

Phenomenological prospects for BGL models

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1 2HDM

2 BGL models

3 Analysis

4 Conclusions

Based on work done in collaboration with:

G.C. Branco, M.N. Rebelo, (Lisbon) & F.J. Botella (Valencia)

[Eur. Phys. J. C76 \(2016\) 3, 161, arXiv:1508.05101](#)

+ L. Pedro (Lisbon/Graz) and A. Carmona (Zurich/CERN)

[JHEP 1407 \(2014\) 078 , arXiv:1401.6147](#)

+ Work in progress



2HDM (I) – Unnecessary reminder

- Instead of a single doublet Φ , two doublets Φ_1 & Φ_2
- Full lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{kin+gauge}} - V(\Phi_1, \Phi_2) + \mathcal{L}_Y$$

[T.D.Lee, PRD 8 (1973), ..., Branco et al., Phys.Rep. 516 (2013)]

- In $\mathcal{L}_{\text{kin+gauge}}$, $(D_\mu \Phi)(D^\mu \Phi)^\dagger \rightarrow \sum_i (D_\mu \Phi_i)(D^\mu \Phi_i)^\dagger$
- Scalar potential, instead of $V(\Phi) = \lambda(v^2 - \Phi^\dagger \Phi)^2$

$$\begin{aligned} V(\Phi_1, \Phi_2) = & \mu_{11}^2 \Phi_1^\dagger \Phi_1 + \mu_{22}^2 \Phi_2^\dagger \Phi_2 + (\mu_{12}^2 \Phi_1^\dagger \Phi_2 + \text{H.C.}) \\ & + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \lambda_4 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) \\ & + (\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{H.C.}) + [(\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2))(\Phi_1^\dagger \Phi_2) + \text{H.C.}] \end{aligned}$$

- Yukawa couplings \mathcal{L}_Y

2HDM (II) – Unnecessary reminder

- Spontaneous symmetry breaking

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 e^{i\alpha_1} \end{pmatrix}, \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 e^{i\alpha_2} \end{pmatrix}$$

$$\sqrt{v_1^2 + v_2^2} = v \simeq 246 \text{ GeV}, \quad \frac{v_2}{v_1} \equiv \tan \beta$$

- Expansion around the minimum of $V(\Phi_1, \Phi_2)$

$$\Phi_j = e^{i\alpha_j} \begin{pmatrix} \varphi_j^+ \\ (v_j + \rho_j + i\eta_j)/\sqrt{2} \end{pmatrix}, \quad j = 1, 2$$

- Rotate to the “Higgs” basis with

$$U \equiv \frac{1}{\sqrt{v_1^2 + v_2^2}} \begin{pmatrix} v_1 e^{-i\alpha_1} & v_2 e^{-i\alpha_2} \\ v_2 e^{-i\alpha_1} & -v_1 e^{-i\alpha_2} \end{pmatrix} = \begin{pmatrix} e^{-i\alpha_1} \cos \beta & e^{-i\alpha_2} \sin \beta \\ e^{-i\alpha_1} \sin \beta & -e^{-i\alpha_2} \cos \beta \end{pmatrix}$$

2HDM (III) – Unnecessary reminder

- Doublets: $\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = U \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$ with $\langle H_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \textcolor{blue}{v} \end{pmatrix}$ and $\langle H_2 \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$
- Components

$$H_1 = \begin{pmatrix} G^+ \\ (v + N^0 + iG^0)/\sqrt{2} \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ (R^0 + iA)/\sqrt{2} \end{pmatrix}$$

- G^0, G^\pm : Goldstone  bosons (longitudinal Z & W^\pm)
- IF the fields in the Higgs basis where the physical (mass eigenstates) scalars ...
 - N^0 , “SM Higgs”
 - additional R^0 scalar & A pseudoscalar,
 - additional H^\pm charged scalar.

... and now, the Yukawa couplings \mathcal{L}_Y

Yukawa couplings (I)

- Yukawa couplings in 2HDM

$$\begin{aligned}\mathcal{L}_Y = & -\bar{Q}_L^0 (\Delta_1 \tilde{\Phi}_1 + \Delta_2 \tilde{\Phi}_2) u_R^0 - \bar{Q}_L^0 (\Gamma_1 \Phi_1 + \Gamma_2 \Phi_2) d_R^0 \\ & - \bar{L}_L^0 (\Sigma_1 \tilde{\Phi}_1 + \Sigma_2 \tilde{\Phi}_2) \nu_R^0 - \bar{L}_L^0 (\Pi_1 \Phi_1 + \Pi_2 \Phi_2) \ell_R^0 + \text{h.c.}\end{aligned}$$

- Quark Yukawa couplings + Mass terms

$$\begin{aligned}\mathcal{L}_Y \supset & -\bar{u}_L^0 \frac{1}{v} (M_u^0 (v + N^0) + N_u^0 R^0 + i N_u^0 A) u_R^0 \\ & - \bar{d}_L^0 \frac{1}{v} (M_d^0 (v + N^0) + N_d^0 R^0 + i N_d^0 A) d_R^0 \\ & - \frac{\sqrt{2}}{v} (\bar{u}_L^0 N_d^0 d_R^0 - \bar{u}_R^0 N_u^{0\dagger} d_L^0) H^+ + \text{H.C.}\end{aligned}$$

where $M_u^0 = \frac{1}{\sqrt{2}} (v_1 \Delta_1 + v_2 e^{i\theta} \Delta_2)$, $M_d^0 = \frac{1}{\sqrt{2}} (v_1 \Gamma_1 + v_2 e^{i\theta} \Gamma_2)$

and $N_u^0 = \frac{1}{\sqrt{2}} (v_2 \Delta_1 - v_1 e^{i\theta} \Delta_2)$, $N_d^0 = \frac{1}{\sqrt{2}} (v_2 \Gamma_1 - v_1 e^{i\theta} \Gamma_2)$

Fermion Masses and Flavour Changing Couplings

- Pictorially

$$\textcolor{red}{M} = v_1 \begin{matrix} \text{Fermion 1} \\ \text{Fermion 2} \end{matrix} + v_2 e^{i\theta} \begin{matrix} \text{Fermion 2} \\ \text{Fermion 1} \end{matrix} = v \begin{matrix} \text{Fermion 1} \\ \text{Fermion 2} \end{matrix}, \quad \textcolor{red}{N} = v_2 \begin{matrix} \text{Fermion 1} \\ \text{Fermion 2} \end{matrix} - v_1 e^{i\theta} \begin{matrix} \text{Fermion 2} \\ \text{Fermion 1} \end{matrix} = v \begin{matrix} \text{Fermion 1} \\ \text{Fermion 2} \end{matrix}$$

- Diagonalisation of mass matrices:

$$U_{uL}^\dagger \textcolor{red}{M_u^0} U_{uR} = M_u \equiv \text{diag} (m_u, m_c, m_t)$$

$$U_{dL}^\dagger \textcolor{red}{M_d^0} U_{dR} = M_d \equiv \text{diag} (m_d, m_s, m_b)$$

- ... gives **flavour changing couplings** with R^0 and A ,
the “non-SM” neutral scalars!

$$U_{uL}^\dagger \textcolor{red}{N_u^0} U_{uR} \equiv \textcolor{red}{N_u} = ?$$

$$U_{dL}^\dagger \textcolor{red}{N_d^0} U_{dR} \equiv \textcolor{red}{N_d} = ?$$

Yukawa couplings (II)

- \mathcal{L}_Y in terms of physical quark fields

$$\begin{aligned}
 \mathcal{L}_Y \supset & -\frac{1}{v} \mathbf{N^0} (\bar{u} M_u u + \bar{d} M_d d) \\
 & - \frac{1}{v} \mathbf{R^0} \left[\bar{u} (\mathbf{N_u} \gamma_R + \mathbf{N_u^\dagger} \gamma_L) u + \bar{d} (\mathbf{N_d} \gamma_R + \mathbf{N_d^\dagger} \gamma_L) d \right] \\
 & + \frac{i}{v} \mathbf{A} \left[\bar{u} (\mathbf{N_u} \gamma_R - \mathbf{N_u^\dagger} \gamma_L) u - \bar{d} (\mathbf{N_d} \gamma_R - \mathbf{N_d^\dagger} \gamma_L) d \right] \\
 & - \frac{\sqrt{2}}{v} \mathbf{H^+} \bar{u} (\mathbf{V N_d} \gamma_R - \mathbf{N_u^\dagger V} \gamma_L) d + \text{h.c.}
 \end{aligned}$$

- Mixing matrix (CKM), $\mathbf{V} = U_{uL}^\dagger U_{dL}$

FCNC (I)

Ways out

- Discrete symmetries & Natural Flavour Conservation
 - [Paschos, Glashow & Weinberg, PRD 15 (1977), ...]
 - Type I: Φ_2 couples to u_R , d_R , e_R
 - Type II: Φ_2 couples to u_R , Φ_1 couples to d_R , e_R
 - Lepton specific: Φ_2 couples to u_R , d_R , Φ_1 couples to e_R
 - Flipped: Φ_2 couples to u_R , e_R , Φ_1 couples to d_R
 - Aligned 2HDM: $\Delta_2 \propto \Delta_1$, $\Gamma_2 \propto \Gamma_1$
 - [Pich & Tuzón, PRD 80 (2009), ...]
 - *Effective* alignment
 - [Serôdio, PLB 700 (2011), Medeiros-Varzielas, PLB 701 (2011)]

FCNC (II)

Alternative:

- suppression factors in FCNC

[Joshi & Rindani, PLB 260 (1991)]

[Antaramian, Hall & Rasin, PRL 69 (1992)]

[Hall & Weinberg, PRD 48 (1993)]

...

- naturally suppressed – i.e. “controlled” – FCNC

[Lavoura, Int.J.Mod.Phys. A9 (1994)]

[Branco, Grimus & Lavoura (BGL), PLB 380 (1996)]

[Botella, Branco & Rebelo, PLB 687 (2010)]

[Botella, Branco, Nebot & Rebelo, JHEP 1110 (2011)]

...

[Bhattacharyya, Das & Kundu, PRD 89 (2014)]

- The general idea: symmetry imposes small FCNC
- In the BGL case:

FCNC proportional to fermion masses & mixings!



Enter BGL models (I)

- Symmetry

$$Q_{Lj}^0 \mapsto e^{i\tau} Q_{Lj}^0 , \quad u_{Rj}^0 \mapsto e^{i2\tau} u_{Rj}^0 , \quad \Phi_2 \mapsto e^{i\tau} \Phi_2$$

with $\tau \neq 0, \pi$ and j is 1 or 2 or 3 (at will)

- Reminder:

$$\mathcal{L}_Y \supset -\bar{Q}_L^0 (\Delta_1 \tilde{\Phi}_1 + \Delta_2 \tilde{\Phi}_2) u_R^0 - \bar{Q}_L^0 (\Gamma_1 \Phi_1 + \Gamma_2 \Phi_2) d_R^0$$

Consider for example $j = 3$:

$$\begin{aligned} \Delta_1 \mapsto \Delta'_1 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\tau} \end{pmatrix} \Delta_1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i2\tau} \end{pmatrix} \\ &\Rightarrow \arg \Delta'_1 - \arg \Delta_1 = \begin{pmatrix} 0 & 0 & +2\tau \\ 0 & 0 & +2\tau \\ -\tau & -\tau & +\tau \end{pmatrix} \end{aligned}$$



Enter BGL models (II)

while

$$\Delta_2 \mapsto \Delta'_2 = e^{-i\tau} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\tau} \end{pmatrix} \Delta_2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i2\tau} \end{pmatrix}$$

$$\Rightarrow \arg \Delta'_2 - \arg \Delta_2 = \begin{pmatrix} -\tau & -\tau & +\tau \\ -\tau & -\tau & +\tau \\ -2\tau & -2\tau & 0 \end{pmatrix}$$

The symmetry requires

Up Yukawas: $\Delta_1 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}$

Down Yukawas: $\Gamma_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}$



Up Yukawas – Example up $j = 3$

- Up Yukawas:

$$\Delta_1 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}$$

- Reminder:

$$M_u^0 = \frac{1}{\sqrt{2}}(v_1 \Delta_1 + v_2 e^{i\theta} \Delta_2), \quad N_u^0 = \frac{1}{\sqrt{2}}(v_2 \Delta_1 - v_1 e^{i\theta} \Delta_2)$$

- For the Up Yukawas, M_u^0 and N_u^0 are simultaneously diagonalised
 \Rightarrow NO FCNC
- The U_{uL} rotation is block diagonal, $U_{uL} = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Down Yukawas – Example up $j = 3$

- Down Yukawas:

$$\Gamma_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}$$

- Reminder:

$$M_d^0 = \frac{1}{\sqrt{2}}(\textcolor{blue}{v}_1 \Gamma_1 + v_2 e^{i\theta} \Gamma_2), \quad N_d^0 = \frac{1}{\sqrt{2}}(\textcolor{blue}{v}_2 \Gamma_1 - v_1 e^{i\theta} \Gamma_2)$$

- For the Down Yukawas

$$U_{dL}^\dagger \textcolor{red}{M}_d^0 U_{dR} = M_d, \quad U_{dL}^\dagger \textcolor{red}{N}_d^0 U_{dR} = ?$$

The BGL magic (I) – Example up $j = 3$

- Rewrite

$$\begin{aligned} \textcolor{red}{N_d^0} &= \frac{1}{\sqrt{2}}(v_2\Gamma_1 - v_1 e^{i\theta}\Gamma_2) = \\ &\quad \underbrace{\frac{v_2}{v_1} \frac{1}{\sqrt{2}}(v_1\Gamma_1 + v_2 e^{i\theta}\Gamma_2)}_{M_d^0} - \frac{v_2}{\sqrt{2}} \left(\frac{v_1}{v_2} + \frac{v_2}{v_1} \right) e^{i\theta}\Gamma_2 \end{aligned}$$

$$U_{dL}^\dagger \textcolor{red}{N_d^0} U_{dR} = \frac{v_2}{v_1} M_d - \frac{v_2}{\sqrt{2}} \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) U_{dL}^\dagger e^{i\theta}\Gamma_2 U_{dR}$$

- Problem with $U_{dL}^\dagger e^{i\theta}\Gamma_2 U_{dR}$
- The solution: if $\Gamma_2 \propto P \textcolor{red}{M}_d^0$ with P some fixed matrix,

$$U_{dL}^\dagger \Gamma_2 U_{dR} \propto U_{dL}^\dagger P \textcolor{red}{M}_d^0 U_{dR} = U_{dL}^\dagger P U_{dL} M_d$$

The BGL magic (II) – Example up $j = 3$

- Which P ?

$$P = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \frac{v_2}{\sqrt{2}} e^{i\theta} \Gamma_2 = P \textcolor{red}{M}_d^0$$

- The final touch: since $U_{uL} = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 1 \end{pmatrix}$,

$$\textcolor{red}{V} = U_{uL}^\dagger U_{dL} \Rightarrow [U_{dL}]_{3i} = \textcolor{red}{V}_{3i}$$

and

$$[U_{dL}^\dagger P U_{dL}]_{ij} = \textcolor{red}{V}_{3i}^* \textcolor{red}{V}_{3j}$$

- Finally

$$[\textcolor{red}{N}_d]_{ij} = [U_{dL}^\dagger \textcolor{red}{N}_d^0 U_{dR}]_{ij} = \textcolor{blue}{t}_\beta [M_d]_{ij} - \left(\textcolor{blue}{t}_\beta + \textcolor{blue}{t}_\beta^{-1} \right) \textcolor{red}{V}_{3i}^* \textcolor{red}{V}_{3j} [M_d]_{jj}$$



Neutral couplings in BGL models – Example up $j = 3$

- Up sector

$$N_u = -t_\beta^{-1} \operatorname{diag}(0, 0, m_t) + t_\beta \operatorname{diag}(m_u, m_c, 0)$$

- Down sector

$$\begin{aligned} N_d &= t_\beta \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} \\ &\quad - (t_\beta + t_\beta^{-1}) \begin{pmatrix} m_d |V_{td}|^2 & m_s V_{td}^* V_{ts} & m_b V_{td}^* V_{tb} \\ m_d V_{ts}^* V_{td} & m_s |V_{ts}|^2 & m_b V_{ts}^* V_{tb} \\ m_d V_{tb}^* V_{td} & m_s V_{tb}^* V_{ts} & m_b |V_{tb}|^2 \end{pmatrix} \end{aligned}$$

It all comes from the symmetry

BGL models - The zoo (I)

- We have seen an example with

$$Q_{Lj}^0 \mapsto e^{i\tau} Q_{Lj}^0 , \quad u_{Rj}^0 \mapsto e^{i2\tau} u_{Rj}^0 , \quad \Phi_2 \mapsto e^{i\tau} \Phi_2$$

and $j = 3$, leading to FCNC in the **DOWN** sector,

controlled by $V_{ti} V_{tk}^*$

- but we can as well choose $j = 1$ or $j = 2$,
then leading to FCNC in the **DOWN** sector
controlled by $V_{ui} V_{uk}^*$ or $V_{ci} V_{ck}^*$
- ... or start with this symmetry

$$Q_{Lj}^0 \mapsto e^{i\tau} Q_{Lj}^0 , \quad d_{Rj}^0 \mapsto e^{i2\tau} d_{Rj}^0 , \quad \Phi_2 \mapsto e^{i\tau} \Phi_2$$

which would lead to FCNC in the **UP** sector, controlled by $V_{ij} V_{kj}^*$

- Models: for the moment 3 up quark + 3 down quark

BGL models - The zoo (II)

- Models

- Quark sector 3 up quark + 3 down quark
- Lepton sector 3 ℓ + 3 ν (Dirac neutrinos)
[N.B. for Majorana ν 's, only half, no ν -FCNC]

- Need **all** for Renormalization Group Evolution, e.g.

$$\begin{aligned} \mathcal{D}\Gamma_k &= a_\Gamma \Gamma_k + \\ &+ \sum_{l=1}^2 \left[3\text{Tr}\left(\Gamma_k \Gamma_l^\dagger + \Delta_k^\dagger \Delta_l\right) + \text{Tr}\left(\Pi_k \Pi_l^\dagger + \Sigma_k^\dagger \Sigma_l\right) \right] \Gamma_l + \\ &+ \sum_{l=1}^2 \left(-2\Delta_l \Delta_k^\dagger \Gamma_l + \Gamma_k \Gamma_l^\dagger \Gamma_l + \frac{1}{2} \Delta_l \Delta_l^\dagger \Gamma_k + \frac{1}{2} \Gamma_l \Gamma_l^\dagger \Gamma_k \right), \end{aligned}$$

BGL models - The zoo (III)

- In the previous example, “model t ”,
 - flavour changing couplings of down quarks with R^0 and A ,
 - flavour conserving couplings of up quarks with R^0 and A ,
- 3 choices of symmetry with down fields,
3 choices of symmetry with up fields
→ 6 different quark models
- In the lepton sector: 6 different choices for neutrinos and charged leptons, overall, **36 models**
[Botella, Branco, Nebot & Rebelo, JHEP(2011) 1102.0520]
- but...

What about flavour changing couplings of the **Higgs**?

The answer: N^0 and R^0 are NOT the physical scalars

Physical neutral scalars

- Neutral mass eigenstates, with only one mixing (no CP),

$$\begin{pmatrix} \text{H} \\ \text{h} \end{pmatrix} = \begin{pmatrix} -c_\alpha & -s_\alpha \\ s_\alpha & -c_\alpha \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} = \begin{pmatrix} -c_{\beta\alpha} & -s_{\beta\alpha} \\ s_{\beta\alpha} & -c_{\beta\alpha} \end{pmatrix} \begin{pmatrix} \text{N}^0 \\ \text{R}^0 \end{pmatrix}$$

with h “the Higgs”

From

$$\begin{aligned} \mathcal{L}_{\bar{q}qN} \supset & -\text{N}^0 \frac{1}{v} [\bar{u} M_u u + \bar{d} M_d d] \\ & - \text{R}^0 \frac{1}{v} [\bar{u} (\text{N}_u \gamma_R + \text{N}_u^\dagger \gamma_L) u + \bar{d} (\text{N}_d \gamma_R + \text{N}_d^\dagger \gamma_L) d] \end{aligned}$$

we arrive to

$$\begin{aligned} \mathcal{L}_{\bar{q}qN} \supset & -\frac{\text{h}}{v} \bar{u} \left((s_{\beta\alpha} M_u - c_{\beta\alpha} \text{N}_u) \gamma_R + (s_{\beta\alpha} M_u - c_{\beta\alpha} \text{N}_u^\dagger) \gamma_L \right) u \\ & - \frac{\text{h}}{v} \bar{d} \left((s_{\beta\alpha} M_d - c_{\beta\alpha} \text{N}_d) \gamma_R + (s_{\beta\alpha} M_d - c_{\beta\alpha} \text{N}_d^\dagger) \gamma_L \right) d \\ & + \frac{\text{H}}{v} \bar{u} \left((c_{\beta\alpha} M_u + s_{\beta\alpha} \text{N}_u) \gamma_R + (c_{\beta\alpha} M_u + s_{\beta\alpha} \text{N}_u^\dagger) \gamma_L \right) u \\ & + \frac{\text{H}}{v} \bar{d} \left((c_{\beta\alpha} M_d + s_{\beta\alpha} \text{N}_d) \gamma_R + (c_{\beta\alpha} M_d + s_{\beta\alpha} \text{N}_d^\dagger) \gamma_L \right) d \end{aligned}$$

Higgs in BGL models

Most salient features

- Flavour changing couplings of the Higgs with up or with down quarks, e.g. $h \rightarrow bs, bd, t \rightarrow hc, hu$
- Flavour changing couplings of the Higgs with neutrinos or with charged leptons, e.g. $h \rightarrow \mu\tau$
- Modified flavour conserving (diagonal) couplings
- Only two new parameters involved, $\tan\beta$ and $\beta - \alpha$
⇒ correlated predictions, magic combination $c_{\beta\alpha}(t_\beta + t_\beta^{-1})$



Analysis

Goal

- Analyse prospects for the previous Higgs flavour changing processes
- Concentrate on $\tan\beta$ and $\alpha - \beta$

Constraints

- From Higgs diagonal couplings: $\gamma\gamma$, WW , ZZ , $t\bar{t}$, $b\bar{b}$, $\tau\bar{\tau}$
Notice that both **decay** and **production** are modified!
- From low-energy flavour physics: more involved since H , A and H^\pm do participate together with h , extended (additional parameters)
Without mixing [Botella et al., JHEP 1407 (2014) 078, arXiv:1401.6147]
- From the scalar sector

Top decays $t \rightarrow hq$ (I)

- Rate

$$\Gamma_{(d_\rho)}(t \rightarrow hq) = \frac{m_t^3}{32\pi v^2} \left(1 - \frac{m_h^2}{m_t^2}\right)^2 |V_{q\rho}|^2 |V_{t\rho}|^2 c_{\beta\alpha}^2 (t_\beta + t_\beta^{-1})^2$$

- Branching ratio

$$\text{Br}_{(d_\rho)}(t \rightarrow hq) = \frac{\Gamma_{(d_\rho)}(t \rightarrow hq)}{\Gamma(t \rightarrow Wb)} = f(x_h, y_W) \frac{|V_{q\rho} V_{t\rho}|^2}{|V_{tb}|^2} c_{\beta\alpha}^2 (t_\beta + t_\beta^{-1})^2$$

where

$$f(x_h, y_W) = \frac{1}{2} (1 - x_h)^2 (1 - 3y_W^2 + 2y_W^3)^{-1},$$

$$x_h = \frac{m_h^2}{m_t^2}, \quad y_W = \frac{M_W^2}{m_t^2}, \quad f(x_h, y_W) \simeq 0.1306$$

Top decays $t \rightarrow hq$ (II)

- Relevant CKM combinations

Model	$t \rightarrow hu$	$t \rightarrow hc$
d	$ V_{ud}V_{td} ^2 (\sim \lambda^6) = 7.51 \cdot 10^{-5}$	$ V_{cd}V_{td} ^2 (\sim \lambda^8) = 4.01 \cdot 10^{-6}$
s	$ V_{us}V_{ts} ^2 (\sim \lambda^6) = 8.20 \cdot 10^{-5}$	$ V_{cs}V_{ts} ^2 (\sim \lambda^4) = 1.53 \cdot 10^{-3}$
b	$ V_{ub}V_{tb} ^2 (\sim \lambda^6) = 1.40 \cdot 10^{-5}$	$ V_{cb}V_{tb} ^2 (\sim \lambda^4) = 1.68 \cdot 10^{-3}$

N.B. $\lambda \simeq 0.22$

- With upper bounds 0.79% and 0.56% from the ATLAS and CMS collaborations in b and s models

$$|c_{\beta\alpha}(t_\beta + t_\beta^{-1})| \lesssim 4.9$$

Higgs decays $h \rightarrow bq$ (I)

- Rate

$$\Gamma_{(u_k)}(h \rightarrow \bar{b}q + b\bar{q}) = c_{\beta\alpha}^2(t_\beta + t_\beta^{-1})^2 |V_{kq}|^2 |V_{kb}|^2 \Gamma_{\text{SM}}(h \rightarrow b\bar{b})$$

with $\text{Br}_{\text{SM}}(h \rightarrow b\bar{b}) = 0.578$

- CKM factors

Model	$h \rightarrow bd$	$h \rightarrow bs$
u	$ V_{ud}V_{ub} ^2 (\sim \lambda^6) = 1.33 \cdot 10^{-5}$	$ V_{us}V_{ub} ^2 (\sim \lambda^8) = 7.14 \cdot 10^{-7}$
c	$ V_{cd}V_{cb} ^2 (\sim \lambda^6) = 8.52 \cdot 10^{-5}$	$ V_{cs}V_{cb} ^2 (\sim \lambda^4) = 1.59 \cdot 10^{-3}$
t	$ V_{td}V_{tb} ^2 (\sim \lambda^6) = 7.90 \cdot 10^{-5}$	$ V_{ts}V_{tb} ^2 (\sim \lambda^4) = 1.61 \cdot 10^{-3}$

N.B. $\lambda \simeq 0.22$

- Best prospects for $h \rightarrow bs$ in c, t models, for $c_{\beta\alpha}^2(t_\beta + t_\beta^{-1})^2 \sim 1$, with $\text{Br}(h \rightarrow bs)$ up to the 10^{-3} level

Leptonic Higgs decays, $h \rightarrow \mu\tau$ (I)

- Rate

$$\Gamma_{(\nu_\sigma)}(h \rightarrow \mu\bar{\tau} + \bar{\mu}\tau) = c_{\beta\alpha}^2(t_\beta + t_\beta^{-1})^2 |U_{\mu\sigma} U_{\tau\sigma}|^2 \Gamma_{\text{SM}}(h \rightarrow \tau\bar{\tau})$$

with $\text{Br}_{\text{SM}}(h \rightarrow \tau\bar{\tau}) = 0.0637$

Mod.	$h \rightarrow e\mu$	$h \rightarrow e\tau$	$h \rightarrow \mu\tau$
ν_1	$ U_{e1} U_{\mu 1} ^2 (\frac{1}{9}) = 0.11$	$ U_{e1} U_{\tau 1} ^2 (\frac{1}{9}) = 0.12$	$ U_{\mu 1} U_{\tau 1} ^2 (\frac{1}{36}) = 0.028$
ν_2	$ U_{e2} U_{\mu 2} ^2 (\frac{1}{9}) = 0.09$	$ U_{e2} U_{\tau 2} ^2 (\frac{1}{9}) = 0.13$	$ U_{\mu 2} U_{\tau 2} ^2 (\frac{1}{9}) = 0.115$
ν_3	$ U_{e3} U_{\mu 3} ^2 = 0.0128$	$ U_{e3} U_{\tau 3} ^2 = 0.0097$	$ U_{\mu 3} U_{\tau 3} ^2 (\frac{1}{4}) = 0.234$

PMNS factors; rough estimates correspond to tri-bimaximal mixing
 (except for $|U_{e3}|$)

Leptonic Higgs decays, $h \rightarrow \mu\tau$ (II)

From LHC run I,

- CMS analysis:

$$\text{Br}(h \rightarrow \mu\bar{\tau} + \tau\bar{\mu}) = (0.84^{+0.39}_{-0.37})\%$$

- ATLAS analysis:

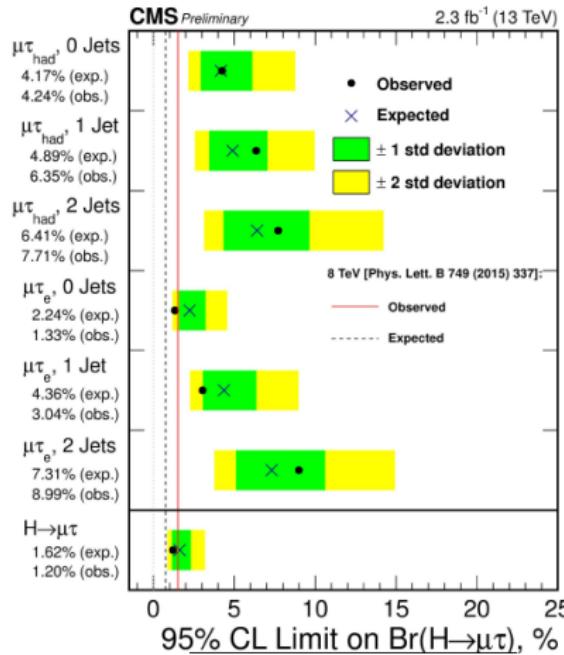
$$\text{Br}(h \rightarrow \mu\bar{\tau} + \tau\bar{\mu}) = (0.77 \pm 0.62)\%$$

- $\text{Br}(h \rightarrow \mu\bar{\tau} + \tau\bar{\mu})$ in the 1% ballpark with

$$c_{\beta\alpha}^2(t_\beta + t_\beta^{-1})^2 \sim 1$$

Leptonic Higgs decays, $h \rightarrow \mu\tau$ (III)

- ... but yet to be seen at LHC@13TeV



Higgs signals (I)

Signal strengths of the form

$$\mu_i^X = \frac{[\sigma(pp \rightarrow h)]_i}{[\sigma(pp \rightarrow h)_{\text{SM}}]_i} \frac{\text{Br}(h \rightarrow X)}{\text{Br}(h \rightarrow X)_{\text{SM}}}$$

- i for the different combinations of production mechanisms
- X for the decay channels

Measurements of the Higgs boson production and decay rates and constraints on its couplings from a combined ATLAS and CMS analysis of the LHC pp collision data at $\sqrt{s} = 7$ and 8 TeV,

[ATLAS & CMS, JHEP 1608 (2016) 045, arXiv:1606.02266]

Higgs signals (II)

Table 8: Best fit values of $\sigma_i B^i$ for each specific channel $i \rightarrow H \rightarrow f$, as obtained from the generic parameterisation with 23 parameters for the combination of the ATLAS and CMS measurements, using the $\sqrt{s} = 7$ and 8 TeV data. The cross sections are given for $\sqrt{s} = 8$ TeV, assuming the SM values for $\sigma_i(7\text{ TeV})/\sigma_i(8\text{ TeV})$. The results are shown together with their total uncertainties and their breakdown into statistical and systematic components. The expected uncertainties in the measurements are displayed in parentheses. The SM predictions [32] and the ratios of the results to these SM predictions are also shown. The values labelled with a “-” are either not measured with a meaningful precision and therefore not quoted, in the case of the $H \rightarrow ZZ$ decay channel for the WH , ZH , and $t\bar{t}H$ production processes, or not measured at all and therefore fixed to their corresponding SM predictions, in the case of the $H \rightarrow bb$ decay mode for the ggF and VBF production processes.

Production process	Decay mode														
	$H \rightarrow \gamma\gamma$ [fb]			$H \rightarrow ZZ$ [fb]			$H \rightarrow WW$ [pb]			$H \rightarrow \tau\tau$ [fb]			$H \rightarrow bb$ [pb]		
	Best fit value	Uncertainty Stat	Uncertainty Syst	Best fit value	Uncertainty Stat	Uncertainty Syst	Best fit value	Uncertainty Stat	Uncertainty Syst	Best fit value	Uncertainty Stat	Uncertainty Syst	Best fit value	Uncertainty Stat	Uncertainty Syst
ggF	Measured	$48.0^{+0.0}_{-0.7}$	$+0.8_{-0.4}^{+0.2}$	-2.3	580^{+170}_{-160}	$+170_{-160}^{+80}$	-46	$3.5^{+0.7}_{-0.7}$	$+0.5_{-0.5}^{+0.5}$	-0.5	1300^{+700}_{-700}	$+400_{-400}^{+500}$	-500	-	-
	Predicted	44 ± 5	-	-	510 ± 60	-	-	4.1 ± 0.5	-	-	$(+700)_{(-700)}$	$(+400)_{(-400)}^{+500}$	-500	-	-
	Ratio	$1.10^{+0.23}_{-0.22}$	$+0.22_{-0.21}^{+0.07}$	-0.05	$1.13^{+0.34}_{-0.31}$	$+0.33_{-0.30}^{+0.09}$	-0.07	$0.84^{+0.17}_{-0.17}$	$+0.12_{-0.12}^{+0.12}$	-0.12	$1.0^{+0.6}_{-0.6}$	$+0.6_{-0.6}^{+0.4}$	-0.4	-	-
VBF	Measured	$4.6^{+0.8}_{-0.8}$	$+1.0_{-0.7}^{+0.6}$	-0.6	3^{+56}_{-25}	$+56_{-25}^{+7}$	-	$0.39^{+0.15}_{-0.15}$	$+0.12_{-0.12}^{+0.12}$	-0.09	125^{+37}_{-37}	$+34_{-34}^{+19}$	-19	-	-
	Predicted	3.60 ± 0.20	-	-	42.2 ± 2.0	-	-	0.341 ± 0.017	-	-	100 ± 6	-	-	0.91 ± 0.04	-
	Ratio	$1.3^{+0.5}_{-0.5}$	$+0.5_{-0.5}^{+0.2}$	-0.2	$0.1^{+1.1}_{-0.6}$	$+1.1_{-0.6}^{+0.2}$	-	$1.2^{+0.6}_{-0.4}$	$+0.4_{-0.3}^{+0.2}$	-0.2	$1.3^{+0.4}_{-0.4}$	$+0.3_{-0.3}^{+0.2}$	-	-	-
WH	Measured	$0.7^{+2.0}_{-0.9}$	$+2.1_{-1.8}^{+0.7}$	-0.2	-	-	-	$0.24^{+0.18}_{-0.18}$	$+0.15_{-0.15}^{+0.10}$	-0.10	-64^{+64}_{-64}	$+55_{-55}^{+32}$	-32	$0.42^{+0.23}_{-0.23}$	$+0.17_{-0.17}^{+0.12}$
	Predicted	1.60 ± 0.09	-	-	-	-	-	$(+0.16)_{(-0.14)}$	$(+0.14)_{(-0.13)}^{(+0.08)}$	(-0.07)	$(+67)_{(-64)}^{(+69)}$	$(+30)_{(-54)}^{(+32)}$	-32	$(+0.22)_{(-0.21)}^{(+0.18)_{(-0.17)}}$	$(+0.12)_{(-0.11)}^{(+0.12)_{(-0.11)}}$
	Ratio	$0.5^{+1.3}_{-1.2}$	$+1.3_{-1.1}^{+0.2}$	-0.2	-	-	-	$1.6^{+1.2}_{-1.0}$	$+1.0_{-0.9}^{+0.8}$	-0.5	$-1.4^{+1.4}_{-1.4}$	$+1.2_{-1.1}^{+0.7}$	-0.8	$1.0^{+0.3}_{-0.5}$	$+0.4_{-0.4}^{+0.3}$
ZH	Measured	$0.5^{+2.9}_{-2.5}$	$+2.8_{-2.4}^{+0.5}$	-0.2	-	-	-	$0.53^{+0.23}_{-0.23}$	$+0.21_{-0.21}^{+0.10}$	-0.10	58^{+56}_{-56}	$+52_{-52}^{+20}$	-20	$0.08^{+0.09}_{-0.09}$	$+0.08_{-0.08}^{+0.04}$
	Predicted	0.94 ± 0.06	-	-	-	-	-	$(+0.17)_{(-0.14)}$	$(+0.16)_{(-0.14)}^{(+0.05)}$	(-0.07)	$(+49)_{(-49)}^{(+48)}$	$(+17)_{(-38)}^{(-17)}$	-38	$(+0.10)_{(-0.09)}^{(+0.09)_{(-0.08)}}$	$(+0.05)_{(-0.04)}^{(+0.05)_{(-0.04)}}$
	Ratio	$0.5^{+3.0}_{-2.5}$	$+3.0_{-2.5}^{+0.5}$	-0.2	-	-	-	$5.9^{+2.6}_{-2.2}$	$+2.3_{-2.1}^{+1.1}$	-0.8	$2.2^{+2.2}_{-1.8}$	$+2.0_{-1.7}^{+0.8}$	-0.6	$0.4^{+0.4}_{-0.4}$	$+0.3_{-0.3}^{+0.2}$
$t\bar{t}H$	Measured	$0.64^{+0.48}_{-0.45}$	$+0.48_{-0.44}^{+0.07}$	-0.04	-	-	-	$0.14^{+0.05}_{-0.04}$	$+0.04_{-0.03}^{+0.03}$	-0.03	-15^{+30}_{-30}	$+26_{-26}^{+15}$	-15	$0.08^{+0.07}_{-0.07}$	$+0.04_{-0.04}^{+0.06}$
	Predicted	0.294 ± 0.035	-	-	-	-	-	$(+0.04)_{(-0.04)}$	$(+0.04)_{(-0.04)}^{(+0.02)}$	(-0.02)	$(+31)_{(-28)}^{(+28)}$	$(+16)_{(-17)}^{(-17)}$	-17	$(+0.07)_{(-0.06)}^{(+0.04)_{(-0.05)}}$	$(+0.05)_{(-0.04)}^{(+0.05)_{(-0.03)}}$
	Ratio	$2.2^{+1.6}_{-1.5}$	$+1.6_{-1.3}^{+0.2}$	-0.1	-	-	-	$5.0^{+1.8}_{-1.7}$	$+1.5_{-1.5}^{+0.8}$	-0.9	$-1.9^{+3.7}_{-3.3}$	$+3.2_{-2.7}^{+1.9}$	-1.8	$1.1^{+1.0}_{-1.0}$	$+0.5_{-0.5}^{+0.8}$



Higgs signals (III)

Table 4: Higgs boson production cross sections σ_i , partial decay widths Γ^f , and total decay width (in the absence of BSM decays) parameterised as a function of the κ coupling modifiers as discussed in the text, including higher-order QCD and EW corrections to the inclusive cross sections and decay partial widths. The coefficients in the expression for Γ_H do not sum exactly to unity because some contributions that are negligible or not relevant to the analyses presented in this paper are not shown.

Production	Loops	Interference	Effective	Resolved
			scaling factor	scaling factor
$\sigma(ggF)$	✓	$t-b$	κ_g^2	$1.06 \cdot \kappa_t^2 + 0.01 \cdot \kappa_b^2 - 0.07 \cdot \kappa_t \kappa_b$
$\sigma(VBF)$	—	—		$0.74 \cdot \kappa_W^2 + 0.26 \cdot \kappa_Z^2$
$\sigma(WH)$	—	—	κ_W^2	
$\sigma(qq/qg \rightarrow ZH)$	—	—	κ_Z^2	
$\sigma(gg \rightarrow ZH)$	✓	$t-Z$		$2.27 \cdot \kappa_Z^2 + 0.37 \cdot \kappa_t^2 - 1.64 \cdot \kappa_Z \kappa_t$
$\sigma(tH)$	—	—	κ_t^2	
$\sigma(gb \rightarrow tHW)$	—	$t-W$		$1.84 \cdot \kappa_t^2 + 1.57 \cdot \kappa_W^2 - 2.41 \cdot \kappa_t \kappa_W$
$\sigma(qq/qb \rightarrow tHq)$	—	$t-W$		$3.40 \cdot \kappa_t^2 + 3.56 \cdot \kappa_W^2 - 5.96 \cdot \kappa_t \kappa_W$
$\sigma(bbH)$	—	—	κ_b^2	
Partial decay width				
Γ^{ZZ}	—	—		κ_Z^2
Γ^{WW}	—	—		κ_W^2
$\Gamma^{\gamma\gamma}$	✓	$t-W$	κ_γ^2	$1.59 \cdot \kappa_W^2 + 0.07 \cdot \kappa_t^2 - 0.66 \cdot \kappa_W \kappa_t$
Γ^{tt}	—	—		κ_t^2
Γ^{bb}	—	—		κ_b^2
$\Gamma^{\mu\mu}$	—	—		κ_μ^2
Total width ($B_{BSM} = 0$)				
Γ_H	✓	—	κ_H^2	$0.57 \cdot \kappa_b^2 + 0.22 \cdot \kappa_W^2 + 0.09 \cdot \kappa_g^2 + 0.06 \cdot \kappa_t^2 + 0.03 \cdot \kappa_Z^2 + 0.03 \cdot \kappa_c^2 + 0.0023 \cdot \kappa_\gamma^2 + 0.0016 \cdot \kappa_{(Z\gamma)}^2 + 0.0001 \cdot \kappa_\tau^2 + 0.00022 \cdot \kappa_\mu^2$

BGL vs. Higgs signals – Modified couplings

$\text{h}\bar{t}t$ coupling ($\times m_t/v$) in BGL models

u,c	t	d_i
$s_{\beta\alpha} + c_{\beta\alpha} t_\beta$	$s_{\beta\alpha} - c_{\beta\alpha} t_\beta^{-1}$	$s_{\beta\alpha} + c_{\beta\alpha} [(1 - V_{td_i} ^2) t_\beta - V_{td_i} ^2 t_\beta^{-1}]$

$\text{h}\bar{b}b$ coupling ($\times m_b/v$) in BGL models

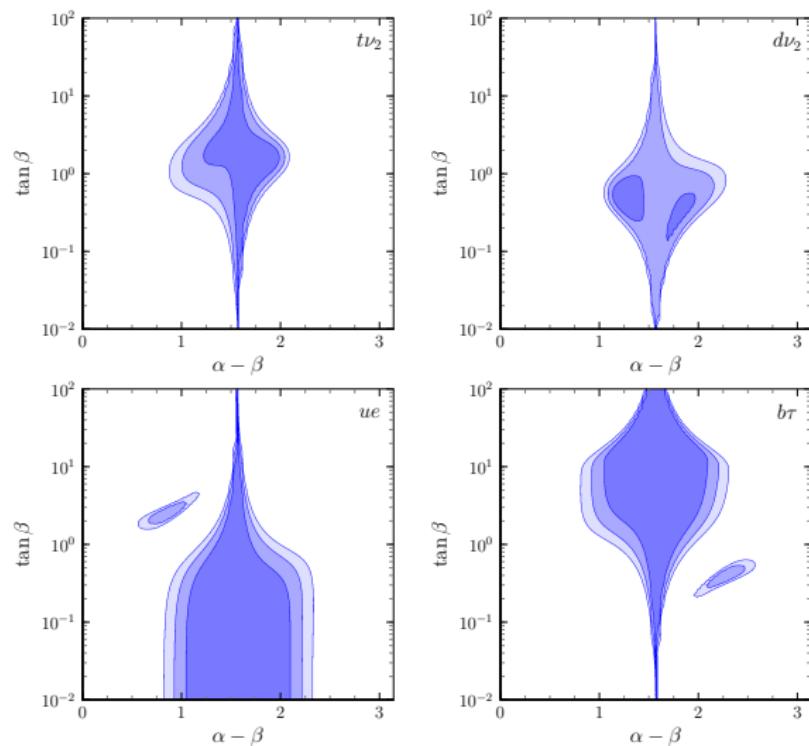
d,s	b	u_i
$s_{\beta\alpha} + c_{\beta\alpha} t_\beta$	$s_{\beta\alpha} - c_{\beta\alpha} t_\beta^{-1}$	$s_{\beta\alpha} + c_{\beta\alpha} [(1 - V_{u_i b} ^2) t_\beta - V_{u_i b} ^2 t_\beta^{-1}]$

$\text{h}\bar{\tau}\tau$ coupling ($\times m_\tau/v$) in BGL models

e, μ	τ	ν_i
$s_{\beta\alpha} + c_{\beta\alpha} t_\beta$	$s_{\beta\alpha} - c_{\beta\alpha} t_\beta^{-1}$	$s_{\beta\alpha} + c_{\beta\alpha} [(1 - U_{\tau i} ^2) t_\beta - U_{\tau i} ^2 t_\beta^{-1}]$

Don't forget that $\text{h}VV \mapsto s_{\beta\alpha} \text{h}VV!$

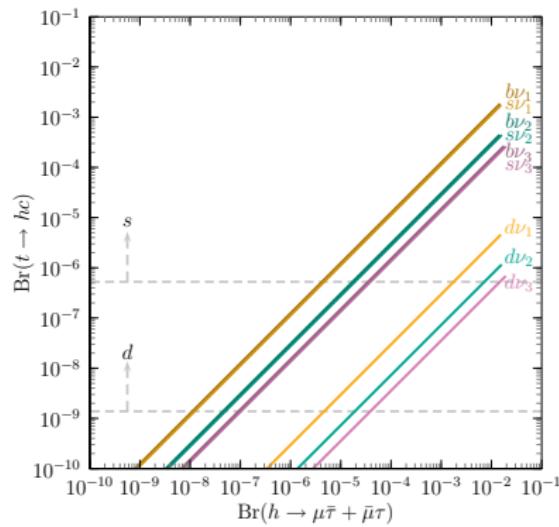
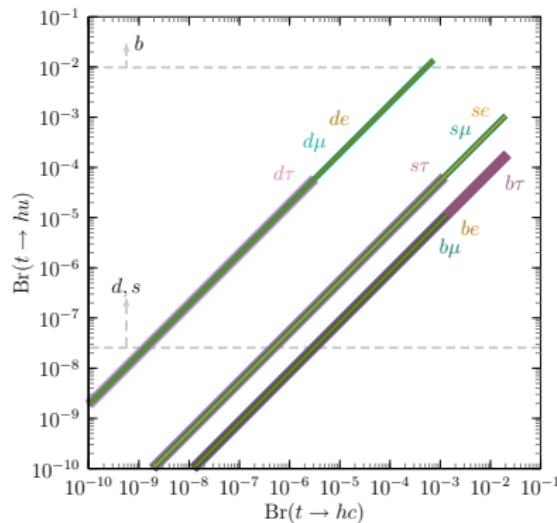




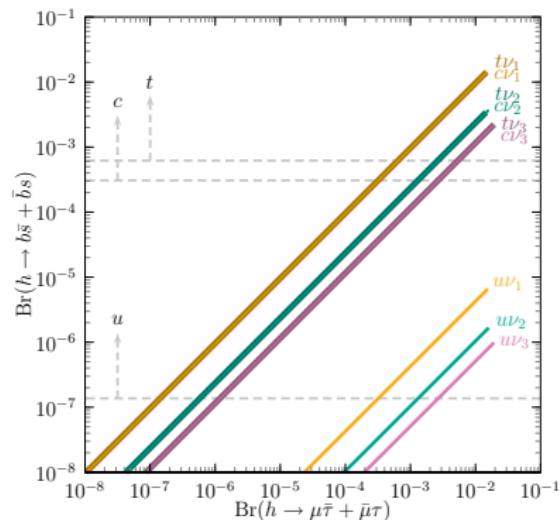
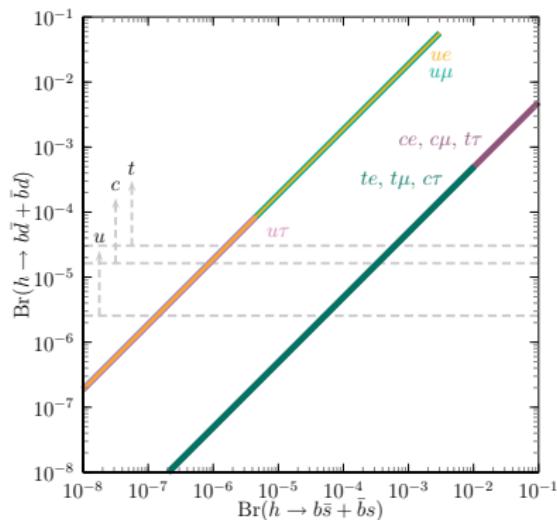
68%, 95% and 99% CL regions in t_β vs $\alpha - \beta$ for a sample of models

N.B. As in [arXiv:1508.05101](https://arxiv.org/abs/1508.05101)

Prospects – $t \rightarrow hq$ & $h \rightarrow \mu\tau$



Prospects – $t \rightarrow hq$ & $h \rightarrow \mu\tau$



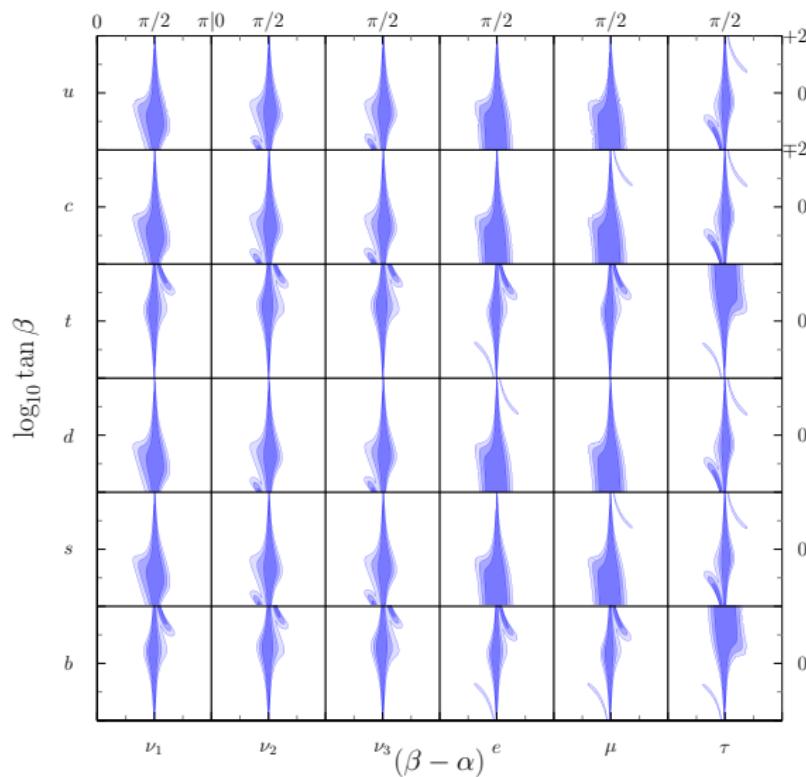
Full analysis

- Involves additional parameters: masses of H^\pm , A, H
- Without mixing in the scalar sector, provides a first understanding of valid t_β ranges

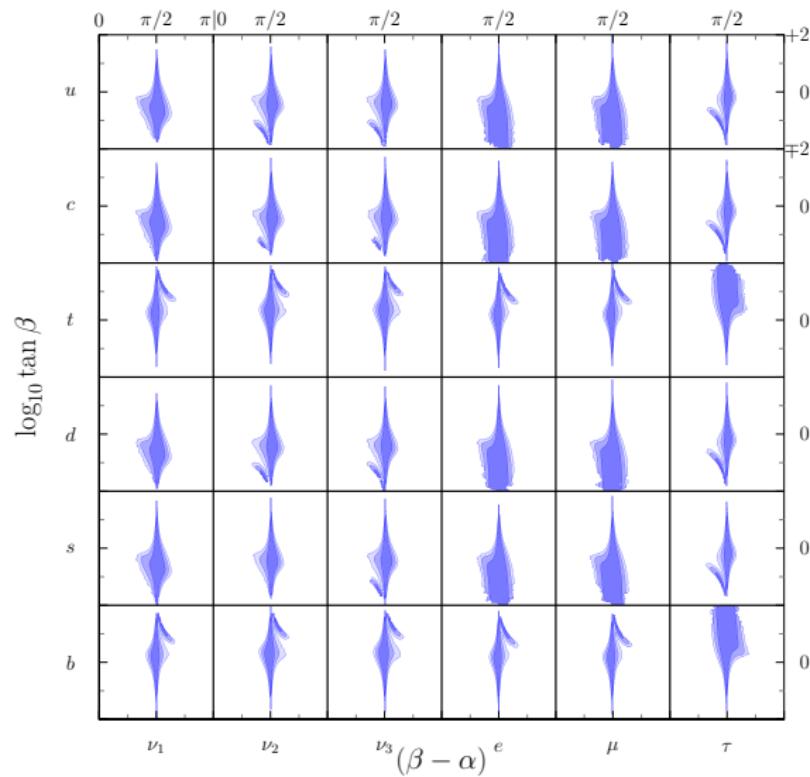
Botella et al., JHEP 1407 (2014) 078, [arXiv:1401.6147](https://arxiv.org/abs/1401.6147)

- With mixing, interplay of different contributions very relevant
- Massive amount of observables (meson mixings, rare decays, ...)
- Scalar sector constraints (positivity, perturbative unitarity, oblique corrections)





Only Higgs signals



+ Scalar potential (Positivity, Perturbative unit., ...) + oblique EW



Summary & Conclusions

- Class of models with reduced parametric freedom: $\tan \beta$ & $\alpha - \beta$:
 ⇒ predictivity & correlations,
 ⇒ importance of flavour diagonal Higgs data to constrain
 flavour changing couplings.
- $t \rightarrow hu$ & $t \rightarrow hc$ branching ratios can saturate current bounds
- Different correlated patterns, $\text{Br}(t \rightarrow hc) > \text{Br}(t \rightarrow hu)$ in s, b models but $\text{Br}(t \rightarrow hc) < \text{Br}(t \rightarrow hu)$ in d models
- $h \rightarrow bs$ & $h \rightarrow bd$ branching ratios within reach of ILC sensitivity
- $h \rightarrow \mu\tau$ branching ratios can match the run I “CMS hint”
- Correlations between flavour changing processes in the quark and lepton sectors

Summary & Conclusions

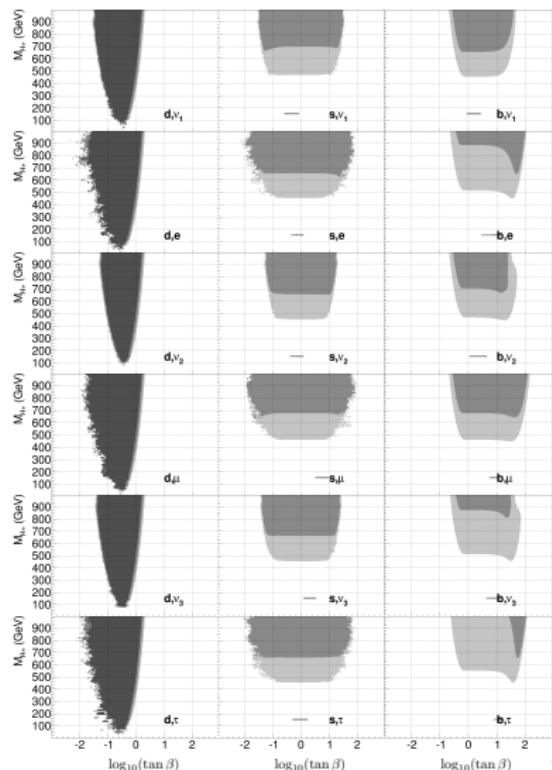
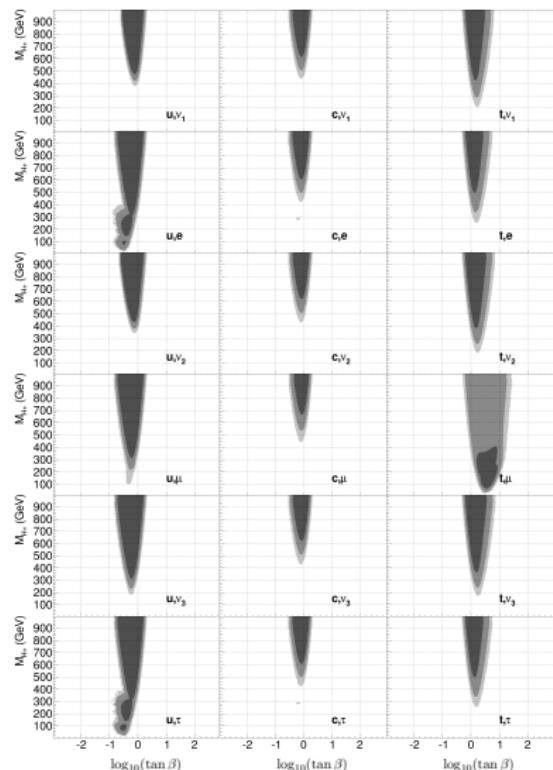
- Word of caution: in some models, low-energy constraints will play an important role
- ... more involved game including all scalars
- Detailed analysis of all models in progress, addressing implications beyond the SM-like Higgs
 - in terms of deviations from SM expectations in different observables (+ correlations)
 - in terms of prospects for the new scalars



Thank you for your attention!

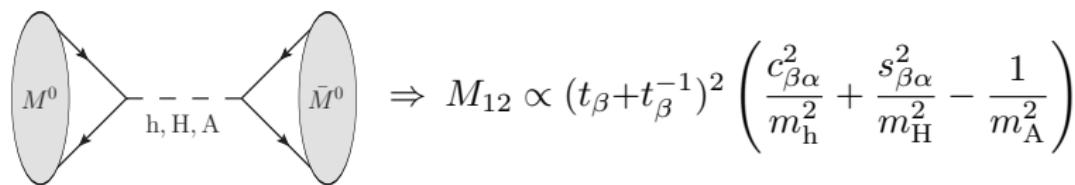


Backup: Old full analysis, No scalar mixing results



Backup: Meson mixings

- Naive bounds from the h contribution alone shown as dashed lines in $t \rightarrow hu, hc, h \rightarrow bs, bd$ plots
- But...



- with $c_{\beta\alpha} \rightarrow 0$, $m_H \simeq m_A$ is safe (favoured in addition by EW precision data, oblique parameters)
- departing from $c_{\beta\alpha} \rightarrow 0$ in $\{c_{\beta\alpha}, m_H, m_A\}$, cancellation is still complete in this subspace

$$(c_{\beta\alpha}^2, m_H^2, m_A^2) = (c^2, M^2, [c^2/m_h^2 + (1 - c^2)/M^2]^{-1})$$

- explicitly check that the naive “h alone bound” sizably overconstrains with respect to the “full” situation



Backup: Perturbative unitarity

- Scalar potential with $\lambda_5 = \lambda_6 = \lambda_7 = 0$
- In terms minimal set of parameters, relations like

$$m_A^2 s_{\beta\alpha} c_{\beta\alpha} = v^2 [s_{2\alpha}(\lambda_2 s_\beta^2 - \lambda_1 c_\beta^2) + (\lambda_3 + \lambda_4)s_{2\beta}c_{2\alpha}]$$

Gunion & Haber, Phys. Rev. D67 (2003) 075019

- Since $c_{\beta\alpha}(t_\beta + t_\beta^{-1}) = \frac{c_{\beta\alpha}}{s_\beta c_\beta}$,

$$\frac{c_{\beta\alpha}s_{\beta\alpha}}{s_\beta c_\beta} = c_{\beta\alpha}(t_\beta + t_\beta^{-1})s_{\beta\alpha} = \frac{v^2}{m_A^2} [s_{2\alpha}(\lambda_2 t_\beta - \lambda_1 t_\beta^{-1}) + 2(\lambda_3 + \lambda_4)c_{2\alpha}]$$

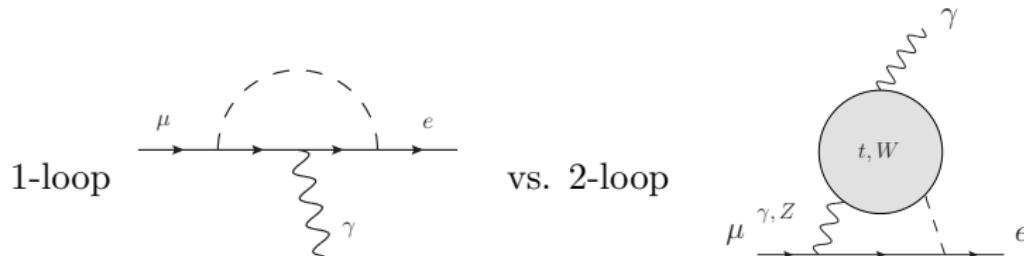
For example, for $m_A \sim v$, $c_{\beta\alpha}(t_\beta + t_\beta^{-1}) \sim \mathcal{O}(1)$ does not challenge naive perturbativity requirements $\lambda_i \leq 4\pi$



Backup: $\mu \rightarrow e\gamma$ (I)

- Chiral suppression at one loop lifted at two loops
[Bjorken & Weinberg, Phys. Rev. Lett. 38 \(1977\) 622](#)
- Barr-Zee contributions
[Barr & Zee, Phys. Rev. Lett. 65 \(1990\) 21](#)

$$\text{Br}(\mu \rightarrow e\gamma)_{\text{2 loop}} = \frac{3}{8} \left(\frac{\alpha}{\pi} \right)^3 (t_\beta + t_\beta^{-1})^2 |U_{ej} U_{\mu j}^*|^2 |A_{(Q)}|^2 \simeq \\ 5.77 \times 10^{-9} (t_\beta + t_\beta^{-1})^2 |U_{ej} U_{\mu j}^*|^2 |A_{(Q)}|^2$$

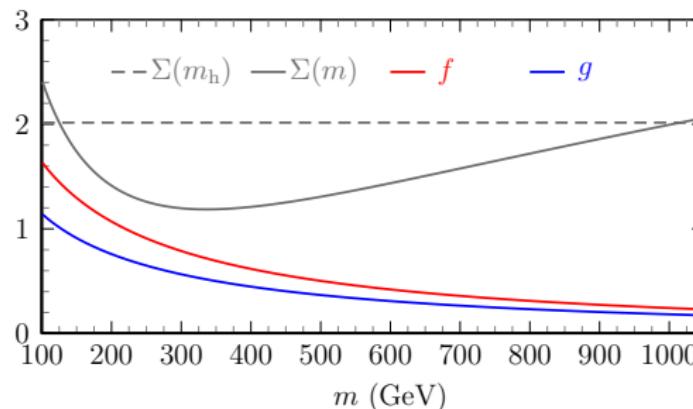


Backup: $\mu \rightarrow e\gamma$ (II)

- $A_{(Q)}$ is the amplitude:

$$A_{(Q)} = c_{\beta\alpha} s_{\beta\alpha} [\Sigma(m_h) - \Sigma(m_H)] + \frac{8}{3} K_t [c_{\beta\alpha}^2 f(z_h) + s_{\beta\alpha}^2 f(z_H) - g(z_A)]$$

$y_X = M_W^2/M_X^2$ and $z_X = m_t^2/M_X^2$ ($X = h, H, A$), and K_t the model dependent $ht\bar{t}$ factor



$f(m_t^2/m^2)$, $g(m_t^2/m^2)$ and $\Sigma(m)$

Backup: $\mu \rightarrow e\gamma$ (III)

- Explicit check

