Revised muon (g-2) in THDMs

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Two-Higgs doublet models

- The Higgs basis:

$$\Phi_1 = \begin{bmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v + S_1 + iG^0\right) \end{bmatrix} \qquad \Phi_2 = \begin{bmatrix} H^+ \\ \frac{1}{\sqrt{2}}\left(S_2 + iS_3\right) \end{bmatrix}$$

- If $\varphi_i^0(x) = \{h(x), H(x), A(x)\} \;\Rightarrow\; \varphi_i^0(x) = \mathcal{R}_{ij} S_j(x)$

- When the potential is CP-conserving:

$$\begin{pmatrix} h \\ H \\ A \end{pmatrix} = \begin{pmatrix} \cos\tilde{\alpha} & \sin\tilde{\alpha} & 0 \\ -\sin\tilde{\alpha} & \cos\tilde{\alpha} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix}$$

- $\tilde{\alpha} \equiv \alpha - \beta$, $v = \sqrt{v_1^2 + v_2^2} \approx 246$ GeV, $\tan\beta \equiv v_2/v_1$.

Yukawa Lagrangian

- The general Yukawa Lagrangian in the Higgs basis:

$$\mathcal{L}_Y = -\frac{\sqrt{2}}{v}\left\{ \bar{Q}'_L\,(M'_d\Phi_1 + Y'_d\Phi_2)\,d'_R + \bar{Q}'_L\,(M'_u\Phi_1 + Y'_u\Phi_2)u'_R + \bar{L}'_L\,(M'_l\Phi_1 + Y'_l\Phi_2)\,l'_R \right\}$$

- with M'_f and Y'_f complex independent matrices (non simultaneously diagonalizable) $\Rightarrow$ tree level FCNCs.
- One usually imposes a discrete $\mathcal{Z}_2$ symmetry on the Higgs doublets: $\phi_1 \to \phi_1\,,\ \phi_2 \to -\phi_2$ (in a generic basis), etc.
- Keeps them under control: BGL models.
- Our approach: impose alignment in the flavour space: $Y'_f \sim M'_f$.

Yukawa Lagrangian

- Now we can simultaneously diagonalize both matrices and:

$$Y_{d,l} = \varsigma_{d,l}\, M_{d,l} \qquad\qquad Y_u = \varsigma_u^*\, M_u$$

- The Yukawa Lagrangian now reads:

$$\mathcal{L}_Y = -\frac{\sqrt{2}}{v}\, H^+ \left\{ \bar{u} \left[\varsigma_d\, V M_d \mathcal{P}_R - \varsigma_u\, M_u^\dagger V \mathcal{P}_L \right] d \,+\, \varsigma_l\, \bar{\nu} M_l \mathcal{P}_R l \right\} - \frac{1}{v} \sum_{\varphi_i^0, f} y_f^{\varphi_i^0}\, \varphi_i^0\, \left[\bar{f}\, M_f \mathcal{P}_R f \right] \,+\, \mathrm{h.c.}$$

[A.Pich, P.Tuzon '09]

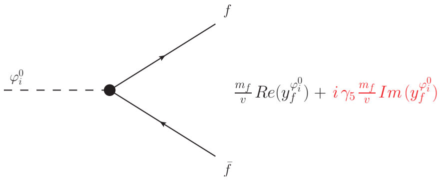
- If the Higgs potential is CP-conserving then the neutral Yukawas read:

$$y_{d,l}^h = \cos\tilde{\alpha} + \varsigma_{d,l}\, \sin\tilde{\alpha} \qquad y_{d,l}^H = -\sin\tilde{\alpha} + \varsigma_{d,l}\, \cos\tilde{\alpha} \qquad y_{d,l}^A = i\, \varsigma_{d,l}$$

$$y_u^h = \cos\tilde{\alpha} + \varsigma_u^*\, \sin\tilde{\alpha} \qquad y_u^H = -\sin\tilde{\alpha} + \varsigma_u^*\, \cos\tilde{\alpha} \qquad y_u^A = -i\, \varsigma_u^*$$

Yukawa Lagrangian

- The complex parameters still allow for new sources of CP-violation in the neutral Yukawa sector:

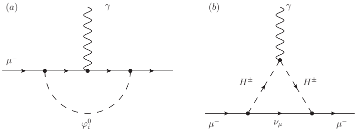


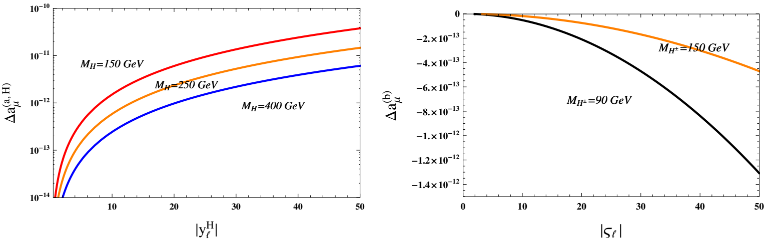
- SM: $Re(y_f^{\varphi_i^0}) = 1$ and $Im(y_f^{\varphi_i^0}) = 0$.
- For real ς_f we can recover the usual $\mathcal{Z}_2$ models:

Model	ς_d	ς_u	ς_l
Type I	$\cot\beta$	$\cot\beta$	$\cot\beta$
Type II	$-\tan\beta$	$\cot\beta$	$-\tan\beta$
Type X	$\cot\beta$	$\cot\beta$	$-\tan\beta$
Type Y	$-\tan\beta$	$\cot\beta$	$\cot\beta$
Inert	0	0	0

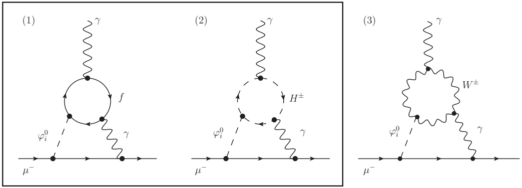
One-loop contributions

$$\boxed{\Delta a_\mu^{exp} \equiv a_\mu^{exp} - a_\mu^{SM} = 262(85) \times 10^{-11}}$$

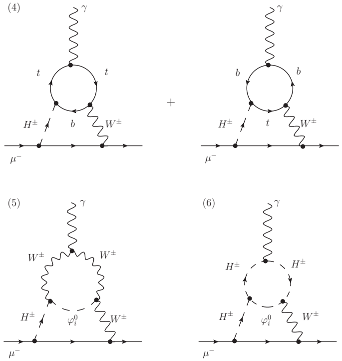




Two-loop contributions



$+\ \gamma \leftrightarrow Z$

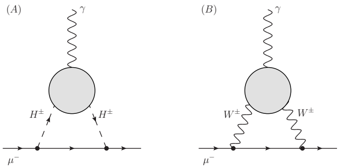


Barr-Zee $\sim \dfrac{\alpha}{\pi}\,\dfrac{M^2}{m_\mu^2}$

$\gamma \to W,\ \varphi_i^0 \to H^\pm$ $\sim$ suppressed by massive W propagator, re-enhanced by ς_f couplings.

Two-loop contributions

- $\gamma \to W$, $\varphi_i^0 \to H^\pm$ $\sim$ suppressed by massive W propagator, re-enhanced by ς_f $\to$ thus the new diagrams can bring potentially sizeable contributions!
- Similar contributions

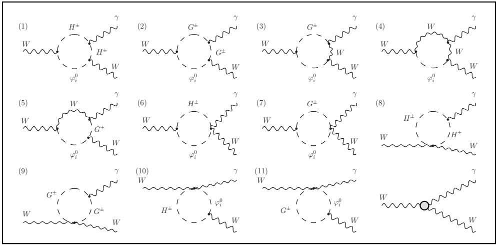


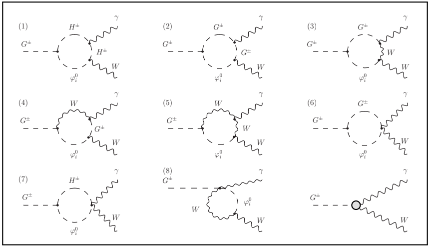
Case (A): relative suppression factor m_μ^2 / M_W^2 with respect to the previous diagrams.

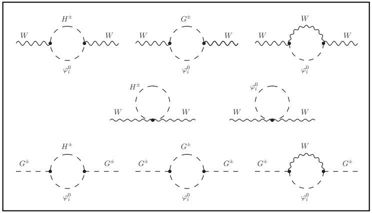
Case (B): No relative suppression with respect to the previous diagrams $\to$ must be calculated.

Two-loop contributions

- Discarding (B)







Two-loop contributions

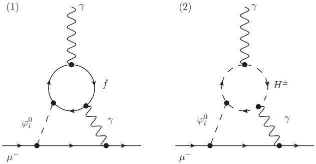
$$\Delta a_\mu = \frac{\alpha}{128\,\pi^2\,s_W^2}\,\frac{m_\mu^2}{v^2}\int_0^1 dx\left(\sum_i \mathcal{R}_{i1}^2\,\mathcal{A}-\mathcal{A}_{\rm SM}+\sum_i|\mathcal{R}_{i2}+i\mathcal{R}_{i3}|^2\,\mathcal{B}+\mathcal{C}\right) \sim \mathcal{O}(10^{-11})$$

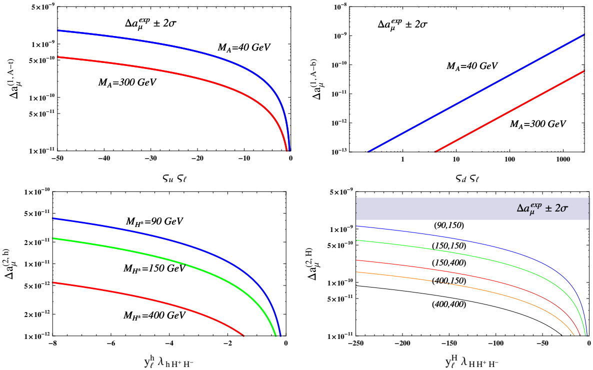
$$\begin{aligned}\mathcal{A} = {} & \frac{7}{3}x(1-x)\ln\frac{\bar a^2(M_W^2)}{M_W^2}-\frac{(2x^2-3x+2)\,M_{\varphi_i^0}^2}{2x(M_W^2-\bar M_x^2)}+\frac{6(x-1)\,\bar M_x^2+(-12x^2+30x-55)\,M_W^2}{6(M_W^2-\bar M_x^2)}+\\ & +\frac{M_{\varphi_i^0}^2\,\ln(\bar M_x^2/M_W^2)}{2xM_W^2(M_W^2-\bar M_x^2)^2}\left(\bar M_x^4\,x(2x-1)-2M_W^4+4M_W^2\bar M_x^2\,x(1-x)\right)+\\ & +\frac{\ln(\bar M_x^2/M_W^2)}{6x\,(M_W^2-\bar M_x^2)^2}\left(\bar M_x^4\,x(16x-9)+M_W^4\,(8x-42)+2M_W^2\bar M_x^2\,(-6x^3+10x^2-30x+21)\right)+\\ & +\frac{x(1-x)}{4M_W^2\,\bar a^2(M_W^2)}\left(M_W^4\,(3x^2-8x-\frac{50}{3})+2x\,M_W^2M_{\varphi_i^0}^2-M_{\varphi_i^0}^4\right),\end{aligned}$$

$$\begin{aligned}\mathcal{B} = {} & \frac{7}{3}x(1-x)\ln\frac{a^2(M_W^2)}{M_W^2}+\frac{1}{2}(2x-1)\frac{M_x^2-2M_W^2(x-1)}{M_W^2-M_x^2}+\frac{(M_{\varphi_i^0}^2-M_{H\pm}^2)(3-2x)}{2(M_W^2-M_x^2)}+\\ & +\frac{M_x^2\ln(M_x^2/M_W^2)}{6(M_W^2-M_x^2)^2}\left(M_W^2 2x(7-6x)+M_x^2(10x-9)\right)-\frac{(M_{\varphi_i^0}^2-M_{H\pm}^2)^2\,x(1-x)}{4M_W^2\,a^2(M_W^2)}+\\ & +\frac{(M_{\varphi_i^0}^2-M_{H\pm}^2)\,M_x^2\ln(M_x^2/M_W^2)}{2M_W^2(M_W^2-M_x^2)^2}\left(4M_W^2(1-x)+M_x^2(2x-1)\right)+\frac{1}{4M_W^2}\left(2(1-x)\,M_{H\pm}^2\ln\frac{a^2(M_W^2)}{M_{H\pm}^2}+2x\,M_{\varphi_i^0}^2\ln\frac{a^2(M_W^2)}{M_W^2}\right),\end{aligned}$$

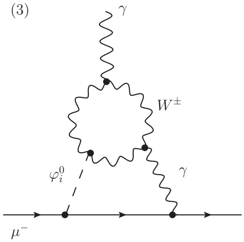
$$\mathcal{C}=\sum_i\left(-\frac{M_{\varphi_i^0}^2}{4M_W^2}\ln\frac{M_{\varphi_i^0}^2}{M_W^2}+\mathcal{R}_{i1}^2\,\frac{1}{4}(-3x^2+4x-6)\ln\frac{\bar a^2(M_W^2)}{\bar a_{\rm SM}^2(M_W^2)}+\mathcal{R}_{i1}^2\,\frac{x\,M_{\varphi_i^0}^2}{2M_W^2}\ln\frac{\bar a^2(M_W^2)}{M_W^2}\right)-\frac{x\,M_\phi^2}{2M_W^2}\ln\frac{\bar a_{\rm SM}^2(M_W^2)}{M_\phi^2}+\frac{1}{6}\,.$$

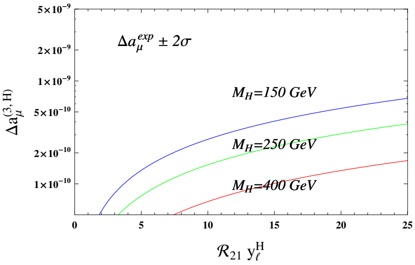
Two-loop contributions





Two-loop contributions

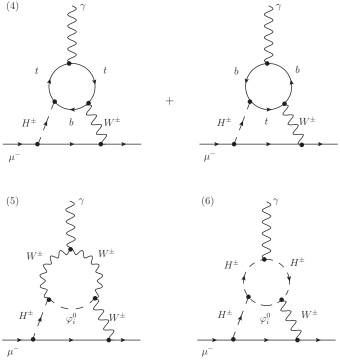


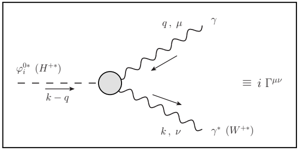


$$\Delta a_\mu^{(3)} = \sum_i \frac{\alpha\, m_\mu^2}{8\,\pi^3\, v^2}\; \mathrm{Re}\left(y_l^{\varphi_i^0}\right) \mathcal{R}_{i1}\, \mathcal{F}^{(3)}\left(\frac{M_W^2}{M_{\varphi_i^0}^2}\right)$$

$$\mathcal{F}^{(3)}(\omega) = \frac{1}{2}\int_0^1 dx\, \frac{x\,[3x(4x-1)+10]\omega - x(1-x)}{\omega - x(1-x)}\, \ln\left(\frac{\omega}{x(1-x)}\right).$$

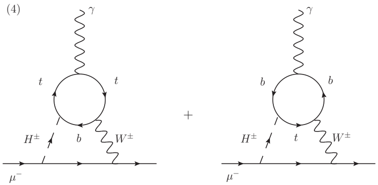
Two-loop contributions

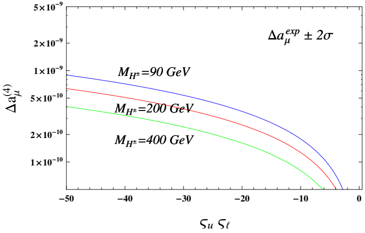




$$i\,\Gamma^{\mu\nu} = i\,(g^{\mu\nu}k\cdot q - k^\mu q^\nu)\,S \,+\, i\,\epsilon^{\mu\nu\alpha\beta}\,k_\alpha\,q_\beta\,\tilde{S} \,+\, Gauge.\ dep.$$

Two-loop contributions





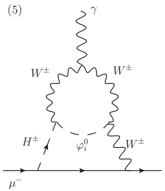
$$\Delta a_\mu^{(4)} = \frac{\alpha\, m_\mu^2\; N_C\, |V_{tb}|^2}{32\,\pi^3\, s_W^2\, v^2\,(M_{H^\pm}^2 - M_W^2)} \int_0^1 dx \left[Q_t x + Q_b(1-x) \right]$$

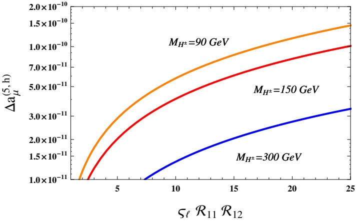
$$\times \left[\mathrm{Re}(\varsigma_d \varsigma_l^*)\, m_b^2 x(1-x) + \mathrm{Re}(\varsigma_u \varsigma_l^*)\, m_t^2 x(1+x) \right] \left[\mathcal{G}\left(\frac{m_t^2}{M_{H^\pm}^2}, \frac{m_b^2}{M_{H^\pm}^2} \right) - \mathcal{G}\left(\frac{m_t^2}{M_W^2}, \frac{m_b^2}{M_W^2} \right) \right]$$

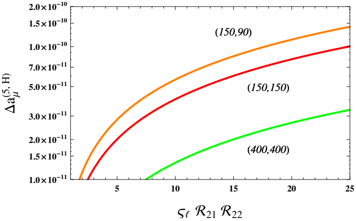
$$\mathcal{G}(\omega^a, \omega^b) = \frac{\ln\left(\dfrac{\omega^a x + \omega^b (1-x)}{x(1-x)} \right)}{x(1-x) - \omega^a x - \omega^b (1-x)} .$$

Result confirmed by analytically by D. Stöckinger et al. *"The muon magnetic moment in the* 2HDM*: complete two-loop result"* arXiv:1607.06292 [hep-ph].

Two-loop contributions

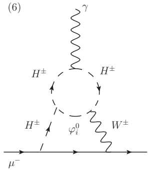


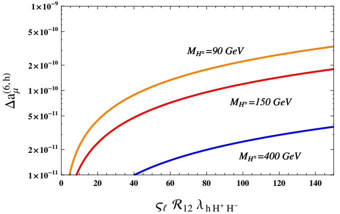


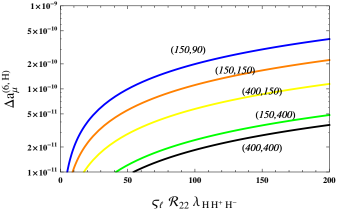


$$\Delta a_\mu^{(5)} = \frac{\alpha\, m_\mu^2}{64\,\pi^3\, s_W^2\, v^2\,(M_{H^\pm}^2 - M_W^2)} \sum_i \mathrm{Re}\left[\varsigma_l^*\, \mathcal{R}_{i1}(\mathcal{R}_{i2} - i\mathcal{R}_{i3})\right] \int_0^1 dx\, x^2$$
$$\times \left[(M_{H^\pm}^2 + M_W^2 - M_{\varphi_i^0}^2)(1-x) - 4M_W^2 \right] \left[\mathcal{G}\left(\frac{M_W^2}{M_{H^\pm}^2}, \frac{M_{\varphi_i^0}^2}{M_{H^\pm}^2}\right) - \mathcal{G}\left(1, \frac{M_{\varphi_i^0}^2}{M_W^2}\right) \right]$$

Two-loop contributions





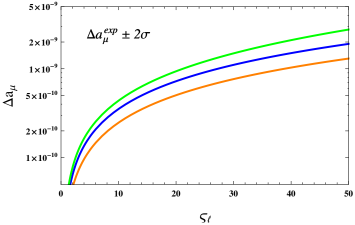


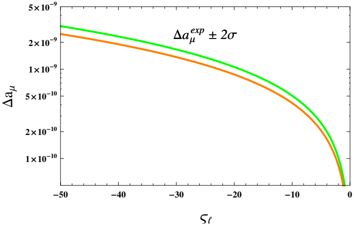
$$\Delta a_\mu^{(6)} = \frac{\alpha\, m_\mu^2}{64\,\pi^3\, s_w^2\,(M_{H^\pm}^2 - M_W^2)} \sum_i \mathrm{Re}\left[\varsigma_l^*\,(\mathcal{R}_{i2} - i\mathcal{R}_{i3})\right] \lambda_{\varphi_i^0 H^+ H^-} \int_0^1 dx\, x^2 (x-1) \times \left[\mathcal{G}\left(1, \frac{M_{\varphi_i^0}^2}{M_{H^\pm}^2}\right) - \mathcal{G}\left(\frac{M_{H^\pm}^2}{M_W^2}, \frac{M_{\varphi_i^0}^2}{M_W^2}\right)\right]$$

D. Stöckinger et al. arXiv:1607.06292: The total bosonic contributions can be of $\mathcal{O}(2..4 \times 10^{-10})$!!

Total contribution

- The dominant contributions $\rightarrow$ mechanisms (3) and (4), other *new* contributions are sub-dominant.
- The total effect





- Left: Δa_μ as a function of ς_l (for positive values) and $\cos\tilde{\alpha} = 0.9$, $\varsigma_u = -0.8$, $\varsigma_d = -20$, $M_h = 125$ GeV, $\lambda_{hH^+H^-} = 0$, $\lambda_{hH^+H^-} = -5$. The remaining masses (in GeV): $M_H = M_{H^\pm} = M_A = 250$ (lower orange curve), 150 (middle blue curve), $M_H = M_{H^\pm} = 150$ and $M_A = 50$ (upper green curve).
- Right: Δa_μ as a function of ς_l (for negative values) and $\cos\tilde{\alpha} = 0.9$, $\varsigma_u = 0.8$, $\varsigma_d = 2$, $M_h = 125$ GeV, $\lambda_{hH^+H^-} = 0$, $\lambda_{hH^+H^-} = 5$ and $M_H = M_{H^\pm} = 250$ GeV and $M_A = 40$ GeV (upper green curve) or $M_H = M_{H^\pm} = 350$ GeV and $M_A = 50$ GeV (lower orange curve).
- **As expected, from the various $\Delta a_\mu^{(i)}$ individual contributions $\rightarrow$ significant contribution for low masses of the scalars (especially for low M_A) and large couplings.**
- We also observe $\rightarrow$ in some cases we do not need the maximum allowed value of $|\varsigma_l|$ to reach the 2σ region of Δa_μ^{exp}; a value $|\varsigma_l| \sim 30$ might just be enough!

Conclusions

- It is a common belief that only a restrained number of diagrams, (1) and (2) can significantly contribute to Δa_μ in 2HDMs.
- In most of the previous analyses a CP-odd scalar in the low-mass range is enough to explain, or reduce, the discrepancy between theory and experiment.
- In this work we have shown that the extra degrees of freedom of the A2HDM given by the ς_f parameters, can also explain this discrepancy in some region of the parameter space.
- If not, they can significantly reduce it in most cases.
- We have also seen that the W loop contribution (3) associated with a heavy scalar H can bring important contributions even if it has a global suppression factor $\mathcal{R}_{21}$.
- This contribution is positive for negative values of ς_l.
- The most interesting case is, however, the fermionic loop contribution (4) with the dominant part given by the top-quark.
- The last two diagrams (5) and (6) are also interesting, as they can sum up to an $\mathcal{O}(10\%)$ of the total contribution.
- Not all of these new contributions can be made simultaneously positive, however the total Δa_μ is positive for most parameter configurations.