Order-4 CP symmetry and its consequences

Igor Ivanov

CFTP, Instituto Superior Técnico, Universidade de Lisboa

Workshop on Multi-Higgs Models, Lisbon, 6-9 September, 2016

based on:
I. P. Ivanov, J. P. Silva, PRD 93, 095014 (2016)
A. Aranda, I. P. Ivanov, E. Jiménez, arXiv:1608.08922

INVESTIGADOR FCT

GOVERNO DA REPÚBLICA PORTUGUESA

PROGRAMA OPERACIONAL POTENCIAL HUMANO

QUADRO DE REFERÊNCIA ESTRATÉGICO NACIONAL

Quest for CP violation

- CP-violation does not follow from the gauge structure of the SM.
- The smallness of CPV might be related to the fermion masses/mixing.
- The gauge structure of SM does not significantly restrict the scalar sector.

Extended scalar sectors offer many opportunities to naturally introduce CPV: explicit (soft/hard), spontaneous, geometrical, mismatching CPs in different sectors, etc.

Without direct experimental indication of New Physics, it makes sense to theoretically explore all possible ways the CPV can appear.

Freedom of defining CP

In QFT, the discrete transformations such as CP are not uniquely defined *a priori* [e.g. Feinberg, Weinberg, 1959].
For example, in NHDM with doublets ϕ_i, $i = 1, \dots, N$, the transformation

$$\phi_i \xrightarrow{CP} X_{ij}\phi_j^* \,, \quad X \in U(N)\,,$$

with any X can play the role of "the CP transformation" [e.g. Branco, Lavoura, Silva, 1999]. The "standard" convention $\phi_i \xrightarrow{CP} \phi_i^*$ is basis-dependent.

I will show that in models with several gauge-blind scalars the freedom of defining CP is even larger. In particular, it can lead to scalars which are *CP*-half-odd:

$$J: \quad \Phi(\mathbf{x}, t) \xrightarrow{CP} i\,\Phi(-\mathbf{x}, t)\,.$$

Notice: (1) no conjugation, (2) CP4: order-4 transformation, $J^2 \neq \mathbb{I}$, $J^4 = \mathbb{I}$.

Freedom of defining CP

In QFT, the discrete transformations such as CP are not uniquely defined *a priori* [e.g. Feinberg, Weinberg, 1959].
For example, in NHDM with doublets ϕ_i, $i = 1, \ldots, N$, the transformation

$$\phi_i \xrightarrow{CP} X_{ij}\phi_j^* \,, \quad X \in U(N)\,,$$

with any X can play the role of "the CP transformation" [e.g. Branco, Lavoura, Silva, 1999]. The "standard" convention $\phi_i \xrightarrow{CP} \phi_i^*$ is basis-dependent.

I will show that in models with several gauge-blind scalars the freedom of defining CP is even larger. In particular, it can lead to scalars which are *CP*-half-odd:

$$J: \quad \Phi(\mathbf{x}, t) \xrightarrow{CP} i\,\Phi(-\mathbf{x}, t)\,.$$

Notice: (1) no conjugation, (2) CP4: order-4 transformation, $J^2 \neq \mathbb{I}$, $J^4 = \mathbb{I}$.

CP4-3HDM

Consider 3HDM with the following potential $V = V_0 + V_1$ (notation: $i \equiv \phi_i$):

$$V_0 = -m_{11}^2(1^\dagger 1) - m_{22}^2(2^\dagger 2 + 3^\dagger 3) + \lambda_1(1^\dagger 1)^2 + \lambda_2\left[(2^\dagger 2)^2 + (3^\dagger 3)^2\right]$$
$$+\lambda_3(1^\dagger 1)(2^\dagger 2 + 3^\dagger 3) + \lambda_3'(2^\dagger 2)(3^\dagger 3) + \lambda_4\left[(1^\dagger 2)(2^\dagger 1) + (1^\dagger 3)(3^\dagger 1)\right] + \lambda_4'(2^\dagger 3)(3^\dagger 2)\,,$$

with all parameters real, and

$$V_1 = \lambda_5(3^\dagger 1)(2^\dagger 1) + \frac{\lambda_6}{2}\left[(2^\dagger 1)^2 - (3^\dagger 1)^2\right] + \lambda_8(2^\dagger 3)^2 + \lambda_9(2^\dagger 3)\left[(2^\dagger 2) - (3^\dagger 3)\right] + h.c.$$

with real $\lambda_{5,6}$ and complex $\lambda_{8,9}$. It is invariant under order-4 CP:

$$J:\ \phi_i \xrightarrow{CP} X_{ij}\phi_j^*\,, \quad X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix}.$$

Its square, $J^2 = XX^* = \mathrm{diag}(1, -1, -1)$ and $J^4 = \mathbb{I}$.
This model has no other symmetries [Ivanov, Keus, Vdovin, 2012].

Physical scalars

CP-conserving minimum: $v_i = (v, 0, 0)$.

Physical scalars: h_{SM}, degenerate $H^{\pm}_{2,3}$, and two pairs of degenerate neutrals: the heavier H and A and the ligher h and a, with masses

$$M^2,\, m^2 = -m^2_{22} + \frac{v^2}{2}\left(\lambda_3 + \lambda_4 \pm \sqrt{\lambda_5^2 + \lambda_6^2}\right)\,.$$

However, these real neutrals are not CP-eigenstates:

$$H \xrightarrow{CP} A, \quad A \xrightarrow{CP} -H\,, \quad h \xrightarrow{CP} -a, \quad a \xrightarrow{CP} h\,.$$

Can we combine them into neutral complex fields which are CP-eigenstates?

Physical scalars

Yes, we can:

$$\Phi = \frac{1}{\sqrt{2}}(H - iA)\,, \quad \varphi = \frac{1}{\sqrt{2}}(h + ia)\,, \quad \Phi \xrightarrow{CP} i\Phi\,, \quad \varphi \xrightarrow{CP} i\varphi\,.$$

Conserved quantum number: not $\mathbb{Z}_2$-parity but the charge q defined modulo 4.

In terms of (Φ, φ), the kinetic term is OK: $|\partial_\mu \Phi|^2 + |\partial_\mu \varphi|^2$. The interactions contains only combinations

$$\varphi^*\varphi\,, \quad \varphi^4\,, \quad (\varphi^*)^4\,, \quad \varphi^2(\varphi^*)^2\,, \quad \text{where } \varphi \text{ stands for } \Phi \text{ or } \varphi,$$

all of which conserve q. Transitions $\varphi^* \to \varphi\varphi\varphi$, $\varphi\varphi \to \varphi^*\varphi^*$ are possible, while $\varphi \to \varphi^*$ are forbidden by q conservation.

Similarly to ZHA vertex in CP-conserving 2HDM, with H and A of opposite CP-parities, we have $Z\Phi\varphi$ vertex, with two scalars of the same CP-properties:

$$\text{instead of} \quad (+1)\cdot(-1) = -1 \quad \text{we have} \quad i \cdot i = -1\,.$$

Physical scalars

Yes, we can:

$$\Phi = \frac{1}{\sqrt{2}}(H - iA)\,, \quad \varphi = \frac{1}{\sqrt{2}}(h + ia)\,, \quad \Phi \xrightarrow{CP} i\Phi\,, \quad \varphi \xrightarrow{CP} i\varphi\,.$$

Conserved quantum number: not $\mathbb{Z}_2$-parity but the charge q defined modulo 4.

In terms of (Φ, φ), the kinetic term is OK: $|\partial_\mu \Phi|^2 + |\partial_\mu \varphi|^2$. The interactions contains only combinations

$$\varphi^*\varphi\,, \quad \varphi^4\,, \quad (\varphi^*)^4\,, \quad \varphi^2(\varphi^*)^2\,, \quad \text{where } \varphi \text{ stands for } \Phi \text{ or } \varphi,$$

all of which conserve q. Transitions $\varphi^* \to \varphi\varphi\varphi$, $\varphi\varphi \to \varphi^*\varphi^*$ are possible, while $\varphi \to \varphi^*$ are forbidden by q conservation.

Similarly to ZHA vertex in CP-conserving 2HDM, with H and A of opposite CP-parities, we have $Z\Phi\varphi$ vertex, with two scalars of the same CP-properties:

$$\text{instead of} \quad (+1)\cdot(-1) = -1 \quad \text{we have} \quad i \cdot i = -1\,.$$

To C, or not to C — that is the question

$\Phi(\mathbf{x}, t) \xrightarrow{CP} i\Phi(-\mathbf{x}, t)$ looks like P transformation rather than CP.

Where did we lose the C in CP?

The origin of CP-half-odd scalars

Back to basics

Single complex scalar field:

$$\phi(\mathbf{x},t) = \int \tilde{dp}\left[a(\mathbf{p})e^{-ipx} + b^\dagger(\mathbf{p})e^{ipx}\right].$$

If we define $\phi \xrightarrow{CP} \phi^* \Rightarrow a(\mathbf{p}) \xrightarrow{CP} b(-\mathbf{p})$, $b(\mathbf{p}) \xrightarrow{CP} a(-\mathbf{p})$, which means

$$b^\dagger|0\rangle \text{ is antiparticle of } a^\dagger|0\rangle\,.$$

Two complex mass-degenerate scalars $\phi_i(x)$, $i = 1, 2$. If $\phi_1 \xrightarrow{CP} \phi_2^*$, $\phi_2 \xrightarrow{CP} \phi_1^*$, then $a_1(\mathbf{p}) \leftrightarrow b_2(-\mathbf{p})$, $b_1(\mathbf{p}) \leftrightarrow a_2(-\mathbf{p})$ under CP, which means

$$b_2^\dagger|0\rangle \quad \text{is antiparticle of} \quad a_1^\dagger|0\rangle\,,$$
$$b_1^\dagger|0\rangle \quad \text{is antiparticle of} \quad a_2^\dagger|0\rangle\,,$$

By choosing X in $\phi_i \xrightarrow{CP} X_{ij}\phi_j^*$, we assign who is antiparticle of whom.

Back to basics

Single complex scalar field:

$$\phi(\mathbf{x}, t) = \int \tilde{dp} \left[a(\mathbf{p}) e^{-ipx} + b^\dagger(\mathbf{p}) e^{ipx} \right] .$$

If we define $\phi \xrightarrow{CP} \phi^* \Rightarrow a(\mathbf{p}) \xrightarrow{CP} b(-\mathbf{p})$, $b(\mathbf{p}) \xrightarrow{CP} a(-\mathbf{p})$, which means

$$b^\dagger|0\rangle \text{ is antiparticle of } a^\dagger|0\rangle .$$

Two complex mass-degenerate scalars $\phi_i(x)$, $i = 1, 2$. If $\phi_1 \xrightarrow{CP} \phi_2^*$, $\phi_2 \xrightarrow{CP} \phi_1^*$, then $a_1(\mathbf{p}) \leftrightarrow b_2(-\mathbf{p})$, $b_1(\mathbf{p}) \leftrightarrow a_2(-\mathbf{p})$ under CP, which means

$$b_2^\dagger|0\rangle \quad \text{is antiparticle of} \quad a_1^\dagger|0\rangle ,$$
$$b_1^\dagger|0\rangle \quad \text{is antiparticle of} \quad a_2^\dagger|0\rangle ,$$

By choosing X in $\phi_i \xrightarrow{CP} X_{ij}\phi_j^*$, we assign who is antiparticle of whom.

Basis change

Within the same $\phi_1 \xrightarrow{CP} \phi_2^*$, $\phi_2 \xrightarrow{CP} \phi_1^*$ example, define η and ξ as

$$\begin{pmatrix} \eta \\ \xi \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \eta \xrightarrow{CP} \eta^*, \; \xi \xrightarrow{CP} -\xi^*.$$

Remember: η and ξ are mass-degenerate.

Combine two CP-even fields $\mathrm{Re}\eta$ and $\mathrm{Im}\xi \to \Phi = \mathrm{Re}\eta - i\,\mathrm{Im}\xi$.

Combine two CP-odd fields $\mathrm{Re}\xi$ and $\mathrm{Im}\eta \to \tilde{\Phi} = \mathrm{Re}\xi - i\,\mathrm{Im}\eta$.

$$\Phi \xrightarrow{CP} \Phi, \quad \tilde{\Phi} \xrightarrow{CP} -\tilde{\Phi}.$$

Conjugation disappeared.

Basis change

One can now describe passage from $(\phi_1, \phi_2) \to (\eta, \xi) \to (\Phi, \tilde{\Phi})$ via

$$\begin{pmatrix} \Phi \\ \tilde{\Phi} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2^* \end{pmatrix} .$$

This basis change "undoes" the conjugation.

This is a norm-preserving non-holomorphic map on $\mathbb{C}^2$, not the usual basis change. It is a valid $O(4)$-rotation in $\mathbb{R}^4$ spanned by $(\mathrm{Re}\eta, \mathrm{Im}\eta, \mathrm{Re}\xi, \mathrm{Im}\xi)$.

If complex fields are gauge-blind (do not carry any non-zero gauge quantum number), the group of basis changes increases from $U(2)$ to $O(4)$.

Basis change

For operators,

$$\Phi = \int \tilde{dp}(a e^{-ipx} + b^\dagger e^{ipx})\,, \quad \tilde\Phi = \int \tilde{dp}(\tilde a e^{-ipx} + \tilde b^\dagger e^{ipx})\,,$$

then passage from a_i, b_i to $a, b, \tilde a, \tilde b$ is given by

$$\begin{pmatrix} a \\ \tilde a \\ b \\ \tilde b \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ b_1 \\ b_2 \end{pmatrix}.$$

We have four different orthogonal one-particle states, all having the same quantum numbers, and we are free to assign who is antiparticle of whom.

In particular, Φ^* produces not the antiparticle of Φ but a different field.

For mass-degenerate gauge-blind scalars,
conjugating or not under *CP* is a matter of basis choice.

In all transformations, we never redefined the *CP* transformation itself.

CP4

CP-half-odd scalars arise in a similar way.

- Starting point: $\phi_1 \xrightarrow{CP} i\phi_2^*$, $\phi_2 \xrightarrow{CP} -i\phi_1^*$.
- Operators: $a_1 \xrightarrow{CP} ib_2\,,\ b_2 \xrightarrow{CP} ia_1$ and $a_2 \xrightarrow{CP} -ib_1\,,\ b_1 \xrightarrow{CP} -ia_2$.
- Basis change:

$$\begin{pmatrix} \Phi \\ \varphi^* \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2^* \end{pmatrix}.$$

- Result: $\Phi \xrightarrow{CP} i\Phi$, $\varphi \xrightarrow{CP} i\varphi$. For operators

$$a_\Phi = (a_1 + b_2)/\sqrt{2} \xrightarrow{CP} ia_\Phi\,, \quad b_\Phi^\dagger = (a_2^\dagger + b_1^\dagger)/\sqrt{2} \xrightarrow{CP} ib_\Phi^\dagger\,.$$

An example of doubling of mass eigenstates beyond Kramers degeneracy (a possibility mentioned e.g. in Weinberg, vol. 1, app. 2C).

Yukawa couplings for CP-half-odd scalars

Yukawas for CP-half-odd scalars

Can CP-half-odd scalars have CP-conserving Yukawa interactions? Yes, provided the CP mixes the fermion families.

Generic Yukawa sector for $N = 3$ fermion generations:

$$-\mathcal{L}_Y = \bar{\psi}_i (A_{ij} + B_{ij}\gamma_5)\psi_j\, \phi + \bar{\psi}_i [(A^\dagger)_{ij} - (B^\dagger)_{ij}\gamma_5]\psi_j\, \phi^* \,.$$

with arbitrary complex A, B. Assume that $\phi \xrightarrow{CP} i\phi$ and $\psi_i \xrightarrow{CP} Y^*_{ij}\psi^c_j$, $Y \in U(3)$. Then, invariance under CP implies

$$iY^\dagger A^T Y = A\,, \quad -iY^\dagger B^T Y = B\,.$$

Although $\mathrm{Tr} A^k = 0$ and $\det A = 0$, we still find Y and non-zero A, B.

Yukawas for CP-half-odd scalars

In the basis, where Y takes its simplest form

$$Y = \begin{pmatrix} e^{i\beta} & 0 & 0 \\ 0 & 0 & e^{i\alpha} \\ 0 & e^{-i\alpha} & 0 \end{pmatrix},$$

the solutions are

$$\alpha = -\frac{\pi}{4} + \pi k \qquad A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & a_{23} \\ 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & b_{32} & 0 \end{pmatrix};$$

$$\alpha = \frac{\pi}{4} + \pi k \qquad A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & a_{32} & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & b_{23} \\ 0 & 0 & 0 \end{pmatrix};$$

$$\alpha = \pm\frac{\pi}{2} \qquad A = \begin{pmatrix} 0 & a_{12} & a_{13} \\ \mp e^{i\beta} a_{13} & 0 & 0 \\ \pm e^{i\beta} a_{12} & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & b_{12} & b_{13} \\ \pm e^{i\beta} b_{13} & 0 & 0 \\ \mp e^{i\beta} b_{12} & 0 & 0 \end{pmatrix}.$$

Yukawas for CP4-3HDM

In CP4-3HDM, the charged lepton sector (here, $\phi_a \equiv \phi_a^0$):

$$-\mathcal{L}_Y = \bar{\ell}_{Li}\Gamma^a_{ij}\ell_{Rj}\,\phi_a + \bar{\ell}_{Ri}(\Gamma^a_{ij})^\dagger\ell_{Lj}\,\phi_a^*\,.$$

Take $\phi_a \xrightarrow{CP} X_{ab}\phi_b^*$, with the same X as before, and $\ell_i \xrightarrow{CP} Y_{ij}^*\ell_j^c$. Then,

$$Y^\dagger\Gamma_1^* Y = \Gamma_1\,,\quad -iY^\dagger\Gamma_2^* Y = \Gamma_3\,,\quad iY^\dagger\Gamma_3^* Y = \Gamma_2\,.$$

We switch to the fermion basis in which

$$Y = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & e^{i\alpha} \\ 0 & e^{-i\alpha} & 0 \end{pmatrix},$$

CP-conservation requires that fermions be degenerate: $m_2 = m_3$.

Yukawas for CP4-3HDM

- case 1: $\alpha = \pm\pi/4 + \pi k$, order-8 transformation.

$$\Gamma_1 = \begin{pmatrix} g_1 & 0 & 0 \\ 0 & g_2 & 0 \\ 0 & 0 & g_2^* \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & g_{23} \\ 0 & g_{32} & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \pm g_{32}^* \\ 0 & \mp g_{23}^* & 0 \end{pmatrix}.$$

- case 2: $\alpha = \pm\pi/2$, order-4 transformation.

$$\Gamma_1 = \begin{pmatrix} g_1 & 0 & 0 \\ 0 & g_2 & g_3 \\ 0 & -g_3^* & g_2^* \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & g_{12} & g_{13} \\ g_{21} & 0 & 0 \\ g_{31} & 0 & 0 \end{pmatrix}, \quad \Gamma_3 = \pm \begin{pmatrix} 0 & -g_{13}^* & g_{12}^* \\ g_{31}^* & 0 & 0 \\ -g_{21}^* & 0 & 0 \end{pmatrix}.$$

Yukawas for CP4-3HDM

Explicitly, in case 1 [notation: $(e,\mu,\tau) = (\ell_1,\ell_2,\ell_3)$]:

$$-\mathcal{L}_Y = (\bar\mu\tau)(g\Phi - \tilde g\varphi) + (\bar\tau\gamma_5\mu)(\tilde g^*\Phi + g^*\varphi) + h.c.,$$

where

$$g = \frac{c_\gamma g_{23} - s_\gamma g_{32}^*}{\sqrt{2}}\,, \quad \tilde g = \frac{s_\gamma g_{23} + c_\gamma g_{32}^*}{\sqrt{2}}\,, \quad \tan 2\gamma = -\lambda_6/\lambda_5\,.$$

Notice that fermion biliears are *CP*-half-odd, and that insertion of γ_5 introduces an extra "*CP*-oddness" as in the usual case:

$$\bar\mu\tau \xrightarrow{CP} -i\bar\mu\tau\,, \quad \bar\tau\mu \xrightarrow{CP} i\bar\tau\mu\,, \quad \bar\mu\gamma_5\tau \xrightarrow{CP} i\bar\mu\gamma_5\tau\,, \quad \bar\tau\gamma_5\mu \xrightarrow{CP} -i\bar\tau\gamma_5\mu\,.$$

CP-half-odd scalars do not have to be inert.

Remarks on phenomenology

- Does a model based on CP4 lead to any phenomenological signal which cannot be mimicked by any usual CP-conserving model?

$$(\mathcal{CP}) a^\dagger_\Phi a^\dagger_\Phi (\mathcal{CP})^{-1} = -a^\dagger_\Phi a^\dagger_\Phi \,.$$

  We get a *CP*-odd pair of two identical bosons $\to$ any pheno signal?

- A variation on inert doublet model based on CP4 instead of $CP + \mathbb{Z}_2$: fully calculable, can be readily tested against collider data and DM searches.
- If CP4 is spontaneous broken, can CP4-3HDM with Yukawas reproduce the quark sector?
- Any specific predictions for the neutrino sector?

Conclusions

- CP4-3HDM is the simplest model featuring CP-half-odd scalars: $\Phi \xrightarrow{CP} i\Phi$. Their origin is the extra freedom of basis change arising in models with mass-degenerate gauge-blind scalars.
- No fine-tuning is required: the gauge blindness, mass degeneracy, and CP-half-oddness appear naturally within CP4-3HDM.
- The CP-half-odd scalars do not have to be inert: they can couple to fermions in CP-conserving way.
- The model is compact, analytically tractable, and can have interesting phenomenological consequences.