

Vector-Like Quarks at the Origin of Light Quark Masses and Mixing

Gustavo C. Branco

Departamento de Física and CFTP/IST

talk given at "Workshop on Multi-Higgs
Models"

September 2016

Collaboration with :

Francisco Botella, Miguel Nebot, M.N. Rebelo
and J.I. Silva Marcos

2

A novel flavour fine-tuning problem in the SM:

Contrary to "conventional wisdom" in the SM, with no extra-symmetries the "natural value" of $|V_{13}|^2 + |V_{23}|^2$ is large, of order one:

$$|V_{13}|^2 + |V_{23}|^2 = O(1)$$

This is to be compared with
the experimental value :

$$|V_{13}|^2 + |V_{23}|^2 \approx 1.6 \times 10^{-3}$$

In order to show the fine-tuning problem, let us consider the extreme chiral (EC) limit, where

$$m_d = m_s = 0 ; m_b \neq 0$$

$$m_u = m_c = 0 ; m_t \neq 0$$

4

In the EC limit the general quark mass matrices can be written:

$$M_d = U_L^{d+} \begin{bmatrix} 0 & 0 \\ 0 & m_b \end{bmatrix} U_R^d$$

$$M_u = U_L^{u+} \begin{bmatrix} 0 & 0 \\ 0 & m_t \end{bmatrix} U_R^u$$

where $U_L^{d,u}$, $U_R^{d,u}$ are arbitrary unitary matrices. The ordering of the eigenvalues in the diagonal matrices has no physical meaning !!

Taking into account that in the EC limit the first 2 generations are massless, one can make an arbitrary redefinition of the light quark masses through a unitary transformation of the type

$$W_{u,d} = \begin{bmatrix} X_{u,d} & 0 \\ 0 & 1 \end{bmatrix}$$

$X_{u,d} \rightarrow 2 \times 2$ unitary matrices.

Under this transformation, V^0 transforms as :

$$V^0 \rightarrow V' = W_u^+ V^0 W_d$$

One has the freedom to choose $X_{u,d}$ at will, to diagonalise the 2×2 upper left sector of V' , leading to $V'_{12} = V'_{21} = 0$.

So, one has :

$$V' = \begin{pmatrix} V'_{11} & 0 & V'_{13} \\ 0 & V'_{22} & V'_{23} \\ V'_{31} & V'_{32} & V'_{33} \end{pmatrix}$$

Unitarity of V' leads then to :

$$V'^{*}_{13} V'_{23} = 0$$

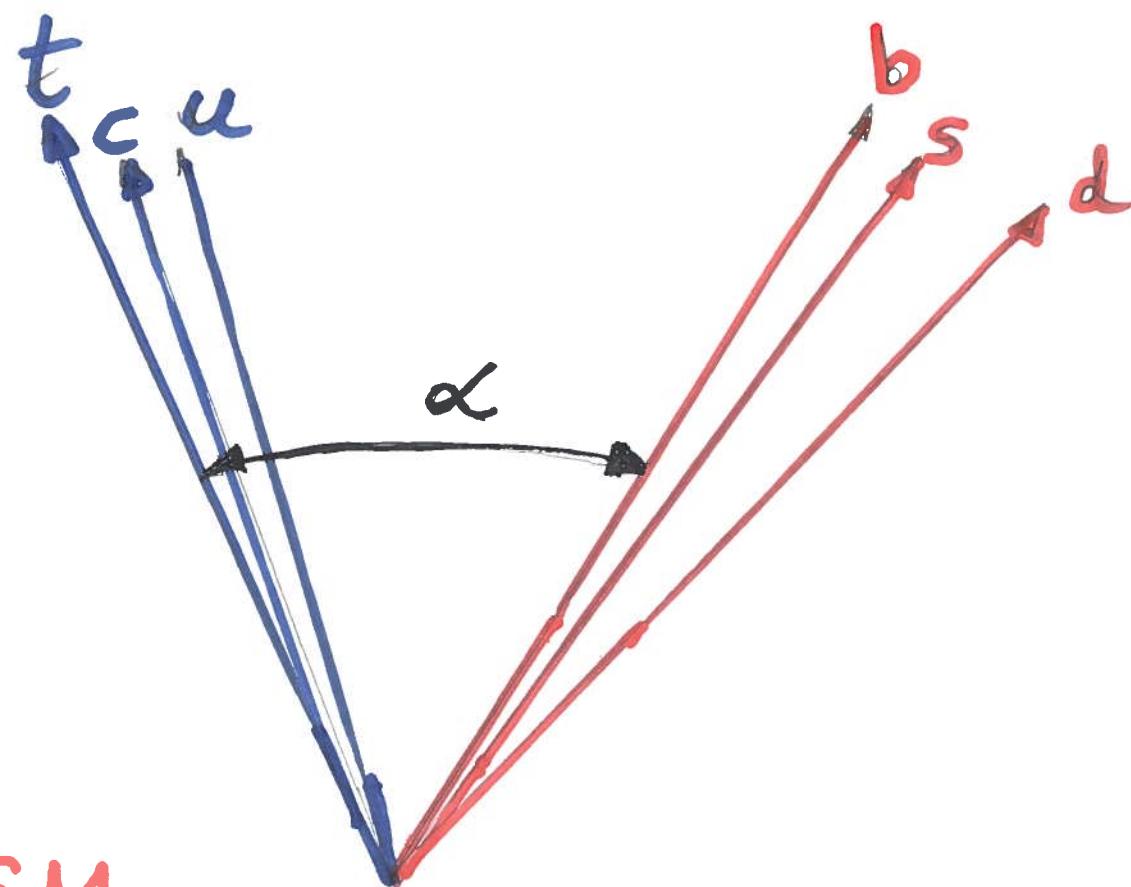
One can then choose, without loss of generality, $V'_{13} = 0$, $V'_{31} = 0$

V_{CKM} becomes then an orthogonal matrix

$$V_{CKM} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\alpha & s_\alpha \\ 0 & -s_\alpha & c_\alpha \end{bmatrix}$$

Important point – This mixing
is meaningful even in the EC
limit and it is arbitrary

Schematic picture of the Novel Fine-Tuning Problem



In the SM,
The angle α is in general of order (1)
and independent of quark mass hierarchy!

Tentative implications

- This novel fine-tuning problem provides further motivation to introduce some flavour symmetry in the SM which could guarantee

$$V_{CKM} \approx 1$$

- The root of the problem is the fact that in SM, y_u, y_d are totally independent complex matrices.

A comment :

This flavour fine-tuning problem in the SM, may provide motivation for considering left-right symmetric theories like e.g

$$SU(2)_L \times SU(2)_R \times U(1)$$

With only one bi-doublet, one has

$$M_u \propto M_d \Rightarrow V_{CKM} = 1$$

11

An invariant measure of alignment in flavour space.

Suppose that one has quark mass matrices M_u, M_d , with hierarchical mass eigenvalues, written in an arbitrary weak-basis (WB)

How can one have an invariant measure of alignment?

Consider the following WB invariant:

$$A \equiv \frac{1}{2} \text{tr } B^2 ; \quad B \equiv h_d - h_u$$

where $h_d = \frac{H_d}{\text{tr}[H_d]} ; \quad h_u = \frac{H_u}{\text{tr}[H_u]}$

where $H_{ud} \equiv (M_{u,d} M_{u,d}^+)$

By construction $\text{tr } h_d = \text{tr } h_u = 1$

In the EC limit, where $M_{u,d}$ have rank one

$$A = |V_{23}|^2 + |V_{13}|^2 .$$

The invariant A gives a measure of the size of the mixing even when the first two generations acquire mass.

$$A \approx |V_{23}|^2 + |V_{13}|^2 + O\left(\frac{m_s}{m_b}\right)^4$$

A simple discrete symmetry, introduced in the context of the SM, which leads to alignment and $V_{CKM}^{\text{CKM}} = \mathbb{1}$

$$Q_L^o \rightarrow e^{i\tau} Q_L^o, \quad ; \quad Q_{L_2}^o \rightarrow \bar{e}^{-2i\tau} Q_{L_2}^o; \quad Q_{L_3}^o \rightarrow \bar{e}^{-i\tau} Q_{L_3}^o$$

$$d_R^o \rightarrow \bar{e}^{i\tau} d_R^o; \quad ; \quad d_{R_2}^o \rightarrow \bar{e}^{-i\tau} d_{R_2}^o \quad d_{R_3}^o \rightarrow \bar{e}^{-i\tau} d_{R_3}^o$$

$$u_R^o \rightarrow e^{i\tau} u_R^o, \quad ; \quad u_{R_2}^o \rightarrow e^{i\tau} u_{R_2}^o; \quad u_{R_3}^o \rightarrow u_{R_3}^o$$

The Yukawa couplings are :

$$\mathcal{L}_Y = - \left[\bar{Q}_{L_i}^{\circ} \phi Y_d d_{R_j}^{\circ} + \bar{Q}_{L_i}^{\circ} \tilde{\phi} Y_u u_{R_j}^{\circ} \right] + h.c.$$

With the discrete symmetry, the couplings become :

$$Y_d = \begin{bmatrix} 0 & 00 \\ 0 & 00 \\ 0 & 0x \end{bmatrix}; Y_u = \begin{bmatrix} 0 & 00 \\ 0 & 00 \\ 0 & 0x \end{bmatrix}$$

How to generate masses for the
light-quark generations?

Possible answer: introduce
vector-like quarks (VLQ)
(VLQ) are "cousins" of right-
-handed neutrinos (ν_R). Both
VLQ and ν_R have gauge invariant
masses.

For example :

$$\overline{D}_L \ M_D \ D_R$$

$$\gamma_R^T C_R^M \gamma_R$$

We know that it is "natural" to consider $M_D, M_R \gg v$

but the actual value is not known

27 Seven reasons to consider vector-like quarks

1. They provide a self-consistent framework with naturally small violations of 3×3 unitarity of V_{CKM} .
2. Lead to naturally small Flavour Changing Neutral currents (FCNC) mediated by Z_μ

NEW Physics in $\left\{ \begin{array}{l} B_d - \bar{B}_d \text{ mixing} \\ B_s - \bar{B}_s \text{ mixing} \\ K^0 - \bar{K}^0 \text{ mixing} \end{array} \right.$

3. Provide the simplest framework to have Spontaneous CP Violation, with a vacuum phase generating a non-trivial CKM phase.
4. Provide New Physics contributions to $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixings
5. Provide a simple solution to the Strong CP problem, which does not require Axions
6. May contribute to the understanding of the observed pattern of fermion masses and mixing.

7. Provide a framework where there is a common origin for all CP violations:

- (i) CP violation in the Quark Sector
- (ii) CP Violation in the Lepton Sector,
detectable through neutrino oscillations

$U_{e3} \neq 0 \rightarrow$ Great News

- (iii) CP violation needed to generate the Baryon Asymmetry of the Universe (BAU) through Leptogenesis.

Comment :

There is *nothing "strange"* in having deviations of 3×3 unitarity. The PMNS matrix in the *leptonic sector* in the context of type-one seesaw (ν SM) is not 3×3 unitary !!

Generation of realistic quark masses and mixings through the introduction of vector-like quarks:

3 $Q = -\frac{1}{3}$ VLQ D_L, D_R

3 $Q = \frac{2}{3}$ VLQ U_L, U_R

We only include terms which are allowed by gauge invariance and the discrete symmetry. Will not introduce soft-breaking terms

Extension of the symmetry to the full Lagrangian.

$$\overset{\circ}{D}_{L_1} \rightarrow e^{-i3\tau} \overset{\circ}{D}_{L_1}; \quad \overset{\circ}{D}_{L_2} \rightarrow \bar{e}^{-2i\tau} \overset{\circ}{D}_{L_2}; \quad \overset{\circ}{D}_{L_3} \rightarrow \bar{e}^{-i\tau} \overset{\circ}{D}_{L_3}$$

$$\overset{\circ}{D}_{R_1} \rightarrow e^{-i2\tau} \overset{\circ}{D}_{R_1}; \quad \overset{\circ}{D}_{R_2} \rightarrow \bar{e}^{-i3\tau} \overset{\circ}{D}_{R_2}; \quad \overset{\circ}{D}_{R_3} \rightarrow \overset{\circ}{D}_{R_3}$$

$$\overset{\circ}{U}_{L_1} \rightarrow \bar{e}^{-i\tau} \overset{\circ}{U}_{L_1}; \quad \overset{\circ}{U}_{L_2} \rightarrow \overset{\circ}{U}_{L_2}; \quad \overset{\circ}{U}_{L_3} \rightarrow \bar{e}^{i\tau} \overset{\circ}{U}_{L_3}$$

$$\overset{\circ}{U}_{R_1} \rightarrow \overset{\circ}{U}_{R_1}; \quad \overset{\circ}{U}_{R_2} \rightarrow \bar{e}^{i\tau} \overset{\circ}{U}_{R_2}; \quad \overset{\circ}{U}_{R_3} \rightarrow e^{i2\tau} \overset{\circ}{U}_{R_3}$$

A complex scalar singlet is also introduced, which transforms as: $S \rightarrow \underline{\bar{e}^i \tau S}$

	$d_R^o(-\tau)$	$d_R^o(-\tau)$	$d_R^o(-2\tau)$	$D_R^o(-2\tau)$	$D_R^o(-3\tau)$	$D_R^o(0)$
$\bar{Q}_{L_1}^o(-\tau)$	•	•	•	•	•	$-\tau$
$\bar{Q}_{L_2}^o(2\tau)$	•	•	•	•	$-\tau$	•
$\bar{Q}_{L_3}^o(\tau)$	•	•	$-\tau$	$-\tau$	•	•
$\bar{D}_{L_1}^o(3\tau)$	•	•	τ	τ	1	•
$\bar{D}_{L_2}^o(2\tau)$	τ	τ	1	1	$-\tau$	•
$\bar{D}_{L_3}^o(\tau)$	1	1	$-\tau$	$-\tau$	•	τ

Similar structure for up sector

Mass terms written in a compact form

$$\mathcal{L}_M = -(\bar{d}_L^o \bar{D}_L^o) M_d \begin{bmatrix} d_R^o \\ D_R^o \end{bmatrix} - (\bar{u}_L^o \bar{U}_L^o) M_u \begin{bmatrix} u_R^o \\ U_R^o \end{bmatrix}$$

$$M_d = \begin{bmatrix} m_d & w_d \\ \dots & \dots \\ X_d & M_d \end{bmatrix}$$

$m_d, w_d \rightarrow$ of order v

$X_d, M_d \rightarrow$ of order $\langle S \rangle^{\frac{1}{2}} V$ and invariant mass terms.

similar for up quarks.

The 6×6 matrix $M_d, (M_u)$ are diagonalized by

$$\begin{bmatrix} d_L^0 \\ D_L^0 \end{bmatrix} = \begin{bmatrix} A_{dL} \\ B_{dL} \end{bmatrix} d_L = U_{dL} d_L \quad \text{similar for up quarks}$$

A_{dL}, B_{dL} are 3×6 matrices

$$U_{dL}^+ M_d U_{dR} = \text{diag}(d_d^2, D_d^2)$$

L.Lavoura, GCB (1986)

L.Lavoura, J.P.Silva
(1993)

charged currents:

$$\mathcal{L}_W = -g/\sqrt{2} (\bar{u}_L^0 \gamma^\mu d_L^0) W_\mu = -g (\bar{u}_L V \gamma^\mu d_L) W_\mu$$

$V = A_{uL}^+ A_{dL}$

, with u_L, d_L running from 1 to 6

$$\mathcal{L}_Z = \frac{g}{\cos \theta_W} \left[\frac{1}{2} (\bar{u}_L W_u \gamma^\mu u_L - \bar{d}_L W_d \gamma^\mu d_L) - \sin^2 \theta_W \cdot \right. \\ \left. \left(\frac{2}{3} \bar{u} \gamma^\mu u - \frac{1}{3} \bar{d} \gamma^\mu d \right) \right] Z_\mu$$

$$W_d = V^* V ; \quad W_u = V V^* ; \quad V \in (A_{uL}^+ A_{dL})$$

$$\mathcal{L}_H = -\frac{h}{v} \left[\bar{d}_L W_d D_d d_R + \bar{u}_L W_u D_u u_R \right] + h.c.$$

There are Z -mediated and h -mediated neutral currents which violate flavour (FCNC) but they are both naturally suppressed.

$$\mathcal{U}_{qL} = \begin{bmatrix} K_q & R_q \\ S_q & T_q \end{bmatrix}$$

with $q = u, d$; K_q, R_q, S_q, T_q are 3×3 matrices

$$K_q K_q^+ = \mathbb{I} - R_q R_q^+$$

$$R_q \approx \frac{(m_q X_q^+ + \omega_q M_q^+)^T T_q}{D_q^2} \approx (m/M)$$

$$K_q^+ K_q = I - S_q^+ S_q$$

$$S_q \approx \left(\frac{X_q m_q^+ + M_q \omega_q^+}{X_q X_q^+ + M_q M_q^+} \right) K_q \approx m/M$$

$U_{L,R}^q$ will be determined through an exact numerical diagonalization. But it is useful to derive an effective Hermitian quark mass matrix

$$K_q^{-1} H_{\text{eff}}^q K_q = \text{diag.}(d^z)$$

$$H_{\text{eff}}^q = (m_q m_q^+ + w_q w_q^+) - (m_q X_q^+ + w_q M_q^+) \left(X_q X_q^+ + M_q M_q^+ \right)^{-1} \times \\ (X_q m_q^+ + M_q w_q^+)$$

Realistic Examples

masses at M_Z scale in GeV

$$m_d = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3, \dots \end{bmatrix} ; m_u = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 185, \dots \end{bmatrix}$$

$$w_d = \begin{bmatrix} 0 & 0 & 0.06 \dots \\ 0 & 0.399 & 0 \\ 0.399 & 0 & 0 \end{bmatrix} ; w_u = \begin{bmatrix} 0 & 0 & 0.009 \dots \\ 0 & 3.7 \dots & 0 \\ 185, \dots & 0 & 0 \end{bmatrix}$$

$$M_d = \begin{bmatrix} 767 & & \\ & 1535 & 0 \\ 0 & & 1842 \end{bmatrix} ; M_u = \begin{bmatrix} 1295 & & \\ & 1481 & 0 \\ 0 & & 2221 \end{bmatrix}$$

$$X_d = \begin{bmatrix} 0 & 0 & -115,.. \\ -262,.. & i 46,.. & 460,.. \\ 486,.. & 0 & 368,.. \end{bmatrix}; X_u = \begin{bmatrix} 0 & 0 & 68,.. \\ -212,.. & -185,.. & 0 \\ 416,.. & 0 & 0 \end{bmatrix}$$

$$|V| = \begin{bmatrix} 0.97... & 0.22... & 0.003... & \cdot & \cdot & \cdot \\ 0.22 & .97.. & 0.04... & \cdot & \cdot & \cdot \\ 0.008 & 0.039 & 0.98 & \cdot & \cdot & \cdot \\ \dots & \dots & \dots & \cdot & \cdot & \cdot \\ \text{very small} & \text{very small} & 0.15 & \cdot & \cdot & \cdot \\ & & & \cdot & \cdot & \cdot \end{bmatrix}$$

Conclusions

- There is a novel fine-tuning problem which provides motivation to introduce a flavour symmetry which leads to

$$\sqrt{V_{CKM}} = \mathbb{I} \text{ in leading order}$$

- Vector-like quarks may generate light quark masses and a realistic

$$\sqrt{V_{CKM}}$$