Modern «theoretical» limitation for parameters of 2HDM and similar models

I. F. Ginzburg
Sobolev Inst. of Mathematics, SB RAS
and Novosibirsk State University
Novosibirsk, Russia

The work was supported by the National Science Centre, Poland,
the HARMONIA project under contract
UMO-2015/18/M/ST2/00518 (2016-2019)

The BSM models contain usually new fields and new coupling parameters. Possible values of these new parameters are studied in a number of papers.

Small fraction of these couplings are determined now from the data (accelerator and cosmology). Other couplings are estimated from the theoretical reasons.

In many cases such «theoretical» estimates are in fact not justified.

We consider as representative example well known Two Higgs doublet model in its simplest version — Inert Doublet Model (IDM), constructed in such a manner to describe both SM and Dark Matter (DM).

Brief description of IDM

In the IDM, the SM with standard Higgs field ϕ_S is supplemented by Higgs field ϕ_D, having no interaction with matter fields and vacuum expectation value (v.e.v.) $\langle\phi_D\rangle = 0$. Model is described by Lagrangian, keeping the D-parity (symmetry under replacement $\phi_D \leftrightarrow -\phi_D$):

$$\mathcal{L} = \mathcal{L}_{gf}^{SM} + \mathcal{L}_Y(f, \phi_S) + \frac{1}{2}(D_\mu\phi_S D_\mu\phi_S^\dagger + D_\mu\phi_D D_\mu\phi_D^\dagger) - V\,.$$

Here $\mathcal{L}_{gf}^{SM}$ is $SU(2)\times U(1)$ SM interaction of gauge bosons and fermions, $\mathcal{L}_Y(f,\phi_S)$ describes Yukawa interaction of fermions f with Higgs field ϕ_S only.

With the ground state $\langle\phi_S\rangle = v/\sqrt{2}$, $\langle\phi_D\rangle = 0$ the standard decomposition of fields has form

$$\phi_S = \begin{pmatrix} G^+ \\ (v + h + iG^0)/\sqrt{2} \end{pmatrix}, \quad \phi_D = \begin{pmatrix} D^+ \\ (D + iD_A)/\sqrt{2} \end{pmatrix}, \quad v = 246\ GeV.$$

Goldstone modes $G^\pm$, G^0 disappear in unitary gauge. h with mass $M_h \approx 125$ GeV is SM-like Higgs boson, D, D_A and $D^\pm$ are physical particles with masses $M_D \leq M_A < M_\pm \equiv M_+$ respectively. The particle D is candidate for DM particle.

Note: there is no correspondence between masses of D-particles and $M_h = 125$.

Possible interactions of D and D_A are identical. Their attribution as scalar and pseudoscalar is only subject of agreement. They have opposite P-parities, but their proper P-parities cannot be fixed.

We use potential V in the form (with real λ_i)

$$V = M_\pm^2 \phi_D^\dagger \phi_D + \frac{1}{2}\left[\left(\lambda_1(\phi_S^\dagger \phi_S)^2 - \frac{v^2}{2}\right)^2 + \lambda_2(\phi_D^\dagger \phi_D)^2\right] +$$
$$+\lambda_3(\phi_S^\dagger \phi_S - \frac{v^2}{2})(\phi_D^\dagger \phi_D) + \lambda_4(\phi_S^\dagger \phi_D)(\phi_D^\dagger \phi_S) + \lambda_5 Re((\phi_S^\dagger \phi_D)^2) + V_0.$$

Provided perturbativity constraints are fulfilled, parameters of this potential are expressed via measurable (in principle) quantities – masses M_h, M_D, M_A, $M_\pm$, v.e.v. v, coupling D^+D^-h (quantity $v\lambda_3$) and $DDDD$ coupling λ_2:

$$\lambda_1 = M_h^2/v^2,\ \lambda_5 = (M_D^2 - M_A^2)/v^2 \le 0,\ \ \lambda_4 + \lambda_5 = 2(M_D^2 - M_\pm^2)/v^2 \le 0.$$

This inert state is realized at

$$\lambda_3 < 2M_\pm^2/v^2 \quad \text{or} \quad \lambda_3 > 2M_\pm^2/v^2 > \lambda_3 - \sqrt{\lambda_1 \lambda_2}\,.$$

The condition $\lambda_4 + \lambda_5 < 0$ guarantees absence of charged vacuum.

Values of parameters

Certainly, all parameters of model should be determined from experiment. However, now that is a dream.
Now we know $M_h = 125$ GeV, $v = 246$ GeV and $\lambda_1 \approx 1/2$. In the foreseeble future one can hope to measure masses of D, D_A, $D^{\pm}$ and couplings of these particles with gauge bosons (gauge couplings). Some of couplings can be limited from cosmology reasons. Other couplings (in our case λ_3 and λ_2) look very difficult for observation. We hope for some help from cosmology data.
So we should use our theoretical knowledge about models.

The standard assumption in the description of such models is: the observable phenomena in theory are described perturbatively, i. e. the first non-vanishing approximation of perturbation theory for these phenomena has reasonable inaccuracy (e. g. $< 10 \div 30\%$). *(This is a tree approximation for most of phenomena and a one-loop approximation for the phenomena which are absent at tree level, e.g. decays $h \to \gamma\gamma$, $h \to Z\gamma$, $h \to gg$.)*

Perturbativity constraints

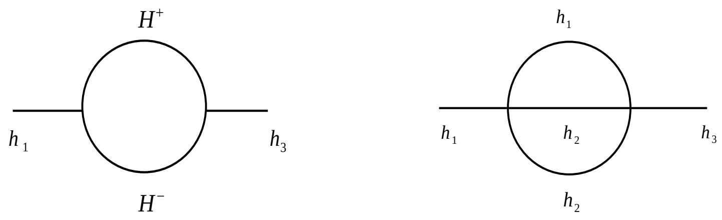
These constraints generally limits values λ_{1-5}. The starting point in obtaining of these constraints is the observation that the effective parameter of perturbative expansion is not λ_i $(i = 1, 2, ...5)$ but λ_i/Δ with $\Delta = 8\pi$ or 4π. The perturbativity condition is written usually in the form $|\lambda_i| < \Delta$.

At $|\lambda_i| \approx \Delta$ perturbative description of physical phenomena is incorrect even at low energies. In particular, the equations, expressing masses and couplings via parameters of Lagrangian, become invalid.

The first non-vanishing approximation of perturbation theory describes physical phenomena with relative inaccuracy k only at $|\lambda_i| < k\Delta$ with $k < 1$. With convention to have accuracy better than 30% we use perturbativity limitation in the form

$$\lambda_{3,4,5} < 4\pi/3 \,.$$

• Radiative corrections to masses M_D, M_A, $M_\pm$ are given by loop diagrams, $\propto (\lambda_{3,4,5}\times\lambda_{3,4,5})/(4\pi)^2$ (left) which are small in the mentioned region and two-loop diagrams $\propto \lambda_2^2/(4\pi)^4$ (right). The latter corrections are small at $(4\pi)^2/2 > \lambda_2 > 4\pi$.



Mass corrections. Left – $h_{1,3} \to DD, D_A D_A, D^\pm D^\pm$, in loop one line is h, another – D, D_A or $D^\pm$ respectively, vertices – λ_3, λ_4 or λ_5 in all possible combinations. Right – all lines are D-particles, vertex – λ_2

Therefore, even at large values λ_2 the correlation between masses of D-particles and λ's are stored. At $\lambda_2 < (4\pi)^2/2$ it is absolutely correct, for higher value of λ_2 it looks correct.
Therefore, the opportunity to have large λ_2, violating perturbativity limitation don't violate perturbativity limitation for other λ's. The perturbative description of many phenomena can coexist with strong interaction in D-sector at $\sqrt{s} > M_A, M_+$.
Neither modern data plus anticipated in the foreseeble future measurements no theoretical reasons cannot give limitation for λ_2.

$\Diamond$ Another example provide the one-loop radiative corrections (RC) to the triple Higgs coupling $g(hhh)$. A. Arhrib et al. present calculations arXiv:1507.03630 [hep-ph] at $M_\pm = M_A = M_D$ ($\lambda_4 = \lambda_5 = 0$). In this case the one-loop corrections to the $g(hhh)$ are described by single parameter λ_3, they reach 180% at $|\lambda_3| \approx \Delta$. With the limitation (**??**) these RC become less than 20%.

If we like to use perturbative estimates for description of masses, one should be $\lambda_{4,5} < 4\pi/3$ and,therefore

$$M_A^2 - M_D^2,\, M_\pm^2 - M_D^2 < 0.5 \cdot 10^6 GeV^2\,.$$

It allows opportunity to have not very heavy D (e.g. $30 \div 50$ GeV) with heavy (not obligatory) D_A, $D^\pm$. With the growth of M_D region of permissible values M_A, $M_\pm$ shrinks.

Positivity constraints

.

These constraint mean that $V > 0$ at large quasi-classical values of fields:

$$\lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 + \lambda_4 + \lambda_5 + \sqrt{\lambda_1 \lambda_2} > 0\,, \quad \lambda_3 + \sqrt{\lambda_1 \lambda_2} > 0\,.$$

This well known form is valid if only perturbativity constraints are fulfilled.

The perturbative unitarity constraints

.

These constraints were constructed as those described entire set of parameters of potential. They guarantee that the tree approximation for D_aD_b and D_ah scattering amplitudes are below the unitary limit for single s-wave in each channel.

These constraints only describes the desire of authors. We cannot expect for observation of mentioned amplitudes in the reasonable time. The corresponding processes are absent in experimental plans. The validity or invalidity of these constraints cannot be checked. Therefore these constraints should not be taken into account in the discussion of field of parameters.

Some conclusions

• The limitations given by perturbative unitarity constraints don't influence for possible values of parameters of model and our opportunity to use simple estimates in description of model.

• One of parameters of potential (λ_2 in our case) can be large enough without violation of perturbativity in other parameters.