SPONTANEOUS CP VIOLATION IN THE 2HDM

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Outline of talk

- Present model
- Highlight some important couplings
- The importance of basis independence
- Conditions for CP violation
- Conditions for Spontaneous CP violation
- The alignment limit
- Summary

The model:
The general 2HDM potential

$$
\begin{aligned}
V(\Phi_1, \Phi_2) &= -\frac{1}{2}\left\{m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}\right]\right\} \\
&\quad + \frac{\lambda_1}{2}(\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) \\
&\quad + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{1}{2}\left[\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.}\right] \\
&\quad + \left\{\left[\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)\right](\Phi_1^\dagger \Phi_2) + \text{h.c.}\right\} \\
&\equiv Y_{a\bar{b}} \Phi_{\bar{a}}^\dagger \Phi_b + \frac{1}{2} Z_{a\bar{b}c\bar{d}} (\Phi_{\bar{a}}^\dagger \Phi_b)(\Phi_{\bar{c}}^\dagger \Phi_d)
\end{aligned}
$$

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The model: Vacuum expectation values

Most general form that conserves electric charge:

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 e^{i\xi_1} \end{pmatrix}, \quad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\xi_2} \end{pmatrix}$$

$$v_1^2 + v_2^2 = v^2 = (246\,\mathrm{GeV})^2$$

The model: Extracting the physical fields

$$\Phi_j = e^{i\xi_j} \begin{pmatrix} \varphi_j^+ \\ (v_j + \eta_j + i\chi_j)/\sqrt{2} \end{pmatrix}, \quad j = 1, 2.$$

Introduce orthogonal states:

$$\begin{pmatrix} G_0 \\ \eta_3 \end{pmatrix} = \begin{pmatrix} v_1/v & v_2/v \\ -v_2/v & v_1/v \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}$$

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} v_1/v & v_2/v \\ -v_2/v & v_1/v \end{pmatrix} \begin{pmatrix} \varphi_1^\pm \\ \varphi_2^\pm \end{pmatrix}.$$

The model: Neutral sector mass matrix and the neutral scalars

Introduce neutral sector rotation matrix:

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}$$

satisfying $R\mathcal{M}^2 R^{\mathrm{T}} = \mathrm{diag}(M_1^2, M_2^2, M_3^2)$

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Important couplings involving gauge bosons: The coupling e_i

$$H_i H_j Z_\mu : \quad \frac{g}{2v\cos\theta_{\mathrm{W}}}\epsilon_{ijk}e_k(p_i - p_j)_\mu,$$

$$H_i Z_\mu Z_\nu : \quad \frac{ig^2}{2\cos^2\theta_{\mathrm{W}}}e_i\, g_{\mu\nu},$$

$$H_i W_\mu^+ W_\nu^- : \quad \frac{ig^2}{2}e_i\, g_{\mu\nu}.$$

$$e_i \equiv v_1 R_{i1} + v_2 R_{i2}$$

$$e_1^2 + e_2^2 + e_3^3 = v^2 = (246\,\mathrm{GeV})^2$$

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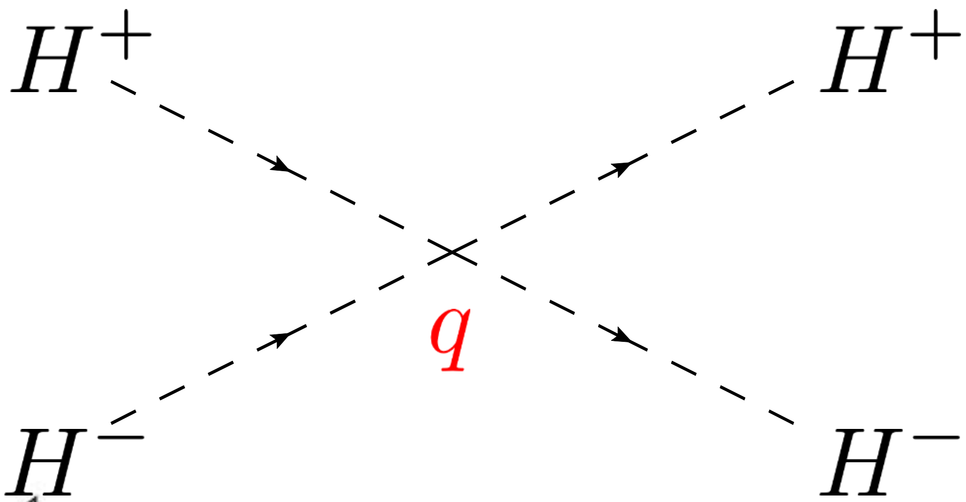
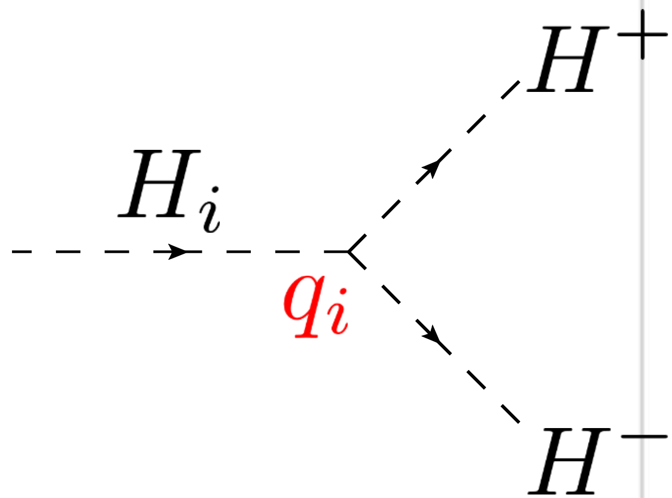
Important couplings involving charged scalars: *q* and q_i

$$q_i \equiv \text{Coefficient}(V, H_i H^- H^+)$$

$$q \equiv \text{Coefficient}(V, H^- H^- H^+ H^+)$$





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Change of basis

- Initial expression of potential is defined with respect to fields Φ_1 and Φ_2 - which defines the basis.
- We can change to a new basis by the following transformation

$$\bar{\Phi}_i = U_{ij}\Phi_j$$

  where *U* is a U(2) matrix.
- Observables must not depend on choice of basis – they should be basis-independent.
- We choose to work in the most general basis.

CP violation

- Building on earlier work by Lavoura&Silva [Phys.Rev.**D50** 4619], Botella&Silva [Phys.Rev.**D51** 3870, Gunion&Haber [Phys.Rev. **D72** 095002], we expressed invariants that control the CP properties of the 2HDM in terms of observables only (couplings and masses).
- The vanishing of these three invariants, Im J_1, Im J_2 and Im J_{30} is a sufficient and necessary condition for CP conservation in the 2HDM.
- Each invariant can be identified with a physical process allowing for its measuring.

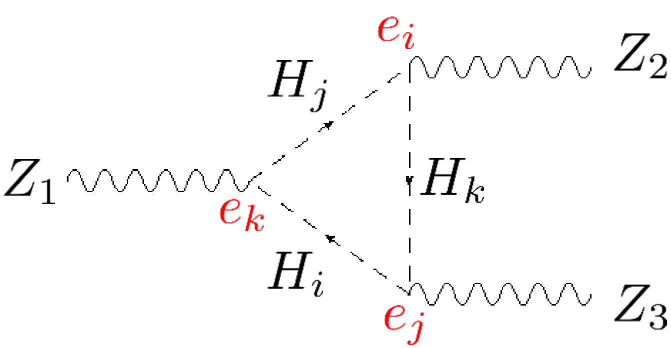
The three invariants Im J_i

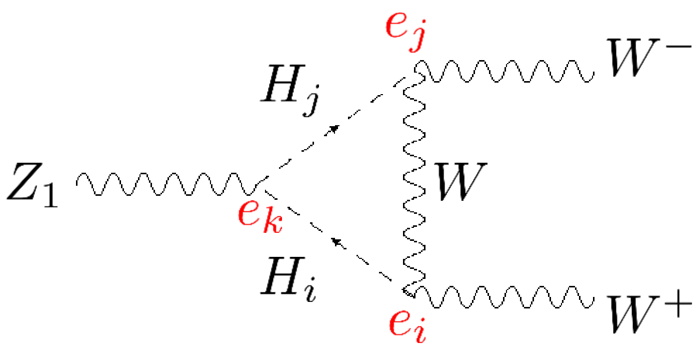
$$
\begin{aligned}
\operatorname{Im} J_1 &= \frac{1}{v^5}\sum_{i,j,k}\epsilon_{ijk}M_i^2 e_i e_k q_j \\
&= \frac{1}{v^5}[e_1e_2q_3(M_2^2-M_1^2)-e_1e_3q_2(M_3^2-M_1^2)+e_2e_3q_1(M_3^2-M_2^2)], \\
\operatorname{Im} J_2 &= \frac{2}{v^9}\sum_{i,j,k}\epsilon_{ijk}e_ie_je_kM_i^4M_k^2 = \frac{2e_1e_2e_3}{v^9}\sum_{i,j,k}\epsilon_{ijk}M_i^4M_k^2 \\
&= \frac{2e_1e_2e_3}{v^9}(M_2^2-M_1^2)(M_3^2-M_2^2)(M_3^2-M_1^2), \\
\operatorname{Im} J_{30} &\equiv \frac{1}{v^5}\sum_{i,j,k}\epsilon_{ijk}q_iM_i^2e_jq_k, \\
&= \frac{1}{v^5}[q_1q_2e_3(M_2^2-M_1^2)-q_1q_3e_2(M_3^2-M_1^2)+q_2q_3e_1(M_3^2-M_2^2)].
\end{aligned}
$$

Each invariant relates to a physical process!

Processes containing Im J_2:

- ZZZ vertex and ZWW vertex

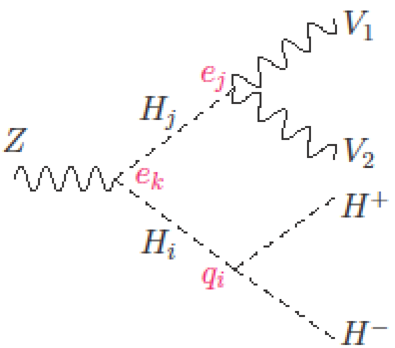


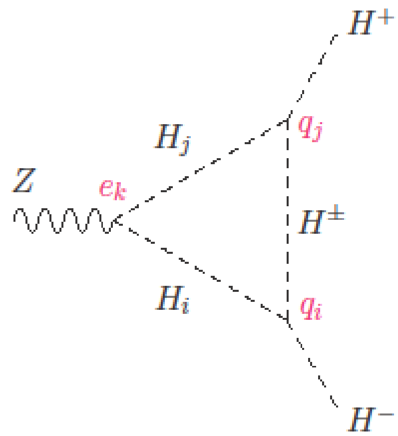


- Summing over all possible combinations of *i*,*j*,*k*, we find $\mathcal{M} \propto \mathrm{Im} J_2$

Processes containing Im J_1 and Im J_3 :

- $Z \to VVH^+H^-$ or $Z \to H^+H^-$





- Summing over all possible combinations of *i,j,k*, we find

$\mathcal{M}$ contains $\mathrm{Im} J_1$ $\qquad$ $\mathcal{M}$ contains $\mathrm{Im} J_3$

Spontaneous CP violation

- Finding CP violation does not tell us whether the CP violation is spontaneous or explicit.
- Four other invariants can be used to check the type of CP violation present in the model.
- Gunion&Haber: I_{Y3Z}, I_{2Y2Z}, I_{3Y3Z}, I_{6Z}.
- (See also work by Branco, Rebelo & Silva-Marcos [Phys.Lett. **B614** 187] for invariants of this type.)

$$I_{Y3Z} = \mathrm{Im}\left[Z^{(1)}_{a\bar{c}} Z^{(1)}_{e\bar{b}} Z_{b\bar{e}c\bar{d}} Y_{d\bar{a}}\right],$$

$$I_{2Y2Z} = \mathrm{Im}\left[Y_{a\bar{b}} Y_{c\bar{d}} Z_{b\bar{a}d\bar{f}} Z^{(1)}_{f\bar{c}}\right],$$

$$I_{3Y3Z} = \mathrm{Im}\left[Z_{a\bar{c}b\bar{d}} Z_{c\bar{e}d\bar{g}} Z_{e\bar{h}f\bar{q}} Y_{g\bar{a}} Y_{h\bar{b}} Y_{q\bar{f}}\right],$$

$$I_{6Z} = \mathrm{Im}\left[Z_{a\bar{b}c\bar{d}} Z^{(1)}_{b\bar{f}} Z^{(1)}_{d\bar{h}} Z_{f\bar{a}j\bar{k}} Z_{k\bar{j}m\bar{n}} Z_{n\bar{m}h\bar{c}}\right].$$

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Spontaneous CP violation

- The invariants I_{Y3Z}, I_{2Y2Z}, I_{3Y3Z}, I_{6Z} are easily calculated in terms of parameters of the potential, but offers little physical insight.
- Can these also be re-expressed in terms of couplings/masses just like the Im J_i ?
- The answer is YES!!!
- Introduce notation

$$
\begin{aligned}
\mathrm{Im}\, J_{11} &\equiv \frac{1}{v^7}\sum_{i,j,k} \epsilon_{ijk} e_i M_i^2 M_j^2 e_k q_j \\
&= \frac{1}{v^7}[e_1 e_2 q_3 M_3^2(M_2^2 - M_1^2) - e_1 e_3 q_2 M_2^2(M_3^2 - M_1^2) + e_2 e_3 q_1 M_1^2(M_3^2 - M_2^2)], \\
d_{ijk} &= \frac{q_1^i M_1^{2j} e_1^k + q_2^i M_2^{2j} e_2^k + q_3^i M_3^{2j} e_3^k}{v^{i+2j+k}}, \\
m_+ &= \frac{M_{H^\pm}^2}{v^2}.
\end{aligned}
$$

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I_{Y3Z} and I_{2Y2Z}

$$
\begin{aligned}
\frac{I_{Y3Z}}{v^2} &= (d_{010}d_{012} - d_{010}d_{101} + 2d_{010}m_+ - 2d_{012}m_+ - d_{022} - 2d_{101}m_+ + d_{200})\,\mathrm{Im}\,J_1 \\
&+ (-d_{012} + 2d_{101} - 4m_+ - 2q)\,\mathrm{Im}\,J_{11} + \left(\frac{d_{101}}{2} - m_+ - q\right)\mathrm{Im}\,J_2 \\
&+ (-d_{012} + d_{101} - 2m_+)\,\mathrm{Im}\,J_{30}, \\
\frac{I_{2Y2Z}}{v^4} &= \left(\frac{d_{010}d_{012}}{2} - \frac{d_{010}d_{101}}{2} + d_{010}m_+ - d_{012}m_+ - \frac{d_{022}}{2} + \frac{d_{111}}{2}\right)\mathrm{Im}\,J_1 \\
&+ \left(-\frac{d_{012}}{2} + \frac{d_{101}}{2} - m_+\right)\mathrm{Im}\,J_{11} + \left(\frac{d_{101}}{4} - \frac{m_+}{2} - \frac{q}{2}\right)\mathrm{Im}\,J_2.
\end{aligned}
$$

Note that both expressions are linear in q and m_+

I_{6Z} and I_{3Y3Z}

$$
\begin{aligned}
I_{6Z} \;=\; & \big(4d_{010}^3 d_{012} - 8d_{010}^2 d_{101}^2 + 16d_{010}^2 q^2 - 12d_{010}^2 d_{022} + 8d_{010}^2 d_{200} - 8d_{010}^2 d_{012} q - 8d_{010}^2 d_{012} m_+ + 16d_{010}^2 q m_+ + 4d_{010} d_{101}^3 + 2d_{010} d_{012} d_{101}^2 - 40 d_{010} d_{012} q^2 \\
& +8d_{010} d_{101} q^2 + 8d_{010} d_{012} m_+^2 - 16 d_{010} q m_+^2 - 4d_{010} d_{012} d_{020} + 4d_{010} d_{012} d_{022} + 12 d_{010} d_{032} - 2d_{010} d_{012}^2 d_{101} + 4d_{010} d_{022} d_{101} + 36 d_{010} d_{101} d_{111} - 16 d_{010} d_{012} d_{200} \\
& -4d_{010} d_{101} d_{200} - 24 d_{010} d_{210} - 4d_{010} d_{012}^2 q + 12 d_{010} d_{101}^2 q - 16 d_{010} d_{020} q + 40 d_{010} d_{022} q - 20 d_{010} d_{012} d_{101} q + 12 d_{010} d_{111} q - 8d_{010} d_{200} q + 8 d_{010} d_{101}^2 m_+ \\
& -16 d_{010} q^2 m_+ + 20 d_{010} d_{022} m_+ - 8 d_{010} d_{200} m_+ - 32 d_{010} d_{012} q m_+ - 4d_{022}^2 - 4d_{200}^2 - 16 d_{020} q^2 + 40 d_{022} q^2 - 8 d_{111} q^2 - 8 d_{012}^2 m_+^2 - 8 d_{012} d_{101} m_+^2 + 16 d_{012} q m_+^2 \\
& +16 d_{101} q m_+^2 - 4 d_{012} d_{020} d_{101} + 2 d_{012} d_{022} d_{101} + 6 d_{012}^2 d_{111} - 8 d_{101}^2 d_{111} + 8 d_{020} d_{111} - 16 d_{022} d_{111} - 4 d_{012} d_{101} d_{111} + 2 d_{012}^2 d_{200} + 4 d_{101}^2 d_{200} + 8 d_{022} d_{200} + 6 d_{012} d_{101} d_{200} \\
& +32 d_{012} d_{020} q + 4 d_{012} d_{022} q + 16 d_{030} q - 64 d_{032} q - 8 d_{020} d_{101} q + 24 d_{022} d_{101} q - 4 d_{012} d_{111} q - 16 d_{101} d_{111} q + 4 d_{012} d_{200} q + 4 d_{101} d_{200} q - 8 d_{101}^3 m_+ - 4 d_{012} d_{101}^2 m_+ + 16 d_{012} q^2 m_+ \\
& -12 d_{032} m_+ - 8 d_{022} d_{101} m_+ - 8 d_{101} d_{111} m_+ + 12 d_{012} d_{200} m_+ + 8 d_{101} d_{200} m_+ + 8 d_{210} m_+ + 8 d_{012}^2 q m_+ - 8 d_{101}^2 q m_+ + 8 d_{022} q m_+ + 16 d_{012} d_{101} q m_+ - 8 d_{200} q m_+\big) \operatorname{Im} J_1 \\
& + \big(-8d_{010}^2 d_{012} + 16 d_{010}^2 q - 16 d_{010} q^2 + 20 d_{010} d_{022} - 8 d_{010} d_{111} - 32 d_{010} d_{012} q + 16 d_{010} d_{101} q + 16 d_{010} d_{012} m_+ - 32 d_{010} q m_+ + 32 d_{012} q^2 - 16 d_{101} q^2 - 16 d_{012} m_+^2 + 32 q m_+^2 + 8 d_{012} d_{020} \\
& \quad -4 d_{012} d_{022} - 16 d_{032} - 4 d_{012}^2 d_{101} + 8 d_{022} d_{101} + 8 d_{012} d_{111} - 40 d_{101} d_{111} + 20 d_{012} d_{200} + 16 d_{210} - 16 d_{101}^2 q - 16 d_{020} q + 24 d_{022} q + 8 d_{012} d_{101} q - 24 d_{111} q + 24 d_{200} q + 4 d_{012}^2 m_+ \\
& \quad -8 d_{101}^2 m_+ + 32 q^2 m_+ - 28 d_{022} m_+ + 16 d_{012} d_{101} m_+ + 8 d_{200} m_+ + 32 d_{012} q m_+ - 32 d_{101} q m_+\big) \operatorname{Im} J_{11} \\
& + \big(8q^3 - 8 d_{010} q^2 + 4 d_{012} q^2 - 8 d_{101} q^2 + 16 q^2 m_+ + 4 d_{010}^2 q - 8 d_{101}^2 q + 8 q m_+^2 - 4 d_{010} d_{012} q - 4 d_{020} q + 4 d_{022} q + 4 d_{010} d_{101} q + 10 d_{200} q - 8 d_{010} q m_+ - 8 d_{101} q m_+ - d_{012} d_{101}^2 \\
& \quad -4 d_{012} m_+^2 - 2 d_{010}^2 d_{012} + 2 d_{012} d_{020} + 4 d_{010} d_{022} - 4 d_{032} + 2 d_{022} d_{101} - 2 d_{010} d_{111} - 4 d_{101}^2 m_+ + 4 d_{010} d_{012} m_+ - 4 d_{022} m_+ - 2 d_{012} d_{101} m_+ + 6 d_{111} m_+ + 4 d_{200} m_+\big) \operatorname{Im} J_2 \\
& + \big(6 d_{012}^3 + 4 d_{010} d_{012}^2 - 8 d_{012}^2 m_+ - 4 d_{010}^2 d_{012} + 2 d_{012} d_{101}^2 - 8 d_{012} m_+^2 + 4 d_{012} d_{020} - 20 d_{012} d_{022} + 12 d_{010} d_{012} d_{101} + 12 d_{012} d_{200} - 24 d_{010} d_{012} q + 8 d_{010} d_{012} m_+ \\
& \quad +32 d_{012} q m_+ + 4 d_{101}^3 + 8 d_{010} d_{101}^2 + 16 q m_+^2 + 8 d_{010} d_{022} + 8 d_{022} d_{101} - 16 d_{010} d_{111} - 24 d_{101} d_{111} - 8 d_{010} d_{200} - 4 d_{101} d_{200} + 8 d_{210} + 8 d_{010}^2 q + 4 d_{101}^2 q - 8 d_{020} q \\
& \quad +28 d_{022} q + 8 d_{010} d_{101} q - 16 d_{111} q - 8 d_{101}^2 m_+ - 8 d_{022} m_+ + 8 d_{111} m_+ + 8 d_{200} m_+ - 16 d_{010} q m_+ - 16 d_{101} q m_+\big) \operatorname{Im} J_{30}
\end{aligned}
$$

$$
\begin{aligned}
\frac{I_{3Y3Z}}{v^6} \;=\; & \Big(\frac{d_{010}^3 d_{012}}{4} - \frac{3 d_{010}^2 d_{022}}{4} + d_{010} d_{012} m_+^2 + \frac{5 d_{010} d_{012} d_{022}}{4} + \frac{d_{010} d_{032}}{4} + \frac{1}{4} d_{010} d_{012}^2 d_{101} + \frac{d_{010} d_{101} d_{111}}{4} - d_{010} d_{012}^2 m_+ + \frac{3}{2} d_{010} d_{022} m_+ - d_{010} d_{111} m_+ - \frac{3 d_{010} d_{012} d_{020}}{4} \\
& \quad - \frac{d_{010} d_{022} d_{101}}{4} - \frac{d_{010} d_{012} d_{111}}{4} + \frac{d_{022} d_{101}^2}{2} - 2 d_{012}^2 m_+^2 - d_{020} m_+^2 + 2 d_{022} m_+^2 - d_{012} d_{101} m_+^2 + \frac{d_{020} d_{022}}{2} + \frac{d_{012} d_{030}}{2} + \frac{d_{012} d_{032}}{4} + \frac{d_{012} d_{020} d_{101}}{2} + \frac{d_{101}^2 d_{111}}{4} \\
& \quad + \frac{d_{022} d_{111}}{2} + \frac{d_{012}^3 m_+}{2} + \frac{1}{2} d_{012} d_{101}^2 m_+ - d_{012} d_{020} m_+ + d_{012}^2 d_{101} m_+ + d_{020} d_{101} m_+ - \frac{5}{2} d_{022} d_{101} m_+ + d_{012} d_{111} m_+ - d_{101} d_{111} m_+ + \frac{1}{2} d_{012} d_{200} m_+ + d_{210} m_+ \\
& \quad - \frac{3 d_{022}^2}{2} - \frac{d_{032} d_{101}}{2} - \frac{d_{020} d_{101}^2}{4} - \frac{d_{012}^2 d_{111}}{4} - \frac{d_{012} d_{101} d_{111}}{4} - \frac{d_{012} d_{101} d_{200}}{4} - \frac{d_{101} d_{210}}{4}\Big) \operatorname{Im} J_1 \\
& + \Big(-\frac{d_{101}^3}{4} + \frac{d_{010} d_{101}^2}{4} + d_{101}^2 m_+ - \frac{d_{012} d_{101}^2}{4} + \frac{d_{022} d_{101}}{2} + \frac{d_{101} d_{200}}{4} + \frac{d_{012} d_{101} q}{2} - d_{010} d_{101} m_+ + d_{012} d_{101} m_+ - \frac{d_{010} d_{012} d_{101}}{2} - \frac{d_{101} d_{111}}{2} - \frac{d_{012}^2 d_{101}}{4} + d_{010} m_+^2 \\
& \quad - 3 d_{012} m_+^2 + \frac{d_{010} d_{022}}{4} + d_{032} + \frac{d_{012} d_{111}}{2} + \frac{d_{012}^2 m_+}{2} + d_{010} d_{012} m_+ - 2 d_{022} m_+ + 2 d_{111} m_+ - d_{200} m_+ - d_{012} q m_+ - \frac{5 d_{012} d_{022}}{4}\Big) \operatorname{Im} J_{11} \\
& + \Big(-\frac{d_{101}^3}{4} + \frac{d_{010} d_{101}^2}{8} + \frac{3 d_{101}^2 m_+}{4} + \frac{d_{010}^2 d_{101}}{4} + \frac{3 d_{022} d_{101}}{4} + \frac{d_{101} d_{200}}{4} + \frac{d_{012} d_{101} q}{2} - \frac{1}{2} d_{010} d_{101} m_+ + \frac{1}{2} d_{012} d_{101} m_+ - \frac{d_{020} d_{101}}{4} - \frac{d_{010} d_{101} q}{4} + \frac{d_{010} m_+^2}{2} - d_{012} m_+^2 \\
& \quad + \frac{d_{012} d_{020}}{4} + \frac{d_{010} d_{022}}{2} + \frac{d_{012}^2 q}{4} + \frac{d_{010} d_{012} q}{4} + \frac{d_{012}^2 m_+}{4} + \frac{5}{4} d_{010} d_{012} m_+ + \frac{d_{020} m_+}{2} + \frac{d_{111} m_+}{2} + \frac{1}{2} d_{010} q m_+ - d_{012} q m_+ - \frac{d_{032}}{2} - \frac{d_{022} q}{2} - \frac{d_{010}^2 m_+}{2} - \frac{3 d_{022} m_+}{2} \\
& \quad - \frac{d_{010}^2 d_{012}}{4} - \frac{d_{012} d_{200}}{4} - \frac{3 d_{200} m_+}{4} - \frac{5 d_{010} d_{111}}{8} - \frac{d_{012} d_{111}}{8}\Big) \operatorname{Im} J_2 \\
& + \Big(\frac{d_{012}^2 d_{101}}{4} - d_{012} m_+^2 + d_{111} m_+ - \frac{d_{022} m_+}{2} - \frac{d_{101} d_{111}}{4}\Big) \operatorname{Im} J_{30}
\end{aligned}
$$

Spontaneous CP violation

- The invariants I_{Y3Z}, I_{2Y2Z}, I_{3Y3Z}, I_{6Z} must all vanish in order to have a CP invariant potential.
- Put $I_{Y3Z} = I_{2Y2Z} = I_{3Y3Z} = I_{6Z} = 0$ and solve for q and $M_{H^\pm}^2$ to get:

$$M_{H^\pm}^2 = \frac{v^2}{2D}[e_1 q_1 M_2^2 M_3^2 + e_2 q_2 M_3^2 M_1^2 + e_3 q_3 M_1^2 M_2^2 - M_1^2 M_2^2 M_3^2],$$

$$q = \frac{1}{2D}[(e_2 q_3 - e_3 q_2)^2 M_1^2 + (e_3 q_1 - e_1 q_3)^2 M_2^2 + (e_1 q_2 - e_2 q_1)^2 M_3^2 + M_1^2 M_2^2 M_3^2]$$

with $D = e_1^2 M_2^2 M_3^2 + e_2^2 M_3^2 M_1^2 + e_3^2 M_1^2 M_2^2$

Spontaneous CP violation (mission accomplished)

Theorem. Let us assume that the quantity

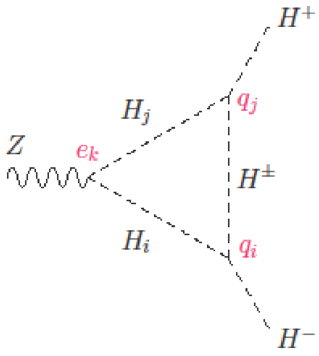
$$D = e_1^2 M_2^2 M_3^2 + e_2^2 M_3^2 M_1^2 + e_3^2 M_1^2 M_2^2 \tag{1}$$

is non-zero[1]. Then, in a charge-conserving general 2HDM, CP is violated spontaneously if and only if the following three statements are satisfied simultaneously:

- At least one of the three invariants $\mathrm{Im}\, J_1$, $\mathrm{Im}\, J_2$, $\mathrm{Im}\, J_{30}$ is nonzero.
- $$M_{H^\pm}^2 = \frac{v^2}{2D}[e_1 q_1 M_2^2 M_3^2 + e_2 q_2 M_3^2 M_1^2 + e_3 q_3 M_1^2 M_2^2 - M_1^2 M_2^2 M_3^2], \tag{2}$$
- $$q = \frac{1}{2D}[(e_2 q_3 - e_3 q_2)^2 M_1^2 + (e_3 q_1 - e_1 q_3)^2 M_2^2 + (e_1 q_2 - e_2 q_1)^2 M_3^2 + M_1^2 M_2^2 M_3^2]. \tag{3}$$

Spontaneous CP violation in the alignment limit

- In the alignment limit $e_1 = v, \quad e_2 = e_3 = 0.$



- Also

$$\mathrm{Im}\, J_1 = \mathrm{Im}\, J_2 = 0,$$
$$\mathrm{Im}\, J_{30} = \frac{q_2 q_3}{v^4}(M_3^2 - M_2^2).$$

- Constraints of theorem simplifies to

$$\begin{aligned} M_{H^\pm}^2 &= \frac{vq_1 - M_1^2}{2}, \\ q &= \frac{1}{2}\left(\frac{q_2^2}{M_2^2} + \frac{q_3^2}{M_3^2} + \frac{M_1^2}{v^2}\right). \end{aligned}$$

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Spontaneous CP violation in the alignment limit

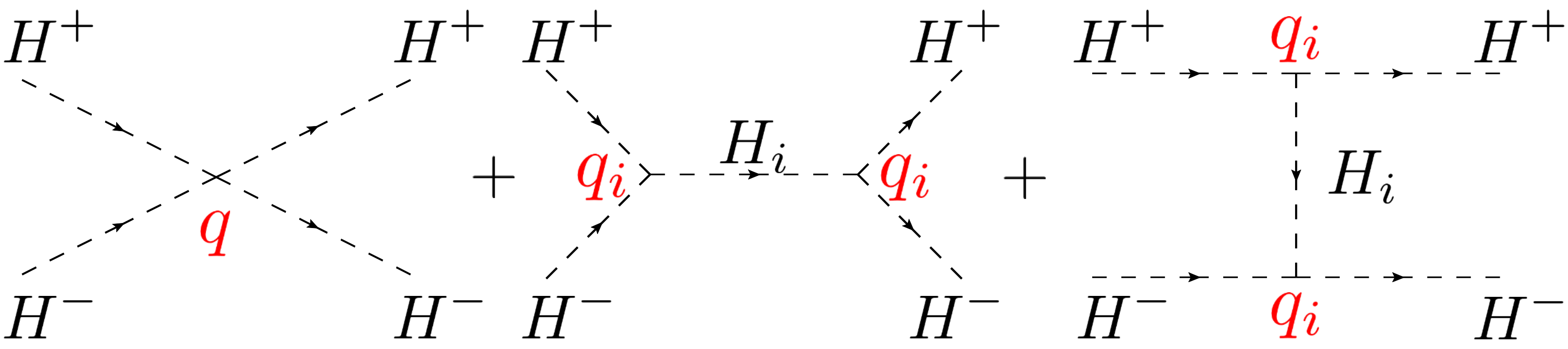
- Rewriting constraints to

$$q_1 = \frac{1}{v}\left(2M_{H\pm}^2 + M_1^2\right),$$

$$q - \frac{1}{2}\left(\frac{q_1^2}{M_1^2} + \frac{q_2^2}{M_2^2} + \frac{q_3^2}{M_3^2}\right) = \frac{-2M_{H\pm}^2(M_1^2 + M_{H\pm}^2)}{v^2 M_1^2}.$$

- First constraint testable by measuring q_1 and compare to $M_{H\pm}^2$ and M_1^2.
- What about the second constraint???

Elastic H^+H^- scattering



Combined amplitude:

$$\mathcal{M} \propto -4iq - \frac{iq_1^2}{s-M_1^2} - \frac{iq_2^2}{s-M_2^2} - \frac{iq_3^2}{s-M_3^2} - \frac{iq_1^2}{t-M_1^2} - \frac{iq_2^2}{t-M_2^2} - \frac{iq_3^2}{t-M_3^2}.$$

Limit: $s \to 0$ and $t \to 0$ yields $\mathcal{M} \propto -4i\left[q - \frac{1}{2}\left(\frac{q_1^2}{M_1^2} + \frac{q_2^2}{M_2^2} + \frac{q_3^2}{M_3^2}\right)\right]$

Lhs. of second condition

Challenge 1: Need colliding beams of charged scalars.

Challenge 2: Limit: $s \to 0$ and $t \to 0$ is impossible to achieve.

Extrapolation and fits needed.

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Summary

- Conditions for Spontaneous CP violation in the 2HDM has been re-expressed in terms of masses and couplings.
- In principle this provides a way to distinguish spontaneous and explicit CP violation in the 2HDM through experiment.
- Realizing such an experiment seems challenging, as seen in the simplified example of the alignment limit.