Unitarity bounds, bounded from below bounds, and the masses and couplings of the scalars in the general 2HDM

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In the two-Higgs-doublet model (2HDM) one may subsume the four quadratic forms $\phi_i^\dagger \phi_j$ $(i, j = 1, 2)$ in

$$K_0 = \phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2, \tag{1}$$

$$K = \begin{pmatrix} \phi_1^\dagger \phi_2 + \phi_2^\dagger \phi_1 \\ -i\, \phi_1^\dagger \phi_2 + i\, \phi_2^\dagger \phi_1 \\ \phi_1^\dagger \phi_1 - \phi_2^\dagger \phi_2 \end{pmatrix}. \tag{2}$$

Under an $SU(2)$ change of basis of (ϕ_1, ϕ_2), K_0 is invariant while K behaves as an $SO(3)$ vector. The quartic part of the scalar potential may be written

$$V_4 = \eta_{00} \left(K_0\right)^2 + 2 K_0\, \eta^T K + K^T E K, \tag{3}$$

where η_{00} is invariant, η is an $SO(3)$ vector (a 3×1 matrix), and E is an $SO(3)$ symmetric tensor (a 3×3 matrix) containing the ten parameters of V_4.

The bounded from below (BFB) (or stability) conditions on V_4 guarantee that $V_4 > 0$ for any configuration of ϕ_1 and ϕ_2. The BFB conditions are analytic; they are invariant under a basis transformation of (ϕ_1, ϕ_2).

The unitarity bounds prescribe that the eigenvalues of the matrices of the scattering amplitudes following from V_4 (in the point-interaction regime) must be smaller in modulus than 4π. The unitarity bounds for a general V_4 (*i.e.* for a 2HDM without non-gauge symmetries) were first derived by Ginzburg & Ivanov (GI). They have also demonstrated that those bounds are invariant under a basis transformation of (ϕ_1, ϕ_2). For instance, some of the quantities that must be smaller (in modulus) than 4π are $I = 2\eta_{00} - 2\,\mathrm{tr}\,E$, and the eigenvalues of the 4×4 matrices

$$\mathcal{M}_1 = \begin{pmatrix} 4\eta_{00} - I & 4\eta^T \\ 4\eta & 4E + I \times 1_{3\times 3} \end{pmatrix}, \tag{4}$$

$$\mathcal{M}_2 = \begin{pmatrix} 12\eta_{00} - I & 12\eta^T \\ 12\eta & 12E + I \times 1_{3\times 3} \end{pmatrix}. \tag{5}$$

One may use the unitarity bounds in any basis for (ϕ_1, ϕ_2) that one wishes—they always have the form derived by GI.

We have used the unitarity bounds in the Higgs basis—the one where only ϕ_1 has vacuum expectation value (VEV) and that VEV is real and positive.

The Higgs basis is fully general—it encompasses all the 2HDMs. Therefore, our results apply to any 2HDM.

(We work in the 2HDM without fermions, *viz.* without Yukawa couplings.)

Our assumptions were:

- The analytical BFB bounds.
- The analytical unitarity bounds.
- ϕ_1 with real VEV $v = 174\,\mathrm{GeV}$, ϕ_2 without VEV.
- The mass matrix of the neutral scalars has one eigenvalue $(125\,\mathrm{GeV})^2$.
- The neutral scalar h_1 with mass $125\,\mathrm{GeV}$ has component $\cos\vartheta_1$ along the real, neutral component of ϕ_1 larger than 0.9.
- The oblique parameter T (given by its contribution from only the extra scalars) is within its 1σ range $-0.04 < T < 0.20$.

The values of the parameters of V_4 in the Higgs basis directly lead to the masses and the couplings of the physical scalars. We have investigated the ranges in the 2HDM of the couplings g_3, g_4, and g_{1CC} defined by

$$\mathcal{L} = \cdots - g_3 \left(h_1 \right)^3 - g_4 \left(h_1 \right)^4 - g_{1CC} h_1 C^+ C^-, \tag{6}$$

where $C^\pm$ are the charged scalars of the 2HDM.

We have studied other models, in particular an extension of the 2HDM through one real scalar field S that is invariant under the gauge symmetry but carries the symmetry $S \to -S$. (This symmetry prevents cubic couplings in the potential that would affect g_3 and g_{1CC}.) We have assumed the scalar field S to have VEV w. The quartic part of the potential is

$$\begin{aligned} V_4 &= \eta_{00} \left(K_0 \right)^2 + 2 K_0\, \eta^T K + K^T E K \\ &\quad + \frac{\psi}{2} S^4 + S^2 \left(\xi_0 K_0 + \xi^T K \right), \end{aligned} \tag{7}$$

where ξ_0 is invariant and ξ is an $SO(3)$ vector (a 3×1 matrix) under a change of basis of (ϕ_1, ϕ_2).

In this model the unitarity conditions have two differences relative to those in the 2HDM:

- The 4×4 matrix $\mathcal{M}_2$ becomes the 5×5 matrix

$$\mathcal{M}_2' = \begin{pmatrix} 6\psi & 2\sqrt{2}\bar{\xi}^T \\ 2\sqrt{2}\bar{\xi} & \mathcal{M}_2 \end{pmatrix}, \quad \bar{\xi} = \begin{pmatrix} \xi_0 \\ \xi \end{pmatrix}. \tag{8}$$

- There are additional unitarity conditions

$$\left| \xi_0 \pm \sqrt{\xi^T \xi} \right| < 2\pi. \tag{9}$$

The BFB conditions are much more awkward in the 2HDM+S. One needs the BFB conditions for the 2HDM and $\psi > 0$, but that does not suffice. For each set of V_4 parameters, we have numerically found the minimum of V_4 in order to find out whether V_4 is always positive or not.

We have also considered another model: the SM with the addition of two real scalar singlets S_1 and S_2:

$$\begin{aligned} V_4 &= \frac{\lambda}{2} \left(\phi^\dagger \phi\right)^2 + \frac{\psi_1}{2} S_1^4 + \frac{\psi_2}{2} S_2^4 + \psi_3 S_1^2 S_2^2 \\ &\quad + \phi^\dagger \phi \left(\xi_1 S_1^2 + \xi_2 S_2^2\right) \qquad (10) \\ &= \frac{1}{2} V^T \begin{pmatrix} \lambda & \xi_1 & \xi_2 \\ \xi_1 & \psi_1 & \psi_3 \\ \xi_2 & \psi_3 & \psi_2 \end{pmatrix} V, \qquad (11) \end{aligned}$$

where $V^T = \begin{pmatrix} \phi^\dagger\phi & S_1^2 & S_2^2 \end{pmatrix}$. Since the quantities in V are positive definite, the BFB conditions in this case are analytical:

$$\begin{aligned}
\lambda &> 0, && (12)\\
\psi_1 &> 0, && (13)\\
\psi_2 &> 0, && (14)\\
a_1 \equiv \xi_1 + \sqrt{\lambda\psi_1} &> 0, && (15)\\
a_2 \equiv \xi_2 + \sqrt{\lambda\psi_2} &> 0, && (16)\\
a_3 \equiv \psi_3 + \sqrt{\psi_1\psi_2} &> 0, && (17)\\
\sqrt{\lambda\psi_1\psi_2} + \xi_1\sqrt{\psi_2} + \xi_2\sqrt{\psi_1} + \psi_3\sqrt{\lambda} & && \\
+\sqrt{2a_1a_2a_3} &> 0. && (18)
\end{aligned}$$

As for unitarity, the quantities that must be smaller in modulus than 4π are λ, $2\xi_1$, $2\xi_2$, $4\psi_3$, and the eigenvalues of

$$\begin{pmatrix} 6\psi_1 & 2\psi_3 & 2\xi_1 \\ 2\psi_3 & 6\psi_2 & 2\xi_2 \\ 2\xi_1 & 2\xi_2 & 3\lambda \end{pmatrix}. \qquad (19)$$

In figure 1 we depict the predictions of the three models for g_3 and g_4, as compared to their SM values.

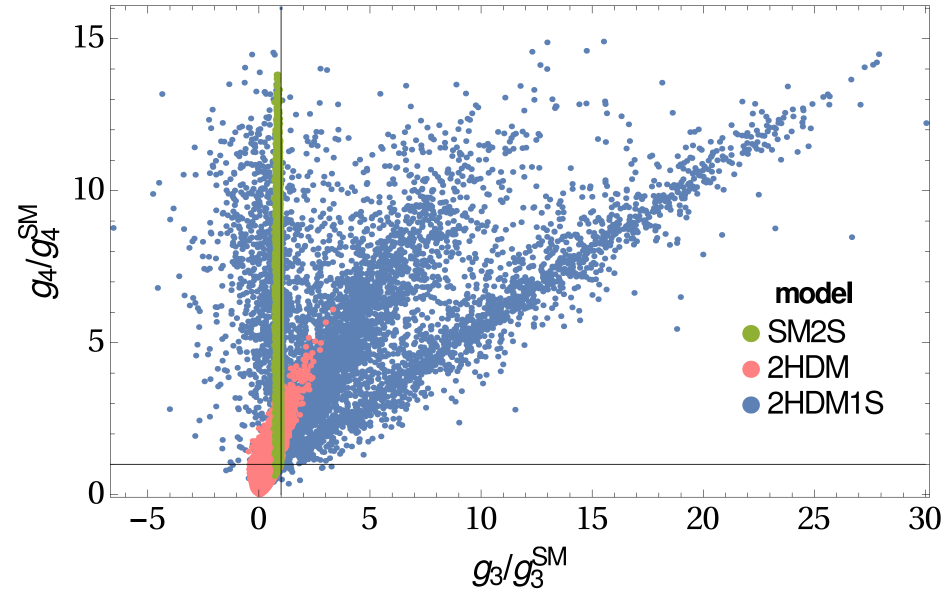

Figure 1: Scatter plot of $g_4\,/g_4^{\rm SM}$ and $g_3\,/g_3^{\rm SM}$.

The models with singlets allow for g_4 up to fifteen times larger than in the SM. The cubic coupling g_3 may also be one order of magnitude larger than in the SM in the 2HDM+S.

In figure 2 we plot the masses of the three new scalars of the 2HDM against $\cos\vartheta_1$.

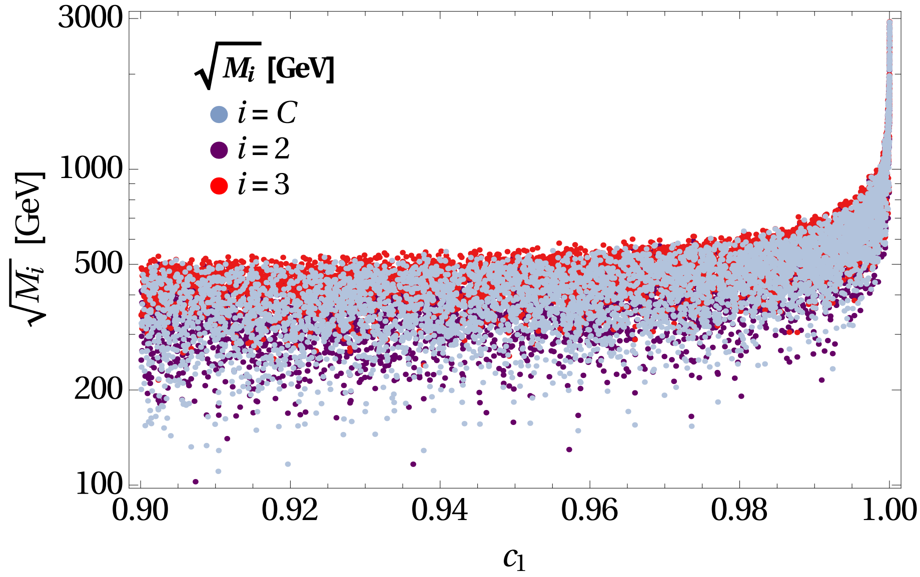

Figure 2: The masses of the new scalars of the 2HDM *versus* $\cos\vartheta_1$.

One sees that, except when $\cos\vartheta_1$ is really very close to 1, the new scalars must be rather light, *viz.* lighter than 700 GeV or even less.

However, this nice result gets spoiled in the 2HDM+S, as one sees in figure 3.

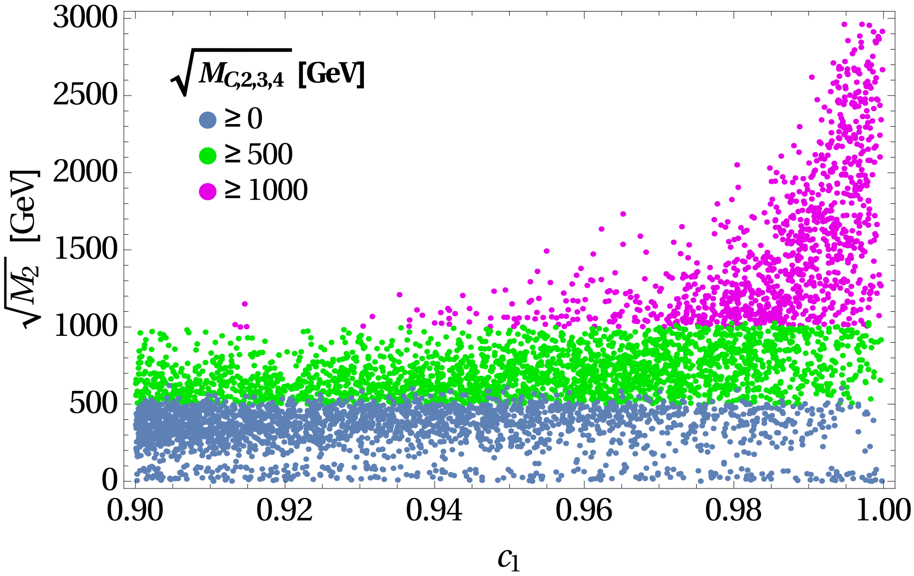

Figure 3: The mass of the lightest non-SM neutral scalar *versus* $\cos \vartheta_1$ in the 2HDM+S.

In the 2HDM+S all the scalars may be heavier than 1 TeV even for $\cos \vartheta_1 \approx 0.92$.

Conclusion

- We have implemented both the unitarity conditions and the BFB conditions in the Higgs basis of the 2HDM. This has allowed us to extract bounds on the masses and couplings of the scalar particles of the general 2HDM. The bounds are independent of any symmetry that a particular 2HDM may possess.
- We have used the same procedure in the 2HDM with the addition of one real singlet, and in the SM in the addition of two real singlets, in both cases with reflection symmetries acting on each of the singlets.
- It may be difficult to generalize our work to more complicated models, both because they might contain too many parameters and because analytical BFB conditions are in general unknown.

THANK YOU!