Basis-invariant conditions for CP4 3HDM

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POPH PROGRAMA OPERACIONAL POTENCIAL HUMANO

QREN QUADRO DE REFERÊNCIA ESTRATÉGICO NACIONAL

1. Model building with exotic CP
2. Basis-invariant conditions: 2HDM example
3. Basis-invariant conditions of CP4
4. Conclusions

CP symmetry in the SM and beyond

- SM: CKM matrix as the only source of *CP*-violation.
- New opportunities in bSM models with extended Higgs sectors.
  - New Higgses may be the source of the complex CKM [T.D.Lee, 1973] and mediate additional *CP*-violation [Weinberg, 1976; Branco, 1979];
  - CP symmetry can be a member of a larger flavour symmetry group; see e.g. [King, 1701.04413];
  - Exotic CP symmetries with consequences for model-building.

Many bSM models with extended Higgs sectors are on the market, with CP playing various roles [Branco, Lavoura, Silva, 1999; Ivanov, 1702.03776].

Freedom of defining CP

In QFT, CP is not uniquely defined *a priori*.

- phase factors $\phi(\vec{r},t) \xrightarrow{CP} e^{i\alpha}\phi^*(-\vec{r},t)$ [Feinberg, Weinberg, 1959],
- with N scalar fields ϕ_i, the general CP transformation is

$$J: \quad \phi_i \xrightarrow{CP} X_{ij}\phi_j^*\,, \quad X \in U(N)\,.$$

  If $\mathcal{L}$ is invariant under such J with whatever X, it is explicitly CP-conserving [Grimus, Rebelo, 1997; Branco, Lavoura, Silva, 1999].

- NB: The "standard" convention $\phi_i \xrightarrow{CP} \phi_i^*$ is basis-dependent!

Freedom of defining CP

$$J: \quad \phi_i \xrightarrow{CP} X_{ij}\phi_j^*\,, \quad X \in U(N)\,,$$

Applying J twice leads to family transformation $J^2 = XX^*$ which may be non-trivial. It may happen than only $J^k = \mathbb{I}$ (k = power of 2).

CP-symmetry does not have to be of order 2

The usual CP = CP2, the first non-trivial is CP4, then CP8, CP16, etc.

What about CP6?
Well, $\mathbb{Z}_6 \simeq \mathbb{Z}_2 \times \mathbb{Z}_3 \quad \Rightarrow \quad \text{CP6} = \text{usual CP} \times \mathbb{Z}_3$.

FAQ on CP4

1. Is it possible to build a multi-Higgs model only with CP4, without usual CP?

   Yes, but with $N \geq 3$ Higgs doublets [Ivanov, Keus, Vdovin, 2012; Ivanov, Silva, 2016].

   2HDM with CP4 (aka maximally *CP*-symmetric model) has accidental symmetries including the usual CP [Maniatis, von Manteuffel, Nachtmann, 2008; Ferreira, Haber, Maniatis, Nachtmann, Silva, 2011].

2. What about higher-order CP?

   No problem. Examples of CP8 and CP16 5HDM in [Ivanov, Laletin, 2018].

3. Do such models really conserve *CP* in terms of observables?

   Yes, they do, see e.g. [Haber, Ogreid, Osland, Rebelo, 2018].

FAQ on CP4

4. Is there an observable which distinguishes CP4 from usual CP?

   Yes, see [Haber, Ogreid, Osland, Rebelo, 2018].

5. Can one extend CP4 to quarks/neutrinos?

   Yes.
   For quark sector, CP4 must be spontaneously broken [Ferreira, Ivanov, Jimenez, Pasechnik, Serodio, 2018].

   For neutrinos, see e.g. a scotogenic model based on CP4 rather than $\mathbb{Z}_2$ [Ivanov, 2018].

Basis-invariant conditions of *CP* conservation in 2HDM

Basis-invariant methods

With N Higgs doublets, there is large freedom of basis changes:
$\phi_a \mapsto U_{ab}\phi_b$, $U \in U(N)$.

A symmetry can be evident in one basis and hidden in another → challenge!

No change of physics content → physics-revelant statements must be basis-invariant → one needs basis-invariant criteria for various phenomena in NHDM such as CP-conservation.

Powerful recipe [Botella, Silva, 1995]:

- write down all couplings as tensors under basis changes,
- take their product and contract all indices → basis invariants J_k,
- find algebraically independent J_k,
- link them to the phenomenon you study.

Explicit *CP* conservation in 2HDM scalar sector

The most general 2HDM potential:

$$V = Y_{ab}(\phi_a^\dagger \phi_b) + Z_{ab,cd}(\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d)\,,$$

or, in the explicit form,

$$\begin{aligned} V &= -\frac{1}{2}\left[m_{11}^2(\phi_1^\dagger\phi_1) + m_{22}^2(\phi_2^\dagger\phi_2) + m_{12}^2(\phi_1^\dagger\phi_2) + m_{12}^{2\,*}(\phi_2^\dagger\phi_1)\right] \\ &+ \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\ &+ \left[\frac{1}{2}\lambda_5(\phi_1^\dagger\phi_2)^2 + \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + \mathrm{h.c.}\right] \end{aligned}$$

It contains $4 + 10 = 14$ free parameters.

General 2HDM scalar sector

Checking explicit CP-conservation [Davidson, Haber, 2005; Gunion, Haber, 2005; Branco, Rebelo, Silva-Marcos, 2005]:

- There exists of a basis with all coefs real → symmetry $\phi_a \to \phi_a^*$.
- Construct invariants with Y_{ab} and $Z_{ab,cd}$ and establish independent ones;
- Basis-invariant criterion: check the following four invariants

$$\mathrm{Im}(Z^{(1)}_{ac} Z^{(1)}_{eb} Z_{be,cd} Y_{da}) = 0\,, \qquad \mathrm{Im}(Y_{ab} Y_{cd} Z_{ba,df} Z^{(1)}_{fc}) = 0\,,$$
$$\mathrm{Im}(Z_{ab,cd} Z^{(1)}_{bf} Z^{(1)}_{dh} Z_{fa,jk} Z_{kj,mn} Z_{nm,hc}) = 0\,,$$
$$\mathrm{Im}(Z_{ac,bd} Z_{ce,dg} Z_{eh,fq} Y_{ga} Y_{hb} Y_{qf}) = 0\,, \quad \text{where} \quad Z^{(1)}_{ac} \equiv Z_{ab,bc}\,.$$

Not very human-friendly, though.

Bilinear space formalism

Geometric constructions in the bilinear space [Nachtmann et al, 2004–2007; Ivanov, 2006–2007; Nishi, 2006–2008] is extremely powerful for the 2HDM scalar sector.

V depends on bilinears $\phi_a^\dagger \phi_b$. Organize them into combinations:

$$r_0 = \phi_a^\dagger \phi_a \equiv \phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2\,, \quad r_i = \phi_a^\dagger \sigma^i_{ab}\phi_b \equiv \begin{pmatrix} 2\mathrm{Re}(\phi_1^\dagger\phi_2) \\ 2\mathrm{Im}(\phi_1^\dagger\phi_2) \\ (\phi_1^\dagger\phi_1) - (\phi_2^\dagger\phi_2) \end{pmatrix}\,,$$

which satisfy $r_0 \geq 0$ and $r_0^2 - r_i^2 \geq 0$.

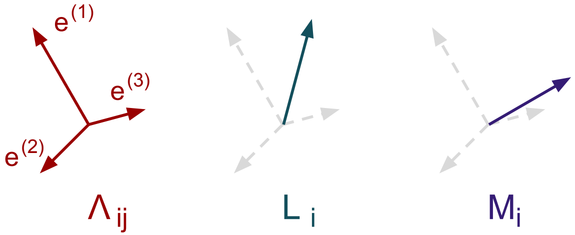
Basis change: an $SO(3)$ rotation; CP-transformation: a mirror reflection.

The general 2HDM Higgs potential is a quadratic form in (r_0, r_i):

$$V = -M_0 r_0 - M_i r_i + \Lambda_{00} r_0^2 + \Lambda_{0i} r_0 r_i + \Lambda_{ij} r_i r_j\,.$$

2HDM Higgs potential

Geometrically, 2HDM scalar sector = two scalars M_0, Λ_{00}, two 3-vectors M_i and $L_i = \Lambda_{0i}$, and a symmetric tensor Λ_{ij}.



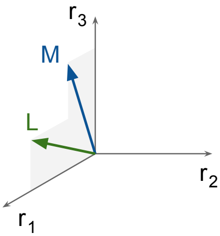
All symmetries are encoded in the orientation of M_i and L_i with respect to eigenvectors of Λ_{ij}.

Direct way to (human-derived) basis-independent quantities.

Explicit CP-conservation in 2HDM

The potential is CP-conserving if it possesses a reflection symmetry, which implies that:

There exists an eigenvector of Λ_{ij} orthogonal to M_i and L_i.



That's the answer.

In algebraic form:

$$(M, M^{(1)}, M^{(2)}) = (M, M^{(1)}, L^{(2)}) = (M, L^{(1)}, L^{(2)}) = (L, L^{(1)}, L^{(2)}) = 0\,,$$

where $M^{(k)} \equiv \Lambda^k M$ and $(a, b, c) = \epsilon_{ijk} a_i b_j c_k$.

Intermezzo

It is not obligatory to use basis invariants!

One can formulate basis-invariant statements in terms of basis-covariant objects, which can be more compact and transparent.

Basis-invariant conditions of CP4 symmetry in 3HDM

CP4 3HDM

The scalar potential of CP4 3HDM [Ivanov, Silva, 2016] $V = V_0 + V_1$ can be written in a suitable basis as (notation: $i \equiv \phi_i$):

$$\begin{aligned} V_0 &= -m_{11}^2(1^\dagger 1 + 2^\dagger 2) - m_{33}^2(3^\dagger 3) + \lambda_1\left[(1^\dagger 1)^2 + (2^\dagger 2)^2\right] + \lambda_2(3^\dagger 3)^2 + \\ &+ \lambda_3(3^\dagger 3)(1^\dagger 1 + 2^\dagger 2) + \lambda_3'(1^\dagger 1)(2^\dagger 2) + \lambda_4\left(|1^\dagger 3|^2 + |2^\dagger 3|\right) + \lambda_4'|1^\dagger 2|^2\,, \end{aligned}$$

with all parameters real, and

$$V_1 = \frac{\lambda_6}{2}\left[(1^\dagger 3)^2 + (3^\dagger 2)^2\right] + \lambda_8(1^\dagger 2)^2 + \lambda_9(1^\dagger 2)\left[(1^\dagger 1) - (2^\dagger 2)\right] + h.c.$$

with real λ_6 and complex $\lambda_{8,9}$. It is invariant under CP4 $J:\ \phi_i \xrightarrow{CP} X_{ij}\phi_j^*$ with

$$X = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad J^2 = \mathrm{diag}(-1,\,-1,\,1)\,, \quad J^4 = \mathbb{I}\,.$$

Bilinears for 3HDM

Bilinear approach for 3HDM:

$$r_0 = \frac{1}{\sqrt{3}}\phi_a^\dagger \phi_a\,, \quad r_i = \phi_a^\dagger (t^i)_{ab}\phi_b\,, \quad i = 1,\dots,8\,,$$

where $t_i = \lambda_i/2$ are $SU(3)$ generators satisfying

$$[t_i, t_j] = i f_{ijk} t_k\,, \quad \{t_i, t_j\} = \frac{1}{3}\delta_{ij}\mathbf{1}_3 + d_{ijk} t_k\,.$$

The potential takes the same form

$$V = -M_0 r_0 - M_i r_i + \Lambda_{00} r_0^2 + L_i r_0 r_i + \Lambda_{ij} r_i r_j\,,$$

with vectors $M_i, L_i \in \mathbb{R}^8$ and an 8×8 matrix Λ_{ij}.

Basis changes $\to SO(8)$ rotations. However, $SU(3) \subset SO(8) \Rightarrow$ matrix Λ_{ij} is not in general diagonalizable by a basis change!

CP2 vs CP4

Standard *CP*, $\phi_a \to \phi_a^*$, corresponds to a particular reflection:

- vectors from $V_+ = (r_3,\, r_8,\, r_1,\, r_4,\, r_6)$ stay unchanged,
- vectors from $V_- = (r_2,\, r_5,\, r_7)$ flip signs.

3HDM potential is explicitly CP2-invariant if there exists a basis in which

- vectors $M_i, L_i \in V_+$,
- Λ_{ij} has the block-diagonal form, with a 5×5 block in V_+ and a 3×3 block in V_-.

CP2 vs CP4

CP4, $\phi_a \to X_{ab}\phi_b^*$, leads in the bilinear space to

$$\begin{aligned} &r_8 \to r_8\,, \quad (r_1, r_2, r_3) \to -(r_1, r_2, r_3) \\ &r_4 \to r_6\,, \quad r_6 \to -r_4\,, \quad r_5 \to -r_7\,, \quad r_7 \to r_5\,. \end{aligned}$$

3HDM potential is explicitly CP4-invariant if there exists a basis in which (1) M_i, L_i are parallel to r_8 and (2) matrix Λ_{ij} is

$$\Lambda_{ij} = \begin{pmatrix} \square_{3\times 3} & 0 & 0 \\ 0 & \square_{4\times 4} & 0 \\ 0 & 0 & \Lambda_{88} \end{pmatrix}$$

with an arbitrary 3×3 block in the subspace (r_1, r_2, r_3) and a specific pattern in the 4×4 block.

CP2 vs CP4

CP2 and CP4 lead to different constraints. You can have

- CP2 without CP4 (usual *CP*-conserving 3HDM),
- CP4 without CP2 (but still perfectly *CP*-conserving),
- both CP2 and CP4 + a bunch of other accidental symmetries.

NB: trying to establish *CP* conservation via *CP*-odd basis invariants $I_k = 0$ as e.g. in [de Medeiros Varzielas, King, Luhn, Neder, 2016] cannot distinguish the usual CP from CP4, CP8, etc.

One MUST go beyond *CP*-odd basis invariants!

Self-alignment

Tensor d_{ijk} defines a non-linear action in the adjoint space. If $a_i \in \mathbb{R}^8$, let

$$b_i = \sqrt{3} d_{ijk} a_j a_k \,.$$

If b_i is parallel to a_i, we say that a_i is self-aligned.

a_i is self-aligned $\Leftrightarrow$ there is a basis in which a_i is along r_8.

Checking self-alignment of a vector is a basis-invariant way of detecting the all-important r_8 direction.

Complete alignment in CP4 3HDM

CP4 3HDM in the adjoint space: $M_i = (0, \dots, M_8)$, $L_i = (0, \dots, L_8)$, and

$$\Lambda_{ij} = \begin{pmatrix} \lambda_4' + \mathrm{Re}\lambda_8 & -\mathrm{Im}\lambda_8 & 2\mathrm{Re}\lambda_9 & \cdot & \cdot & \cdot & \cdot & \cdot \\ -\mathrm{Im}\lambda_8 & \lambda_4' - \mathrm{Re}\lambda_8 & -2\mathrm{Im}\lambda_9 & \cdot & \cdot & \cdot & \cdot & \cdot \\ 2\mathrm{Re}\lambda_9 & -2\mathrm{Im}\lambda_9 & 2\lambda_1 - \lambda_3' & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \lambda_4 + \lambda_6 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \lambda_4 - \lambda_6 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_4 + \lambda_6 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_4 - \lambda_6 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \Lambda_{88} \end{pmatrix}.$$

Define $K_i = d_{ijk}\Lambda_{jk}$ and $K_i^{(n)} = d_{ijk}(\Lambda^n)_{jk} \to$ all $K_i^{(n)} = (0, \dots, 0, K_8^{(n)})$.

In short, all vectors M_i, L_i, $K_i^{(n)}$ are parallel and self-aligned.

Complete alignment in CP4 3HDM

Complete alignment:

In CP4 3HDM, all adjoint space vectors constructed from arbitrary number of M_i, L_i, and Λ_{ij} and connected via arbitrarily complicated network of invariant tensors δ_{ij}, d_{ijk}, f_{ijk} are parallel and possess the self-alignment property.

Is this property unique to CP4 3HDM?

Complete alignment in S_3 3HDM

No!

CP-violating S_3 3HDM has potential $V = V_0 + V_{S_3}$ with

$$V_{S_3} = \lambda_5(\phi_1^\dagger\phi_3)(\phi_2^\dagger\phi_3) + \lambda' \left[(\phi_2^\dagger\phi_1)(\phi_3^\dagger\phi_1) + (\phi_1^\dagger\phi_2)(\phi_3^\dagger\phi_2)\right] + h.c.$$

and it also exhibits complete alignment:

$$\Lambda_{ij} = \begin{pmatrix} \lambda_4' & \cdot & \cdot & \mathrm{Re}\lambda' & \mathrm{Im}\lambda' & \mathrm{Re}\lambda' & \mathrm{Im}\lambda' & \cdot \\ \cdot & \lambda_4' & \cdot & \mathrm{Im}\lambda' & -\mathrm{Re}\lambda' & -\mathrm{Im}\lambda' & \mathrm{Re}\lambda' & \cdot \\ \cdot & \cdot & 2\lambda_1 - \lambda_3' & \cdot & \cdot & \cdot & \cdot & \cdot \\ \mathrm{Re}\lambda' & \mathrm{Im}\lambda' & \cdot & \lambda_4 & \cdot & \lambda_5 & \cdot & \cdot \\ \mathrm{Im}\lambda' & -\mathrm{Re}\lambda' & \cdot & \cdot & \lambda_4 & \cdot & -\lambda_5 & \cdot \\ \mathrm{Re}\lambda' & -\mathrm{Im}\lambda' & \cdot & \lambda_5 & \cdot & \lambda_4 & \cdot & \cdot \\ \mathrm{Im}\lambda' & \mathrm{Re}\lambda' & \cdot & \cdot & -\lambda_5 & \cdot & \lambda_4 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \Lambda_{88} \end{pmatrix},$$

in spite of absence of the nice block-diagonal structure.

Complete alignment

Complete alignment signals the presence of 2D irrep, not a higher-order CP.

One needs to look deeper into the properties of Λ_{ij} to distinguish CP4 3HDM from S_3 3HDM.

CP4 3HDM

The defining feature of CP4 3HDM is complete alignment and the block-diagonal structure

$$\Lambda_{ij} = \begin{pmatrix} \square_{3\times 3} & 0 & 0 \\ 0 & \square_{4\times 4} & 0 \\ 0 & 0 & \Lambda_{88} \end{pmatrix}$$

That is, three eigenvectors of Λ_{ij} belong to the (r_1, r_2, r_3) subspace.

Vectors from this subspace can be recognized in the basis-invariant way!
If $v_i^{(8)}$ is the eigenvector along r_8, then

$$a_i \in (r_1, r_2, r_3) \quad \Leftrightarrow \quad f_{ijk} a_j v_k^{(8)} = 0\,.$$

That is, a_i is f-orthogonal to $v_i^{(8)}$.

Necessary and sufficient conditions for CP4 in 3HDM

A basis-invariant algorithm for recognizing the presence of CP4 in 3HDM.

Write down M_i, L_i, Λ_{ij}. Calculate eigenvectors of Λ_{ij}.

The model possesses CP4 if and only if

- there exists a self-aligned eigenvector: $d_{ijk} v_j^{(8)} v_k^{(8)}$ is parallel to $v_i^{(8)}$;
- there exist three eigenvectors which are f-orthogonal to $v_i^{(8)}$: $f_{ijk} v_j^{(\alpha)} v_k^{(8)} = 0$.
- M_i, L_i, $K_i = d_{ijk}\Lambda_{jk}$, and $K_i^{(2)} = d_{ijk}(\Lambda^2)_{jk}$ are aligned with $v_i^{(8)}$.

Conclusions

- CP4 3HDM is the simplest model exhibiting higher-order *CP* symmetry. It possesses remarkables structural properties, leads to unusual phenomenology, which is worth exploring in detail.
- Detecting the presence of CP4 without usual CP in a basis-invariant way is a challenging task. The usual approach based on *CP*-odd invariants $I_k = 0$ cannot recognize it.
- Using the bilinear approach, we established basis-invariant necessary and sufficient conditions for the presence of CP4 in 3HDM.

Lessons

There exist physically distinct forms of *CP* symmetry.

To recognize them, one must go beyond *CP*-odd basis invariants.