From E_8-inspired trinification to a L-R symmetric theory

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FCT Fundação para a Ciência e a Tecnologia MINISTÉRIO DA CIÊNCIA, TECNOLOGIA E ENSINO SUPERIOR

LUND UNIVERSITY

Outline

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Trinification models (Glashow, Georgi and De Rujula 1984)

- LR gauge interactions and well motivated by E_6
- $\mathrm{SU}(3)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{R} \times \mathrm{SU}(3)_\mathrm{C}$ with $\mathbb{Z}_3 \rightarrow$ gauge unification

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- All matter can be elegantly arranged in bi-fundamental representations for each generation

$$\mathbf{27}^i = \left(\mathbf{3}, \overline{\mathbf{3}}, \mathbf{1}\right)^i \otimes \left(\mathbf{1}, \mathbf{3}, \overline{\mathbf{3}}\right)^i \otimes \left(\overline{\mathbf{3}}, \mathbf{1}, \mathbf{3}\right)^i \equiv \boldsymbol{L} \otimes \boldsymbol{Q_R} \otimes \boldsymbol{Q_L}$$

- The model can accomodate any quark and lepton masses and mixing angles **(Sayre et al. 2006)**
- Naturally light neutrinos via, e.g. radiative seesaw **(Cauet et al. 2011)**

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- The model can accomodate any quark and lepton masses and mixing angles **(Sayre et al. 2006)**
- Naturally light neutrinos via, e.g. radiative seesaw **(Cauet et al. 2011)**
- Well motivated as low energy versions of $E_8 \times E_8$ heterotic string theory **(Gross et al. 1985)**, E_6 orbifold **(Braam et al. 2010)** or $N = 8$ supergravity **(Cremmer et al. 1979)**.

Issues of standard trinification

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Trinification-based models were left as the least developed GUT scenarios

Our proposal

Novel solution including

1. A global $\mathrm{SU}(3)_{\mathrm{F}}$ family symmetry **inspired** by E_8
2. **Unification of the Higgs and lepton sectors** via a common chiral supermultiplet

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Low energy completion

- non-SUSY multi-scalar and multi-fermion models

Outline

The Model — Chiral supermultiplet representations

$\mathbf{SU(3)_F} \times \mathbf{E_6}$ is a maximal subgroup of E_8

- Branching rules for the fundamental representation of E_8 (Slansky)

$$\mathbf{248} = (\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{78}) \oplus (\mathbf{3}, \mathbf{27}) \oplus (\overline{\mathbf{3}}, \overline{\mathbf{27}})$$

- Branching rules for the adjoint representation of E_6 down to trinification

$$\mathbf{78} = (\mathbf{8}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{8}) \oplus (\mathbf{3}, \mathbf{3}, \overline{\mathbf{3}}) \oplus (\overline{\mathbf{3}}, \overline{\mathbf{3}}, \mathbf{3})$$

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Matter content of our E_8-inspired model in red

$$[\mathrm{SU}(3)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{R} \times \mathrm{SU}(3)_\mathrm{C}] \times \mathbb{Z}_3 \times \mathbf{SU(3)_F}\,,$$

- $\mathbb{Z}_3$ is a cyclic permutation symmetry that enables **gauge coupling unification**

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Chiral Supermultiplet Fields					
Superfield		$\mathrm{SU}(3)_\mathrm{C}$	$\mathrm{SU}(3)_\mathrm{L}$	$\mathrm{SU}(3)_\mathrm{R}$	$\mathrm{SU}(3)_\mathrm{F}$
Lepton	$(L^i)^l{}_r$	$\mathbf{1}$	$\mathbf{3}^l$	$\bar{\mathbf{3}}_r$	$\mathbf{3}^i$
Right-Quark	$(Q_R^i)^r{}_x$	$\bar{\mathbf{3}}_x$	$\mathbf{1}$	$\mathbf{3}^r$	$\mathbf{3}^i$
Left-Quark	$(Q_L^i)^x{}_l$	$\mathbf{3}^x$	$\bar{\mathbf{3}}_l$	$\mathbf{1}$	$\mathbf{3}^i$
Colour-adjoint	Δ_C^a	$\mathbf{8}^a$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Left-adjoint	Δ_L^a	$\mathbf{1}$	$\mathbf{8}^a$	$\mathbf{1}$	$\mathbf{1}$
Right-adjoint	Δ_R^a	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{8}^a$	$\mathbf{1}$
Flavour-adjoint	Δ_F^a	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{8}^a$

Gauge Supermultiplet Fields					
Superfield		$\mathrm{SU}(3)_\mathrm{C}$	$\mathrm{SU}(3)_\mathrm{L}$	$\mathrm{SU}(3)_\mathrm{R}$	$\mathrm{SU}(3)_\mathrm{F}$
Gluon	$G_C^{\mu a},\ \lambda_C^a$	$\mathbf{8}^a$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Left-Gluon	$G_L^{\mu a},\ \lambda_L^a$	$\mathbf{1}$	$\mathbf{8}^a$	$\mathbf{1}$	$\mathbf{1}$
Right-Gluon	$G_R^{\mu a},\ \lambda_R^a$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{8}^a$	$\mathbf{1}$

Fundamental tri-triplets:

$$\left(L^i\right)^l{}_r = \begin{pmatrix} H_{11} & H_{12} & \nu_L \\ H_{21} & H_{22} & e_L \\ \nu_R & e_R & \phi \end{pmatrix}^i, \left(Q_R^i\right)^r{}_x = \begin{pmatrix} u_R^{\bar{1}} & u_R^{\bar{2}} & u_R^{\bar{3}} \\ d_R^{\bar{1}} & d_R^{\bar{2}} & d_R^{\bar{3}} \\ D_R^{\bar{1}} & D_R^{\bar{2}} & D_R^{\bar{3}} \end{pmatrix}^i, \left(Q_L^i\right)^x{}_l = \begin{pmatrix} u_L^1 & d_L^1 & D_L^1 \\ u_L^2 & d_L^2 & D_L^2 \\ u_L^3 & d_L^3 & D_L^3 \end{pmatrix}^i$$

$\mathbb{Z}_3$ cyclic permutations:

$$L \xrightarrow{\mathbb{Z}_3} Q_\mathrm{L}\,,$$
$$Q_\mathrm{L} \xrightarrow{\mathbb{Z}_3} Q_\mathrm{R}\,,$$
$$Q_\mathrm{R} \xrightarrow{\mathbb{Z}_3} L.$$

We refer to the model as **Supersymmetric Higgs-Unified Trinification**

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SHUT

Superpotential

$$W = \sum_{A=L,R,C} \left(\lambda_{78} d_{abc} \Delta_A^a \Delta_A^b \Delta_A^c + \mu_{78} \delta_{ab} \Delta_A^a \Delta_A^b\right) + \left(\lambda_1 d_{abc} \Delta_F^a \Delta_F^b \Delta_F^c + \mu_1 \delta_{ab} \Delta_F^a \Delta_F^b\right)$$
$$+ \lambda_{27} \varepsilon_{ijk} \left(Q_L^i\right)^x{}_l \left(Q_R^j\right)^r{}_x \left(L^k\right)^l{}_r \,, \quad \text{with} \;\; d_{abc} = 2\mathrm{Tr}\left[\{T_a, T_b\} T_c\right]$$

- i, j and k $\to$ flavour indices
- x, l and r $\to$ colour, left-chirality and right-chirality respectively
- a, b and c $\to$ adjoint indices.
- **Universal Yukawa coupling for chiral quarks at unification scale, λ_{27}**

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    - **Why then $\mathbf{E_8}$?**
    - Minimal SUSY trinification with $\mathrm{SU}(3)_{\mathrm{F}}$ and Higgs-lepton unification does not have a stable vacuum $\to$ **$\mathbf{SU(3)_C}$ and $\mathbf{SU(2)_L}$ fully broken at GUT scale**

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- **SHUT is the minimal working model!**

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Symmetry breaking

Scalar potential

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(1) D-terms

$$
\begin{aligned}
V_{\mathfrak{D}} = -\frac{1}{2} g_U^2 \Bigg\{ \sum_c & \left(\tilde{\Delta}_L^{a*} f_{abc} \tilde{\Delta}_L^b\right) \left(\tilde{\Delta}_L^{d*} f_{dec} \tilde{\Delta}_L^e\right) \\
& -\mathrm{i} \left(\tilde{\Delta}_L^{a*} f_{abc} \tilde{\Delta}_L^b\right) \left[\left(\tilde{L}_i^*\right)^{r_1}{}_{l_1} \left(T^c\right)^{l_1}{}_{l_2} \left(\tilde{L}^i\right)^{l_2}{}_{r_1} - \left(\tilde{Q}_L^i\right)^{x_1}{}_{l_3} \left(T^c\right)^{l_3}{}_{l_2} \left(\tilde{Q}_{Li}^*\right)^{l_2}{}_{x_1} \right] \Bigg\} \\
& + \frac{1}{2} g_U^2 \left[T^a\right]^{l_1}{}_{l_2} \left[T_a\right]^{l_3}{}_{l_4} \Bigg[\left(\tilde{L}_i^*\right)^{r_1}{}_{l_1} \left(\tilde{L}^i\right)^{l_2}{}_{r_1} \left(\tilde{L}_j^*\right)^{r_2}{}_{l_3} \left(\tilde{L}^j\right)^{l_4}{}_{r_2} \\
& \qquad + \left(\tilde{Q}_L^i\right)^{x_1}{}_{l_1} \left(\tilde{Q}_{Li}^*\right)^{l_2}{}_{x_1} \left(\tilde{Q}_L^j\right)^{x_2}{}_{l_3} \left(\tilde{Q}_{Lj}^*\right)^{l_4}{}_{x_2} \\
& \qquad - 2 \left(\tilde{L}_i^*\right)^{r_1}{}_{l_1} \left(\tilde{L}^i\right)^{l_2}{}_{r_1} \left(\tilde{Q}_L^j\right)^{x_2}{}_{l_3} \left(\tilde{Q}_{Lj}^*\right)^{l_4}{}_{x_2} \Bigg] \\
& \qquad + (\mathbb{Z}_3 \text{ permutations})
\end{aligned}
$$

- $g_L = g_R = g_C \equiv g_U$
- D-term interactions between adjoint and fundamental scalars

(2) Soft SUSY-breaking terms

$$
\begin{aligned}
V_{\text{soft}}^{\text{gauge}} = {} & m_{\mathbf{27}}^2 \left[\left(\tilde{L}^i\right)^l{}_r \left(\tilde{L}_i^*\right)^r{}_l \right] + \delta_{ab} \left[b_{\mathbf{78}} \tilde{\Delta}_L^a \tilde{\Delta}_L^b + m_{\mathbf{78}}^2 \tilde{\Delta}_L^{*a} \tilde{\Delta}_L^b + c.c \right] \\
& + d_{abc} \left[A_{\mathbf{78}} \tilde{\Delta}_L^a \tilde{\Delta}_L^b \tilde{\Delta}_L^c + C_{\mathbf{78}} \tilde{\Delta}_L^{*a} \tilde{\Delta}_L^b \tilde{\Delta}_L^c + c.c. \right] \\
& + A_G \left[\tilde{\Delta}_L^a \left(T_a\right)_{l_1}^{l_2} \left(\tilde{L}_i^*\right)^r{}_{l_1} \left(\tilde{L}^i\right)^{l_2}{}_r + c.c. \right] \\
& + A_{\mathbf{27}} \left[\varepsilon_{ijk} \left(\tilde{Q}_L^i\right)^x{}_l \left(\tilde{Q}_R^j\right)^r{}_x \left(\tilde{L}^k\right)^l{}_r + c.c. \right] + \mathbb{Z}_3 \text{ permutations}
\end{aligned}
$$

$$
\begin{aligned}
V_{\text{soft}}^{\text{global}} = {} & \delta_{ab} \left[b_{\mathbf{1}}^2 \tilde{\Delta}_F^a \tilde{\Delta}_F^b + m_{\mathbf{1}}^2 \tilde{\Delta}_F^{*a} \tilde{\Delta}_F^b + c.c \right] + A_{\mathbf{1}} d_{abc} \left[\tilde{\Delta}_F^a \tilde{\Delta}_F^b \tilde{\Delta}_F^c + c.c. \right] \\
& + A_F \left[\tilde{\Delta}_F^a \left(T_a\right)_j^i \left(\tilde{L}_i^*\right)^r{}_l \left(\tilde{L}^j\right)^l{}_r + c.c. \right] + \mathbb{Z}_3 \text{ permutations}
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- The only fully diagonal $\mathrm{SU}(3)$ generator is T^8

$$T^8_{\mathrm{A}} = \frac{1}{2\sqrt{2}} \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \hline 0 & 0 & -2 \end{array} \right) ,$$

- $\langle \tilde{\Delta}^8_{L,R} \rangle = v$ and $\langle \tilde{\Delta}^8_F \rangle = v_F$ we break SHUT symmetry to a rank-6 LR effective model:

$$[\mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{SU}(3)_{\mathrm{L}} \times \mathrm{SU}(3)_{\mathrm{R}}] \times \mathbb{Z}_3 \times \mathrm{SU}(3)_{\mathrm{F}} \longrightarrow$$
$$\mathrm{SU}(3)_{\mathrm{C}} \times [\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}} \times \mathrm{U}(1)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{R}}] \times \mathbb{Z}_2 \times \mathrm{SU}(2)_{\mathrm{F}} \times \mathrm{U}(1)_{\mathrm{F}}$$

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$$\mathrm{SU}(3)_{\mathrm{C}} \times [\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}} \times \mathrm{U}(1)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{R}}] \times \mathbb{Z}_2 \times \mathrm{SU}(2)_{\mathrm{F}} \times \mathrm{U}(1)_{\mathrm{F}}$$

Minimization:

1. **Positive mass spectrum for the full scalar sector→STABLE MINIMUM**
2. 8 gauge goldstones in the **adjoint sector**

$$\left| D^{\mu} \left\langle \tilde{\Delta}^b_{L,R} \right\rangle \right|^2 = \frac{3}{4} g_U^2 v^2 \sum_{a=4}^{7} \eta_{\mu\nu} G^{\mu a}_{L,R} G^{\nu a}_{L,R} \,,$$

3. 4 flavour goldstones (absorbed by flavour gauge bosons or decouple according to Burgess [hep-ph/9812468])

Higgs-slepton and squark masses

# of real d.o.f.'s		$(mass)^2$	Scalar components
8	**2 doublets**	$m_{27}^2 - \frac{1}{\sqrt{6}}\left(A_G v + 2A_F v_F\right)$	$\tilde{\nu}_R^{(3)}, \tilde{e}_R^{(3)}, \tilde{\nu}_L^{(3)}, \tilde{e}_L^{(3)}$
2	**1 singlet**	$m_{27}^2 - \frac{1}{\sqrt{6}}\left(4A_G v + 2A_F v_F\right)$	$\tilde{\phi}^{(3)}$
8	**1 bi-doublet**	$m_{27}^2 + \frac{1}{\sqrt{6}}\left(2A_G v - 2A_F v_F\right)$	$H_{11}^{(3)}, H_{21}^{(3)}, H_{12}^{(3)}, H_{22}^{(3)}$
4	**2 singlets**	$m_{27}^2 - \frac{1}{\sqrt{6}}\left(4A_G v - A_F v_F\right)$	$\tilde{\phi}^{(1,2)}$
16	**4 doublets**	$m_{27}^2 - \frac{1}{\sqrt{6}}\left(A_G v - A_F v_F\right)$	$\tilde{\nu}_R^{(1,2)}, \tilde{e}_R^{(1,2)}, \tilde{\nu}_L^{(1,2)}, \tilde{e}_L^{(1,2)}$
16	**2 bi-doublets**	$m_{27}^2 + \frac{1}{\sqrt{6}}\left(2A_G v + A_F v_F\right)$	$H_{11}^{(1,2)}, H_{21}^{(1,2)}, H_{12}^{(1,2)}, H_{22}^{(1,2)}$
24		$m_{27}^2 + \frac{1}{\sqrt{6}}\left(A_G v - 2A_F v_F\right)$	$\tilde{u}_L^{(3)}, \tilde{d}_L^{(3)}, \tilde{u}_R^{(3)}, \tilde{d}_R^{(3)}$
12		$m_{27}^2 - \frac{1}{\sqrt{6}}\left(2A_G v + 2A_F v_F\right)$	$\tilde{D}_L^{(3)}, \tilde{D}_R^{(3)}$
48		$m_{27}^2 + \frac{1}{\sqrt{6}}\left(A_G v + A_F v_F\right)$	$\tilde{u}_L^{(1,2)}, \tilde{d}_L^{(1,2)}, \tilde{u}_R^{(1,2)}, \tilde{d}_R^{(1,2)}$
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$$\left(\begin{array}{cc|c} H_{11} & H_{12} & \nu_L \\ H_{21} & H_{22} & e_L \\ \hline \nu_R & e_R & \phi \end{array}\right)^{(1,2|3)}, \quad \left(\begin{array}{ccc} u_R^{\bar{1}} & u_R^{\bar{2}} & u_R^{\bar{3}} \\ d_R^{\bar{1}} & d_R^{\bar{2}} & d_R^{\bar{3}} \\ \hline D_R^{\bar{1}} & D_R^{\bar{2}} & D_R^{\bar{3}} \end{array}\right)^{(1,2|3)}, \quad \left(\begin{array}{cc|c} u_L^1 & d_L^1 & D_L^1 \\ u_L^2 & d_L^2 & D_L^2 \\ u_L^3 & d_L^3 & D_L^3 \end{array}\right)^{(1,2|3)},$$

Effective non-SUSY multi-Higgs models

A plethora of effective L-R non-SUSY models

Light eigenstates	model label
$\tilde{\phi}^{(3)}$	Model 1 A
$\tilde{\phi}^{(1,2)}$	Model 1 B
$\tilde{\phi}^{(3)}$, $\tilde{\phi}^{(1,2)}$	Model 1 A+B
$H^{(1,2\|3)}$, $\tilde{E}_{L,R}^{(1,2\|3)}$, $\tilde{\phi}^{(1,2\|3)}$, $\tilde{D}^{(1,2\|3)}$, $\tilde{u}_{L,R}^{(1,2\|3)}$, $\tilde{d}_{L,R}^{(1,2\|3)}$	Model 1 C , Model 1 C $\not{\mathbb{Z}}_3$
$H^{(3)}$	Model 2 A
$H^{(3)}$, $\tilde{E}_{L,R}^{(3)}$, $\tilde{\phi}^{(3)}$, $\tilde{D}^{(3)}$, $\tilde{u}_{L,R}^{(3)}$, $\tilde{d}_{L,R}^{(3)}$	Model 2 B , Model 2 B $\not{\mathbb{Z}}_3$
$H^{(1,2)}$, $\tilde{E}_{L,R}^{(1,2)}$, $\tilde{\phi}^{(1,2)}$, $\tilde{D}^{(1,2)}$, $\tilde{u}_{L,R}^{(1,2)}$, $\tilde{d}_{L,R}^{(1,2)}$	Model 3 , Model 3 $\not{\mathbb{Z}}_3$
$H^{(1,2)}$	Model 4
$H^{(1,2)}$, $H^{(3)}$	Model 5

- $\not{\mathbb{Z}}_3$ denotes softly broken $\mathbb{Z}_3$ symmetry in the scalar sector
  - if preserved need to be radiatively broken by $\phi^{(1,2)}$ or $\phi^{(3)}$ VEVs

Fermion masses

Scalar-fermion terms

$$\mathcal{L}^{\text{fermion}} = \mathcal{L}^{\text{fermion}}_{\mathcal{F}} + \mathcal{L}^{\text{fermion}}_{\mathcal{D}} + \mathcal{L}^{\text{fermion}}_{\text{soft}} \qquad (\text{F} - \text{terms from the superpotential})$$

- No F-term interactions mixing adjoint and fundamental sectors

D-terms

$$\mathcal{L}^{\text{fermion}}_{\mathcal{D}} = -\sqrt{2} g_U \left[\left(\tilde{L}^*_i\right)^r{}_{l_1} \left(T^a\right)^{l_1}{}_{l_2} \left(L^i\right)^{l_2}{}_r \tilde{\lambda}^a_L + \left(\tilde{L}^*_i\right)^{r_1}{}_l \left(T^a\right)^{r_2}{}_{r_1} \left(L^i\right)^l{}_{r_2} \tilde{\lambda}^a_R \right.$$
$$\left. - \mathrm{i} f_{abc} \tilde{\Delta}^{*b}_L \Delta^c_L \tilde{\lambda}^a_L \right] + (\mathbb{Z}_3 \text{ permutations}) \,.$$

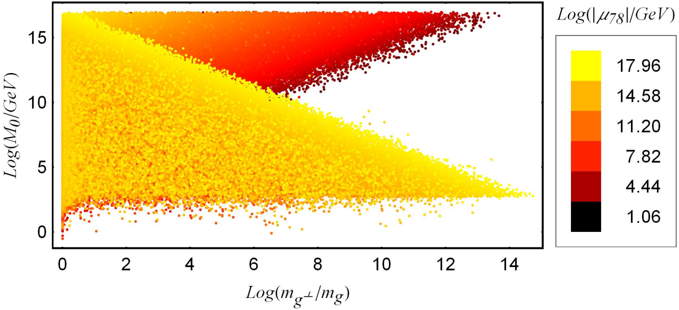
Soft SUSY-breaking terms

$$\mathcal{L}^{\text{fermion}}_{\text{soft}} = -\frac{1}{2} M_0 \delta_{ab} \tilde{\lambda}^a_L \tilde{\lambda}^b_L - M'_0 \delta_{ab} \tilde{\lambda}^a_L \Delta^b_L + h.c. + (\mathbb{Z}_3 \text{ permutations}) \,,$$

- **The model naturally contains Dirac gaugino mass terms**

# of Weyl spinors	$(mass)^2$	Fermionic components
81	0	$\phi^{(1,2\|3)}, \tilde{H}^{(1,2\|3)}, e_{L,R}^{(1,2\|3)}, \nu_{L,R}^{(1,2\|3)}$
1	$\frac{1}{6}\left(v_F^2\lambda_{\mathbf{1}}^2 - 2\sqrt{6}v_F\lambda_{\mathbf{1}}\mu_{\mathbf{1}} + 6\mu_{\mathbf{1}}^2\right)$	$\Delta_F^8 \equiv \mathcal{S}_F$
3	$\frac{1}{6}\left(v_F^2\lambda_{\mathbf{1}}^2 + 2\sqrt{6}v_F\lambda_{\mathbf{1}}\mu_{\mathbf{1}} + 6\mu_{\mathbf{1}}^2\right)$	$\Delta_F^{1,2,3} \equiv T_F$
4	$\frac{1}{24}\left(v_F^2\lambda_{\mathbf{1}}^2 - 4\sqrt{6}v_F\lambda_{\mathbf{1}}\mu_{\mathbf{1}} + 24\mu_{\mathbf{1}}^2\right)$	$\Delta_F^{4,5,6,7} \equiv \tilde{\mathcal{H}}_F$
8 **low scale gluinos**?	$\frac{1}{2}\left(X_C^{\mathbf{8}} - \sqrt{Y_C^{\mathbf{8}} + Z_C^{\mathbf{8}}}\right)$	$c_{\theta_{\mathbf{8}}}\tilde{\lambda}_C^a - s_{\theta_{\mathbf{8}}}\Delta_C^a \equiv \tilde{g}^a$
8	$\frac{1}{2}\left(X_C^{\mathbf{8}} + \sqrt{Y_C^{\mathbf{8}} + Z_C^{\mathbf{8}}}\right)$	$s_{\theta_{\mathbf{8}}}\tilde{\lambda}_C^a + c_{\theta_{\mathbf{8}}}\Delta_C^a \equiv \tilde{g}_\perp^a$
2	$\frac{1}{24}\left(X_{L,R}^{\mathbf{1}} - \sqrt{Y_{L,R}^{\mathbf{1}} + Z_{L,R}^{\mathbf{1}}}\right)$	$c_{\theta_{\mathbf{1}}}\tilde{\lambda}_{L,R}^8 - s_{\theta_{\mathbf{1}}}\Delta_{L,R}^8 \equiv \mathcal{S}_{L,R}$
2	$\frac{1}{24}\left(X_{L,R}^{\mathbf{1}} + \sqrt{Y_{L,R}^{\mathbf{1}} + Z_{L,R}^{\mathbf{1}}}\right)$	$s_{\theta_{\mathbf{1}}}\tilde{\lambda}_{L,R}^8 + c_{\theta_{\mathbf{1}}}\Delta_{L,R}^8 \equiv \mathcal{S}_{L,R}^\perp$
6	$\frac{1}{24}\left(X_{L,R}^{\mathbf{3}} - \sqrt{Y_{L,R}^{\mathbf{3}} + Z_{L,R}^{\mathbf{3}}}\right)$	$c_{\theta_{\mathbf{3}}}\tilde{\lambda}_{L,R}^{1,2,3} - s_{\theta_{\mathbf{3}}}\Delta_{L,R}^{1,2,3} \equiv T_{L,R}$
6	$\frac{1}{24}\left(X_{L,R}^{\mathbf{3}} + \sqrt{Y_{L,R}^{\mathbf{3}} + Z_{L,R}^{\mathbf{3}}}\right)$	$s_{\theta_{\mathbf{3}}}\tilde{\lambda}_{L,R}^{1,2,3} + c_{\theta_{\mathbf{3}}}\Delta_{L,R}^{1,2,3} \equiv T_{L,R}^\perp$
8	$\frac{1}{48}\left(X_{L,R}^{\mathbf{2}} - \sqrt{Y_{L,R}^{\mathbf{2}} + Z_{L,R}^{\mathbf{2}}}\right)$	$\rho_1\Delta_{L,R}^{4,6} + \rho_2\Delta_{L,R}^{5,7} + \rho_3\tilde{\lambda}_{L,R}^{4,6} + \rho_4\tilde{\lambda}_{L,R}^{5,7} \equiv \tilde{\mathcal{H}}_{L,R}$
8	$\frac{1}{48}\left(X_{L,R}^{\mathbf{2}} + \sqrt{Y_{L,R}^{\mathbf{2}} + Z_{L,R}^{\mathbf{2}}}\right)$	$\overline{\rho}_1\Delta_{L,R}^{4,6} + \overline{\rho}_2\Delta_{L,R}^{5,7} + \overline{\rho}_3\tilde{\lambda}_{L,R}^{4,6} + \overline{\rho}_4\tilde{\lambda}_{L,R}^{5,7} \equiv \tilde{\mathcal{H}}_{L,R}^\perp$

- $X_A^{\boldsymbol{R}}$, $Y_A^{\boldsymbol{R}}$ and $Z_A^{\boldsymbol{R}}$ are functions of the theory parameters
- **Massless SM fermions→ due to $\mathrm{SU}(3)_{\mathrm{F}}$**



Par	range		
v	10^5	–	10^{17}
μ_{78}	-10^{17}	–	10^{17}
M'_0	0	–	10^{17}
M_0	0	–	10^{17}
g_U	0	–	1.2
λ_{78}	-6	–	6

- $v\,, M'_0\,, M_0\,, \mu_{78}$ in GeV
- $v \sim \mu_{78}$

- **Gauge adjoint fermions can also be naturally very light**
- **Mass hierarchy up to 15 orders of magnitude**
- **Similar for remaining gauge adjoint fermions**

- **Red region:** $M_0\,, M'_0 \gg v\,, \mu_{78}$

$$m^2_{\tilde{g}} \sim \mu^2_{78} - \frac{1}{M_0} M'^2_0 \mu_{78}\,,$$
$$m^2_{\tilde{g}^\perp} \sim 4M_0^2 + 2M'^2_0 + \frac{1}{M_0} M'^2_0 \mu_{78}\,.$$

- **Yellow region:** $M_0\,, M'_0 \ll v\,, \mu_{78}$

$$m^2_{\tilde{g}} \sim 4M_0^2 - \frac{4M_0 M'^2_0}{\mu_{78}}\,,$$
$$m^2_{\tilde{g}^\perp} \sim \mu^2_{78} + 2M'^2_0 + \frac{4M_0 M'^2_0}{\mu_{78}}\,.$$

Outline

- **SHUT model: $\mathrm{E_8}$-inspired SUSY T-GUT model containing family $\mathrm{SU(3)_F}$**

  > Solved the multi-TeV lepton mass problem

  > Elegantly unified Higgs with leptons, gauge and Yukawa couplings

  - Low scale Yukawa structure radiatively generated (work ongoing)

  > Little amount of parameters

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	$\mathrm{U(1)_W}$	$\mathbf{U(1)_B}$
$L\,,\,\tilde{L}$	$+1$	$\mathbf{0}$
$Q_L\,,\,\tilde{Q}_L$	$-1/2$	$\mathbf{+1/3}$
$Q_R^c,\,\tilde{Q}_R$	$-1/2$	$\mathbf{-1/3}$
$G_{\mathrm{L,R,C}}\,,\lambda_{\mathrm{L,R,C}}$	0	$\mathbf{0}$
$\Delta_{\mathrm{L,R,C,F}}\,,\,\tilde{\Delta}_{\mathrm{L,R,C,F}}$	0	$\mathbf{0}$

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$\Delta_{\mathrm{L,R,C,F}}\,,\,\tilde{\Delta}_{\mathrm{L,R,C,F}}$	0	$\mathbf{0}$

- **Several EFT multi-Higgs models to be studied**