

# Renormalisation of the NMSSM

Example of a theory with many Higgses, many parameters: meaning of parameters, mixings, schemes,...

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Work done with G. Bélanger, V. Bizouard, G. Chalons, PRD 2015 and in Progress

When asked about what to talk about?

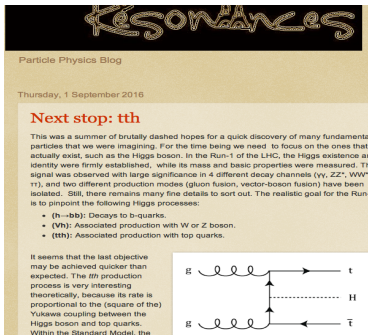
Answer: 750 if confirmed (despite the fact I had nothing appealing) OR NMSSM (the "safe" bet)

## When asked about what to talk about?

*In the comments section you're welcome to lash out on the entire BSM community - we made a wrong call so we deserve it. Please, however, avoid personal attacks (unless on me).  
Alternatively, you can also give us a hug :)*

# When asked about what to talk about?

Could have looked up ResonAAnces...



**Resonances**

Particle Physics Blog

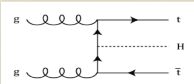
Thursday, 1 September 2016

### Next stop: $t\bar{t}H$

This was a summer of brutally dashed hopes for a quick discovery of many fundamental particles that we were imagining. For the time being we need to focus on the ones that actually exist, such as the Higgs boson. In the Run-1 of the LHC, the Higgs existence and identity were firmly established, while its mass and basic properties were measured. The signal was observed with large significance in 4 different decay channels ( $\gamma\gamma$ ,  $ZZ^*$ ,  $WW^*$ ,  $\tau\tau$ ), and two different production modes (gluon fusion, vector-boson fusion) have been isolated. Still, there remains many fine details to sort out. The realistic goal for the Run-2 is to pinpoint the following Higgs processes:

- ( $h \rightarrow b\bar{b}$ ): Decays to b-quarks.
- ( $Vh$ ): Associated production with W or Z boson.
- ( $t\bar{t}h$ ): Associated production with top quarks.

It seems that the last objective may be achieved quicker than expected. The  $t\bar{t}h$  production process is very interesting theoretically, because its rate is proportional to the (square of the) Yukawa coupling between the Higgs boson and top quarks. Within the Standard Model, the



But that was what I talked about, here, 2 years ago...

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- ▶ Scheme/Scale dependence for a many-parameter set-up

## NMSSM: The Higgs Sector

3 Higgs superfields : two  $SU(2)_L$  doublets  $\hat{H}_u$  and  $\hat{H}_d$ , as in the MSSM, and one additional gauge singlet  $\hat{S}$

$$\hat{H}_u = \begin{pmatrix} \hat{H}_u^+ \\ \hat{H}_u^0 \end{pmatrix}, \quad \hat{H}_d = \begin{pmatrix} \hat{H}_d^0 \\ \hat{H}_d^- \end{pmatrix}, \quad \hat{S}.$$

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$\mathbb{Z}_3 \rightarrow$  Higgs superpotential involves 2 dimensionless couplings  $\lambda$  and  $\kappa$ ,

$$W_{Higgs} = -\lambda \hat{S} \hat{H}_d \cdot \hat{H}_u + \frac{1}{3} \kappa \hat{S}^3, \quad \text{Higgs sector and neutralino (chargino) sector}$$

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$$\begin{aligned} -\mathcal{L}_{soft, scalar} &= m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 \\ &\quad + \lambda A_\lambda H_u \cdot H_d S + \frac{1}{3} \kappa A_\kappa S^3 + h.c \end{aligned}$$

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$$\begin{aligned} V_{Higgs} &= |\lambda(H_u^+ H_d^- - H_u^0 H_d^0) + \kappa S^2|^2 + (m_{H_u}^2 + |\lambda S|^2) (|H_u^0|^2 + |H_u^+|^2) \\ &\quad + (m_{H_d}^2 + |\lambda S|^2) (|H_d^0|^2 + |H_d^+|^2) + \frac{g^2 + g'^2}{8} (|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2)^2 \\ &\quad + \frac{g^2}{2} |H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2 + m_S^2 |S|^2 + (\lambda A_\lambda (H_u^+ H_d^- - H_u^0 H_d^0) S + \frac{1}{3} \kappa A_\kappa S^3 + h.c). \end{aligned}$$

## The Higgs fields

$$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} = \begin{pmatrix} v_d + \frac{h_d^0 + ia_d^0}{\sqrt{2}} \\ h_d^- \end{pmatrix},$$
$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} = \begin{pmatrix} h_u^+ \\ v_u + \frac{h_u^0 + ia_u^0}{\sqrt{2}} \end{pmatrix},$$
$$S = s + \frac{h_s^0 + ia_s^0}{\sqrt{2}}.$$

As in the MSSM  $\tan \beta \equiv t_\beta = v_u/v_d$  and  $v^2 = v_u^2 + v_d^2$ , ( $v_u = s_\beta v$ ,  $c_\beta v$ ) and  $M_W^2 = g^2 v^2/2$ . The non vanishing value of the vev of  $S$  also gives a solution to the so-called  $\mu$ -problem of the MSSM, by generating this parameter dynamically:

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we can also define:

$$\Lambda_v = \lambda v \quad m_\kappa = \kappa s = (\kappa/\lambda)\mu$$



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$$V_{mass} = \frac{1}{2} \begin{pmatrix} h_d^0 & h_u^0 & h_s^0 \end{pmatrix} M_S^2 \begin{pmatrix} h_d^0 \\ h_u^0 \\ h_s^0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} a_d^0 & a_u^0 & a_s^0 \end{pmatrix} M_P^2 \begin{pmatrix} a_d^0 \\ a_u^0 \\ a_s^0 \end{pmatrix} + \begin{pmatrix} h_d^- & h_u^- \end{pmatrix} M_{\pm}^2 \begin{pmatrix} h_d^+ \\ h_u^+ \end{pmatrix}.$$

Charged Higgs, as in the MSSM, almost

$$M_{\pm}^2 = \frac{1}{2} \begin{pmatrix} \frac{\tau_{h_u^0}}{v_u} & 0 \\ 0 & \frac{\tau_{h_d^0}}{v_d} \end{pmatrix} + \frac{s_{2\beta}}{2} \underbrace{\left( \frac{2\mu}{s_{2\beta}} (A_\lambda + m_\kappa) + (M_W^2 - \Lambda_v^2) \right)}_{M_A^2 = M_{A,\text{MSSM}}^2} \begin{pmatrix} 1/t_\beta & 1 \\ 1 & t_\beta \end{pmatrix}$$

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = U_\beta \begin{pmatrix} h_d^\pm \\ h_u^\pm \end{pmatrix},$$

with (here quite simple)

$$U_\beta = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}.$$

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Issues with definition of  $\beta$ . **Here it is just a rotation matrix, change of basis.**

**Observables are basis independent.**

# Mass mixing in the CP-even Higgs system

$$\left\{ \begin{array}{lcl} M_{S_{11}}^2 & = & \frac{\mathcal{T}_{h_d^0}}{2v_d} + M_Z^2 c_\beta^2 + M_A^2 s_\beta^2, \\ M_{S_{22}}^2 & = & \frac{\mathcal{T}_{h_u^0}}{2v_u} + M_Z^2 s_\beta^2 + M_A^2 c_\beta^2, \\ M_{S_{33}}^2 & = & \frac{\mathcal{T}_{h_s^0}}{2s} + \Lambda_v^2 A_\lambda \frac{c_\beta s_\beta}{\mu} + m_\kappa (A_\kappa + 4m_\kappa), \\ M_{S_{12}}^2 & = & M_{S_{21}}^2 = (\Lambda_v^2 - \frac{M_Z^2}{2}) s_{2\beta} - M_A^2 s_\beta c_\beta, \\ M_{S_{13}}^2 & = & M_{S_{31}}^2 = \Lambda_v (2\mu c_\beta - (A_\lambda + 2m_\kappa) s_\beta), \\ M_{S_{23}}^2 & = & M_{S_{32}}^2 = \Lambda_v (2\mu s_\beta - (A_\lambda + 2m_\kappa) c_\beta), \end{array} \right.$$

$$\begin{pmatrix} h_1^0 \\ h_2^0 \\ h_3^0 \end{pmatrix} = S_h \begin{pmatrix} h_d^0 \\ h_u^0 \\ h_s^0 \end{pmatrix},$$

$$M_Z^2 \left( c_{2\beta}^2 + \frac{\Lambda_v^2}{M_Z^2} s_{2\beta}^2 \right) \equiv M_Z^2 \left( 1 + \left( \frac{\Lambda_v^2}{M_Z^2} - 1 \right) s_{2\beta}^2 \right)$$

# The underlying parameters

$$\underbrace{t_\beta, \lambda, \kappa, \mu, A_\lambda, A_\kappa, (t_{H_d}, t_{H_u}, t_S)}_{\text{in } \tilde{\chi} \text{ sector also}} \quad \text{and} \quad \underbrace{g, g', v \rightarrow e, M_W, M_Z}_{\text{SM}}$$

## Charginos and Neutralinos, the link with the Higgs sector

- The mass matrix for the charginos reads, in the basis  $(\tilde{W}^-, \tilde{H}_d^-)$

$$X = \begin{pmatrix} M_2 & \sqrt{2}M_W s_\beta \\ \sqrt{2}M_W c_\beta & \mu \end{pmatrix},$$

$t_\beta$  dep. very weak:

Measurement of the charginos masses reconstruct  $M_2$  and  $\mu$  although assignment ambiguous unless one

has an idea about "higgsino"/wino content

## Charginos and Neutralinos, the link with the Higgs sector

The  $5 \times 5$  neutralino matrix in the basis

$$\psi_n^{RT} = \psi_n^{LT} = \psi^{0T} = (-i\tilde{B}^0, -i\tilde{W}_3^0, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{S}^0)$$

$$Y = \begin{pmatrix} M_1 & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta & 0 \\ 0 & M_2 & M_Z c_W c_\beta & -M_Z c_W s_\beta & 0 \\ -M_Z s_W c_\beta & M_Z c_W c_\beta & 0 & -\mu & -\Lambda_V s_\beta \\ M_Z s_W s_\beta & -M_Z c_W s_\beta & -\mu & 0 & -\Lambda_V c_\beta \\ 0 & 0 & -\Lambda_V s_\beta & -\Lambda_V c_\beta & 2m_\kappa \end{pmatrix},$$

$t_\beta$  and  $\lambda$  intertwined. If  $\lambda$  small  $t_\beta$  extraction difficult.



## Renormalisation: Parameters and counterterms. See my talk here in ...2009

- From  $\mathcal{G}_P = \underbrace{t_\beta, \lambda, \kappa, \mu, A_\lambda, A_\kappa, (M_{H_U}^2, M_{H_d}^2, M_S^2 \rightarrow t_{H_d}, t_{H_U}, t_S)}_{\text{in } \tilde{\chi} \text{ sector also}}, M_1, M_2 \text{ and } \underbrace{g, g', v \rightarrow e, M_W, M_Z}_{\text{SM}}$

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- ▶ this means that mass mixing will appear: non diagonal transition  $A_i^0 Z^0, A_i^0 G^0, h_1 h_2, \dots (\delta m_{h_i h_j}, \delta m_{H^\pm G^\pm}^2)$  and diagonal masses shifted  $(\delta m_{h_i}^2)$ .

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- ▶ No need to apply shifts to the diagonalising matrices ( $S_H, U(\beta), \dots$ ), these are renormalised (no shift), same with gauge-fixing (not physical)

$$\begin{pmatrix} G^+ \\ H^+ \end{pmatrix}_0 = U(\beta) \begin{pmatrix} h_d^+ \\ h_u^+ \end{pmatrix}_0 \quad \text{implies also} \quad \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = U(\beta) \begin{pmatrix} h_d^+ \\ h_u^+ \end{pmatrix}.$$

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▶

$$\begin{cases} \hat{\Sigma}_{h_i^0 h_i^0}(\rho^2) &= \Sigma_{h_i^0 h_i^0}(\rho^2) - \delta m_{h_i^0}^2 + (\rho^2 - m_{h_i^0}^2) \delta Z_{h_i^0} \\ \hat{\Sigma}_{h_i^0 h_j^0}(\rho^2) &= \Sigma_{h_i^0 h_j^0}(\rho^2) - \delta m_{h_i^0 h_j^0}^2 + \frac{1}{2}(\rho^2 - m_{h_i^0}^2) \delta Z_{h_i^0 h_j^0} + \frac{1}{2}(\rho^2 - m_{h_j^0}^2) \delta Z_{h_j^0 h_i^0} \end{cases}$$

## Conditions: To define wave function renormalisation constants and the counterterms for the underlying parameters

- ▶ Mixing vanishes between physical states when these are on-shell, *essentially* (to solve for  $\delta Z_{ij}$ 's)
- ▶ Residue at the pole (mass) of the propagator is 1
- ▶ The other conditions are set by using/choosing a (minimum/sufficient) set of physical masses as input parameters except  $\alpha_{em}$ . Only two point-functions are needed  
(in the present implementation of the NMSSM).
- ▶ which minimum set?

## Solving for a coupled system of counterterms

$$\begin{pmatrix} \delta \text{input}_1 \\ \dots \\ \dots \\ \delta \text{input}_8 \end{pmatrix} = \mathcal{P}_{8,\text{param.}} \begin{pmatrix} \delta M_1 \\ \delta M_2 \\ \delta \kappa \bullet \\ \delta \mu \bullet \\ \delta \lambda \\ \delta t_\beta \\ \delta A_\lambda \\ \delta A_\kappa \end{pmatrix} + \mathcal{R}_{n,\text{residual}},$$

$\chi^\pm, \chi_i^0, h_i^0, A_i^0, H^\pm$

$\mathcal{R}_{8,\text{residual}}$  counterterms such as gauge couplings, etc (SM)



## Finding the counterterms. Inverting the system

Best to break up the system.  $\mathcal{P}_{n,\text{param.}} = \mathcal{P}_{m,\text{param.}} \oplus \mathcal{P}_{p,\text{param.}} \oplus \dots$ ,  $m + p + \dots = n$

At each step, if possible, avoid a choice such that  $\text{Det}(\mathcal{P}_{m,\text{param.}}) \rightarrow 0$  (like picking up a wino-like neutralino to reconstruct  $M_1$ ).

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Easiest set up:

- ▶ a mixed  $\overline{\text{DR}}$ , with  $t_\beta$  extracted independently from the Higgs sector (through wave function renormalisation condition)  $\delta t_\beta / t_\beta = \left[ \frac{1}{2} (\delta Z_{H_u} - \delta Z_{H_d}) \right]_\infty$ ,

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- ▶ Then  $M_1, \kappa, (\lambda)$  from neutralinos

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Best to break up the system.  $\mathcal{P}_{n,\text{param.}} = \mathcal{P}_{m,\text{param.}} \oplus \mathcal{P}_{p,\text{param.}} \oplus \cdots$ ,  $m + p + \cdots = n$

Easiest set up:

- ▶ a mixed  $\overline{\text{DR}}$ , with  $t_\beta$  extracted independently from the Higgs sector (through wave function renormalisation condition)  $\delta t_\beta / t_\beta = \left[ \frac{1}{2} (\delta Z_{H_u} - \delta Z_{H_d}) \right]_\infty$ ,
- ▶ Then  $\mu, M_2$  from charginos (always)
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$$\mathcal{P}_8 = \mathcal{P}_{1,t_\beta} \oplus \mathcal{P}_{2,\chi_{1,2}^\pm} \oplus \mathcal{P}_{3,\chi^0} \oplus \mathcal{P}_{2,A_1^0,A_2^0} \quad \text{OR}$$

$$\mathcal{P}_8 = \mathcal{P}_{1,t_\beta} \oplus \mathcal{P}_{2,\chi_{1,2}^\pm} \oplus \mathcal{P}_{2,\chi^0} \oplus \mathcal{P}_{3,H^\pm,A_1^0,A_2^0(h^0)}$$

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Or,

Go all On-Shell,  $8 \times 8$ , *identified?*:  $\mathcal{P}_{8; \chi_{1,2}^{\pm}, \chi_{1,2,3}^0, H^{\pm}, A_1^0, h^0}$  8 Masses

$H^{\pm} \rightarrow A_{\lambda}$  only

$A^0, h_i^0 \rightarrow A_{\lambda}$  and  $A_{\kappa}, A_{\bar{\kappa}}$  sensitive to singlet

$\lambda, t_{\beta}$  weak from  $\chi^{\pm}, \chi^0$  better from Higgs masses.

Variants (that all take the chargino masses as input):

$OS_{ijkA_1A_2H^+}$  with the masses of 3  $\chi^0$  preferably  $\tilde{b}, \tilde{h}, \tilde{s}$ -dominated

$OS_{ijh_{\alpha}A_1A_2H^+}$  (only 2 neutralinos)

$OS_{ih_{\alpha}h_{\beta}A_1A_2H^+}$  (only one neutralino,  $\tilde{b}$ )

## Finding the counterterms. Inverting the system

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Or,

Go all  $\overline{\text{DR}}$



## Schemes. Scales. OS vs $\overline{\text{DR}}$ vs mixed schemes

Renormalisation: a definition of the underlying parameters to get rid of infinities.

- The infinities: Most loop calculations (1, 2, 3-point functions), regularization introduces  $\tilde{C}_{UV} = 2/\epsilon - \gamma_E + \ln(4\pi/\bar{\mu}^2) = 2/\epsilon - \gamma_E + \ln(4\pi) + \ln(1/\bar{\mu}^2)$

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$$\delta p_i/p_i = \beta_{p_i}(C_{UV} + \ln(Q_{p_i}/\bar{\mu})) \quad (\beta_{p_i} = \partial p_i / \partial \ln 1/\bar{\mu})$$

Note  $\ln(Q_{p_i}) = \ln(\tilde{Q}_{p_i}) + F(Q'_{p_i})$ ,  $Q_{p_i}$  is **scheme dependent** but  $\beta_{p_i}$  **universal**

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good choice of  $Q_{p_i}$ ,  $\bar{\mu}$  especially if large  $\beta$ ,  $\kappa$ . In full  $\overline{\text{DR}}$   $\mu$  dep. trackable.

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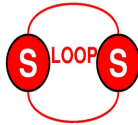
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- ▶ mixed scheme, say  $p_0$   $\overline{\text{DR}}$  (the rest of  $p_i$  is OS). In the inversion of OS scheme,  $p_0$  may enter as residual (good choice?)

$$\frac{\delta O^{\text{mixed}}}{O} = \frac{\delta O^{\text{OS}}}{O} + \beta_{p_0} \left( -\kappa_0 \ln(Q_0 / \bar{\mu}) + \sum_{i \neq 0} \kappa_{i0} \ln(\tilde{Q}_i / \bar{\mu}) \right)$$



**LanHEP**  
MSSM Lagrangian  
Counterterms  
Renormalisation schemes

**MicrOMEGAs**  
@ Tree-Level

**FormCalc/FeynArts/LoopTools**  
Computation of one-loop processes  
(cross-sections, decays, mass corrections)  
**+ In-house 1-loop routines**  
not yet automatised

**MicrOMEGAs**  
@ One-Loop

### SLOOPS

An automatic code for calculation of **loops** diagrams for  $SM$  and  $BSM$  processes with application to **colliders**, **astrophysics** and **cosmology**.

- ▶ **Automatic** derivation of the CT Feynman rules and **computation** of the CT's
- ▶ Models **renormalized**: **SM**, **MSSM**, **NMSSM**, **xSM** (w/ & w/o singlet vev)
- ▶ Modularity between different renormalisation schemes.
- ▶ **Non-linear** gauge fixing.
- ▶ Checks: results **UV**, **IR** finite and **gauge** independent.



# Application to Higgs decays in the NMSSM

**Point A** ( $Q_{\text{susy}} = 1117.25\text{GeV}$ ,  $m_t = 173\text{GeV}$ ,  $m_{h_1^0} = 125.45\text{GeV}$ (1-loop OS))

$M_1$	700	$\lambda$	0.1	$A_\kappa$	0	$m_{\tilde{Q}_3}$	1740	$m_{\tilde{D}, \tilde{U}_{1,2}}$	1000
$M_2$	1000	$\kappa$	0.1	$A_t$	4000	$m_{\tilde{U}_3}$	800	$m_{\tilde{L}_3}$	1000
$M_3$	1000	$\mu$	120	$A_b$	1000	$m_{\tilde{D}_3}$	1000	$m_{\tilde{I}_3}$	1000
$t_\beta$	10	$A_\lambda$	150	$A_l$	1000	$m_{\tilde{Q}_{1,2}}$	1000	$m_{\tilde{L}, \tilde{I}_{1,2}}$	1000

$\lambda A_\lambda = 15\text{GeV}$ ,  $A_t/A_\lambda \sim 27$

**Point B** ( $Q_{\text{susy}} = 753.55\text{GeV}$ ,  $m_t = 146.94\text{GeV}$ ,  $m_{h_1^0} = 124.44\text{GeV}$ (1-loop OS))

$M_1$	120	$\lambda$	0.67	$A_\kappa$	0	$m_{\tilde{Q}_3}$	750	$m_{\tilde{D}, \tilde{U}_{1,2}}$	1500
$M_2$	300	$\kappa$	0.2	$A_t$	1000	$m_{\tilde{U}_3}$	750	$m_{\tilde{L}_3}$	1500
$M_3$	1500	$\mu$	200	$A_b$	1000	$m_{\tilde{D}_3}$	1500	$m_{\tilde{I}_3}$	1500
$t_\beta$	1.92	$A_\lambda$	405	$A_l$	1000	$m_{\tilde{Q}_{1,2}}$	1500	$m_{\tilde{L}, \tilde{I}_{1,2}}$	1500

$\lambda A_\lambda = 271\text{GeV}$ ,  $A_t/A_\lambda \sim 2.5$

# Applications. Two scenarios

		Point A	Point B
$h_1^0$	$h_d^0$	1.1%	22.5%
	$h_u^0$	<b>98.6%</b>	<b>67.4%</b>
	$h_s^0$	0.3%	10.1%
$h_2^0$	$h_d^0$	0.1%	0.%
	$h_u^0$	0.3%	12.5%
	$h_s^0$	<b>99.6%</b>	<b>87.5%</b>
$h_3^0$	$h_d^0$	<b>98.8%</b>	<b>77.5%</b>
	$h_u^0$	1.1%	19.7%
	$h_s^0$	0.1%	2.8%
$A_1^0$	$a_d^0$	0%	1.8%
	$a_u^0$	0%	0.5%
	$a_s^0$	<b>100%</b>	<b>97.7%</b>
$A_2^0$	$a_d^0$	<b>99.0%</b>	<b>76.9%</b>
	$a_u^0$	1.0%	20.8%
	$a_s^0$	0.0%	2.3%

Point A:  $h_u, h_s, h_d, a_s, a_d$

Point B:  $h_u, h_s, h_d, a_s, a_d$

		Point A	Point B
$\bar{\chi}_1^0$	$\bar{B}^0$	-	<b>56.6%</b>
	$\bar{W}^0$	-	32.3%
	$\bar{h}^0$	<b>98.4%</b>	10.3%
	$\bar{S}^0$	0.77%	0.8%
$\bar{\chi}_2^0$	$\bar{B}^0$	-	4.0%
	$\bar{W}^0$	-	2.6%
	$\bar{h}^0$	<b>99.5%</b>	19.3%
	$\bar{S}^0$	-	<b>74.0%</b>
$\bar{\chi}_3^0$	$\bar{B}^0$	-	10.1%
	$\bar{W}^0$	-	-
	$\bar{h}^0$	0.9%	<b>78.9%</b>
	$\bar{S}^0$	<b>99.1%</b>	11.0%
$\bar{\chi}_4^0$	$\bar{B}^0$	<b>99.6%</b>	18.1%
	$\bar{W}^0$	-	12.3%
	$\bar{h}^0$	-	<b>55.8%</b>
	$\bar{S}^0$	-	13.7%
$\bar{\chi}_5^0$	$\bar{B}^0$	-	11.2%
	$\bar{W}^0$	<b>99.3%</b>	<b>52.8%</b>
	$\bar{h}^0$	0.69%	35.7%
	$\bar{S}^0$	-	0.4%

Point A:  $\bar{h}, \bar{h}, \bar{s}, \bar{b}, \bar{w}$

Point B:  $\bar{b}, \bar{s}, \bar{h}, \bar{h}, \bar{w}$

Beware. B much more mixing, A quite pure

## Point A. $\beta$ 's and counterterms

( $\beta$  in units of  $10^{-3}$ )  $\beta_\mu = -11.4$ ,  $\beta_{t_\beta} = 16.9$ ,  $\beta_\lambda = -11.7$ ,  $\beta_\kappa = -0.76$ ,  $\beta_{\mathbf{A}_\lambda} = -1097.4$

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$$\begin{aligned}
 16\pi^2 \frac{1}{h_t^2} \frac{dh_t^2}{dt} &= 6h_t^2, & 16\pi^2 \frac{1}{\lambda^2} \frac{d\lambda^2}{dt} &= 3h_t^2, \\
 16\pi^2 \frac{1}{\mu^2} \frac{d\mu^2}{dt} &= 3h_t^2, & 16\pi^2 \frac{1}{A_t} \frac{dA_t}{dt} &= 6h_t^2, \\
 16\pi^2 \frac{1}{A_\lambda} \frac{dA_\lambda}{dt} &= 3h_t^2 \frac{A_t}{A_\lambda} \quad (+ 4\lambda^2), \\
 \frac{1}{2} 16\pi^2 \frac{1}{A_t} \frac{dA_t}{dt} &= 6h_t^2, & (16\pi^2 \frac{1}{A_\kappa} \frac{dA_\kappa}{dt} &= 6\kappa + 6\lambda^2 \frac{A_\lambda}{A_\kappa}).
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Finite parts computed at  $\bar{\mu} = Q_{\text{susy}} = 1117.25 \text{ GeV}$ .

$$(\delta\mu/\mu, \delta t_\beta/t_\beta, \delta\lambda/\lambda)_{\text{finite}} = \overbrace{(-2.42\%, 0, 62.26\%)}^{t_{134A_1A_2}}; \overbrace{(-1.57\%, -80.69\%, -7.88\%)}^{OS_{34h_2A_1A_2H^+}}$$

$$(\delta\kappa/\kappa, \delta A_\lambda/A_\lambda, \delta A_\kappa)_{\text{finite}} = \overbrace{(64.01\%, -5.49\%, 0.65)}^{t_{134A_1A_2}}; \overbrace{(-6.01\%, 134\%, 0.66)}^{OS_{34h_2A_1A_2H^+}}.$$

## $\beta$ 's and counterterms

( $\beta$  in units of  $10^{-3}$ )

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Point B

$$\beta_\mu = -14.25, \beta_{t_\beta} = 17.63, \beta_\lambda = -20.45, \beta_\kappa = -18.57, \beta_{\mathbf{A}_\lambda} = -122.7$$

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### Point B

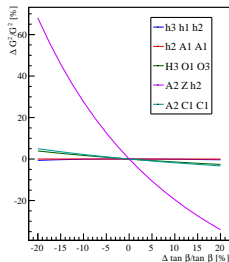
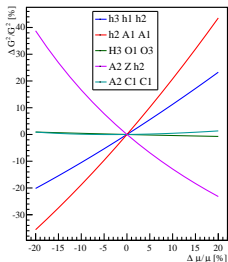
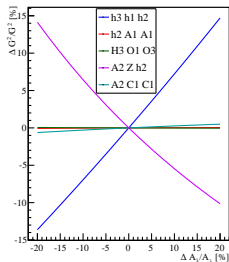
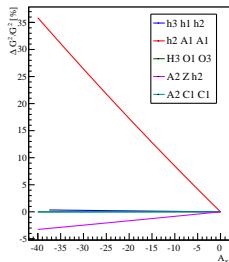
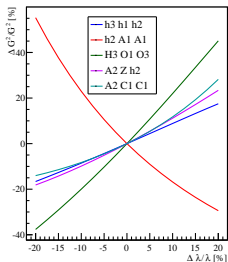
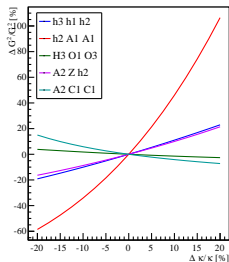
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"Finite parts" computed at  $\bar{\mu} = Q_{\text{susy}} = 753.55 \text{ GeV}$

$$(\delta\mu/\mu, \delta t_\beta/t_\beta, \delta\lambda/\lambda)_{\text{finite}} = \overbrace{(-1.04\%, 0, 3.71\%)}^{t_{123A_1A_2}}; \overbrace{(-1.63\%, 6.49\%, 5.94\%)}^{OS_{34h_2A_1A_2H^+}}$$

$$(\delta\kappa/\kappa, \delta A_\lambda/A_\lambda, \delta A_\kappa)_{\text{finite}} = \overbrace{(3.25\%, 6.85\%, 10.84)}^{t_{123A_1A_2}}; \overbrace{(6.05\%, 3.40\%, 11.54)}^{OS_{34h_2A_1A_2H^+}}.$$

# Point A, parametric dependence





Point A, singlet-like:  $h_2^0, A_1^0, \chi_3^0$ ,  $Q_{\text{SUSY}} = 1117$ ,  $m_{h_2^0} = 240$ ,  $m_{h_3^0, A_2^0, H^\pm} \sim 570$ ,  $\tilde{\chi}_i^0 = (\tilde{h}, \tilde{h}, \tilde{s}, \tilde{b}, \tilde{w})$

---

units width=GeV/100

$t_{134A_1A_2, \mu=m_{h_i \rightarrow}}$

$OS_{34h_2A_1A_2H^+}$

$\overline{\text{DR}}, \mu = m_{h_i \rightarrow}$

$\overline{\text{DR}} Q_{\text{SUSY}}$

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Point A, singlet-like:  $h_2^0, A_1^0, \chi_3^0$ ,  $Q_{\text{SUSY}} = 1117$ ,  $m_{h_2^0} = 240$ ,  $m_{h_3^0, A_2^0, H^\pm} \sim 570$ ,  $\tilde{\chi}_i^0 = (\tilde{h}, \tilde{h}, \tilde{s}, \tilde{b}, \tilde{w})$

units width=GeV/100	$t_{134A_1A_2, \mu=m_{h_i \rightarrow}}$	$OS_{34h_2A_1A_2H^+}$	$\overline{DR}, \mu = m_{h_i \rightarrow}$	$\overline{DR} Q_{\text{SUSY}}$
$h_3^0 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$	4.61 (1%)	4.03 (-11%)	4.13 (-9%)	4.21 (-7.4%)
$A_2^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$	4.22 (28%)	3.72 (13%)	3.30 (0.3%)	3.24 (-1.6%)
$A_2^0 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$	4.96 (-10%)	5.43 (-1.5%)	5.15 (-6%)	5.06 (-8%)

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units width=GeV/100	$t_{134A_1A_2, \mu=m_{h_i \rightarrow}}$	$OS_{34h_2A_1A_2H^+}$	$\overline{\text{DR}}, \mu = m_{h_i \rightarrow}$	$\overline{\text{DR}} Q_{\text{SUSY}}$
$h_2^0 \rightarrow A_1^0 A_1^0$	10.9 (128%)	4.21 (-12%)	4.80 (0.4%)	4.77 (-0.4%)

Point A, singlet-like:  $h_2^0, A_1^0, \chi_3^0$ ,  $Q_{\text{SUSY}} = 1117$ ,  $m_{h_2^0} = 240$ ,  $m_{h_3^0, A_2^0, H^\pm} \sim 570$ ,  $\tilde{\chi}_i^0 = (\tilde{h}, \tilde{h}, \tilde{s}, \tilde{b}, \tilde{w})$

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$h_3^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_3^0$	7.80 (122%)	3.41 (-3%)	3.58 (2%)	3.52 (0.3%)
$h_3^0 \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_3^0$	7.64 (126%)	2.19 (-35%)	3.47 (3%)	3.42 (1.1%)
$A_2^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_3^0$	5.62 (130%)	1.69 (-31%)	2.63 (8%)	2.59 (6.2%)
$A_2^0 \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_3^0$	6.71 (122%)	2.87 (-5%)	3.01 (-0.4%)	2.96 (-1.9%)
$H^+ \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_3^0$	14.4 (125%)	5.24 (-18%)	6.57 (3%)	6.47 (1.1%)

Point A, singlet-like:  $h_2^0, A_1^0, \chi_3^0$ ,  $Q_{\text{SUSY}} = 1117$ ,  $m_{h_2^0} = 240$ ,  $m_{h_3^0, A_2^0, H^\pm} \sim 570$ ,  $\tilde{\chi}_i^0 = (\tilde{h}, \tilde{h}, \tilde{s}, \tilde{b}, \tilde{w})$

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$h_3^0 \rightarrow h_1^0 h_2^0$	4.76 (116%)	3.95 (79%)	3.35 (52%)	2.17 (-1.7%)

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With  $A_\kappa = 0$  (at tree-level), this interaction stems solely from the term  $\kappa^2 S^4$ . The trilinear  $\kappa^2 S \propto (\kappa s)^2 / s \propto \lambda / \mu (\kappa s)^2$ .  $2\kappa s$  sets the mass of the singlino. The percentage correction 128% in the  $t$  scheme and  $-12\%$  in the OS scheme extremely well approximated by the  $\lambda$  dependence ( $\sim 2\delta\lambda/\lambda$ ) of the counterterm.

$$\beta_\mu = -11.40, \beta_{t_\beta} = 16.9, \beta_\lambda = -11.65, \beta_\kappa = -0.76, \beta_{A_\lambda} = -1097.4$$

Finite parts computed at  $\bar{\mu} = Q_{\text{SUSY}} = 1117.25 \text{ GeV}$ .

$$(\delta\mu/\mu, \delta t_\beta/t_\beta, \delta\lambda/\lambda)_{\text{finite}} = \overbrace{(-2.42\%, 0, 62.26\%)}^{t_{134A_1A_2}}; \overbrace{(-1.57\%, -80.69\%, -7.88\%)}^{OS_{34h_2A_1A_2H^+}}$$

$$(\delta\kappa/\kappa, \delta A_\lambda/A_\lambda, \delta A_\kappa)_{\text{finite}} = \overbrace{(64.01\%, -5.49\%, 0.65\%)}^{t_{134A_1A_2}}; \overbrace{(-6.01\%, 134\%, 0.66\%)}^{OS_{34h_2A_1A_2H^+}}.$$

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- The other decays of the Higgses (CP-even, CP-odd or charged ) into neutralinos/charginos involving the mostly singlet  $\chi_3^0$  require mixing (through  $\lambda$  for these processes to proceed. Once this is identified, the results ( $\sim 2\delta\lambda/\lambda$ ) are very similar to the one obtained for  $h_2 \rightarrow A_1^0 A_1^0$ .

Point A, singlet-like:  $h_2^0, A_1^0, \chi_3^0$ ,  $Q_{\text{SUSY}} = 1117$ ,  $m_{h_2^0} = 240$ ,  $m_{h_3^0, A_2^0, H^\pm} \sim 570$ ,  $\tilde{\chi}_i^0 = (\tilde{h}, \tilde{h}, \tilde{s}, \tilde{b}, \tilde{w})$

units width=GeV/100	$t_{134A_1A_2, \mu=m_{h_i \rightarrow}}$	$OS_{34h_2A_1A_2H^+}$	$\overline{\text{DR}}, \mu = m_{h_i \rightarrow}$	$\overline{\text{DR}} Q_{\text{SUSY}}$
$h_3^0 \rightarrow h_1^0 h_2^0$	4.76 (116%)	3.95 (79%)	3.35 (52%)	2.17 (-1.7%)

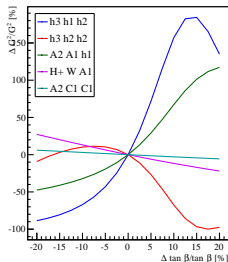
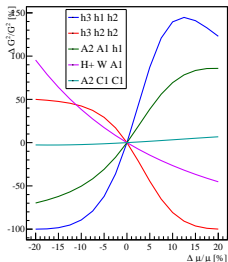
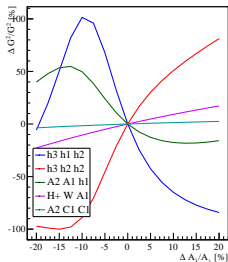
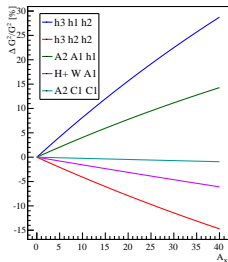
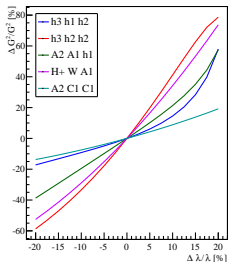
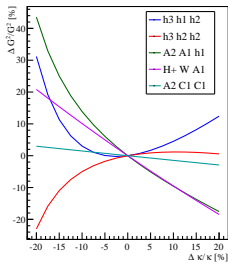
with  $\lambda$  small  $h_1 h_2 h_3 \sim h_u^0 h_d^0 h_s^0$ , the coupling can be read off directly from the potential (before diagonalisation):

$$\lambda A_\lambda + 2\kappa\mu$$

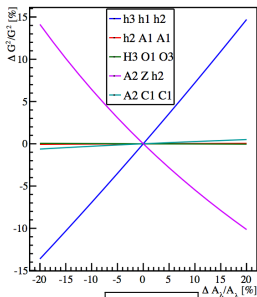
the differences between the schemes explained by the values of the counterterms. Here  $Q_{\text{SUSY}}$  is a good scale in  $\overline{\text{DR}}$ . but the scale dependence is very large.



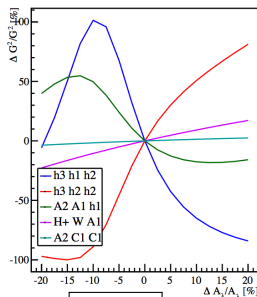
## Point B: Strong parametric dependence



## Point B Strong parametric dependence



Point A



Point B

## Point B. Mixing large, difficult to discuss in terms of the (almost) "pure" states

	$OS_{12h_2A_1A_2H^+}$	$\overline{DR}$	$\overline{DR} Q_{SUSY}$
$h_3^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0$	(14%)	(5%)	(3%)
$h_3^0 \rightarrow A_1^0 Z$	(3%)	(-3%)	(-8 %)
$h_3^0 \rightarrow h_2^0 h_1^0$	(-25%)	(-106%)	(-50%)
$h_3^0 \rightarrow h_2^0 h_2^0$	(6%)	(13%)	(-28%)
$A_2^0 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$	(7%)	(2%)	(1%)
$A_2^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$	(32%)	(2%)	(2%)
$A_2^0 \rightarrow Zh_2^0$	(12%)	(-16%)	(-9%)
$A_2^0 \rightarrow A_1^0 h_1^0$	(-0.3%)	(-32%)	(-17%)
$H^+ \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0$	(6%)	(10%)	(8%)
$H^+ \rightarrow W^+ h_2^0$	(11%)	(-18%)	(-10%)
$H^+ \rightarrow W^+ A_1^0$	(2%)	(-3%)	(-9%)
$H^+ \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^0$	(21%)	(9%)	(9%)

• OS does a good job.

• Again  $h_3 h_2 h_1$  large scheme dependence. The coupling is not totally controlled by  $A_\lambda$  which runs

less, **however the parametric dependence on  $A_\lambda$  is quite large**

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When new particles are discovered, not only their masses will be measured but the way they are produced, the strengths of their production and decays offer an important handle that may not need the reconstruction of the whole spectrum
- ▶ This is technically much more challenging, but it is possible (at least in some manifestations). In the MSSM  $A^0 \rightarrow \tau\bar{\tau}$  was shown to be an **excellent** input for  $t_\beta$ , see my 2009 talk.