A maximally CP symmetric two-Higgs-doublet model

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Multi-Higgs Models, Lisbon 2016

- Standard Model
  one doublet

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$$

  physical Higgs boson: ρ
- Gauge invariant, renormalizable Higgs potential

$$V_{\text{SM}} = -\mu^2(\varphi^\dagger\varphi) + \lambda(\varphi^\dagger\varphi)^2$$

- THDM
  two doublets

$$\varphi_1 = \begin{pmatrix} \varphi_1^+ \\ \varphi_1^0 \end{pmatrix}, \quad \varphi_2 = \begin{pmatrix} \varphi_2^+ \\ \varphi_2^0 \end{pmatrix}$$

  physical Higgs bosons: ρ', h', h'', $H^\pm$.
- T. D. Lee introduced THDM to achieve CP violation.

- THDM Higgs potential

H. E. Haber and R. Hempfling, PRD 48 (1993)

$$
\begin{aligned}
V = {} & m_{11}^2(\varphi_1^\dagger\varphi_1) + m_{22}^2(\varphi_2^\dagger\varphi_2) - \left[m_{12}^2(\varphi_1^\dagger\varphi_2) + h.c.\right] \\
& + \frac{\lambda_1}{2}(\varphi_1^\dagger\varphi_1)^2 + \frac{\lambda_2}{2}(\varphi_2^\dagger\varphi_2)^2 \\
& + \lambda_3(\varphi_1^\dagger\varphi_1)(\varphi_2^\dagger\varphi_2) + \lambda_4(\varphi_1^\dagger\varphi_2)(\varphi_2^\dagger\varphi_1) \\
& + \left[\frac{\lambda_5}{2}(\varphi_1^\dagger\varphi_2)^2 + \lambda_6(\varphi_1^\dagger\varphi_1)(\varphi_1^\dagger\varphi_2) + \lambda_7(\varphi_2^\dagger\varphi_2)(\varphi_1^\dagger\varphi_2) + h.c.\right],
\end{aligned}
$$

with m_{11}^2, m_{22}^2, $\lambda_{1,2,3,4}$ real and m_{12}^2, $\lambda_{5,6,7}$ complex.

Bilinears

O. Nachtmann, A. Manteuffel, MM **EPJC 48** (2006),
C. Nishi **PRD 74** (2006)

Introduce matrix of Higgs doublets $\varphi_i = \begin{pmatrix} \varphi_i^+ \\ \varphi_i^0 \end{pmatrix}$, $i = 1, 2$

$$\phi = \begin{pmatrix} \varphi_1^{\mathrm{T}} \\ \varphi_2^{\mathrm{T}} \end{pmatrix} = \begin{pmatrix} \varphi_1^+ \, \varphi_1^0 \\ \varphi_2^+ \, \varphi_2^0 \end{pmatrix}$$

- Arrange gauge invariant scalar products into Hermitian 2×2 matrix

$$\underline{K} := \begin{pmatrix} \varphi_1^\dagger \varphi_1 & \varphi_2^\dagger \varphi_1 \\ \varphi_1^\dagger \varphi_2 & \varphi_2^\dagger \varphi_2 \end{pmatrix}.$$

- NHDM

  Introduce matrix of Higgs doublets $\varphi_i = \begin{pmatrix} \varphi_i^+ \\ \varphi_i^0 \end{pmatrix}$, $i = 1, \ldots, n$

$$\phi = \begin{pmatrix} \varphi_1^{\mathrm{T}} \\ \vdots \\ \varphi_n^{\mathrm{T}} \end{pmatrix} = \begin{pmatrix} \varphi_1^+ \, \varphi_1^0 \\ \vdots \\ \varphi_n^+ \, \varphi_n^0 \end{pmatrix}$$

- Arrange all $SU(2)_L \times U(1)_Y$ invariants into hermitian $n \times n$ matrix

$$\underline{K} = \phi\phi^\dagger = \begin{pmatrix} \varphi_1^\dagger \varphi_1 & \cdots & \varphi_n^\dagger \varphi_1 \\ \vdots & \ddots & \vdots \\ \varphi_1^\dagger \varphi_n & \cdots & \varphi_n^\dagger \varphi_n \end{pmatrix}$$

- $\underline{K} = \phi\phi^\dagger$ is hermitian, positive semidefinite with rank ≤ 2.
- Basis for $\underline{K}$ are Gell-Mann matrices λ_α, $\lambda_0 = \sqrt{\frac{2}{n}} \mathbb{1}_n$,

$$\underline{K} = \frac{1}{2} K_\alpha \lambda_\alpha, \qquad \alpha = 0, 1, \ldots, n^2 - 1$$

- Example THDM: 4 real coefficients - bilinears

$$\underline{K} = \frac{1}{2}(K_0 \mathbb{1}_2 + K_a \sigma_a), \qquad a = 1, 2, 3.$$

- Inversion reads

$$\varphi_1^\dagger \varphi_1 = (K_0 + K_3)/2, \qquad \varphi_1^\dagger \varphi_2 = (K_1 + iK_2)/2,$$
$$\varphi_2^\dagger \varphi_2 = (K_0 - K_3)/2, \qquad \varphi_2^\dagger \varphi_1 = (K_1 - iK_2)/2.$$

- In terms of

$$K_0, \qquad \boldsymbol{K} \equiv \begin{pmatrix} K_1 \\ \vdots \\ K_{n^2-1} \end{pmatrix}$$

the most general potential can now be written

$$V = \xi_0 K_0 + \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{K} + \eta_{00} {K_0}^2 + 2 K_0\, \boldsymbol{\eta}^{\mathrm{T}} \boldsymbol{K} + \boldsymbol{K}^{\mathrm{T}} E \boldsymbol{K}$$

- with real parameters

$$\xi_0,\ \eta_{00},\ \boldsymbol{\xi},\ \boldsymbol{\eta},\ E = E^{\mathrm{T}}$$

- One-to-one correspondance between Higgs-boson doublets and Hermitean matrix $\underline{K}$ with rank ≤ 2.
- Stability, Stationarity in nHDM studied
  O. Nachtmann, MM **JHEP 02** (2015)

- THDM: Translation from conventional notation to bilinear space

$$\xi_0 = \frac{1}{2}(m_{11}^2 + m_{22}^2)\,, \quad \boldsymbol{\xi} = \frac{1}{2}\begin{pmatrix} -2\mathrm{Re}(m_{12}^2) \\ 2\mathrm{Im}(m_{12}^2) \\ m_{11}^2 - m_{22}^2 \end{pmatrix},$$

$$\eta_{00} = \frac{1}{8}(\lambda_1 + \lambda_2) + \frac{1}{4}\lambda_3\,, \quad \boldsymbol{\eta} = \frac{1}{4}\begin{pmatrix} \mathrm{Re}(\lambda_6 + \lambda_7) \\ -\mathrm{Im}(\lambda_6 + \lambda_7) \\ \frac{1}{2}(\lambda_1 - \lambda_2) \end{pmatrix},$$

$$E = \frac{1}{4}\begin{pmatrix} \lambda_4 + \mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_5) & \mathrm{Re}(\lambda_6 - \lambda_7) \\ -\mathrm{Im}(\lambda_5) & \lambda_4 - \mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_6 - \lambda_7) \\ \mathrm{Re}(\lambda_6 - \lambda_7) & -\mathrm{Im}(\lambda_6 - \lambda_7) & \frac{1}{2}(\lambda_1 + \lambda_2) - \lambda_3 \end{pmatrix}.$$

Change of basis

- Consider the following unitary mixing of the doublets

$$\begin{pmatrix} \varphi_1'(x)^{\mathrm{T}} \\ \vdots \\ \varphi_n'(x)^{\mathrm{T}} \end{pmatrix} = U \begin{pmatrix} \varphi_1(x)^{\mathrm{T}} \\ \vdots \\ \varphi_n(x)^{\mathrm{T}} \end{pmatrix}$$

- Bilinears transform as

$$K_0' = K_0, \qquad \boldsymbol{K}' = R(U)\boldsymbol{K},$$

with $U^\dagger \lambda_a U = R_{ab}(U)\lambda_b$, $R \in SO(n^2-1)$,
proper rotations in $\boldsymbol{K}$-space.

- Under a change of basis $K_0' = K_0$, $\boldsymbol{K}' = R(U)\boldsymbol{K}$ potential remains invariant if

$$\xi_0' = \xi_0, \quad \eta_{00}' = \eta_{00},$$
$$\boldsymbol{\xi}' = R\,\boldsymbol{\xi}, \quad \boldsymbol{\eta}' = R\,\boldsymbol{\eta}, \quad E' = R\,E\,R^{\mathrm{T}}.$$

Symmetries

- Symmetry desirable to restrict nHDM.
- Symmetries easily formulated in terms of bilinears.

$$V = \xi_0 K_0 + \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{K} + \eta_{00} {K_0}^2 + 2 K_0 \, \boldsymbol{\eta}^{\mathrm{T}} \boldsymbol{K} + \boldsymbol{K}^{\mathrm{T}} E \boldsymbol{K}$$

- Transformation $K_0 \to K_0$, $\boldsymbol{K} \to \bar{R} \boldsymbol{K}$, $\bar{R} \in O(n^2 - 1)$ is symmetry of potential iff

$$\boldsymbol{\xi} = \bar{R} \, \boldsymbol{\xi}, \qquad \boldsymbol{\eta} = \bar{R} \, \boldsymbol{\eta}, \qquad E = \bar{R} \, E \, \bar{R}^{\mathrm{T}}$$

- $\bar{R} \in O(n^2 - 1)$, keeping kinetic terms invariant.
  I. F. Ginzburg, M. Krawczyk, PRD 72 (2005)
  I. P. Ivanov and C. C. Nishi, PRD 82 (2010)
  MM, O. Nachtmann, JHEP 11 151 (2011)
  V. Keus, S.F. King, S. Moretti, JHEP 1401 (2014)
  B. Grzadkowski, MM, J. Wudka, JHEP 1111 (2011)
  P. M. Ferreira, H. E. Haber, MM, O. Nachtmann, J. P. Silva, Int.J.Mod.Phys. A26 (2011)

CP symmetry

- CP transformation of the doublet fields

$$\varphi_i(x) \longrightarrow \varphi_i^*(x'), \qquad i = 1, \dots n, \quad x = (t, \boldsymbol{x})^{\mathrm{T}}, \quad x' = (t, -\boldsymbol{x})^{\mathrm{T}}$$

- In terms of bilinears

$$K_0(x) \longrightarrow K_0(x'), \qquad \boldsymbol{K}(x) \longrightarrow \bar{R}\, \boldsymbol{K}(x')$$

- $\bar{R}$ in terms of (generalized) Gell-Mann matrices

$$\lambda_a^{\mathrm{T}} = \bar{R}_{ab} \lambda_b, \qquad a, b \in \{1, \dots, n^2 - 1\}.$$

THDM: $\bar{R} = \mathrm{diag}(1, -1, 1),$

3HDM: $\bar{R} = \mathrm{diag}(1, -1, 1, 1, -1, 1, -1, 1)$

- CP symmetry conditions

$$\xi = \bar{R}\,\xi, \qquad \eta = \bar{R}\,\eta, \qquad E = \bar{R}\,E\,\bar{R}^{\mathrm{T}}.$$

- Spontaneous CP violation: potential has (explicit) CP symmetry not respected by vacuum

$$\bar{R}\langle \boldsymbol{K} \rangle \neq \langle \boldsymbol{K} \rangle$$

- Basis invariant formulation given in THDM.
C. Nishi **PRD 74** (2006),
MM, A. Manteuffel, O. Nachtmann, EPJC57 (2008)
P. M. Ferreira, H. E. Haber, MM, O. Nachtmann, J. P. Silva, Int.J.Mod.Phys. A26 (2011)

Generalized CP transformations

G.Ecker, W.Grimus, W.Konetschny, **NPB 191** (1981)

$$\varphi_i(x) \xrightarrow{\mathrm{CP_g}} U_{ij}\, \varphi_j^*(x')$$

- In the THDM the bilinears transform as

C. Nishi **PRD 74** (2006),
O. Nachtmann, A. Manteuffel, MM **EPJC 57** (2007), O. Nachtmann, MM **JHEP 0905** (2009),
P. Ferreira, J. Silva, **PR D83** (2011)

$$K_0(x) \xrightarrow{\mathrm{CP_g}} K_0(x'), \quad \boldsymbol{K}(x) \xrightarrow{\mathrm{CP_g}} \bar{R}\boldsymbol{K}(x')$$

with improper rotation $\bar{R}$.

- Requiring $\bar{R}^2 = \mathbb{1}_3$ there are two types

(i) $\bar{R} = -\mathbb{1}_3$, point reflection

(ii) $\bar{R} = R^{\mathrm{T}}\, \bar{R}_2\, R$, orthogonal equivalent to $\bar{R}_2$ reflection

- Type (ii): consider diagonal matrices $\bar{R}$

$$\begin{aligned} \text{CP}^{(ii)}_{g,1} &: \quad \bar{R} = \text{diag}(-1, \quad 1, \quad 1) \\ \text{CP}^{(ii)}_{g,2} &: \quad \bar{R} = \text{diag}(\quad 1, -1, \quad 1) \\ \text{CP}^{(ii)}_{g,3} &: \quad \bar{R} = \text{diag}(\quad 1, \quad 1, -1) \end{aligned}$$

- In terms of the original $\varphi_i(x)$ this reads

$$\varphi_i(x) \xrightarrow{\text{CP}_g} W_{ij}\ \varphi_j^*(x')$$

with the 2×2 matrix W given by

CP_g	$\text{CP}_g^{(i)}$	$\text{CP}^{(ii)}_{g,1}$	$\text{CP}^{(ii)}_{g,2}$	$\text{CP}^{(ii)}_{g,3}$
W	ϵ	σ^3	$\mathbb{1}_2$	σ^1

Maximally CP invariant model

- Potential invariant under point reflections

$$\boldsymbol{K}(x) \xrightarrow{\mathrm{CP}_g^{(i)}} -\boldsymbol{K}(x')$$

$$V = \xi_0 K_0 + \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{K} + \eta_{00} {K_0}^2 + 2 K_0\, \boldsymbol{\eta}^{\mathrm{T}} \boldsymbol{K} + \boldsymbol{K}^{\mathrm{T}} E \boldsymbol{K},$$

- that is we have to have

$$\boldsymbol{\xi} = \boldsymbol{\eta} = 0$$

- Potential automatically invariant under 3 plane reflections.

Maximally CP invariant model

- Potential invariant under point reflections

$$\boldsymbol{K}(x) \xrightarrow{\mathrm{CP}_g^{(i)}} -\boldsymbol{K}(x')$$

$$V = \xi_0 K_0 + \cancel{\boldsymbol{\xi}^\mathrm{T} \boldsymbol{K}} + \eta_{00} {K_0}^2 + \cancel{2 K_0 \boldsymbol{\eta}^\mathrm{T} \boldsymbol{K}} + \boldsymbol{K}^\mathrm{T} E \boldsymbol{K},$$

- that is we have to have

$$\boldsymbol{\xi} = \boldsymbol{\eta} = 0$$

- Potential automatically invariant under 3 plane reflections.

- We consider the THDM with the Higgs potential

$$
\begin{aligned}
V(\varphi_1, \varphi_2) = {} & m_{11}^2 \left(\varphi_1^\dagger \varphi_1 + \varphi_2^\dagger \varphi_2 \right) \\
& + \frac{1}{2} \lambda_1 \left((\varphi_1^\dagger \varphi_1)^2 + (\varphi_2^\dagger \varphi_2)^2 \right) \\
& + \lambda_3 (\varphi_1^\dagger \varphi_1)(\varphi_2^\dagger \varphi_2) + \lambda_4 (\varphi_1^\dagger \varphi_2)(\varphi_2^\dagger \varphi_1) \\
& + \frac{1}{2} \lambda_5 \left((\varphi_1^\dagger \varphi_2)^2 + (\varphi_2^\dagger \varphi_1)^2 \right),
\end{aligned}
$$

- Parameters m_{11}^2, λ_1, λ_3, λ_4, λ_5 are real.
- Potential invariant under $\varphi_1 \to -\varphi_1$.

Yukawa couplings in the MCPM

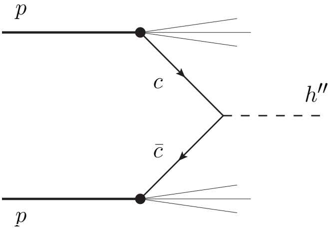
- We require the Yukawa couplings to respect all point and plane reflection symmetries.
- At least two families for non-vanishing couplings.
- Yukawa couplings

$$\mathscr{L}_{\mathrm{Yuk},l}(x) = -y_3 \left\{ \bar{l}_{3\,R}\, \varphi_1^\dagger \begin{pmatrix} \nu_{3\,L} \\ l_{3\,L} \end{pmatrix} - \bar{l}_{2\,R}\, \varphi_2^\dagger(x) \begin{pmatrix} \nu_{2\,L} \\ l_{2\,L} \end{pmatrix} \right\} + h.c.$$

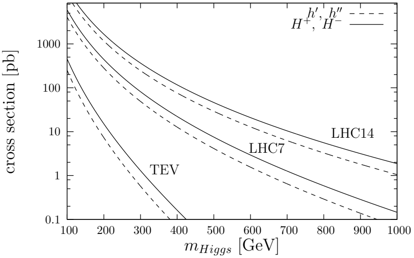
- Via EWSB, Yukawa coupling fixed by lepton mass.
- Yukawa coupling of 2nd family proportional to 3rd family mass.

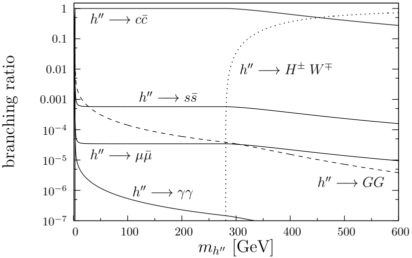
- Drell–Yan Higgs production dominant

O. Nachtmann, MM, JHEP 0905



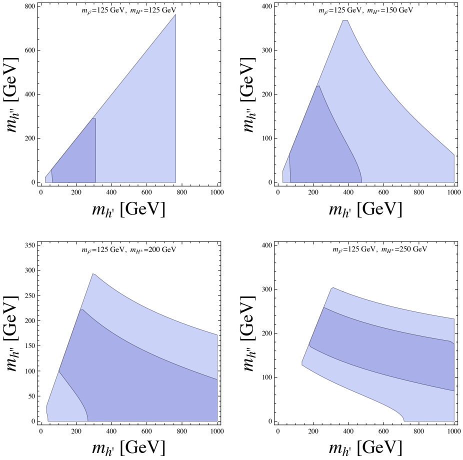
- Very large cross section at LHC.





Oblique parameters

- Check agreement with electroweak measurements.
  Oblique parameters O. Nachtmann, MM, arxiv:1106.1436 [hep-ph]



Conclusion

- Bilinears are powerful tool in nHDM Higgs potential.
- Basis-, CP transformations have simple geometric picture.
- Generalized CP transformations studied.
- *Maximally CP-invariant model* (MCPM).
- Family replication in the MCPM.