ONE-LOOP CORRECTIONS TO FERMION MASSES AND FLAVOR SYMMETRIES

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Particles and Interactions

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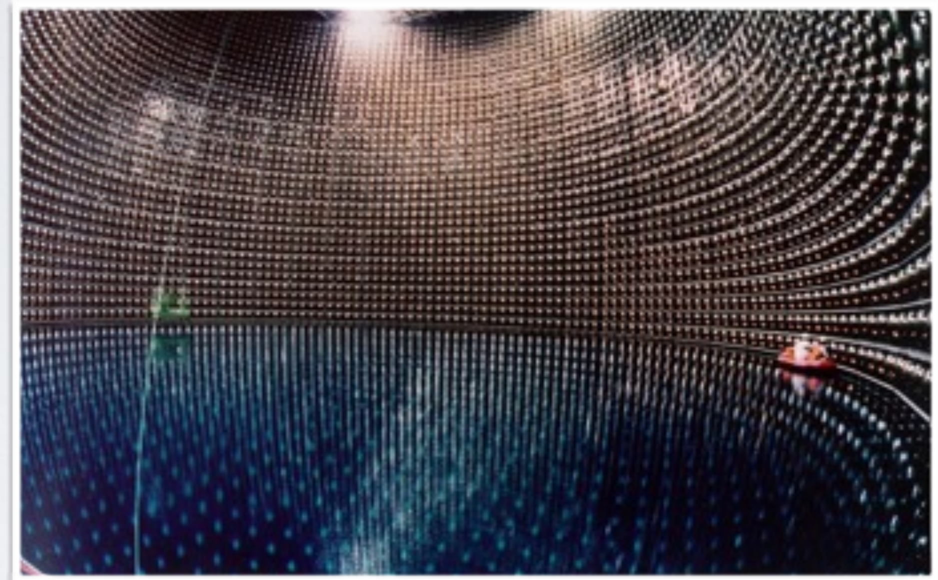
INTRODUCTION

- Standard Model of particle physics arguably only contains massless left(right)-handed electron, muon and tau (anti-)neutrinos

- Measurement of ν-oscillations:
  - Confirmation of non-vanishing mass differences:

$$P_{\nu_\alpha \to \nu_\beta} = \sum_{i,j} U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j} e^{-\mathrm{i}\Delta m^2_{ij} \frac{L}{2E}}$$

- Possibly far reaching implications of mass generating mechanisms:
  - e.g. lepton number violation, composition of dark matter



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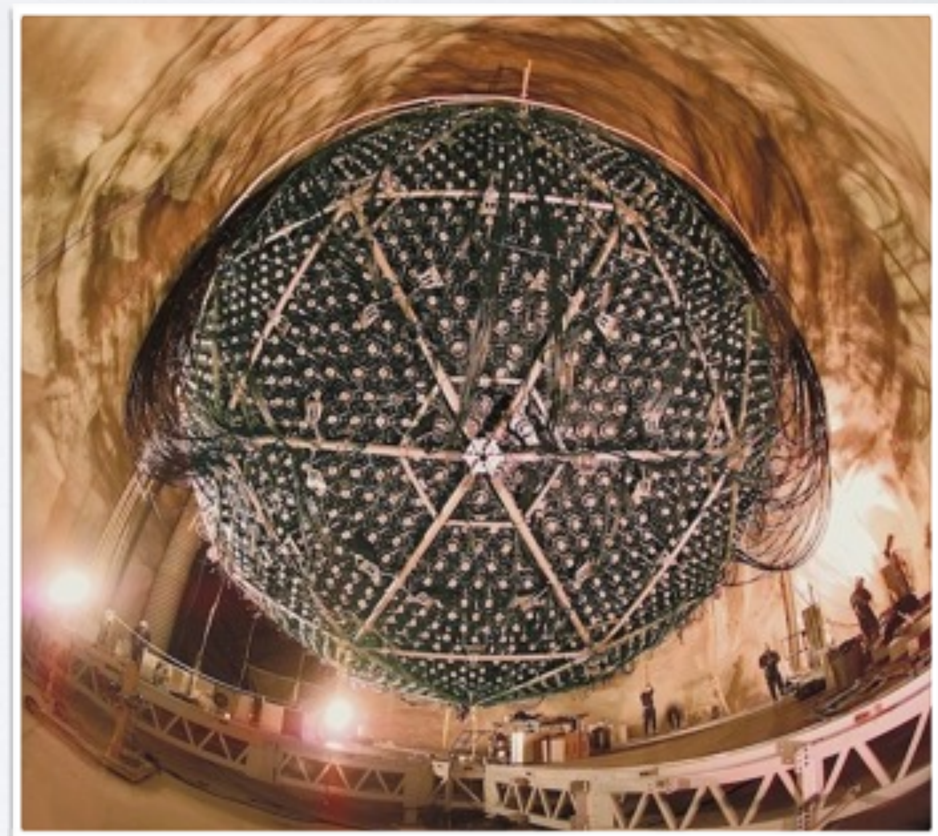

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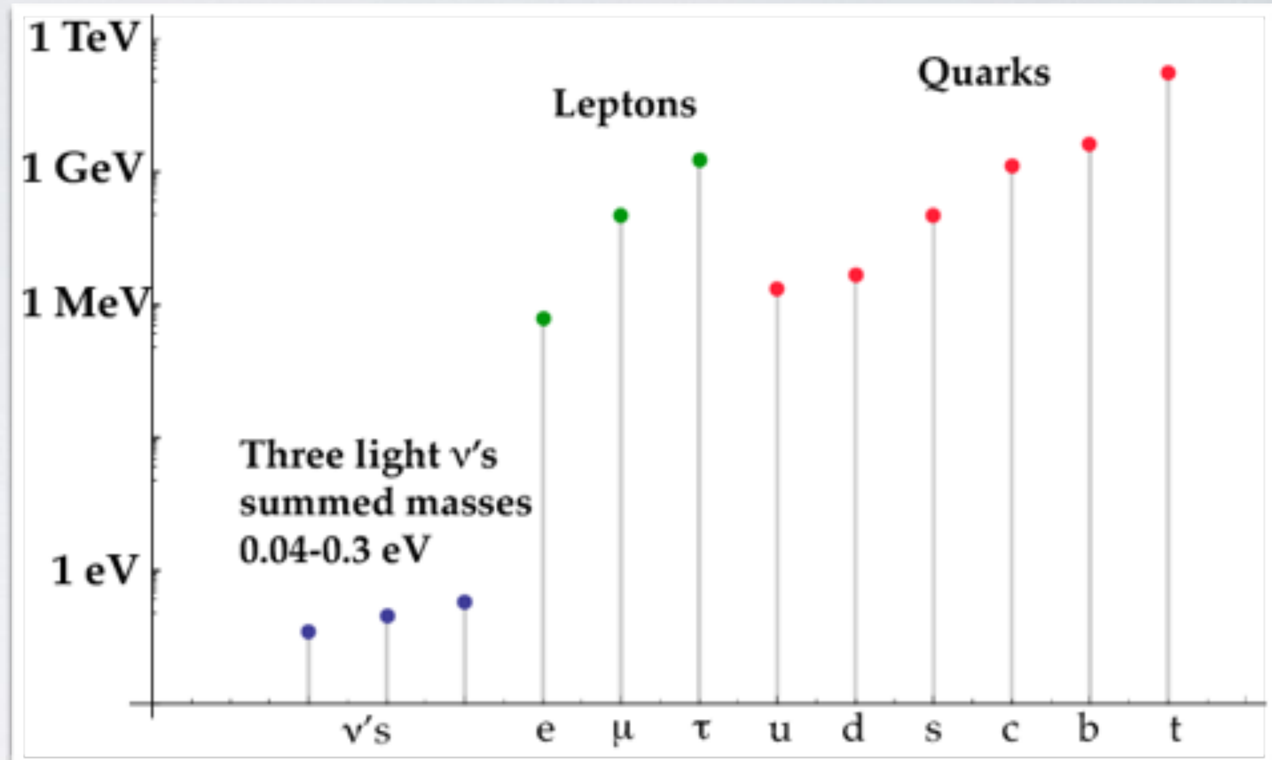
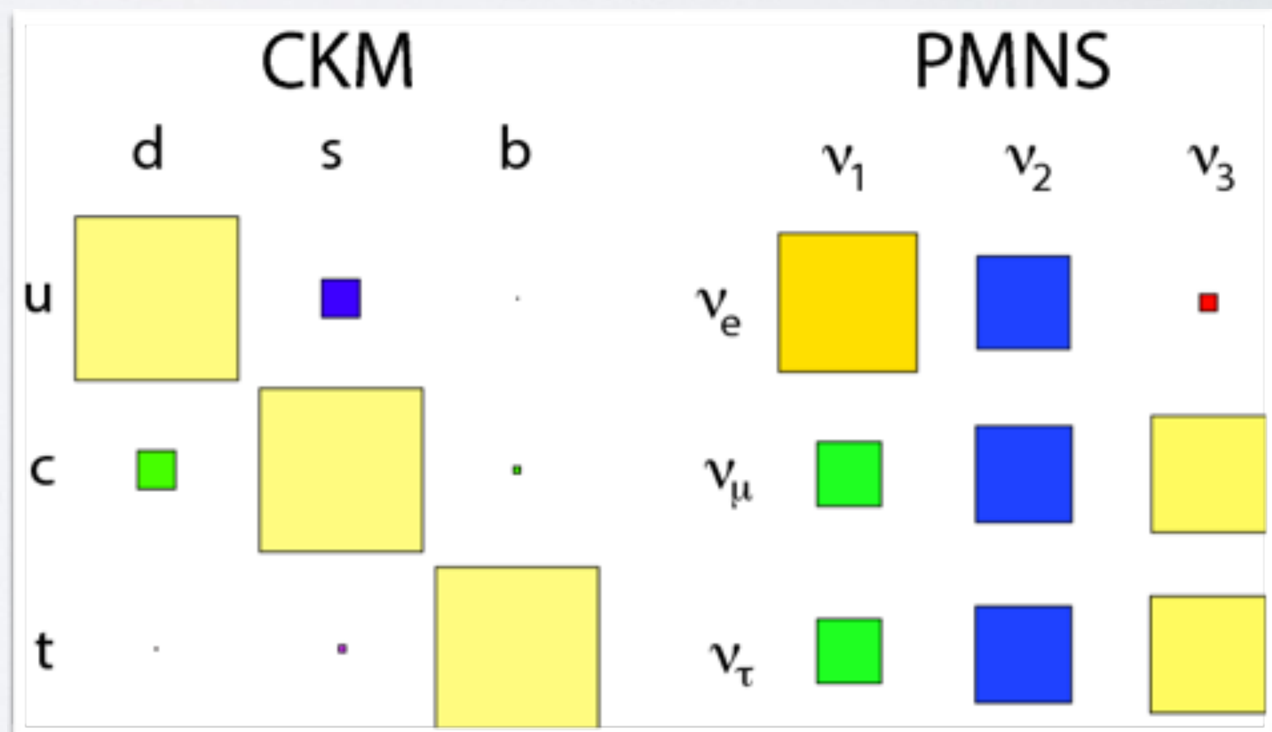
SOME OPEN QUESTIONS

On the theory side:

1. Smallness of ν masses
2. Mild hierarchy in ν mass spectrum vs. strong hierarchy in spectra of charged leptons
3. Mixing angles in lepton mixing matrix U_{PMNS} (especially vs. V_{CKM})





http://arxiv.org/abs/arXiv:1212.6374

NEUTRINO MASS TERMS

- In order to build three gauge invariant (Yukawa) mass terms, necessarily need to introduce at least **three right-handed ν's** **[arXiv:0905.0221]**

$$\mathcal{L}_{m,\mathrm{Dirac}} = y(\overline{\nu}_L \phi^0 - \bar{l}_L \phi^-)\nu_R + \mathrm{h.c.}$$
$$\Rightarrow m_{\mathrm{D}} = y\langle\phi_0\rangle$$

- If νs are of **Majorana** nature, also need to include:

$$\mathcal{L}_{m,\mathrm{Maj}} = M\overline{\nu}_R \nu_R^c + \mathrm{h.c.}$$

  - **New scalars** needed to describe these in terms of Yukawa couplings

- ν masses are at least 10^6 times smaller than electron mass
  - $y \lesssim 10^{-11}$
  - Seems unnaturally small
  - Mechanisms avoiding tiny Yukawa couplings often introduce **new scalars**, e.g. Type II Seesaw

FLAVOR SYMMETRIES

- Attempt to describe/explain structure of U_{PMNS} via symmetries of the mass matrix
- Use combination of discrete symmetries to approximate U_{PMNS}, e.g. **μ-τ symmetry**

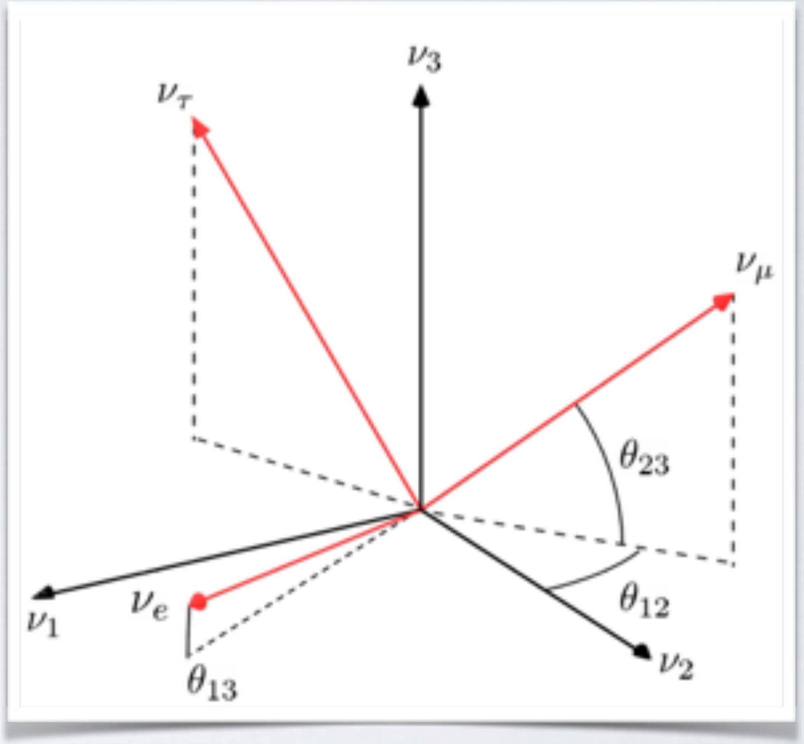
[Phys. Lett. B 579 (2004), 113-122]

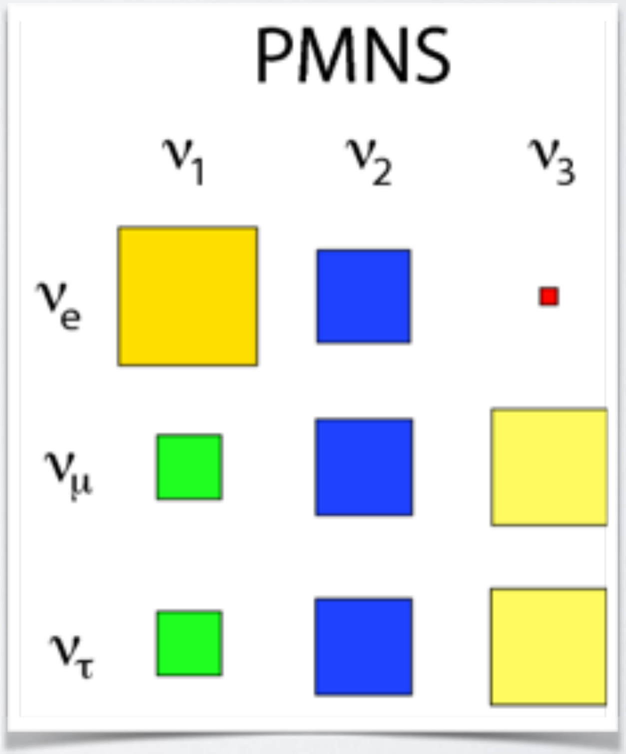
$$S = \begin{matrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{matrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$S \mathcal{M}_\nu S = \mathcal{M}_\nu^*$$

$$\Rightarrow |U_{\mu i}| = |U_{\tau i}| \quad \forall i$$

$$\Rightarrow \delta = \pm \frac{\pi}{2}, \theta_{23} = 45^\circ.$$





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ONE-LOOP CORRECTIONS

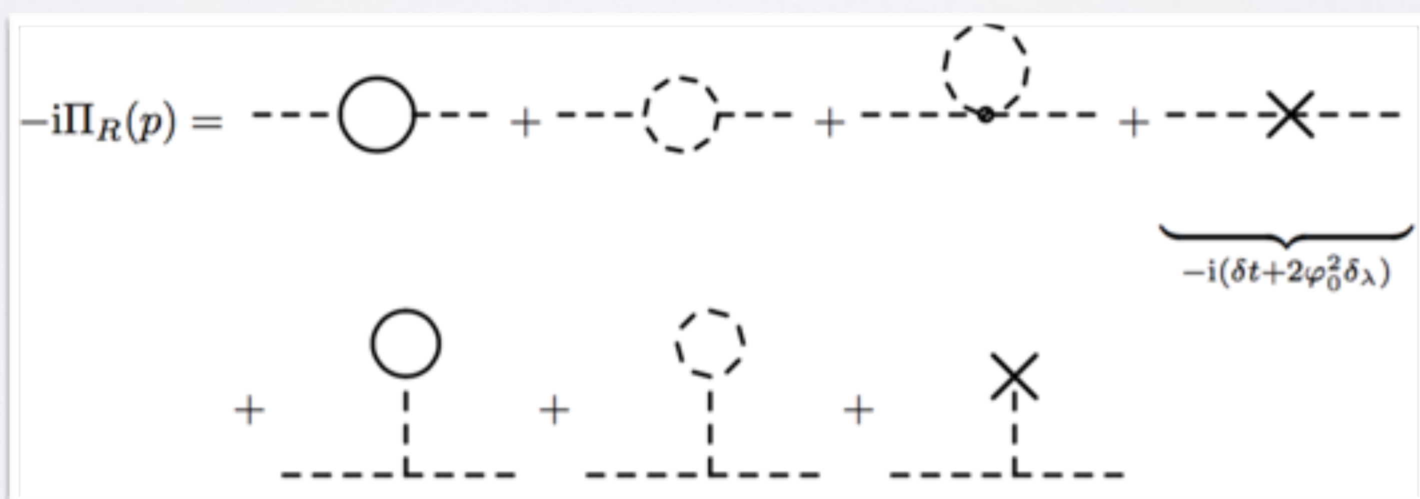
- **Starting point:** Determine one-loop mass corrections in general framework, starting from toy model with a Majorana/Dirac fermion and a real scalar field: **[arXiv:1406.7795]**

$$\mathcal{L}_{\text{toy}} = i\overline{\chi}_L \gamma_\mu \partial^\mu \chi_L + \left(\frac{1}{2} y \chi_L^T C^{-1} \chi_L \phi + \text{h.c.}\right) + \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V(\phi)$$

$$V(\phi) = \frac{1}{2}\mu^2 \phi^2 + \frac{1}{4}\lambda \phi^4$$

  - Find an appropriate renormalization scheme, i.e. choose proper parameter set, investigate role of tadpoles, …

$$\{Z_1, Z_2, y, \lambda, t\} \text{ vs. } \{Z_1, Z_2, m_\chi, M_\phi, t\}$$



GENERALIZATION

- **Idea:** generalize toy model to n_h real scalar and n_f Majorana/Dirac fields

$$\mathcal{L}_{\text{gen}} = \mathrm{i}\overline{\chi}_{lL}\partial\!\!\!/\chi_{lL} + \left(\frac{1}{2}\chi^T_{lL}C^{-1}(Y_k)_{ll'}\chi_{l'L}\phi_k + \text{h.c.}\right) + \frac{1}{2}(\partial_\nu\phi_i)(\partial^\nu\phi_i) - V(\phi)$$

$$V(\phi) = \frac{1}{2}(\mu^2)_{ij}\phi_i\phi_j - \frac{1}{4}\lambda_{ijkl}\phi_i\phi_j\phi_k\phi_l$$

  - Easily generalize one-particle results, e.g. the fermion self-energy (see backup slides)
  - Apply to specific models known from the literature
  - Investigate flavor symmetries, one-loop corrections to mixing angles: **Are the predictions for mixing angles stable under radiative corrections?**

UNEXPECTED ENCOUNTERS

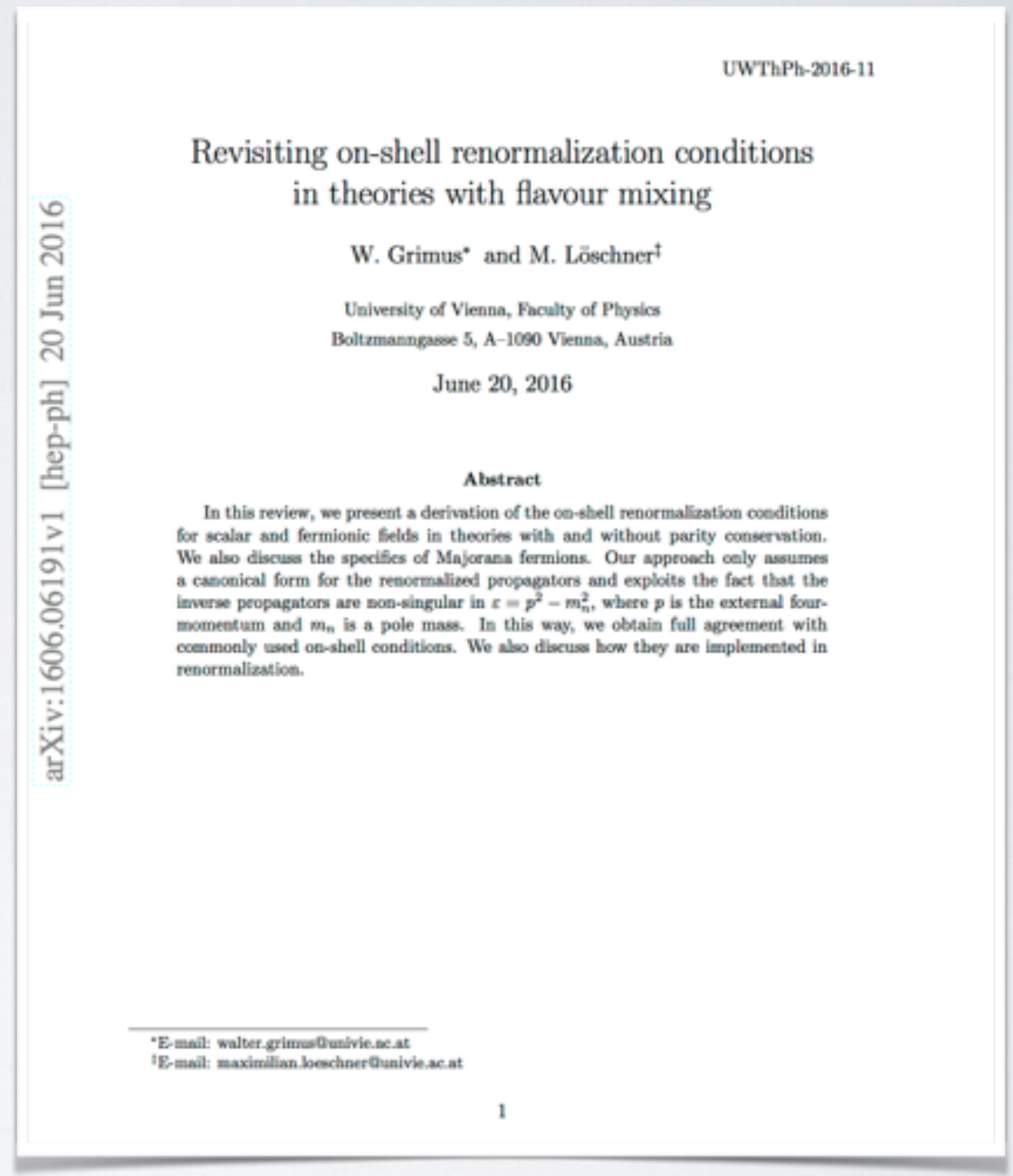
$$S \xrightarrow{p^2 \to m^2} \frac{1}{\not{p} - m} + \tilde{S}$$

- **On-shell conditions** are needed for calculating the field strength renormalization constants

- The derivation of these conditions for theories with mixing seemed a bit vague for the general reader of the relevant literature

  - Review on the derivation and use of **on-shell conditions in theories with flavor mixing**:

  - Conditions for: real scalar fields, Dirac & Majorana fermions in theories with and without CP conservation; comparison of #free parameters vs #conditions, …



UWThPh-2016-11

arXiv:1606.06191v1 [hep-ph] 20 Jun 2016

Revisiting on-shell renormalization conditions in theories with flavour mixing

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Abstract

In this review, we present a derivation of the on-shell renormalization conditions for scalar and fermionic fields in theories with and without parity conservation. We also discuss the specifics of Majorana fermions. Our approach only assumes a canonical form for the renormalized propagators and exploits the fact that the inverse propagators are non-singular in $\varepsilon = p^2 - m_n^2$, where p is the external four-momentum and m_n is a pole mass. In this way, we obtain full agreement with commonly used on-shell conditions. We also discuss how they are implemented in renormalization.

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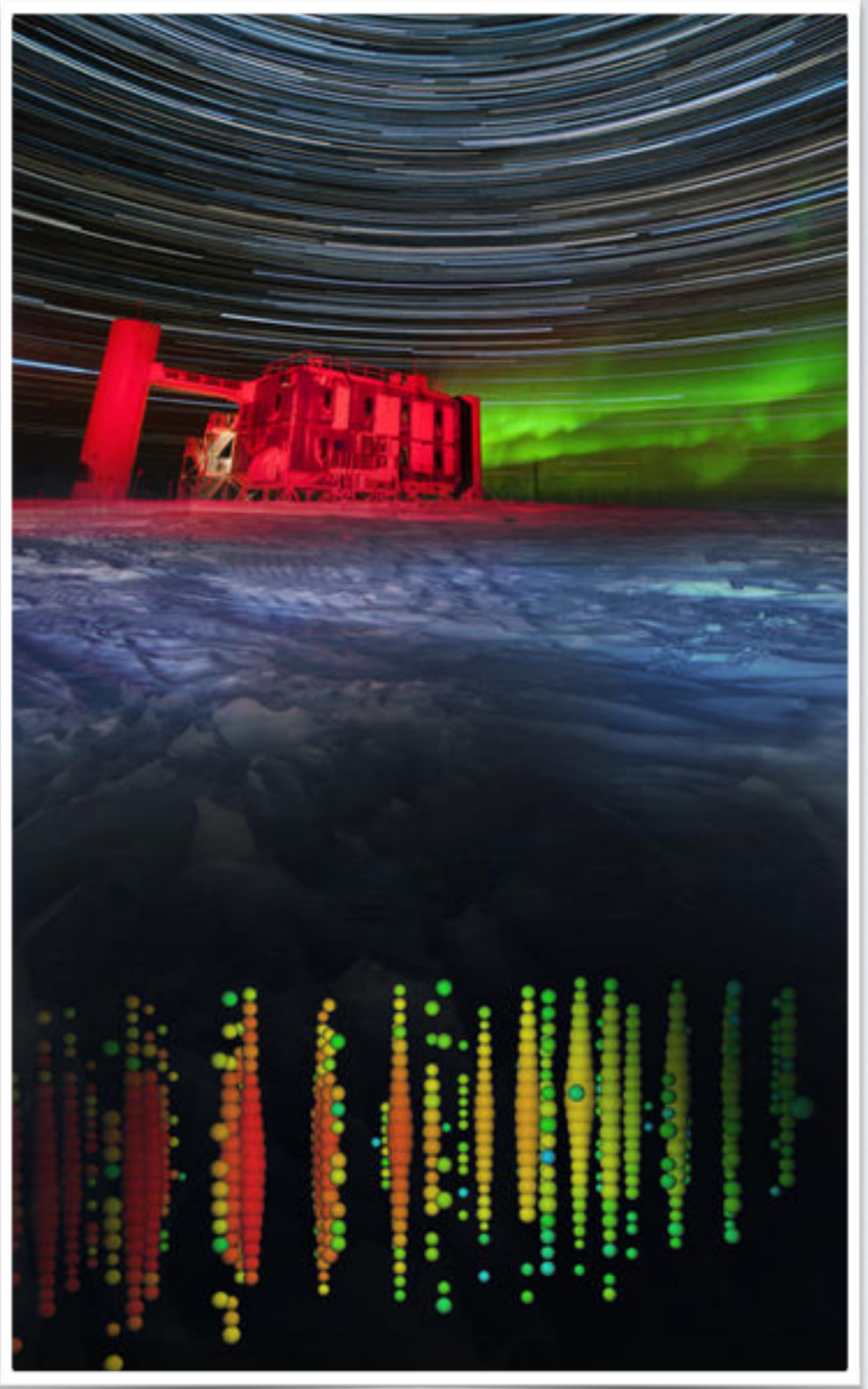
[Int. J. Mod. Phys. A 31, 1630038 (2016)]

*„[...] the discovery of neutrino oscillations implying mass and mixing can be regarded as one of the greatest discoveries in physics in the last two decades, not least because it provides **the only laboratory evidence for physics beyond the Standard Model** […]“*

arXiv:1511.03831[hep-ph]

THANKS!



https://icecube.wisc.edu/gallery/press/view/1964

Backup Slides

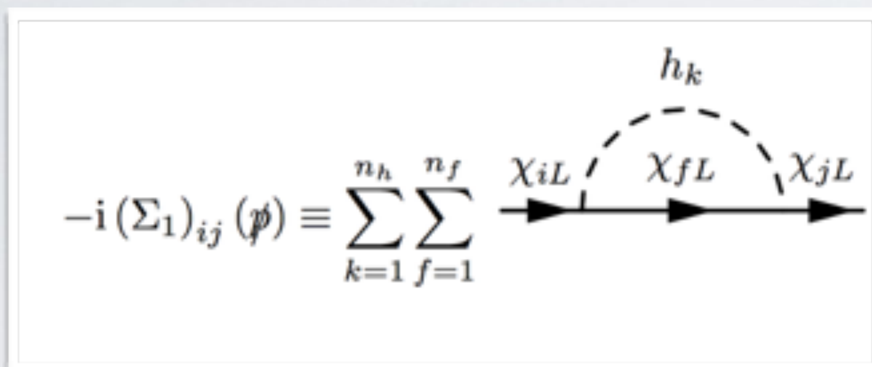
OUTLOOK

- **Generalize the toy model and one-loop results** to arbitrary # of fermion and scalar fields
- Study effects of (discrete) **flavor symmetries**
- Investigate analytically the **stability of tree level predictions** for mixing angles and masses
- Finally: **apply findings to promising models** known from the literature and produce **numerical results** for corrections to masses and mixing angles

EXEMPLARY RESULT

- **Recent result:** Yukawa coupling renormalization constants properly cancel divergencies in fermionic self-energy

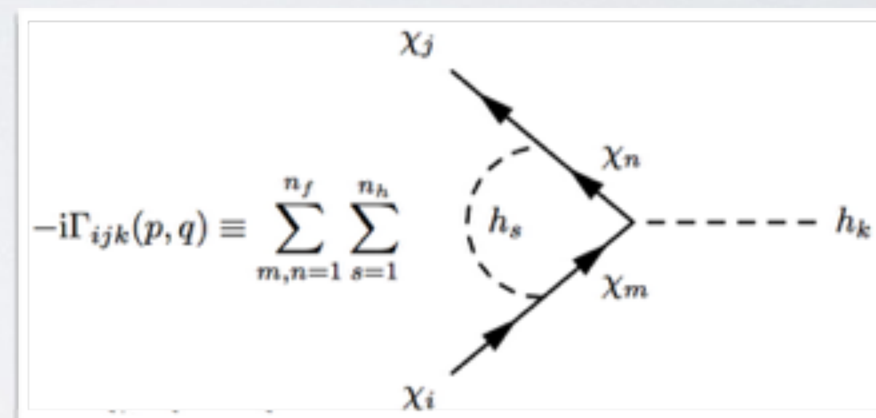
fermionic self-energy:



divergent part prop. to masses:

$$B_\infty^{ij}(p^2) = -\frac{c_\infty}{16\pi^2}\sum_{f=1}^{n_f}\sum_{s=1}^{n_h}(\hat{y}_s)^{if}(\hat{y}_s)^{fj}m_f$$

Yukawa-vertex correction



renorm. constants

$$(\delta\hat{y}_k)_{ij} = \frac{c_\infty}{16\pi^2}\sum_{m,n=1}^{n_f}\sum_{s=1}^{n_h}(\hat{y}_s)_{im}(\hat{y}_s)_{jn}(\hat{y}_k)_{mn}$$

$$\Rightarrow \sum_{k=1}^{n_h} v_k\,(\delta\hat{y}_k)^{ij} = -B_\infty^{ij}(p^2)$$

GENERALIZATION

- Fermion self-energy easily generalizable for the toy model with an arbitrary number of particles:

$$-\mathrm{i}\Sigma_1(\not{p}) = \frac{\mathrm{i}y^2}{16\pi^2}\left(\left[\frac{\not{p}}{2}+m\right]C_\infty - \int\limits_0^1 \mathrm{d}x[(1-x)\not{p}+m]\ln\left(\frac{\Delta(p^2)}{\mathcal{M}^2}\right)\right)$$

$$\Downarrow$$

$$-\mathrm{i}\left(\Sigma_1\right)_{ij}(\not{p}) = \sum_{k=1}^{n_h}\sum_{f=1}^{n_f}\frac{\mathrm{i}(\hat{y}_k)_{if}(\hat{y}_k)_{fj}}{16\pi^2}\left(\left[\frac{\not{p}}{2}+m_f\right]C_\infty - \int\limits_0^1 \mathrm{d}x[(1-x)\not{p}+m_f]\ln\left(\frac{\Delta_{kf}(p^2)}{\mathcal{M}^2}\right)\right)$$

$$C_\infty = \frac{2}{\epsilon}-\gamma_E+\ln 4\pi, \quad \Delta_{kf}(p^2) = x((1-x)p^2-m_f^2)+(1-x)M_k^2$$

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$$\Rightarrow m_{\mathrm{D}} = y\langle\phi_0\rangle$$

- ν masses are at least 10^6 times smaller than electron mass
  - $y \lesssim 10^{-11}$
  - seems unnaturally small
- Can also introduce Majorana mass term

$$\mathcal{L}_{m,\mathrm{Maj}} = M\overline{\nu}_R \nu_R^c + \mathrm{h.c.}$$

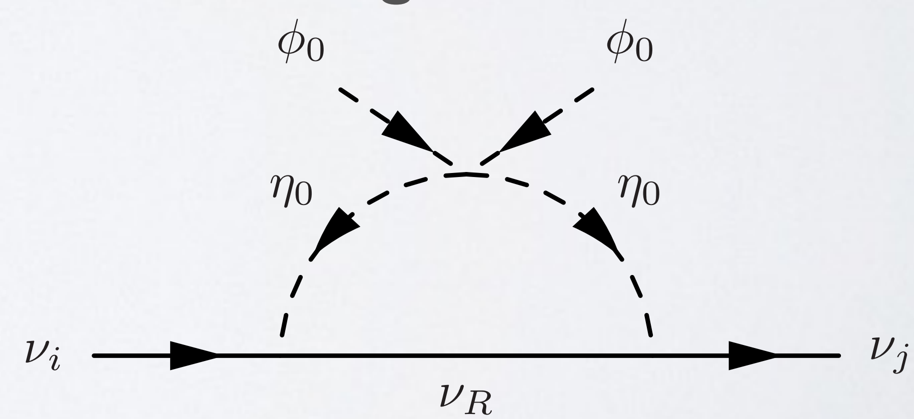
Mechanisms that avoid tiny Yukawa couplings:

- **Type I Seesaw mechanism**

$$M_\nu = \begin{matrix} \nu_L \\ \nu_R \end{matrix} \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix}$$

$$\Rightarrow m_1 \simeq \frac{m_D^2}{M}, \quad m_2 \simeq M$$

- **Radiative mass generation**

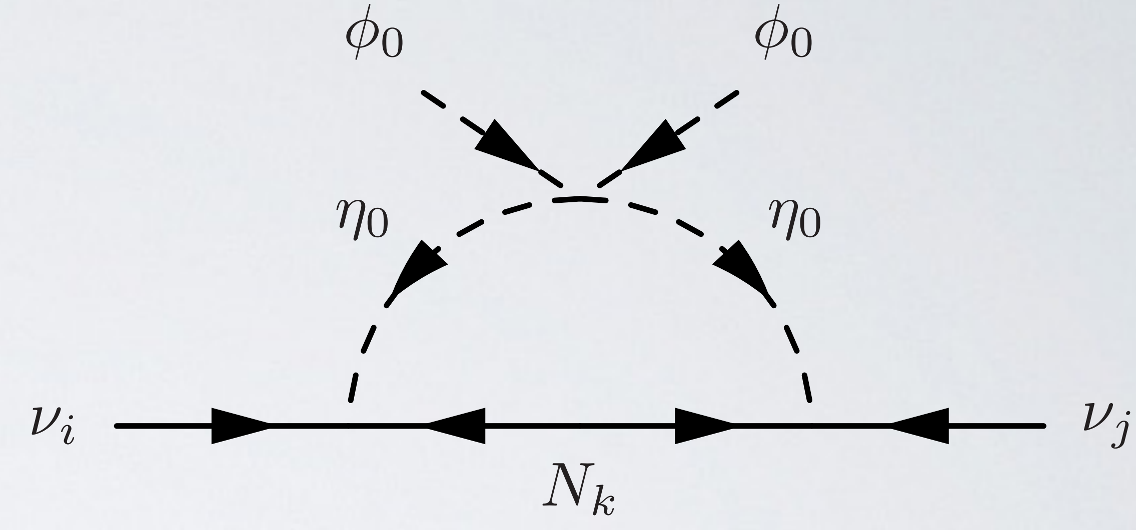


RADIATIVE MASS GENERATION

Example: **Scotogenic Model**

[arXiv:1408.4785]

- Extend SM by three right-handed ν's and second scalar doublet
- Impose exact Z_2 -symmetry: all SM particles even, new particles odd
  - $y(\nu\phi^0 - l\phi^+)N$ forbidden
  - $y(\nu\eta^0 - l\eta^+)N$ allowed

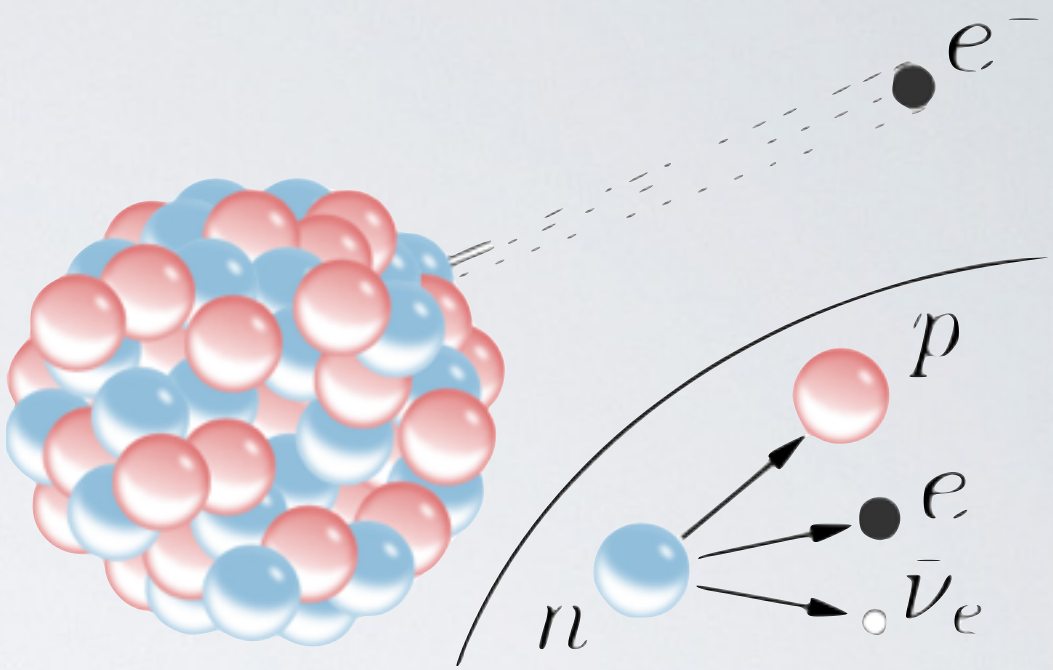


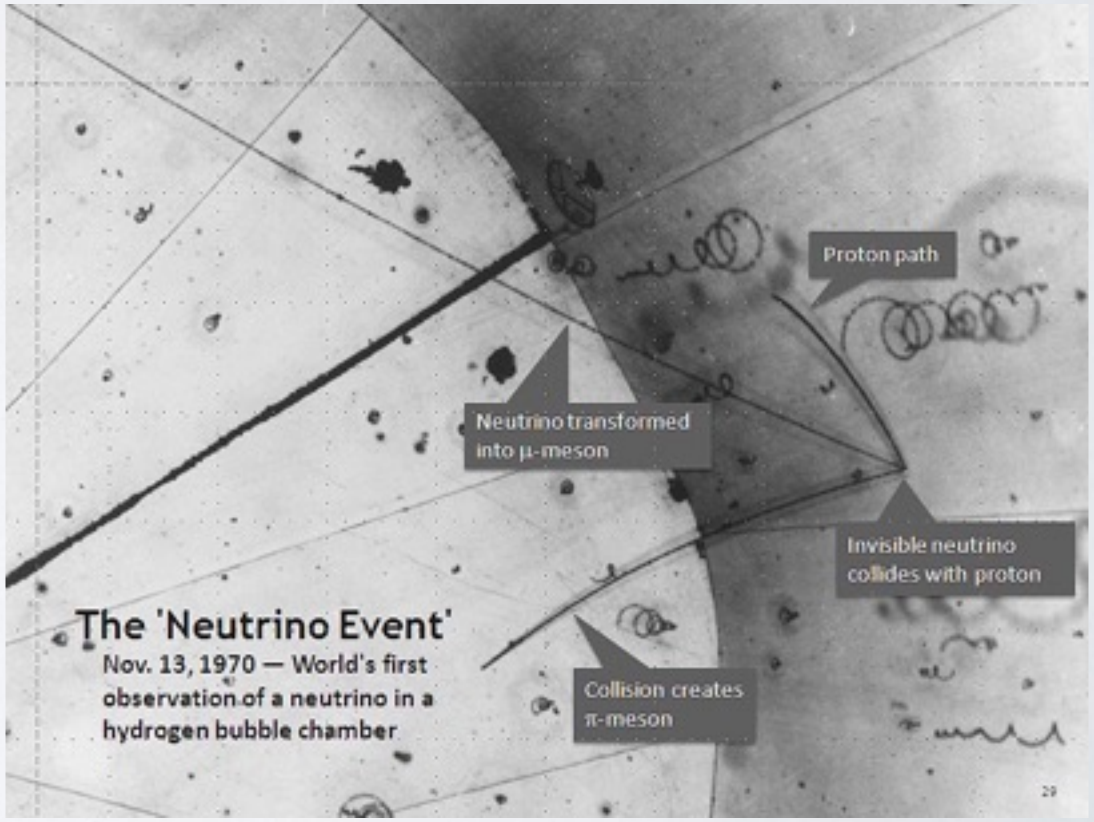
$$N_i\,; \quad (\eta^+, \eta^0) \text{ with } \langle\eta^0\rangle = 0$$

- Vanishing VEV of second scalar doublet
  - No tree level Dirac mass
  - One-loop Majorana mass
- Possible dark matter candidate

HISTORY

- Neutrinos originally postulated by Wolfgang Pauli in 1930
- First direct detection in Cowan–Reines neutrino experiment in 1956
- Two further flavors of neutrinos measured in 1962 and 2000
- Feynman and Gell-Mann: only left(right)-handed (anti-)particles take part in weak interaction
- Standard model of particle physics inherits only massless left(right)-handed electron-, muon- and tau (anti-)neutrinos





SOME OPEN QUESTIONS

Experiment

1. Value of the CP-violating phase in the mixing matrix
2. Normal or inverted mass hierarchy
3. Absolute mass scale of the lightest neutrinos
4. Dirac or Majorana nature

On the theory side:

1. Smallness of ν masses
2. Strong hierarchy in mass spectra of charged leptons
3. Mild hierarchy in ν spectrum
4. One small and two large mixing angles in lepton mixing matrix