mu^2} U^2 \right] \\ & + \frac{1}{m^2}\left[-\frac{1}{60}(P_\mu G'_{\mu\nu})^2 - \frac{1}{90} G'_{\mu\nu} G'_{\nu\sigma} G'_{\sigma\nu} - \frac{1}{12}(P_\mu U)^2 - \frac{1}{6}U^3 - \frac{1}{12} U G'_{\mu\nu} G'_{\mu\nu} \right] \\ & + \frac{1}{m^4}\Bigg[\frac{1}{24}U^4 + \frac{1}{12}U(P_\mu U)^2 + \frac{1}{120}(P^2 U)^2 + \frac{1}{24}U^2 G'_{\mu\nu} G'_{\mu\nu} \\ & \qquad - \frac{1}{120}[(P_\mu U), (P_\nu U)] G'_{\mu\nu} - \frac{1}{120}[U[U, G'_{\mu\nu}], ?] \Bigg] \\ & + \frac{1}{m^6}\left[-\frac{1}{60}U^5 - \frac{1}{20}U^2 (P_\mu U)^2 - \frac{1}{30}(U P_\mu U)^2 \right] + \frac{1}{m^8}\left[\frac{1}{120}U^6 \right] \Bigg\},\end{aligned}$$

Case of non-degenerate masses: [Drozd, Ellis, Quevillon, You (2014-2015)].

Formula much more complicated.

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- Compute these explicit gauge-invariant quantities and take the trace.
- They generate the following $SU(3) \times SU(2) \times U(1)$-invariant operators:

$\mathcal{O}_{WW} = g^2 \lvert H_1\rvert^2 W^i_{\mu\nu} W^{i\,\mu\nu}$	$\mathcal{O}_H = \frac{1}{2}\left(\partial_\mu \lvert H_1\rvert^2\right)^2$
$\mathcal{O}_{BB} = g'^2 \lvert H_1\rvert^2 B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_T = \frac{1}{2}\left(H_1^\dagger \overleftrightarrow{D}_\mu H_1\right)^2$
$\mathcal{O}_{WB} = 2gg'(H_1^\dagger \tau^i H_1) W^i_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_R = \lvert H_1\rvert^2 \lvert D_\mu H_1\rvert^2$
	$\mathcal{O}_D = \lvert D^2 H_1\rvert^2$
	$\mathcal{O}_6 = \lvert H_1\rvert^6$
	$\mathcal{O}_{2W} = -\frac{1}{2}(D^\mu W^i_{\mu\nu})^2$
$\mathcal{O}_{3W} = \frac{1}{3!} g \epsilon^{ijk} W^{i\ \nu}_{\mu} W^{j\ \rho}_{\nu} W^{k\ \mu}_{\rho}$	$\mathcal{O}_{2B} = -\frac{1}{2}(\partial^\mu B_{\mu\nu})^2$

Table: Generated CP-conserving dimension-6 bosonic operators.

Result for the 2HDM: $D = 6$ EFT with 1-loop effects

$$
\begin{aligned}
\mathcal{L}_{\rm eff} &= \mathcal{L}_{\rm SM} + \Delta\mathcal{L}_{\text{eff, tree}} + \Delta\mathcal{L}_{\text{eff, 1-loop}} \\
&= \mathcal{L}_{\rm SM} + \frac{1}{Y_2} c_6^{\rm tree} \mathcal{O}_6 + \frac{1}{(4\pi)^2 Y_2} \left(c_{2B}\mathcal{O}_{2B} + c_{2W}\mathcal{O}_{2W} + c_{3W}\mathcal{O}_{3W} \right. \\
&\quad + c_{WB}\mathcal{O}_{WB} + c_{BB}\mathcal{O}_{BB} + c_{WW}\mathcal{O}_{WW} + c_6\mathcal{O}_6 + c_H\mathcal{O}_H \\
&\quad \left. + c_R\mathcal{O}_R + c_T\mathcal{O}_T \right) + \cdots ,
\end{aligned}
$$

with the following Wilson coefficients:

$$
c_6^{\rm tree} = |Z_6|^2 , \quad c_{2W} = c_{3W} = \frac{g^2}{60} , \quad c_{2B} = \frac{g'^2 Y_\Phi^2}{15} = 4\frac{g'^2}{g^2} c_{2W} Y_\Phi^2 , \quad c_{WB} = \frac{Z_4 Y_\Phi}{12} ,
$$

$$
c_{WW} = \frac{2Z_3 + Z_4}{48} , \quad c_{BB} = \frac{2Z_3 + Z_4}{12} Y_\Phi^2 = 4 c_{WW} Y_\Phi^2 , \quad c_R = \frac{Z_4^2 + |Z_5|^2}{6} - 3(Z_6 Z_7^* + \text{h.c.}) ,
$$

$$
c_T = \frac{Z_4^2 - |Z_5|^2}{12} , \quad c_H = \frac{(2Z_3 + Z_4)^2 + |Z_5|^2}{12} + 3(Z_6 Z_7^* + \text{h.c.}) ,
$$

$$
\begin{aligned}
c_6 = &-\frac{Z_3^3 + (Z_3 + Z_4)^3 + 3(Z_3 + Z_4)|Z_5|^2}{6} + 3Z_2|Z_6|^2 \\
&+ 3(Z_3 + Z_4)(Z_6 Z_7^* + \text{h.c.}) + 3(Z_5 Z_6 Z_7 + \text{h.c.})
\end{aligned}
$$

"Mixed" loops (1/2)

We have so far only considered the heavy fields Φ and integrated over them: for 1-loop generated operators this accounts only for those induced by loops of heavy fields only.

What about operators that can be induced by loops of "mixed" heavy Φ & light φ fields? $\Rightarrow$ adapt the method to take into account φ?

$$S[\varphi_c + \rho, \Phi_c + \eta] = S[\varphi_c, \Phi_c] + \frac{\delta S}{\delta \varphi}[\varphi_c, \Phi_c]\rho + \frac{\delta S}{\delta \Phi}[\varphi_c, \Phi_c]\eta$$
$$+ \frac{1}{2}\frac{\delta^2 S}{\delta \varphi^2}[\varphi_c, \Phi_c]\rho^2 + \frac{1}{2}\frac{\delta^2 S}{\delta \Phi^2}[\varphi_c, \Phi_c]\eta^2 + \frac{\delta^2 S}{\delta \varphi \delta \Phi}[\varphi_c, \Phi_c]\rho\eta + \mathcal{O}(\{\rho, \eta\}^3)\,.$$

Define $\Sigma = (\Phi, \varphi)$, $\Sigma_c = (\Phi_c, \varphi_c)$. $\Sigma = \Sigma_c + \sigma$, $\sigma = (\rho, \eta)$.

$$S[\Sigma_c + \sigma] = S[\Sigma_c] + \frac{\delta S}{\delta \sigma}[\Sigma_c] \cdot \sigma + \frac{1}{2}\sigma^T \cdot \frac{\delta^2 S}{\delta \Sigma^2}[\Sigma_c] \cdot \sigma + \mathcal{O}(\sigma^3)$$

“Mixed” loops (2/2)

The quadratic term $\propto \sigma^T \cdot \frac{\delta^2 S}{\delta \Sigma^2}[\Sigma_c] \cdot \sigma$ will be of the form:

$$\begin{pmatrix} \Phi & \varphi \end{pmatrix} \begin{pmatrix} A & B \\ B^T & C \end{pmatrix} \begin{pmatrix} \Phi \\ \varphi \end{pmatrix}$$

- The A part of the matrix (Φ^2 term) corresponds to the heavy 1-loop contributions seen earlier.
- The D part (φ^2 term) corresponds to light 1-loop contributions (not included for the EFT).
- The off-diagonal B part: $\Phi\varphi$ terms, correspond to 1-loop contributions having both heavy & light fields (“mixed” loops), that need to be included in the EFT as well, plus extra contributions coming from insertions of light fields in heavy loops.

Complicated because φ can be all the SM gauge fields and the fermions.

Outline

Conclusions (1/2)

- I have quickly reviewed the effective 1-loop action formalism to obtain an EFT with 1-loop effects automatically taken into account (simpler than finding & computing all the 1-loop diagrams and guess where they came from).
- Should support automation.

Because I considered interesting to get an EFT at $D = 6$ with 1-loop effects for a general 2HDM model, I tried to apply it:

- The case where 1-loop effects coming only from the integration of the heavy field was already done by Henning, Lu, Murayama.
- I repeated their computations & cross-checked with their result (OK).

Conclusions (2/2)

- 1-loop effects generated from a mixture of both heavy & light fields is not done yet for the 2HDM to my knowledge (Henning, Lu, Murayama, and Quevillon & al. do the exercise for a SM supplemented by a scalar triplet only). Indeed this requires taking into account all the fields (both Higgs doublets, vector and fermionic fields), compute the large coupling matrices, and evaluate the traces.
- I plan to investigate this last point.

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