Real representations and Gauge invariance in the presence of the Higgs mechanism

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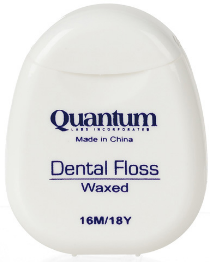
UNI GRAZ

“The federal government recommended flossing in 1979-2015.



In 2016 the government acknowledged the effectiveness of flossing had never been researched, as required.”

(Associated Press)

(Feynman 1973)

All the people are doing the same ritual brush, brush, brush—for no good reason? Think about it.

Summary

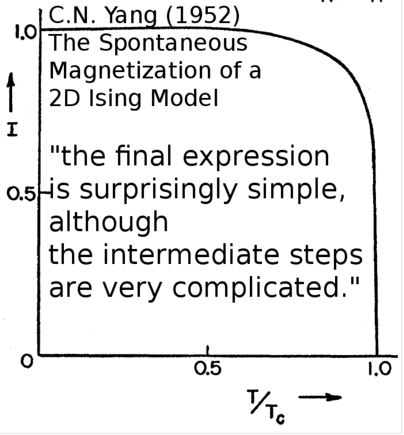
- The mean field approximation ($\phi = < \phi > + \varphi$) fails already in the Ising model.
- Complex vector spaces are not a generalization of real vector spaces (custodial symmetry, position operator).
- Study multi-Higgs models with no assumptions on gauge symmetry breaking.

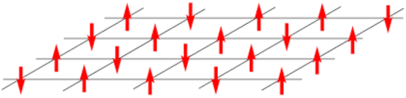
Developing a framework:

gauge symmetry breaking vs. LHC data.

$$I=\left[\frac{1+x^2}{(1-x^2)^2}(1-6x^2+x^4)^{\frac{1}{2}}\right]^{\frac{1}{4}}.$$

$$x=\exp[-(1/kT)\{V_{\uparrow\downarrow}-V_{\uparrow\uparrow}\}]$$





2–D Ising Model

(Strocchi (2005)) 1D: mean field approximation predicts spontaneous symmetry breaking $\neq$ exact solution

2D: analytic description of spontaneous symmetry breaking$\Rightarrow$ disjoint phases.

Possible for a global symmetry
in a system with infinite size.

Local gauge transformation affects
a small sized system near each space-time point $\Rightarrow$ open problem

Mainstream (non-perturbative) view:
't Hooft (1980) "the words *spontaneous breakdown* are formally not correct for local gauge theories. The vacuum *never* breaks local gauge invariance."

Englert (2014)"strictly speaking there is no spontaneous symmetry breaking of a local symmetry.
One uses perturbation theory to select at zero coupling a scalar field configuration from global SSB; but this preferred choice is only a convenient one."

gauge symmetry breaking vs. lattice: ✓

gauge symmetry breaking vs. LHC data?

If the Higgs was found, why should I care?

Infer from experimental data if a quantum model exists mathematically.

Model X predicts y=2 is often problematic.

For instance:

('t Hooft (2002)): *Little experimental support for functional integrals in d>4. Consequences for model building are often underestimated.*

Multi-Higgs: spontaneous breaking of global symmetries?

Help from lattice(Lewis and Woloshyn (2010a); Maas and Pedro (2016))

Higgs potential

Holomorphic functions $\frac{\partial f(z,z^*)}{\partial z^*} = 0$
central objects of study in complex analysis

The Higgs potential is not an holomorphic function
$\frac{\partial V(\phi_j,\phi_j^*)}{\partial \phi^*} \neq 0.$

No advantage in the Higgs field being a complex vector space
$V(\phi_j, \phi_j^*) = V(Re(\phi_j), Im(\phi_j))$

Standard Higgs field (4 real components) $V(\phi, \phi^*) = V(\phi^\dagger \phi)$
$\Rightarrow$ Custodial symmetry $SO(4)/SU(2)_L \simeq SO(3)$

Classical minimization: no limit on the order of the potential $\Rightarrow$ effective field theory for multi-Higgs models (Crivellin et al. (2016))

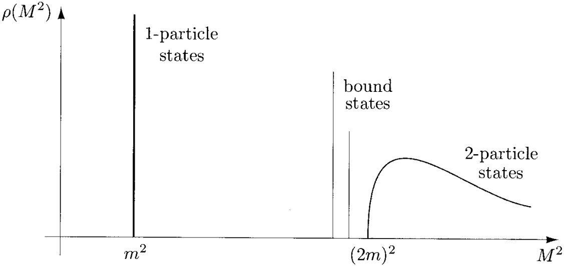
Consequences from non-perturbative definition of spontaneous breakdown

Example: Global symmetry $G/SU(2)_L$ for one-Higgs-doublet cannot be explicitly broken $\Rightarrow$ no spontaneous symmetry breaking of $G/SU(2)_L$

More examples with CP Branco and Ivanov (2016)

Multi-Higgs: Evaluating vevs of $SU(2)_L$-invariant observables,
$\Rightarrow$ no assumptions on gauge symmetry breaking.

Reformulation of electroweak perturbation theory



$$\Delta(p) = \int_0^\infty d\mu^2 \rho(\mu^2) \frac{1}{p^2 - \mu^2 + i\epsilon},$$

Källén–Lehmann spectral representation of two-point functions

Asymptotic states at LHC: masses$\ll 125\ GeV$

Fröhlich, Morchio, and Strocchi (1981)Electron-higgs composite operator Ψ:

$$< \Psi\overline{\Psi} >= electron + electron.(higgs\ boson)$$

In QCD there are also excited hadron states $\Rightarrow$ reformulation not profound.
Neither the first reformulation (Kobe (1983) AC Stark shift)

Obrigado

- The mean field approximation ($\phi =< \phi > +\varphi$) fails already in the Ising model.
- Complex vector spaces are not a generalization of real vector spaces (custodial symmetry, position operator).
- Study multi-Higgs models with no assumptions on gauge symmetry breaking.
- *what are the EM waves?* $\Rightarrow$Maxwell: ~~aether~~

Developing a framework:

gauge symmetry breaking vs. LHC data.

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Prelude

Spontaneous symmetry breaking (SSB) $\rightarrow$ disjoint phases in a system (local interactions, e.g. the Ising model or gauge theories)

Expectation value $\omega_{J,N} : \mathcal{A} \rightarrow \mathbb{R}$ (positive linear functional)

J: intensity of external source breaking a group of symmetries G.
$\mathcal{A}$: set of observables.

Finite size N: continuous expectation values

$$\begin{cases} \omega_{J,N}(A - g(A)) = 0 & \text{if } J = 0 \\ \lim_{J\rightarrow 0} \omega_{J,N}(A - g(A)) = 0 & \end{cases}$$

any observable $A \in \mathcal{A}$ and any transformation $g \in G$.

Definition 1. Spontaneous symmetry breaking when:

$$\lim_{J\to 0}\{\lim_{N\to\infty}\omega_{J,N}(A-g(A))\}\neq 0$$

for some $A\in\mathcal{A}$ and some $g\in G$.

Limit of a convergent sequence of continuous functions is not necessarily continuous.

Other definitions in statistical mechanics are not based on explicit symmetry breaking.

SSB possible for a global symmetry in a system with infinite size.

Elitzur (1975) “a spontaneous breaking of local symmetry for a symmetrical gauge theory without gauge fixing is impossible.”

local gauge transformation affects
only a small sized system near each space-time point.

Outline

Higgs potential in multi-Higgs-doublet models

Confinement

Gauge-invariant operators in 2HDM (no $U(1)_Y$)

Majorana construction

Observable states of 2HDM

Spontaneous symmetry breaking in 2HDMs

The FMS mechanism

$Spin(4)$ symmetric 2HDM for the lattice

1 Higgs potential

Holomorphic functions $\frac{\partial f(z,z^*)}{\partial z^*} = 0$
central objects of study in complex analysis

The Higgs potential is not an holomorphic function
$\frac{\partial V(\phi_j,\phi_j^*)}{\partial \phi^*} \neq 0$.

No advantage in the Higgs field being a complex vector space
$V(\phi_j, \phi_j^*) = V(Re(\phi_j), Im(\phi_j))$

Complex irreducible representations of $G \times H$ are a direct product of complex irreducible representations of G and of H.
Not the case for real irreducible representations.
(wiki/Representation_theory_of_finite_groups,arXiv:1309.5280)

Standard Higgs field (4 real components) $V(\phi, \phi^*) = V(\phi^\dagger \phi)$

$SO(4) \simeq (SU(2)_R \times SU(2)_L)/Z_2$ (generators τ^j and σ^j)

$SU(2)_L$ gauge symmetry
Global symmetry $SO(4)/SU(2)_L \simeq SO(3)$ (E.g. $\phi^\dagger D_\mu \tau^j \phi$)

N-Higgs-doublets (4N real components):

- different global symmetry $G/SU(2)_L$
- charged scalars;
- mixing between neutral scalar particles;
- Spontaneous/explicit global symmetry violation in the Higgs potential;
- Rich flavour phenomenology (e.g. meson decays, oscillations)

Definition 2. (Electroweak symmetry breaking)

After perturbative gauge-fixing,
the Higgs vev minimizes the Higgs potential.

The symmetries broken by the Higgs vev
are the spontaneously broken symmetries.

Perturbation theory can only deal with small perturbations of the Higgs field $\to$ non-null Higgs vev.

Challenge: spontaneous breaking of global symmetries
in the presence of the Higgs mechanism

- Find an absolute minimum of the potential $\phi = \frac{v}{\sqrt{2}}\phi_0$;
- Projector P_0 on the $SU(2)_L$-orbit of ϕ_0 such that $P_0\phi_0 = \phi_0$;
- Modify the Higgs potential $W = V + \epsilon U$,
  $\epsilon > 0$ is arbitrarily small and $U = -v^2\phi^\dagger P_0\phi + (\phi^\dagger\phi)^2$;
- The absolute minima of $W = V + \epsilon U$ is the $SU(2)_L$-orbit of ϕ_0;

The perturbation theory then implies that in the limit $\epsilon \to 0$, there are finite vevs breaking the global symmetries $\Rightarrow$ SSB by Def. 1

Evaluating vevs of $SU(2)_L$-invariant observables,
we make no assumptions about SSB of gauge symmetry.

Classical minimization: no limit on the order of the potential $\Rightarrow$ effective field theory, no assumptions on the ultra-violet completion (appropriate for experimental data Eichhorn et al. (2015))

Consistency: Let $p(\phi)$ be a G_f-invariant polynomial in the Higgs field ϕ.

If any G_f-invariant Higgs potential is necessarily G-invariant,
the observable $p(\phi)$ must also be invariant under G,
since $p(\phi)$ can appear in a G_f-invariant Higgs potential.
If G_f is a classical$\times$finite group $\Rightarrow$ No spontaneous symmetry breaking of G/G_f since all G_f-invariant observables are also G-invariant.

Example: Global symmetry $G/SU(2)_L$ for one-Higgs-doublet cannot be explicitly broken $\Rightarrow$ no spontaneous symmetry breaking of $G/SU(2)_L$

More examples with CP Branco and Ivanov (2016)

2 Confinement

Options for Electroweak Theory:

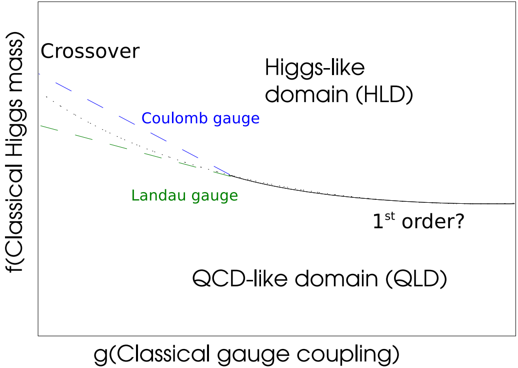
1) Define the theory with gauge fixing (standard in perturbation theory), Gribov (1978); Singer (1978) non-perturbative ambiguity, the local non-abelian gauge-fixing condition is insufficient

2) gauge-invariant gauge charge, e.g. dressed elementary operators (photons are neutral), non-abelian (global) gauge charges cannot be (locally) gauge-invariant Haag (1992).

3) Fröhlich, Morchio, and Strocchi (1981): FMS mechanism inspired in the confinement mechanism, effectively matches gauge fixing+perturbation theory under some assumptions

4) Technicolor 5) ? (next) ...

Maas and Mufti (2015) SU(2) Yang-Mills-Higgs on the lattice phase diagram



$\frac{1}{g} \propto$ gauge coupling
$f \propto m_h^2 v^2$ dashed lines: break global subgroup remaining after incomplete gauge-fixing.

Osterwalder and Seiler (1978)

Fradkin and Shenker (1979)

Caudy and Greensite (2008)

Seiler (2015)

Bonati et al. (2010) “hints that the above transitions are not related to confinement”

Englert (2005) “Electric-magnetic dualities suggest that, at some fundamental level, confinement is a condensation of magnetic monopoles and constitutes the magnetic dual of the BEH mechanism”

However Englert does not cite FMS mechanism

3 Gauge-invariant operators in 2HDM (no $U(1)_Y$)

$SU(2)_L$ Higgs doublets ϕ_1, ϕ_2

gauge field W^j_μ with $j, k, l = 1, 2, 3$

Higgs Potential $V(\phi_1, \phi_2)$,

coupling constant g,

$$\mathcal{L} \equiv ((D^\mu \phi_1)^\dagger (D_\mu \phi_1) + ((D^\mu \phi_2)^\dagger (D_\mu \phi_2) - V(\phi_1, \phi_2) - \frac{1}{4} W^j_{\mu\nu} W^{j\mu\nu}$$

$$D_\mu \equiv \partial_\mu + igW^j_\mu \frac{\sigma^j}{2}$$

$$W^j_{\mu\nu} \equiv -\frac{i}{g} \mathrm{tr}([D_\mu, D_\nu]\sigma^j) = \partial_\mu W^j_\nu - \partial_\nu W^j_\mu - g\epsilon^{jkl} W^k_\mu W^l_\nu$$

Levi-Civita ϵ^{jkl}, Pauli matrices in gauge space σ^j

Karassiov (1992) (+ general Wineman and Pipkin (1964))

Any polynomial of ϕ_1, ϕ_2 which is gauge invariant is a polynomial on

$\phi^*_{1a}\phi_1{}^a$, $\phi^*_{2a}\phi_2{}^a$, $\phi^*_{2a}\phi_1{}^a$; $\epsilon_{ab}\phi_1{}^a\phi_2{}^b$; $\epsilon_{ab}\phi_1{}^{a*}\phi_2{}^{b*}$

$\phi^*_{jb} \equiv (\phi_j{}^b)^*$ $a, b = 1, 2$ are gauge indices.

Also parallel transport $U(x, y, C)$ from y to x along line C.

for infinitesimal line elements

$$U(x, y, C) \approx (1 + D_\mu(x)dl_1^\mu)(1 + D_\nu(x)dl_2^\nu)...(1 + D_\alpha(x)dl_n^\alpha)$$

$dl_1, dl_2, ..., dl_n$ (n finite) are infinitesimal Lorentz vectors forming C by concatenation.

Set of primitive (algebraically ind.) gauge-invariant operators for 2HDM:

- $tr(U(x,x,C'))$ Giles (1981)
- $\phi_j^\dagger(x)U(x,y,C)\phi_k(y)$
- $\phi_j^\dagger(x)U(x,y,C)\overline{\phi}_k(y)$
- $\overline{\phi}_j^\dagger(x)U(x,y,C)\phi_k(y)$
- $\overline{\phi}_j^\dagger(x)U(x,y,C)\overline{\phi}_k(y)$

$\overline{\phi}_j{}^a(x) \equiv \epsilon^{ab}\phi^*_{jb}(x)$,

indices $j,k=1,2$ are Higgs flavor indices,

4 Majorana construction

Shirokov (2015): A^a, B^a are $2^n \times 2^n$ complex unitary matrices

$$A^a A^b + A^b A^a = 2g^{ab}1$$
$$B^a B^b + B^b B^a = 2g^{ab}1$$

$a \in \{1, ..., 2n\}$, $n < 4$, $g \equiv \mathrm{diag}(-1, ..., +1, ...)$ (n entries -1 and n $+1$)
Generalized Pauli's theorem:

1. $B^a = SA^aS^{-1}$. S is unitary and unique up to a phase;
2. there is a basis where all A^a are real;
3. Clifford algebra generated by A^a is isomorphic to the algebra of $2^n \times 2^n$ matrices.

Majorana spinors: 2^n complex vectors u satisfying $\Theta u = u$.
Θ: anti-linear involution commuting with A^a, unique up to a phase.

$n = 3$: 8-dimensional Majorana spinor ϕ (Pilaftsis (2012)).

Generators of $SU(2)_L$: $i\sigma^j \equiv \epsilon^{jkl} A_k A_l$ $(j, k, l = 1, 2, 3)$

$\Sigma_j \equiv A^{j+3}$ $(j = 1, 2, 3)$, $\Sigma_4 \equiv A^1 A^2 A^3$ and $\Sigma_5 \equiv \Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4 = -A^7$.

$1, \Sigma_a$ $(a, b = 1, ..., 5)$: basis of hermitian matrices conserved by $SU(2)_L$.
Σ_a anti-commute with each other.

$[\Sigma_a, \Sigma_b]$: basis of skew-hermitian matrices conserved by $SU(2)_L$, generators of $Spin(5)$ (double cover of $SO(5)$).

Rewrite set of primitive gauge-invariant operators:

- $\phi^\dagger(x) U(x, y, C) \phi(y)$ (singlet under $SO(5)$);
- $\phi^\dagger(x) U(x, y, C) \Sigma_a \phi(y)$ (**5** representation of $SO(5)$);
- $\phi^\dagger(x) U(x, y, C) [\Sigma_a, \Sigma_b] \phi(y)$ (**10** representation of $SO(5)$);

5 Observable states of 2HDM

Higgs potential (basis-invariant formalism O'Neil (2009)):

$$V(\phi) = \mu_a \phi^\dagger \Sigma_a \phi + \frac{1}{2}\lambda_{ab}(\phi^\dagger \Sigma_a \phi)(\phi^\dagger \Sigma_b \phi)$$

parameters of the potential $\Rightarrow$ background fields (spurions)

Ivanov (2006); Botella et al. (2013)

μ_0, λ_{00} singlets,
μ_a, λ_{0a} are 5-dim representations of $SO(5)$
λ_{ab} is a tensor of $SO(5)$

Lagrangian invariant under gauge $SU(2)_L$ and background $Spin(5)$.

Let $V(\phi = \frac{v}{\sqrt{2}}\phi_0)$ be absolute minimum, ($v \equiv$ vev, $\phi_0^\dagger\phi_0 = 1$).

by reparametrization $\Sigma_5\phi_0 = \phi_0$ ($Spin(5) \to Spin(4)$)

$H_1 \equiv \frac{1+\Sigma_5}{2}\phi$ $H_2 \equiv \Sigma_4\frac{1-\Sigma_5}{2}\phi$, at the minimum $H_2 = 0$.

isomorphism $Spin(4) \simeq (SU(2)_{R1} \times SU(2)_{R2})$

$SU(2)_{R1}$ generators $\Sigma_j\Sigma_4(1+\Sigma_5)/2$

$SU(2)_{R2}$ generators $\Sigma_j\Sigma_4(1-\Sigma_5)/2$

after (suitable) gauge fixing, constant $\frac{v}{\sqrt{2}}\phi_0$ minimizing the potential $i\sigma_j\phi_0 = \Sigma_4\Sigma_j\phi_0$ $(j = 1, 2, 3)$,

ϕ^0 conserves $SO(3) \times Spin(3) \simeq (SU(2)_{R1} \times SU(2)_{R2})/Z_2$,

generators $(\Sigma_4\Sigma_j(1 + \Sigma_5)/2 - i\sigma_j)$ and $\Sigma_4\Sigma_j(1 - \Sigma_5)$, respectively.

ϕ_0 fixes a system of gauge coordinates.

4 projections $\phi_0 \phi_0^\dagger$ and $-\Sigma_4 \Sigma_j \phi_0 \phi_0^\dagger \Sigma_4 \Sigma_j$ (fixed $j = 1, 2, 3$) sum to 1 and decompose the 4 dim real spinor space of $SU(2)_L$

subspace $\propto \phi_0$: $\phi_0^\dagger H_1$, $\phi_0^\dagger H_2$.

subspace $\propto \Sigma_4 \Sigma_j \phi_0$: would-be Goldstone bosons $\phi_0^\dagger \Sigma_4 \Sigma_j H_1$ and $\phi_0^\dagger \Sigma_4 \Sigma_j H_2$.

3 projections for the triplet of $SU(2)_L$,

subspace $\propto (\phi_0 \otimes \Sigma_4 \Sigma_j \phi_0 - \Sigma_4 \Sigma_j \phi_0 \otimes \phi_0)$: $\phi_0^\dagger D_\mu \Sigma_4 \Sigma_j \phi_0 = \frac{g}{2} W_\mu^j$.

expand $\sqrt{2}\phi = v\phi_0 + \varphi$

Assuming the fluctuations φ small in average compared to v

$J(SU(2)_{R1})$	$J(SU(2)_{R2})$	Operator	Expansion
0	0	$H_1^\dagger H_1$	$\frac{v^2}{2} + v\phi_0^\dagger \varphi$
1/2	1/2	$H_1^\dagger \Sigma_a \Sigma_4 H_2$	$\frac{v}{2}\phi_0^\dagger \Sigma_a \varphi$
1	0	$H_1^\dagger D_\mu \Sigma_j \Sigma_4 H_1$	$\frac{gv^2}{4} W_\mu^j$

($j = 1, 2, 3$ and $a = 1, 2, 3, 4$, first terms in expansion only)

other primitive invariants involving ≤ 1 covariant derivative expand to ≥ 2 elementary fields at leading order, since the vev contribution to H_2 is null.

Possible to use further covariant derivatives, but cannot expand to 1 elementary field, as there are none with other Lorentz quantum numbers.

6 Spontaneous symmetry breaking in 2HDMs

If absolute minimum not unique (up to gauge transformations), fixing $\Sigma_5\phi_0 = \phi_0$ is in conflict with a global symmetry.

If the symmetry is spontaneously broken, then such a would-be global symmetry of the model is explicitly broken by an infinitesimal parameter.

It may occur in 2HDMs (lattice Lewis and Woloshyn (2010b)) depending on the Higgs potential.

Finite lattice , no spontaneous symmetry breaking:
estimate the results for the infinite-volume limit and then extrapolate the estimates to $J \to 0$

Lewis and Woloshyn (2010b): Done for continuous symmetry breaking in a 2HDM.

7 The FMS mechanism

Frohlich, Morchio, and Strocchi (1981) (group-theory) correspondence:

$$H_1 \Leftrightarrow \frac{v}{\sqrt{2}}\phi_0(\text{fixes gauge coordinate system})$$

$$\text{gauge} - \text{invariant states} \Leftrightarrow \text{elementary gauge} - \text{dependent fields}$$

one-to-one (set of primitive states), except for would-be Goldstone bosons

$\phi_0^\dagger \Sigma_4 \Sigma_j H_1$ disappear from the spectrum,

$\Sigma_4 \Sigma_j$ is skew-adjoint so $H_1^\dagger \Sigma_4 \Sigma_j H_1 = 0$.

Complete expansion:

$$2H_1^\dagger H_1 = v^2 + 2v\phi_0^\dagger \varphi + \varphi^\dagger \varphi$$

$v\phi_0^\dagger \varphi$, $\varphi^\dagger \varphi$ same quantum numbers, to distinguish:

- approximately by the energy spectrum
- or in perturbation theory.

recall KLN theorem sum all initial and final states (incl. soft photons) with same quantum number in a energy window $\Rightarrow$ infrared finite.

Assuming $\varphi^\dagger\varphi \approx$ scattering state, energy spectrum $\gtrsim 2m_H \neq m_H$.

In perturbation theory, for asymptotic state the mass is on-shell m_H, so contribution from $\varphi^\dagger\varphi$ negligible.

For intermediate states, since the (gauge-invariant) Lagrangian is the same no deviations expected.

Calculating the spectrum and testing the FMS mechanism in the lattice is an extension of Maas (2013); Wurtz and Lewis (2013); Maas and Mufti (2014, 2015)

We must still account for precision electroweak observables
In any case, Frohlich et al. (1981)
"standard perturbation expansion cannot be asymptotic to gauge-dependent correlation functions."

8 $Spin(4)$ symmetric 2HDM for the lattice

$$V(\phi) = \mu_0 \phi^\dagger \phi + \mu_5 \phi^\dagger \Sigma_5 \phi$$
$$+ \frac{1}{2}\lambda_{00}(\phi^\dagger\phi)^2 + \lambda_{05}(\phi^\dagger\phi)(\phi^\dagger\Sigma_5\phi) + \frac{1}{2}\lambda_{55}(\phi^\dagger\Sigma_5\phi)^2$$

To avoid breaking the $Spin(4)$ group, $\pm\Sigma_5\phi_0 = \phi_0$. For $\lambda_{05} = 0$:

1. ("control sample") $\mu_5 > 0$, $\lambda_{55} = 0$, realistic Bhupal Dev and Pilaftsis (2014) Maximally-Symmetric 2HDM.

2. $\mu_5 \to 0$ with $\mu_5 > 0$ and $\lambda_{55} \neq 0$, spontaneous symmetry breaking of the discrete Z_4.

3. $\mu_5 \to 0$ with $\mu_5 > 0$ and $\lambda_{55} = 0$, spontaneous symmetry breaking of the continuous $Spin(5) \to Spin(4)$. 4 massless Goldstone bosons.

$< H_1^\dagger(y)H_1(y)H_1^\dagger(x)H_1(x) >$ and $< H_2^\dagger(y)H_2(y)H_2^\dagger(x)H_2(x) >$

After gauge fixing, we can expand them as:

$$< H_1^\dagger(y)H_1(y)H_1^\dagger(x)H_1(x) > \approx \frac{v^4}{4} + \frac{v^2}{2} < \varphi^\dagger(y)\phi_0\phi_0^\dagger\varphi(x) > + ...$$
$$< H_2^\dagger(y)H_2(y)H_2^\dagger(x)H_2(x) > = < \varphi_2^\dagger(y)\varphi_2(y)\varphi_2^\dagger(x)\varphi_2(x) >$$

where $\varphi_2 \equiv \phi_0^\dagger \Sigma_4 \varphi$.
Neglecting interactions, energy spectrum $\gtrsim m_h$ and $\gtrsim 2m_H$.

$\mu_5 \to 0$, check if Z_4 symmetry is recovered.
If $\mu_5 = 0$ by definition the correlations are Z_4 symmetric.

9 Summary

Assuming gauge symmetry breaking or using only complex representations of groups is not enough to study the phenomenology of multi-Higgs-doublet models

For multi-Higgs-doublets, the FMS mechanism justifies that the spectrum is well described by the gauge-dependent elementary states.

If not, the physical states would, as in QCD, require non-perturbative methods, even at weak coupling.

The assumptions:
the field fluctuations around the vacuum are small in average
and there is spontaneous symmetry breaking of the global symmetry when the gauge orbit minimizing the Higgs potential is not unique.

To confirm the FMS mechanism and assumptions requires non-perturbative calculations, next step.

(Addition of photons and fermions in 1601.02006)

Next?

Classical electrodynamics: gauge-invariant local states

Quantum U(1) gauge: either
gauge-invariant non-local states or gauge-dependent local states

Quantum SU(2) gauge: gauge-invariant local states (Higgs mechanism)
Quantum SU(3) gauge: gauge-invariant local states (confinement)

To me, we need to look for gauge-invariant local states in U(1).
Implies probabilities instead of amplitudes Weinberg (2014), in-in formalism
And to work with phaseless (real) operators Pedro (2013)

In the mean time, gauge-dependent local states in U(1)

option 1) ok for abelian Higgs mechanism
(Ginzburg-Landau Superconductivity),

the Higgs mechanism is based in the fact:

breaking local gauge symmetries $\neq$ global symmetries

the Goldsone theorem does not apply
the Nambu-Goldstone bosons may be absent.

Englert (2014) “The vacuum is no more degenerate and strictly speaking there is no spontaneous symmetry breaking of a local symmetry.[...]

The disappearance of the NG boson is thus an immediate consequence of local symmetry. The above argument (Englert, 2005) was formalized much later (Elitzur, 1975)”

10 Introducing Photons

$U(1)_Y$ gauge symmetry with generator $\Sigma_1\Sigma_2$:

background symmetry: $(U(1)_Y \times Spin(3)) \rtimes Z_4$
custodial $Spin(3)$ generators $\Sigma_3\Sigma_4$, $\Sigma_3\Sigma_5$, $\Sigma_4\Sigma_5$
Z_4 generated by the charge reversal transformation $\phi \to \Sigma_2\Sigma_3\phi$.

The $U(1)_Y \times Spin(3)$ is a normal subgroup.
Any transformation is the product of:
element of $U(1)_Y \times Spin(3)$ and element of Z_4.

Parity and charge reversal are conserved separately in the absence of fermions
under charge reversal $B_\mu \to -B_\mu$.

Neutral vacuum condition: ϕ_0 aligned along linear combination of $\Sigma_{3,4,5}$.

11 Introducing Fermions

Quark field Q_L, $\Sigma_1\Sigma_2 Q_L = iQ_L$, $\Sigma_5 Q_L = Q_L$ and transforming under $SU(2)_L$ as ϕ.

Most general Yukawa couplings with the quarks:

$$-\mathcal{L}_{Y_Q} = \overline{Q_L}\,\Gamma_d\phi\, d_R + \overline{Q_L}\,\Sigma_3\Sigma_1\Gamma_u\phi\, u_R + \text{h.c.}$$
$$\Gamma_w \equiv \Gamma_{w\,0} + \Gamma_{w\,1}\Sigma_3\Sigma_4 + \Gamma_{w\,2}\Sigma_4\Sigma_5 + \Gamma_{w\,3}\Sigma_5\Sigma_3)$$

Γ_{wa} self-conjugate and acting as real scalars on ϕ
$w = u, d$ and $a = 0, 1, 2, 3$.

The custodial $Spin(3)$ group acts on ϕ and $\Gamma_w^\dagger$ in the same way,
the product $\Gamma_w\phi$ is $Spin(3)$ invariant.

By reparametrization of Γ_w, $\Sigma_5\phi_0 = \phi_0$. In this basis
$H_1 \equiv \frac{1-i\Sigma_1\Sigma_2}{2}\frac{1+\Sigma_5}{2}\phi$,
$H_2 \equiv \Sigma_4\Sigma_5\frac{1-i\Sigma_1\Sigma_2}{2}\frac{1-\Sigma_5}{2}\phi$,
$\widetilde{H}_j \equiv \Sigma_3\Sigma_1 H_j^*$.

$$
\begin{aligned}
-\frac{v}{\sqrt{2}}\mathcal{L}_{Y_Q} \qquad &= \overline{Q_L}\ H_1 M_d d_R + \overline{Q_L}\ H_2 N_d^0 d_R \\
&+ \overline{Q_L}\ \widetilde{H}_1 M_u u_R + \overline{Q_L}\ \widetilde{H}_2 N_u^0 u_R + \text{h.c.},
\end{aligned}
$$

where $M_w \equiv \Gamma_{w0} + i\Gamma_{w1}$, $N_w^0 \equiv \Gamma_{w3} + i\Gamma_{w4}$.

Majorana masses in seesaw I (νMSM) gauge singlets $\Rightarrow$ nonperturbative $\checkmark$

- $H_1^\dagger i D_\mu \Sigma_1 \Sigma_3 H_1$ (W_μ^+)
- $\cos\theta_W H_1^\dagger i D_\mu H_1 - \sin\theta_W \frac{gv^2}{4} B_\mu$ (Z_μ)
- $\mathcal{A}_\mu \equiv \sin\theta_W H_1^\dagger i D_\mu H_1 + \cos\theta_W \frac{gv^2}{4} B_\mu$ (A_μ)
- $H_1^\dagger H_1$ (h)
- $H_1^\dagger \Sigma_4 H_2$ (R)
- $H_1^\dagger \Sigma_3 H_2$ (I)
- $H_1^\dagger \Sigma_1 H_2$ (H^+)

$H_1^\dagger Q$ (d_L)

$\tilde{H}_1^\dagger Q$ (u_L)

$H_1^\dagger L$ (e_L)

$\tilde{H}_1^\dagger L$ (ν_L)

$$\Sigma_5 \phi_0 = \phi_0$$
$$H_1 \equiv \frac{1 - i\Sigma_1\Sigma_2}{2} \frac{1+\Sigma_5}{2} \phi$$
$$H_2 \equiv \Sigma_4 \Sigma_5 \frac{1 - i\Sigma_1\Sigma_2}{2} \frac{1-\Sigma_5}{2} \phi$$

Yang (1952) “It is the purpose of the present paper to calculate the spontaneous magnetization (i.e. , the intensity of magnetization at zero external field) of a two-dimensional Ising model of a ferromagnet.”

Spontaneous as particular case of Explicit symmetry breaking.
Other *equivalent* (for the Ising model) definitions:

“The spontaneous magnetization I per atom is exactly the usual long-range order parameter s which may be defined as the average of the absolute value of the total spin of the lattice divided by the number of atoms.

That I is equal to s is easily seen-from the fact that the introduction of a vanishingly weak positive magnetic field merely cuts out all states of the lattice for which the total spin is negative.”

't Hooft (1980) "the words "spontaneous breakdown" are formally not correct for local gauge theories. The vacuum *never* breaks local gauge invariance because it itself is gauge invariant.

The neutral intermediate vector boson is the "meson"

$$\phi^\dagger D_\mu \phi = i\frac{gv^2}{4}W^3_\mu + \text{total derivative} + \text{higher orders}$$

The $W^\pm_\mu$ are obtained from the "baryons" $\epsilon_{ij}\phi^i D_\mu \phi^j$, and the Higgs particle can also be obtained from $\phi^\dagger\phi$.

Is there no fundamental difference then between a theory with spontaneous breakdown and a theory with confinement? Sometimes there is. In the above example the Higgs was a faithful representation of $SU(2)$. This is why the above procedure worked."

Fröhlich et al. (1981) "the relevant feature is the structure of the residual group." (defined by the minimizing orbit of the Higgs potential)

Implications for one Higgs doublet from lattice sim.:

Maas and Mufti (2015) $m_h < m_W \Rightarrow$ non-perturbative effects rule

Gies and Sondenheimer (2015) (top-bottom-Higgs system)

non-perturbative effects affect (in)stability of the Higgs potential

...and a big unknown

mostly unexplored, due to technology and mathematical limitations

Careful, non-perturbative effects for small couplings are common

e.g. Hydrogen energy levels

despite that for larger couplings, larger non-perturbative effects expected

Background symmetries

background field or spurion:

- fixed when minimizing the action
- non-trivial representation of background symmetries

when calculating observables, spurions $\Rightarrow$ numerical values.

observables invariant under group of background symmetries

Ivanov (2006) reparametrization

Haber and Surujon (2012) basis transformation that do not change the Lagrangian's functional form or spurion analysis

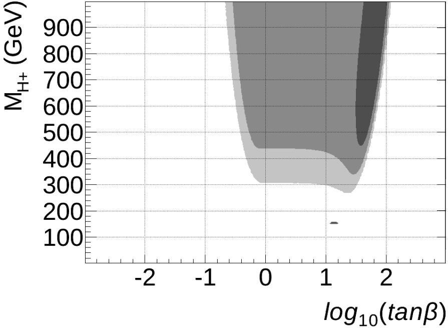
Botella et al. (2013) weak-basis transformations

Georgi (2009) spurions as source fields

11.1 A contribution for a systematic search for FCNCs

BGL analysis code: cftp.ist.utl.pt/∼ leonardo

Test on 2HDM type II:



Giac and Ginac (C++ algebra systems) available (also Flavour Kit, etc.);

Using LLVM the code generation of GiNaC can be improved;

Library containing many known formulas for decays important for FCNC;(contribution)

Library making global fits, from models, from formulas, from experimental data (contribution)

Library containing the experimental data distributions.

(CERN-based ROOSTATS-like package for FCNCs?)

Beyond the Standard Model

Branco and Emmanuel-Costa (2014) the simplest scheme to break spontaneously:

$$SU(3)_C \times SU(2)_L \times U(1)_Y \to SU(3)_C \times U(1)_{em}$$

$\Rightarrow$ One Higgs Doublet

Ellis (2013) QCD, ElectroWeak, Flavour $\Rightarrow$ Experiments ✓

ν masses and mixing, baryon asymmetry,
dark matter, CMB fluctuations

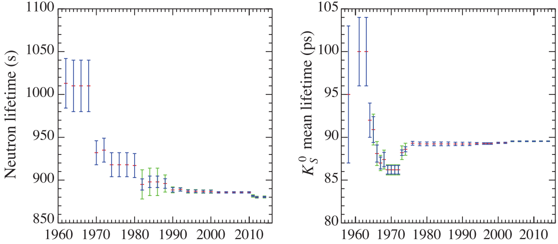
$\Rightarrow$ New Physics

Altarelli (2014) νMSM(3 ν_R, Seesaw I)+inflaton field $\Rightarrow$ Experiments ✓

gravity;cosmological const.(dark energy);hierarchy;strong CP;
arbitrariness;meta-stability;non-perturbative definition;
accidental suppression of FCNCs,EDMs,p^+ decay

Contributions from Social Sciences

PDG (2014)



Wanke (2013) *"significant jumps, pointing either to a common systematic shift or to the effect of biased analyses."*

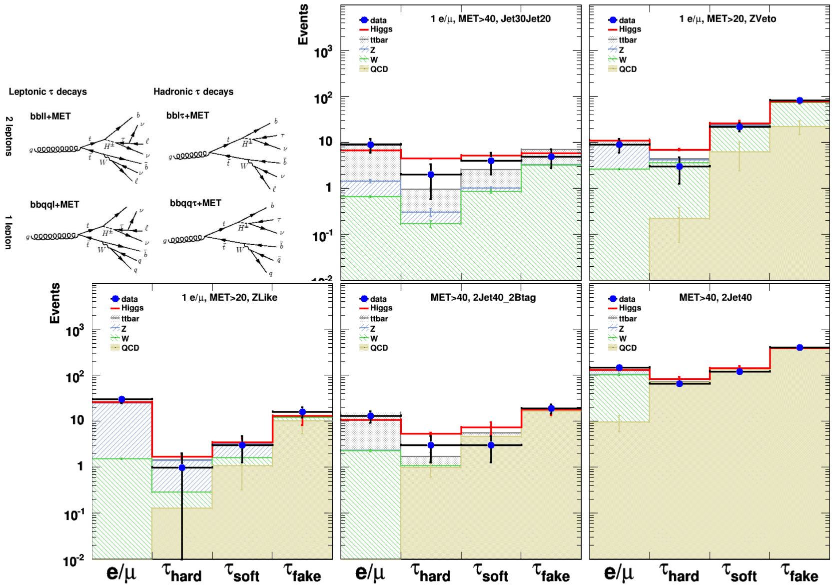
Borrelli (2013) presented at CERN *"As far as theorists are concerned, the role of personal skills seems to be a major factor in the choice of models to work on."*

Kahnemann(2002)

Nobel in Economics Lecture

"people rely on a limited number of heuristic principles which reduce the complex tasks of assessing probabilities and predicting values to simpler judgmental operations. In general, these heuristics are quite useful, but sometimes they lead to severe and systematic errors."

Correlations are important in data analysis



to attribute to new physics a deviation in data

Top-down and Bottom-up approaches

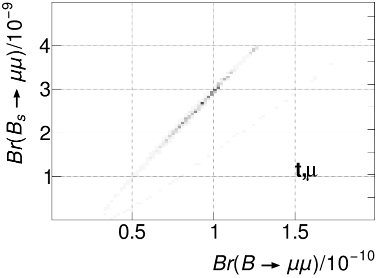
Buras and Girrbach (2014)

models (e.g. BGL) vs. effective field theory (e.g. MFV)

- correlations between observables

  low/high energy, all flavours, hadronic/leptonic
- less sensitive to free parameters
- patterns of flavour violation
- may differ from the SM and MFV

For BGL:



Extend the scalar sector to study Higgs mechanism and SM problems.
But constrain FCNCs, Flavour and CP violation pattern accounted by SM.