

Spontaneous CP violation, nonzero θ_{13} and leptogenesis in Type-II seesaw



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based on: Biswajit Karmakar and A.S., PRD 93 (2016) 013006

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Outline

Introduction : discrete roadmap for lepton mixing

The basic set-up : type- I + II seesaw

Neutrino masses, mixing and δ_{CP}

Leptogenesis : role of triplet and RH neutrinos

Conclusion

Introduction

Questions :

- (a) Why lepton mixing is so different from the that of the quarks?:
[hint toward a new symmetry in the lepton sector]

- (b) Smallness of neutrino mass:
[hint toward a new scale: seesaw mechanism]

- (c) Origin of CP violation in lepton sector:
[can there be a spontaneous CP violation?]

Neutrino Parameters : where do we stand

Neutrino flavor eigenstates and mass eigenstates are related by

- $|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle$

Pontecorvo-Maki-Nakagawa-Sakata parametrization

$$U_{PMNS} = \begin{bmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13}e^{-i\delta} \\ -S_{12}C_{23} - C_{12}S_{13}S_{23}e^{i\delta} & C_{12}C_{23} - S_{12}S_{13}S_{23}e^{i\delta} & C_{13}S_{23} \\ S_{12}S_{23} - C_{12}S_{13}C_{23}e^{i\delta} & C_{12}S_{23} - S_{12}S_{13}C_{23}e^{i\delta} & C_{13}C_{23} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{bmatrix}$$

here $C_{ij} = \cos \theta_{ij}$ and $S_{ij} = \sin \theta_{ij}$.

PDG, Chin. Phys. C 38, 090001 (2014)

Neutrino Parameters

- Three mixing angles (θ_{12} , θ_{23} and θ_{13})
- Two mass squared differences, namely solar mass-squared difference (Δm_{\odot}^2) and atmospheric mass-squared difference (Δm_A^2). At-least, two different neutrino mass spectrum is possible: Normal hierarchy (NH) [$m_1 < m_2 < m_3$] and inverted hierarchy (IH) [$m_3 < m_1 < m_2$]
- Dirac phase (δ)
- Two Majorana phases (α_{21} and α_{31})

● Neutrino Oscillation Parameters:

Forero et al., PRD 90 (2014) 9, 093006

Parameter	Best Fit	1σ range	3σ range
$\Delta m^2_\odot [\times 10^{-5} \text{eV}^2]$	7.60	7.42 – 7.79	7.11 – 8.18
$ \Delta m^2_A [\times 10^{-3} \text{eV}^2]$	2.48	2.41 – 2.53	2.30 – 2.65
	2.38	2.32 – 2.43	2.20 – 2.54
$\sin^2 \theta_{12}$	0.323	0.307 – 0.339	0.278 – 0.375
$\sin^2 \theta_{23}$	0.567	0.439 – 0.599	0.392 – 0.643
	0.573	0.530 – 0.598	0.403 – 0.640
$\sin^2 \theta_{13}$	0.0234	0.0214 – 0.0254	0.0177 – 0.0294
	0.0240	0.0221 – 0.0259	0.0183 – 0.0297
δ/π	1.34	0.96 – 1.98	0 – 2
	1.48	1.16 – 1.82	0 – 2

There is a recent hint for Dirac CP Phase → $\delta \approx 270^\circ$

- Absolute neutrino mass : $\sum_i m_{\nu_i} < 0.23 \text{ eV}$ (95% CL)

PLANCK Collaboration, 1303:5076

- $|m_{ee}| < (0.14 - 0.28) \text{ eV}$
 $< (0.19 - 0.45) \text{ eV}$

KamLAND-Zen, arXiv:1409.0077

EXO-200, arXiv:1402.6956

A discrete roadmap towards lepton masses and mixing

Prior to 2012 : Tribimaximal(TBM) mixing and $\theta_{13} = 0$

Global analysis of neutrino oscillation data suggests $\sin^2 \theta_{12} = 1/3$ and $\sin^2 \theta_{23} = 1/2$.
 With $\theta_{13} = 0$, neutrino mixing matrix takes the form

$$U_{TB} = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

[Harrison, Perkins, Scott, hep-ph/9904297](#)

2012 and afterwards: $\sin \theta_{13} \approx 0.155 \rightarrow \theta_{13} \approx 9^\circ$ [from result of Daya Bay ,
 DOUBLE-CHOOZ, RENO, T2K]

Origin of TBM structure:

- Non-Abelian discrete symmetry: they have multi-dimensional representation and hence can relate one flavor to another.
- A_4 turns out to be simplest possibility to accommodate 3-families as it has 4-irreducible representations: $1, 1', 1'', 3$.

[Ma,Rajasekharan PRD64; Altarelli,Feruglio NPB741](#)

The basic set-up

Neutrino masses and mixing

- ① A general Type-II seesaw mechanism
- ② Standard Model + 3 RH neutrinos (gauge singlets) + **only one scalar triplet** (Δ)
- ③ pure type-I seesaw provides TBM and **scalar triplet contribution generates** θ_{13}

CP violation

- ① Source of CP violation : **Spontaneous CP violation** [Branco, Parada, Rebelo hep-ph/0307119](#)
- ② This unique source of CP violation generates Dirac , Majorana CP phases and CP violation required for matter-antimatter asymmetry of the universe
[Branco, Felipe, Joaquim, Rev. Mod. Phys 84 \(2012\)](#); [Branco, Felipe, Joaquim, Serodio, PRD 86 \(2012\)](#)
- ③ Complex vev of a single flavon serves the purpose
 - [A] magnitude of complex vev \Rightarrow adequate θ_{13}
 - [B] phase of complex vev \Rightarrow ensures CP violation of the theory

Leptogenesis

- ① Decay of RH neutrinos only: lepton asymmetry vanishes
- ② Decay of scalar triplet ($M_R > M_\Delta$) : RH neutrinos in loop \Rightarrow successful leptogenesis scenario
- ③ Decay of RH neutrinos ($M_\Delta > M_R$) : Triplets in loop \Rightarrow constrained leptogenesis scenario
[Hambye, Senjanovic, PLB 582, 73 \(2004\)](#)

- Generic Type-II seesaw:

$$\begin{aligned} m_\nu &= m_\nu^I + m_\nu^{II} \\ &= -m_D^T M_R^{-1} m_D + m_\nu^{II} \end{aligned}$$

- Symmetries and Particle Content

Field	e_R	μ_R	τ_R	L	N_R	H	Δ	ϕ_S	ϕ_T	ξ	ξ'	S
A_4	1	$1''$	$1'$	3	3	1	1	3	3	1	$1'$	1
Z_4	-1	-1	-1	i	i	1	$-i$	-1	$-i$	-1	i	-1
Z_3	ω	ω	ω	ω	ω	1	ω^2	ω	1	ω	ω^2	1

- A_4 multiplication rule:

$$3 \times 3 = 1 + 1' + 1'' + 3_A + 3_S; 1' \times 1' = 1''; 1' \times 1'' = 1; 1'' \times 1'' = 1'$$

If (a_1, a_2, a_3) and (b_1, b_2, b_3) are two triplets under A_4 , then

$$1 \sim a_1 b_1 + a_2 b_3 + a_3 b_2, 1' \sim a_3 b_3 + a_1 b_2 + a_2 b_1, 1'' \sim a_2 b_2 + a_3 b_1 + a_1 b_3$$

$$3_S = \frac{1}{3}(2a_1 b_1 - a_2 b_3 - a_3 b_2, 2a_3 b_3 - a_1 b_2 - a_2 b_1, 2a_2 b_2 - a_1 b_3 - a_3 b_1) \text{ and}$$

$$3_A = \frac{1}{2}(a_2 b_3 - a_3 b_2, a_1 b_2 - a_2 b_1, a_3 b_1 - a_1 b_3)$$

- Diagonal Charged Lepton Sector:

$$\mathcal{L}_I = \frac{y_e}{\Lambda} (\bar{L} \phi_T) H e_R + \frac{y_\mu}{\Lambda} (\bar{L} \phi_T)' H \mu_R + \frac{y_\tau}{\Lambda} (\bar{L} \phi_T)'' H \tau_R,$$

Λ : cut-off scale of the theory, $\langle \phi_S \rangle = (u_S, u_S, u_S)$ and $\langle \phi_T \rangle = (u_T, 0, 0)$

- Type-I seesaw contribution

$$\mathcal{L}_I = y \bar{L} \tilde{H} N_R + x_A \xi \overline{N_R^c} N_R + x_B \phi_S \overline{N_R^c} N_R,$$

$$m_D = Y_D v = yv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}; M_R = \begin{pmatrix} a + 2b/3 & -b/3 & -b/3 \\ -b/3 & 2b/3 & a - b/3 \\ -b/3 & a - b/3 & 2b/3 \end{pmatrix}$$

$$a = 2x_A \langle \xi \rangle = 2x_A u_\xi, b = 2x_B u_S$$

$$\begin{aligned} m_\nu^I &= -m_D^T M_R^{-1} m_D \\ &= -k \begin{pmatrix} \frac{3+\alpha}{3(1+\alpha)} & \frac{\alpha}{3(1+\alpha)} & \frac{\alpha}{3(1+\alpha)} \\ \frac{\alpha}{3(1+\alpha)} & -\frac{\alpha(2+\alpha)}{3(1-\alpha^2)} & \frac{3+\alpha-\alpha^2}{3(1-\alpha^2)} \\ \frac{\alpha}{3(1+\alpha)} & \frac{3+\alpha-\alpha^2}{3(1-\alpha^2)} & -\frac{\alpha(2+\alpha)}{3(1-\alpha^2)} \end{pmatrix} \text{ where } \alpha = b/a \text{ and } k = v^2 y^2 / a \end{aligned}$$

- TBM Mixing:

Here, m_ν^I can be diagonalized by TBM Mixing matrix

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

- U_{TB} also diagonalizes M_R

$$M_R^{\text{diag}} = U_R^T M_R U_R = \text{diag}(a+b, a, |a-b|) \quad \text{with} \quad U_R = U_{TB}$$

● Triplet contribution : Type-II seesaw

$$\mathcal{L}_{II} = \frac{1}{\Lambda^2} \Delta L^T L (x_1 S + x'_1 S^*) \xi'$$

with $\langle \xi' \rangle = u_{\xi'}$, $\boxed{\langle S \rangle = v_S e^{i\alpha_S}}$ \Leftarrow Only source of CP violation



Yukawa matrix for triplet : $Y_\Delta = h \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad h = \frac{1}{\Lambda^2} u_{\xi'} v_S (x_1 e^{i\alpha_S} + x'_1 e^{-i\alpha_S})$

● Complete Scalar Potential:

$$V = V_S + V_H + V_\Delta + V_{SH} + V_{S\Delta} + V_{\Delta H},$$

where

$$\begin{aligned}
 V_S &= \mu_S^2 (S^2 + S^{*2}) + m_S^2 S^* S + \lambda_1 (S^4 + S^{*4}) + \lambda_2 S^* S (S^2 + S^{*2}) + \lambda_3 (S^* S)^2, \\
 V_H &= m_H^2 H^\dagger H + \lambda_4 (H^\dagger H)^2, \\
 V_\Delta &= M_\Delta^2 \text{Tr}(\Delta^\dagger \Delta) + \lambda_5 [\text{Tr}(\Delta^\dagger \Delta)]^2, \\
 V_{SH} &= \lambda_6 (S^* S) H^\dagger H + \lambda_7 (S^2 + S^{*2})(H^\dagger H), \\
 V_{S\Delta} &= \text{Tr}(\Delta^\dagger \Delta)[\lambda_8 (S^2 + S^{*2}) + \lambda_9 S^* S], \\
 V_{\Delta H} &= \lambda_{10} (H^\dagger H) \text{Tr}(\Delta^\dagger \Delta) + \lambda_{11} (H^\dagger \Delta^\dagger \Delta H) + \left(-\frac{\mu}{\Lambda} \tilde{H}^T \Delta \tilde{H} \phi_S \phi_T + h.c. \right).
 \end{aligned}$$

- After Minimizing of Scalar Potential:

$$\langle \Delta^0 \rangle \equiv u_\Delta = \eta \frac{v^2}{M_\Delta^2} \quad \text{and} \quad \eta = \frac{\mu}{\Lambda} u_S u_T$$

- Triplet contribution:

$$m_\nu^{II} = \begin{pmatrix} 0 & 0 & d \\ 0 & d & 0 \\ d & 0 & 0 \end{pmatrix}, \quad d = 2h u_\Delta = 2h\eta \frac{v^2}{M_\Delta^2}.$$

- Effective Neutrino Mass Matrix:

$$m_\nu = m_\nu^I + m_\nu^{II} = -k \begin{pmatrix} \frac{3+\alpha}{3(1+\alpha)} & \frac{\alpha}{3(1+\alpha)} & \frac{\alpha}{3(1+\alpha)} \\ \frac{\alpha}{3(1+\alpha)} & -\frac{\alpha(2+\alpha)}{3(1-\alpha^2)} & \frac{3+\alpha-\alpha^2}{3(1-\alpha^2)} \\ \frac{\alpha}{3(1+\alpha)} & \frac{3+\alpha-\alpha^2}{3(1-\alpha^2)} & -\frac{\alpha(2+\alpha)}{3(1-\alpha^2)} \end{pmatrix} + \begin{pmatrix} 0 & 0 & d \\ 0 & d & 0 \\ d & 0 & 0 \end{pmatrix}.$$

- Diagonalization of Effective Neutrino Mass Matrix:

$$m'_\nu = U_{TB}^T m_\nu U_{TB} = \begin{pmatrix} -\frac{d}{2} - \frac{k}{(1+\alpha)} & 0 & \frac{\sqrt{3}d}{2} \\ 0 & d-k & 0 \\ \frac{\sqrt{3}d}{2} & 0 & \frac{d}{2} + \frac{k}{(1-\alpha)} \end{pmatrix}$$

- To diagonalize m'_ν : Additional rotation required $\Rightarrow U_1^T m'_\nu U_1 = m_\nu^{\text{diag}}$

with $U_1 = \begin{pmatrix} \cos \theta & 0 & \sin \theta e^{-i\psi} \\ 0 & 1 & 0 \\ -\sin \theta e^{i\psi} & 0 & \cos \theta \end{pmatrix}$

- Effectively:

$$(U_{TB} U_1)^T m_\nu U_{TB} U_1 = \text{diag}(m_1 e^{i\gamma_1}, m_2 e^{i\gamma_2}, m_3 e^{i\gamma_3})$$

$m_{i=1,2,3}$: real and positive eigenvalues, $\gamma_{i=1,2,3}$: phases associated to these mass eigenvalues.

- Diagonalizing Matrix: $U_\nu \iff U_{PMNS}$

$$U_\nu = U_{TB} U_1 U_m = \begin{pmatrix} \sqrt{\frac{2}{3}} \cos \theta & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}} e^{-i\psi} \sin \theta \\ -\frac{\cos \theta}{\sqrt{6}} + \frac{e^{i\psi} \sin \theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{\cos \theta}{\sqrt{2}} - \frac{e^{-i\psi} \sin \theta}{\sqrt{6}} \\ -\frac{\cos \theta}{\sqrt{6}} - \frac{e^{i\psi} \sin \theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{\cos \theta}{\sqrt{2}} - \frac{e^{-i\psi} \sin \theta}{\sqrt{6}} \end{pmatrix} U_m$$

with $U_m = \text{diag}(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2})$

- Majorana Phases

$$\alpha_{21} = (\gamma_1 - \gamma_2) \text{ and } \alpha_{31} = (\gamma_1 - \gamma_3)$$

Evaluation of neutrino mixing parameter

Comparing U_ν and U_{PMNS}

$$\sin \theta_{13} = \sqrt{\frac{2}{3}} |\sin \theta|, \quad \sin^2 \theta_{12} = \frac{1}{3(1 - \sin^2 \theta_{13})},$$

$$\sin^2 \theta_{23} = \frac{1}{2} + \frac{1}{\sqrt{2}} \sin \theta_{13} \cos \delta, \quad \delta = \arg[(U_1)_{13}]$$

Connection of triplet contribution with nonzero θ_{13}

- Triplet contribution = $d = |d|e^{i\phi_d}$ where $\beta = |d|/k$ and $k = v^2 y^2/a$

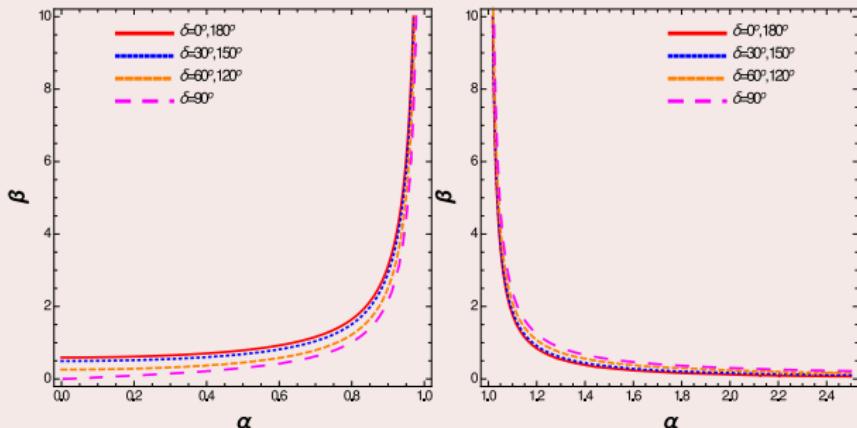
$$\tan 2\theta = \frac{\sqrt{3}}{\alpha} \frac{[1 - (1 - \alpha^2) \cos^2 \phi_d]^{1/2}}{\frac{2}{\beta(1 - \alpha^2)} + \cos \phi_d} \quad \text{and} \quad \tan \delta = \tan \psi = (\tan \phi_d)/\alpha.$$

↑

$$\langle S \rangle = v_S e^{i\alpha_S} \quad \implies \quad \tan \phi_d = \frac{(x_1 - x'_1)}{(x_1 + x'_1)} \tan \alpha_S,$$

- θ_{13} and δ : depends on α, β, ϕ_d
- Cases A: $\alpha < 1$ and Cases B: $\alpha > 1$

Contour plots for $\sin^2 \theta_{13} = 0.0234$ in the $\alpha - \beta$ plane for various δ



- Typical contour plot for $\sin^2 \theta_{13}$ with a specific δ value coincides with $|\pi - \delta|$ values.
- Example: $\delta = 30^\circ$ is repeated for $\delta = 150^\circ, 210^\circ, 330^\circ$.



Light neutrino masses and Majorana phase

- Real, positive light neutrino masses:

$$\begin{aligned} m_1 &= k \left[\left(\frac{\alpha}{(1-\alpha^2)} - \frac{p}{k} \right)^2 + \left(\frac{q}{k} \right)^2 \right]^{1/2}, \\ m_2 &= k \left[1 + \beta^2 - 2\beta \cos \phi_d \right]^{1/2}, \\ m_3 &= k \left[\left(\frac{\alpha}{(1-\alpha^2)} + \frac{p}{k} \right)^2 + \left(\frac{q}{k} \right)^2 \right]^{1/2}, \end{aligned}$$

where p and q are defined as,

$$\begin{aligned} \left(\frac{p}{k} \right)^2 &= \frac{1}{2} \left(\frac{A}{k^2} + \sqrt{\frac{A^2}{k^4} + \frac{B^2}{k^4}} \right) \quad , \quad \left(\frac{q}{k} \right)^2 = \frac{1}{2} \left(-\frac{A}{k^2} + \sqrt{\frac{A^2}{k^4} + \frac{B^2}{k^4}} \right); \\ \frac{A}{k^2} &= \beta^2 \cos 2\phi_d + \beta \frac{\cos \phi_d}{1-\alpha^2} + \frac{1}{(1-\alpha^2)^2} \quad , \quad \frac{B}{k^2} = \beta^2 \sin 2\phi_d + \beta \frac{\sin \phi_d}{1-\alpha^2}. \end{aligned}$$

- Majorana phases:

$$\begin{aligned} \alpha_{21} &= \tan^{-1} \left[\frac{q/k}{p/k + \frac{\alpha}{(\alpha^2-1)}} \right] - \tan^{-1} \left[\frac{\beta \sin \phi_d}{\beta \cos \phi_d - 1} \right], \\ \alpha_{31} &= \pi + \tan^{-1} \left[\frac{q/k}{p/k + \frac{\alpha}{(\alpha^2-1)}} \right] - \tan^{-1} \left[\frac{q/k}{p/k + \frac{\alpha}{(1-\alpha^2)}} \right]. \end{aligned}$$

Constraining Parameters Involved

- Parameters α, β can be constrained by low energy neutrino oscillation data through ratio of solar and atmospheric mass squared difference defined as

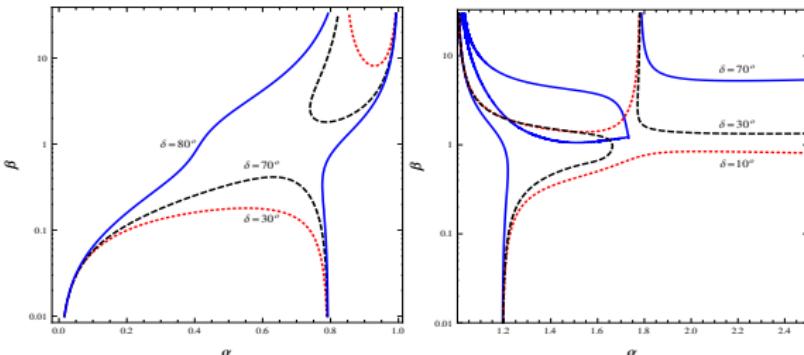
$$r = \frac{\Delta m_{\odot}^2}{|\Delta m_A^2|} \simeq 0.03, \quad \Delta m_{\odot}^2 = \Delta m_{21}^2 \equiv m_2^2 - m_1^2 = 7.6 \times 10^{-5} \text{ eV}^2,$$

$$|\Delta m_A^2| = |\Delta m_{31}^2| \equiv |m_3^2 - m_1^2| = 2.48 \times 10^{-3} \text{ eV}^2.$$

- Hence using above relations we get

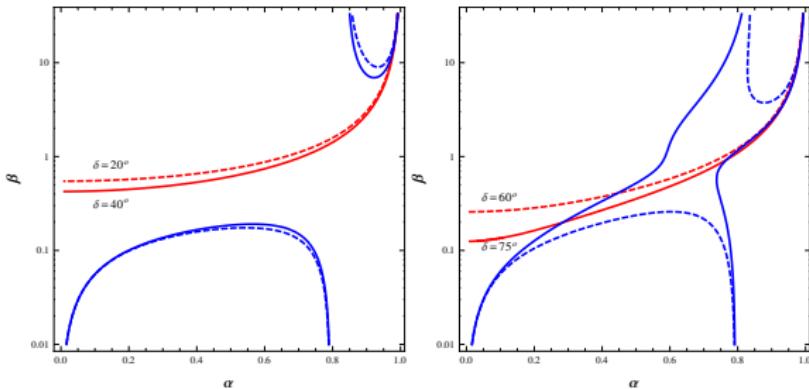
$$r = \frac{(1 - \alpha^2)}{4\alpha} \frac{k}{p} \left[1 + \beta^2 - 2\beta \cos \phi_d - \left(\frac{\alpha}{(1 - \alpha^2)} - \frac{p}{k} \right)^2 - \left(\frac{q}{k} \right)^2 \right].$$

- Contour plots for $r = 0.03$ in the $\alpha - \beta$ plane for various δ for $\alpha < 1$ and $\alpha > 1$



Constraining Parameters Involved: $\alpha < 1$

- Getting solutions for α and β by plotting $\sin^2 \theta_{13} = 0.0234$ and $r = 0.03$ contours together



- α, β values at the intersection points of the r and $\sin^2 \theta_{13}$ contours

δ	α	β	$\sum m_i$ (eV)
$20^\circ(160^\circ, 200^\circ, 340^\circ)$	0.99	28.26	0.0714
$40^\circ(140^\circ, 220^\circ, 320^\circ)$	0.99	20.94	0.0709
$60^\circ(120^\circ, 240^\circ, 300^\circ)$	0.98	11.16	0.0701
$75^\circ(105^\circ, 255^\circ, 285^\circ)$	0.94	3.70	0.0691
	0.77	0.93	0.0734
	0.29	0.20	0.1333
$80^\circ(100^\circ, 260^\circ, 280^\circ)$	0.16	0.11	0.1835
$82^\circ(98^\circ, 262^\circ, 278^\circ)$	0.12	0.09	0.2137
$85^\circ(95^\circ, 265^\circ, 275^\circ)$	0.07	0.05	0.2827

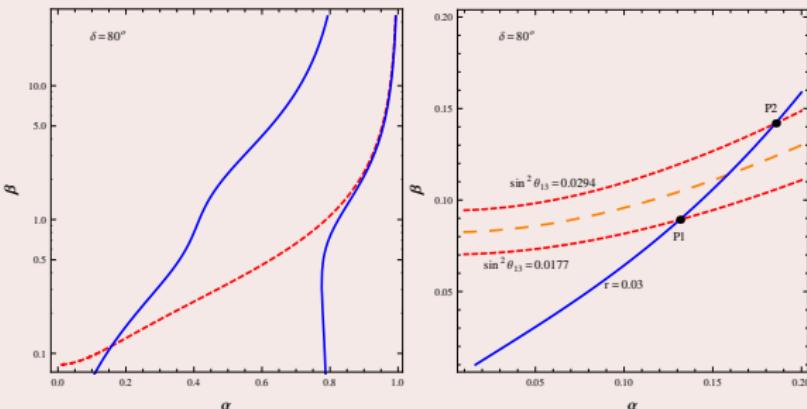
Constraining Parameters Involved: $\alpha < 1$

- Each δ values are repeated at $|\pi - \delta|$
- Accepted solutions : $\alpha, \beta < 1$



Allowed Dirac CP phases: $72^\circ - 82^\circ, 98^\circ - 108^\circ, 252^\circ - 262^\circ, 278^\circ - 288^\circ$ out of $0 - 360^\circ$

$\delta = 80^\circ (\equiv 100^\circ, 260^\circ, 280^\circ)$ and Neutrino Parameters

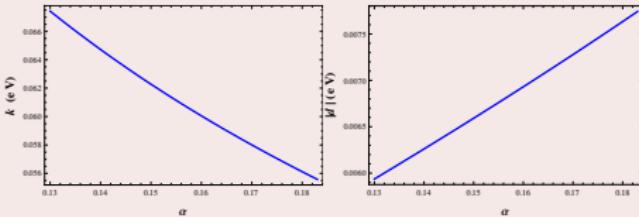


Reference Point P1 (α, β) = (0.13, 0.09)
 Reference Point P2 (α, β) = (0.18, 0.14)

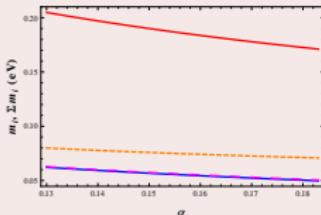
Prediction of neutrino parameters: $\alpha < 1$

- We solve for δ , β from $\sin^2 \theta_{13} = 0.0234$ and $r = 0.03$ constraints
- To find d and k , we must satisfy $|\Delta m_{atm}^2|$ and Δm_{\odot}^2 independently

k vs α and $|d|$ vs α plots for $\delta = 80^\circ (\equiv 100^\circ, 260^\circ, 280^\circ)$



Light neutrino masses vs α plots for $\delta = 80^\circ (\equiv 100^\circ, 260^\circ, 280^\circ)$

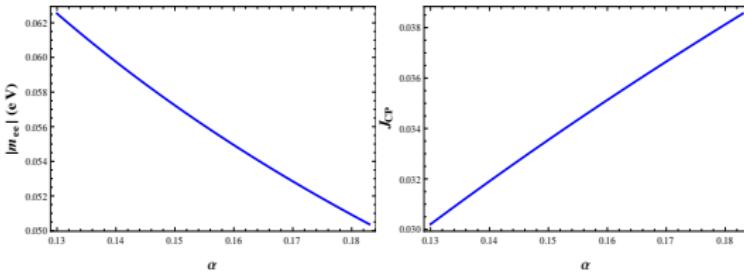


m_1 = blue continuous, m_2 = magenta large dashed, m_3 = orange dashed and Σm_i = red continuous line

Prediction of neutrino parameters: $\alpha < 1$

- Effective neutrino mass parameter $|m_{ee}|$ and Jarlskog invariant for $\delta = 80^\circ$

$$\begin{aligned} |m_{ee}| &= \left| m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{i\alpha_{21}} + m_3 s_{13}^2 e^{i(\alpha_{31}-2\delta)} \right|. \\ J_{CP} &= \text{Im}[U_{\mu 3} U_{e3}^* U_{e2} U_{\mu 2}^*] \\ &= \frac{1}{8} \cos \theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \sin \delta. \end{aligned}$$



Predictions: $0.050 \leq |m_{ee}| \leq 0.062$ eV and $0.03 < |J_{CP}| < 0.04$

Case B: $\alpha > 1$

Solutions for different δ

δ	α	β	$\sum m_i(\text{eV})$
$10^\circ(170^\circ, 190^\circ, 350^\circ)$	1.43	0.36	0.0791
$30^\circ(150^\circ, 210^\circ, 330^\circ)$	1.39	0.45	0.0798
$40^\circ(140^\circ, 220^\circ, 320^\circ)$	1.36	0.53	0.0799
$50^\circ(130^\circ, 230^\circ, 310^\circ)$	1.32	0.64	0.0794
$60^\circ(120^\circ, 240^\circ, 300^\circ)$	1.26	0.83	0.0776
$70^\circ(110^\circ, 250^\circ, 290^\circ)$	1.17	1.13	0.0739
$73^\circ(107^\circ, 253^\circ, 287^\circ)$	1.07	3.02	0.0696

$$\delta = 0^\circ - 63^\circ, (117^\circ - 180^\circ, 180^\circ - 243^\circ, 297^\circ - 360^\circ)$$

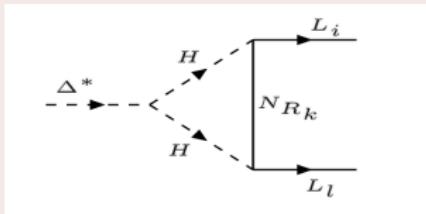
Leptogenesis

Possible Scenarios

- ① Decay of RH neutrinos only: lepton asymmetry vanishes
- ② Option (I), decay of scalar triplet ($M_R > M_\Delta$) : RH neutrinos in loop \Rightarrow successful leptogenesis scenario
- ③ Option (II), decay of RH neutrinos ($M_\Delta > M_R$) : Triplets in loop \Rightarrow constrained leptogenesis scenario

[Hambye,Senjanovic, PLB 582, 73 \(2004\)](#)

Option (I): decay of scalar triplet ($M_R > M_\Delta$)





Leptogenesis

- Decay of Scalar Triplet:

$$\begin{aligned}\Gamma_T &= \Gamma_{\Delta^* \rightarrow LL} + \Gamma_{\Delta^* \rightarrow HH} \\ &= \frac{M_\Delta}{8\pi} \left[\sum_{ij} |(Y_\Delta)_{ij}|^2 + \frac{|\eta|^2}{M_\Delta^2} \right].\end{aligned}$$

- Lepton Asymmetry Parameter: [O'Donnell,Sarkar, PRD 49, 2118 \(1994\)](#); [Hambye,Senjanovic, PLB 582, 73 \(2004\)](#)

$$\begin{aligned}\epsilon_\Delta &= 2 \frac{\Gamma(\Delta^* \rightarrow L + L) - \Gamma(\Delta \rightarrow \bar{L} + \bar{L})}{\Gamma(\Delta^* \rightarrow L + L) + \Gamma(\Delta \rightarrow \bar{L} + \bar{L})}, \\ &= \frac{1}{8\pi} \sum_k M_{Rk} \frac{\sum_{il} \text{Im}[(\hat{Y}_D^*)_{ki} (\hat{Y}_D^*)_{kl} (Y_\Delta)_{il} \eta^*]}{\sum_{ij} |(Y_\Delta)_{ij}|^2 M_\Delta^2 + |\eta|^2} \log(1 + M_\Delta^2/M_{Rk}^2).\end{aligned}$$

Here i, j denote the flavor indices, $\hat{Y}_D = U_R^T Y_D$ in the basis where RH neutrino mass matrix is diagonal

- Masses of RH neutrinos:

$$\begin{aligned}M_{R_1} &= \frac{v^2 y^2}{k} (1 + \alpha), \\ M_{R_2} &= \frac{v^2 y^2}{k}, \\ M_{R_3} &= \left| \frac{v^2 y^2}{k} (1 - \alpha) \right|.\end{aligned}$$

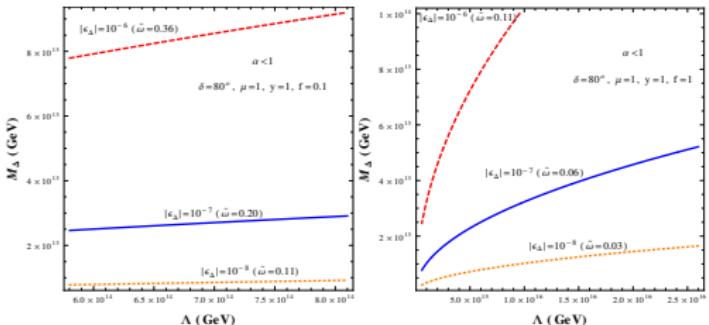
Leptogenesis: for $\delta = 80^\circ$ and $\alpha < 1$

- Lepton Asymmetry Parameter: Our Estimation (with $M_\Delta < M_{Ri} < \Lambda$)

$$\epsilon_\Delta = -\frac{M_\Delta^2}{8\pi v^2 (1-\alpha^2)} \frac{\alpha^2}{\left[3\tilde{\omega}^2 \frac{v_S^2}{\Lambda^2} \left(x_1^2 + x_1'^2 + 2x_1 x_1' \cos 2\alpha_S \right) M_\Delta^2 + (\mu \tilde{\omega}^2 \Lambda)^2 \right]} k \mu \tilde{\omega}^3 v_S (x_1 - x_1') \sin \alpha_S$$

- Here $\tilde{\omega} = v_f / \Lambda$, where v_f is considered to be the common vev of all flavons except S -field's vev $\langle S \rangle = v_S e^{i\alpha_S S}$.
- α_S is the only source of CP-violation.
- Defining $v_S / \Lambda = f \tilde{\omega}$ where f serves as a relative measure of the vevs, the asymmetry parameter (becomes $\tilde{\omega}$ independent),

$$\epsilon_\Delta = -\frac{\alpha^2}{8\pi v^2 (1-\alpha^2)} \frac{k f (x_1 - x_1') \sin \alpha_S (\mu \Lambda / M_\Delta^2)}{\left[(3f^2 / M_\Delta^2) (x_1^2 + x_1'^2 + 2x_1 x_1' \cos 2\alpha_S) + (\mu \Lambda / M_\Delta^2)^2 \right]}.$$



with $x_1 = 0.5$, $x_1' = 1$ and $|d| = 2fv^2\tilde{\omega}^4 \frac{\mu\Lambda}{M_\Delta^2} (x_1 + x_1') \cos \alpha_S \sec \phi_d$.

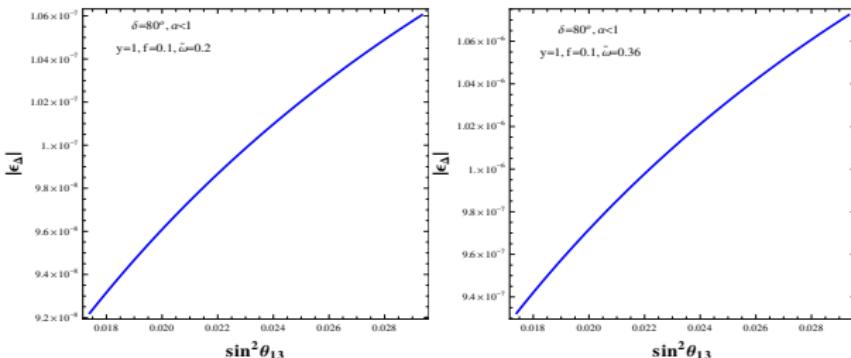
Leptogenesis: for $\delta = 80^\circ$ and $\alpha < 1$

ϵ_Δ vs $\sin^2 \theta_{13}$ for $\delta = 80^\circ$ and $\alpha < 1$

- Approximated Lepton Asymmetry Parameter : For $M_\Delta < \Lambda$

$$\epsilon_\Delta \simeq -\frac{\alpha^2}{8\pi v^2(1-\alpha^2)} kf(x_1 - x'_1) \sin \alpha_S \frac{M_\Delta^2}{\mu \Lambda}$$

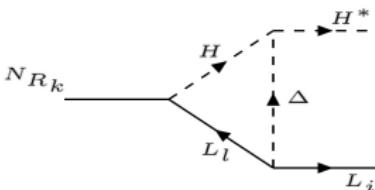
- Substituting $\mu \Lambda / M_\Delta^2$ from $|d|$ in ϵ_Δ above



- Thus, in this framework neutrino oscillation data imposes an upper bound on the lepton asymmetry
- We have performed similar analysis for $\alpha > 1$ case too.

Leptogenesis: Decay of RH Neutrinos

- **Decay of RH neutrinos:** $M_{R_i} < M_\Delta$



- **Lepton Asymmetry Parameter:**

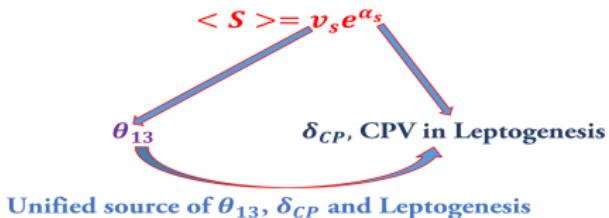
$$\begin{aligned}\epsilon_{N_1} &= \frac{3}{16\pi v^2} M_{R_1} \frac{\sum_{il} \text{Im}[(\hat{Y}_D)_{1i} (\hat{Y}_D)_{1l} (m_\nu^{H*})_{il}]}{\sum_i |(\hat{Y}_D)_{1i}|^2}, \\ &= \frac{3M_{R_1}}{2} \frac{1}{16\pi v^2} |d| \sin \phi_d, \\ \epsilon_{N_2} &= -3M_{R_2} \frac{1}{16\pi v^2} |d| \sin \phi_d \quad \text{and} \quad \epsilon_{N_3} = \mp \frac{3M_{R_3}}{2} \frac{1}{16\pi v^2} |d| \sin \phi_d, \text{ '+' for } \alpha > 1\end{aligned}$$

- **Total Baryon Asymmetry:**

$$\left| \frac{n_B}{s} \right| = 1.48 \times 10^{-3} \sum_i \epsilon_{N_i} D_{N_i}, \text{ where } D_{N_i} : \text{respective efficiency factor}$$

Due to specific flavor structure of M_{R_i} in our model, $\sum_i \epsilon_{N_i} = 0$, for same efficiency factor.

Unique Source of CP Violation



Conclusion

- We have adopted a generic Type-II seesaw with three RH neutrinos and only one scalar triplet
- A_4 symmetry helps in realizing desired flavor structure
- Spontaneous CP violation : turns out to be unique source of CP violation of the model
- Scalar triplet contribution generates desired amount nonzero θ_{13}
- Predictions on Dirac, Majorana phases and Jarlskog invariant
- Predictions on effective neutrino mass parameter and absolute neutrino masses
- Lepton asymmetry realized through decay of scalar triplets and connection with θ_{13}



Thank You