One-loop lepton masses and the Multi-Higgs Doublet Model

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$\int dk\,\Pi$

Doktoratskolleg

Particles and Interactions

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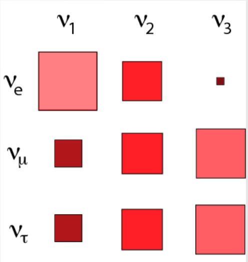
Der Wissenschaftsfonds.

Motivation: renormalizable mass models

An instructive example

- An **abundance of renormalizable ν-mass models** tries to explain observed masses and mixings
- *e.g.* μ-τ-symmetry [GL03]: invariance under non-standard CP-transformations such as $\nu_{L,\alpha} \to \mathrm{i} S_{\alpha\beta}\gamma^0 C\,\overline{\nu}^T_{L,\beta}$ provokes **maximal atmospheric mixing**, other symmetries provoke $m_\mu \ll m_\tau$ (3HDM with heavy Majoranas)

$$S = \begin{array}{c} \\ \nu_e \\ \nu_\mu \\ \nu_\tau \end{array} \begin{array}{c} \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \end{array}, \qquad \mathcal{M}_{\text{light}} = \begin{pmatrix} a & r & r^* \\ r & s & b \\ r^* & b & s^* \end{pmatrix},$$



Source: [St13]

$$S^T \mathcal{M}_{\text{light}} S = \mathcal{M}^*_{\text{light}}$$

$$U^T_{\text{PMNS}}\,\mathcal{M}_{\text{light}}\,U_{\text{PMNS}} = \operatorname{diag}(m_{\nu,1}, m_{\nu,2}, m_{\nu,3})$$

$$\Rightarrow |U_{\mu i}| = |U_{\tau i}| \quad \forall i \quad \text{or } \theta_{23} = 45^\circ,\ \delta = \pm\frac{\pi}{2}.$$

Model setup

Test stability of tree-level predictions using a general model, later apply specific parameter choices/symmetries/seesaw mechanism: *arXiv:1807.00725 [hep-ph]*

$$\mathcal{L} = \mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{Kin}} + \mathcal{L}_{\text{S}} + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\text{Maj}} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}}.$$

- $SU(2)_L \times U(1)_Y$ with n_H scalar doublets

$$\mathcal{L}_{\text{S}} = (D_\mu \Phi_a)^\dagger D^\mu \Phi_a - \mu_{ab}^2 \Phi_a^\dagger \Phi_b - \lambda_{abcd} \left(\Phi_a^\dagger \Phi_b\right) \left(\Phi_c^\dagger \Phi_d\right)$$

- Mass generation via Yukawa interaction and Majorana mass term

$$\mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\text{Maj}} = -\bar{e}_R \Phi_k^\dagger \Gamma_k \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} - \bar{\nu}_R \tilde{\Phi}_k^\dagger \Delta_k \begin{pmatrix} \nu_L \\ l_L \end{pmatrix} - \frac{1}{2} \bar{\nu}_R M_R \nu_R^c + \text{H.c.}$$

$$\Gamma_k \in \mathbb{C}^{n_H \times (n_L \times n_L)}, \quad \Delta_k \in \mathbb{C}^{n_H \times (n_R \times n_L)}$$

- Gauge fixing with R_ξ-gauge (avoids scalar-vector boson mixing @ tree-level)

One-loop calculation

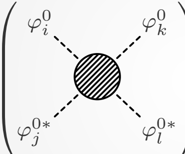
Main goal: one-loop masses (& later mixing angles)

- Potentially far more parameters than process-independent physical observables
  $\Rightarrow$ $\overline{\text{MS}}$-scheme somewhat unavoidable
- **$\overline{\text{MS}}$-Renormalization of scalar sector:** $\{\delta\mu_{ij}^2, \delta\tilde{\lambda}_{ijkl}, \delta v_k\}$
- **$\overline{\text{MS}}$-Renormalization of leptonic sector:** $\{\delta\Delta_k, \delta\Gamma_k, \delta M_R\}$
- **Gauge-dependence discussion:** How to show gauge-parameter independence of one-loop masses?
- **Treatment of tadpole contributions**

Renormalization of scalar sector
Quartic coupling

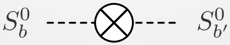
- Determine $\delta\tilde{\lambda}_{ijkl}$ from $\langle\Omega|\, T\varphi_i^0\varphi_j^{0*}\varphi_k^0\varphi_l^{0*}\,|\Omega\rangle$ in **unbroken phase** to simply save some computational effort
- Using dimensional regularisation in $d = 4-\varepsilon$, this is:

$2i\,\delta\tilde{\lambda}_{ijkl} \equiv \Big(\Big)_{c_\infty}$



$$, \quad c_\infty = \frac{2}{\varepsilon} + \gamma_E + \ln(4\pi)$$

- Serves as input for mass counterterm in **broken phase**:

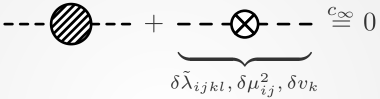


Sufficient for our purposes: $\delta\tilde{\lambda}_{ijkl} = \delta\lambda_{ijkl} + \delta\lambda_{ilkj}$

Renormalization of scalar sector

$\delta\mu^2$ and δv

- Knowing $\delta\tilde{\lambda}_{ijkl}$, find $\delta\mu^2_{ij}$ and δv_k by demanding finite scalar self-energy:



- Need independent δv_k for $\xi_V \neq 0$ [SSV13]. Result: (note the absence of ξ_A)

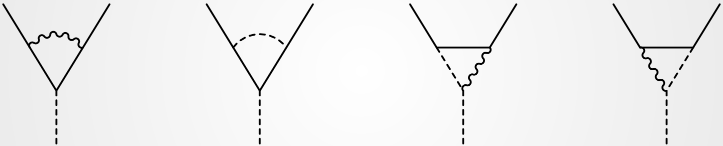
$$\delta v_k = \frac{c_\infty}{16\pi^2}\left(\frac{g^2\xi_W}{2} + \frac{g^2\xi_Z}{4c_W^2}\right) v_k$$

- Check finiteness of scalar one-point function:

Renormalization of leptonic sector

Yukawa couplings

- Determine Yukawa counterterms $\delta\Gamma_k$ and $\delta\Delta_k$ from divergencies in vertex corrections, again in **unbroken phase**:

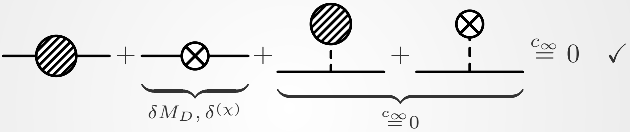


- Find remarkably simple result for neutral leptons

$$\delta\Delta_k = -\frac{c_\infty}{16\pi^2}\left[\left(\frac{g^2\xi_W}{2} + \frac{g^2\xi_Z}{4c_W^2}\right)\Delta_k + \Delta_j\Gamma_k^\dagger\Gamma_j\right],$$

Finite one-loop lepton masses

- Inserting Yukawa- and VEV-counterterms in the lepton mass-counterterms gives finite self-energies
- For neutrinos: $\delta M_D = \frac{1}{\sqrt{2}} \left(\delta v_k \Delta_k + v_k \delta \Delta_k\right)$ + wave-function renorm. $\delta^{(x)}$



- Note: $\delta M_R = 0$ at one-loop!
- Eventually calculate finite mass shifts (for neutrinos) via

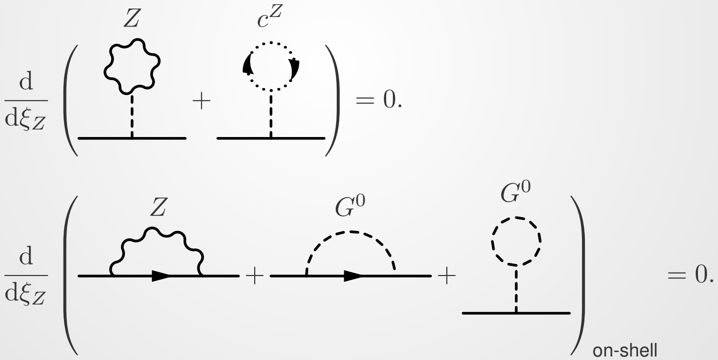
$$\Sigma(p) = \not{p}\left(\Sigma_L^{(A)}(p^2)\gamma_L + \Sigma_R^{(A)}(p^2)\gamma_R\right) + \Sigma_L^{(B)}(p^2)\gamma_L + \Sigma_R^{(B)}(p^2)\gamma_R.$$

$$\Delta m_i = m_i \left(\Sigma_{\nu L}^{(A)}\right)_{ii}(m_i^2) + \mathrm{Re}\left(\Sigma_{\nu L}^{(B)}\right)_{ii}(m_i^2) \quad (i = 1, \ldots, n_L + n_R)$$

- Presentation of **full results for leptonic self-energies (including tadpoles)** in analytic form. (Fits on a page)

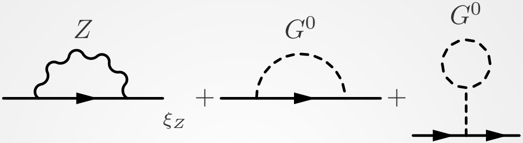
Gauge dependencies

- Consistency check: **gauge-parameter independence of one-loop masses**
- Can analytically show this for on-shell lepton self-energies, *i.e.* here: when $\not{p} \to \hat{m}_\nu$ [Wei73]



Gauge dependencies

Full ξ_Z-dependence of self energies contained in:



$$
\begin{aligned}
\propto \int \frac{\mathrm{d}^d k}{(2\pi)^d} \Big\{ & -\frac{1}{2} \left[(\not{p} - \hat{m}_\nu) F_{LR}^2 + F_{RL}^2 (\not{p} - \hat{m}_\nu) \right] \\
& + (\not{p} - \hat{m}_\nu) F_{LR} \Delta^{(\nu)}(p-k) F_{RL} (\not{p} - \hat{m}_\nu) \\
& + (\not{p} - \hat{m}_\nu) F_{LR} \Delta^{(\nu)}(p-k) (F_{RL} \hat{m}_\nu - \hat{m}_\nu F_{LR}) \\
& + (\hat{m}_\nu F_{LR} - F_{RL} \hat{m}_\nu) \Delta^{(\nu)}(p-k) F_{RL} (\not{p} - \hat{m}_\nu) \Big\} \, \Delta^{G^0}(k, \xi_Z)
\end{aligned}
$$

Gauge dependencies

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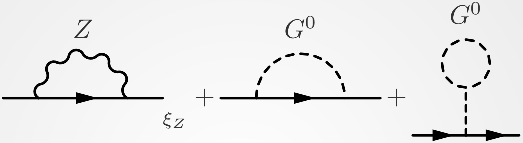


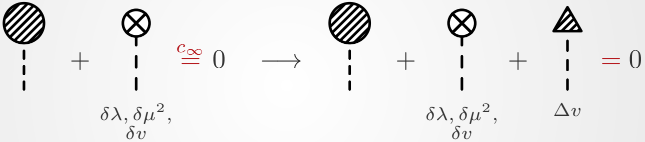
$$\propto \int \frac{\mathrm{d}^d k}{(2\pi)^d} \Big\{ -\frac{1}{2}\left[(\not{p} - \hat{m}_\nu) F_{LR}^2 + F_{RL}^2 (\not{p} - \hat{m}_\nu) \right]$$
$$+ (\not{p} - \hat{m}_\nu) F_{LR} \Delta^{(\nu)}(p-k) F_{RL} (\not{p} - \hat{m}_\nu)$$
$$+ (\not{p} - \hat{m}_\nu) F_{LR} \Delta^{(\nu)}(p-k) (F_{RL}\hat{m}_\nu - \hat{m}_\nu F_{LR})$$
$$+ (\hat{m}_\nu F_{LR} - F_{RL}\hat{m}_\nu) \Delta^{(\nu)}(p-k) F_{RL} (\not{p} - \hat{m}_\nu) \Big\} \Delta^{G^0}(k, \xi_Z)$$

$\Rightarrow$ **Gauge-parameter independent one-loop masses**

Terms of these types do not contribute to mass corrections, only shifts of the propagator residues $\to$ field strength renormalization

Finite tadpole contributions

- We have seen: need **(finite) tadpole contributions for gauge-parameter independence** of one-loop masses
- Can introduce finite VEV shifts Δv_k to absorb these $\Rightarrow$ **vanishing one-point function**



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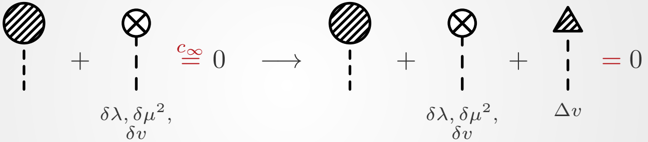
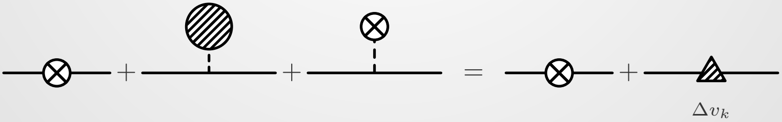


- Equivalence of inserting all tadpole contributions for a given observable versus making the shift $v_k \rightarrow v_k + \Delta v_k$ in the corresponding Lagrangian terms (here: lepton masses)



Finite tadpole contributions

Charged leptons

- **Finite tadpole-shifts introduce finite mass-shifts:**

$$= \frac{-i}{\sqrt{2}} \sum_{b=2}^{2n_H} \left[G_b \gamma_L + G_b^\dagger \gamma_R \right] \times \frac{i}{-M_b^2} \times \frac{i}{2} M_b^2 \left(\Delta v_i^* V_{ib} + V_{ib}^* \Delta v_i \right)$$

$$\Downarrow$$

$$= -\frac{i}{\sqrt{2}} \Big(W_R^\dagger \underbrace{\Gamma_k \Delta v_k^*}_{\Delta M_l} W_L \gamma_L + W_L^\dagger \Gamma_k^\dagger \Delta v_k W_R \gamma_R \Big)$$

For take away

- An **abundance of renormalizable neutrino mass** models available, often with **many new scalars**
- Want to check **perturbative stability of mass and mixing predictions** in generally applicable way
  $\Rightarrow$ **multi-Higgs doublet SM**
- Results: **arXiv:1807.00725 [hep-ph]** (to be published soon)
  - $\overline{\text{MS}}$-renormalization of scalar and leptonic sector in **broken** phase via renorm. of parameters of **unbroken** theory
  - Clear treatment of tadpole renormalization and finite contributions
  - Gauge-parameter independence of one-loop masses analytically shown
  - Rather compact analytic results for complete one-loop lepton mass corrections
- **Current work:** numerical evaluation for μ-τ symmetry model
- **Future work:** study radiative corrections to mixing angles

[St13] S. Stone, *New physics from flavour*, PoS ICHEP **2012** (2013) 033 [arXiv:1212.6374 [hep-ph]].

[GL03] W. Grimus and L. Lavoura, *A Nonstandard CP transformation leading to maximal atmospheric neutrino mixing*, Phys. Lett. B **579** (2004) 113 [hep-ph/0305309].

[FGL17] M. Fox, W. Grimus and M. Löschner, *Renormalization and radiative corrections to masses in a general Yukawa model*, arXiv:1705.09589 [hep-ph].

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[SSV13] M. Sperling, D. Stöckinger and A. Voigt, *Renormalization of vacuum expectation values in spontaneously broken gauge theories*, JHEP **1307** (2013) 132 [arXiv:1305.1548 [hep-ph]].