Western Norway
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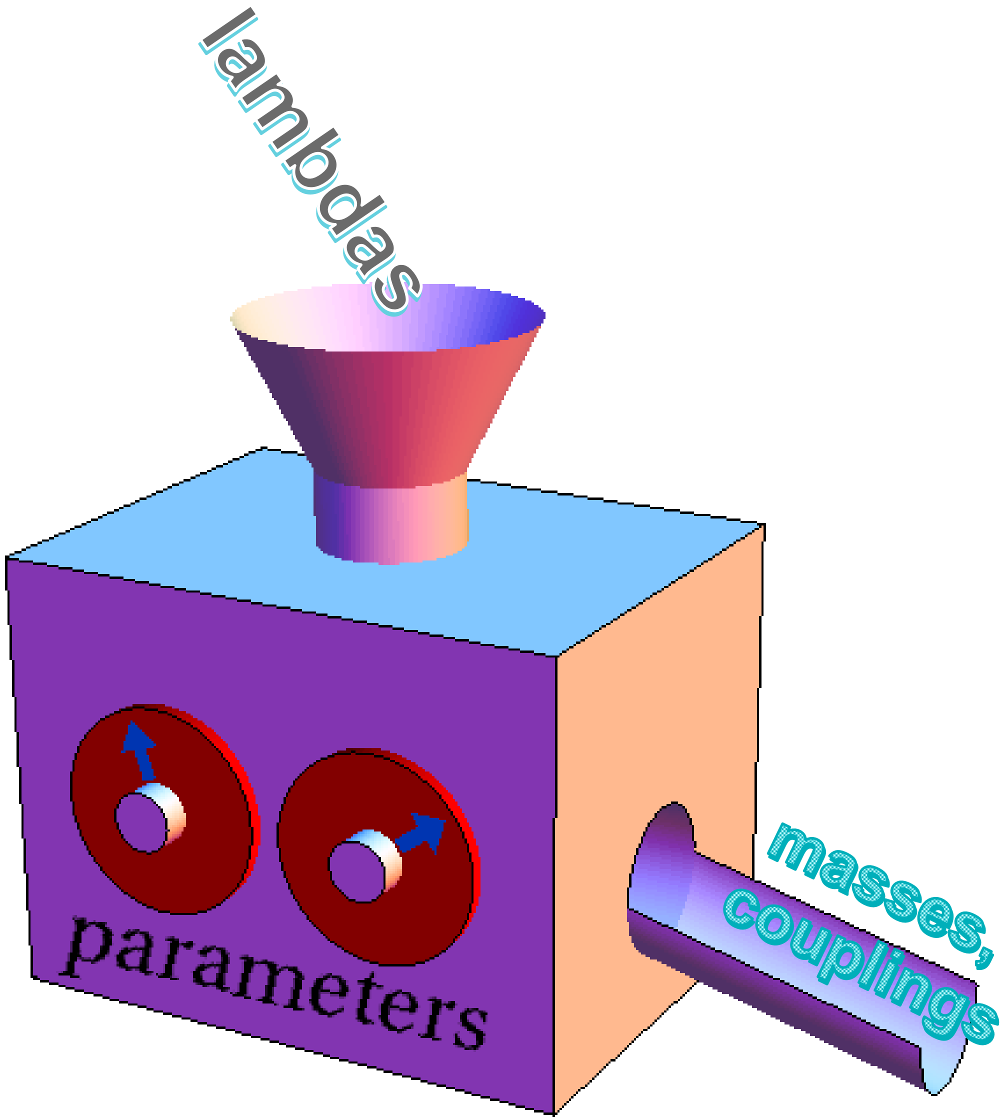
Physical parametrization of the 2HDM

Talk given at
“Workshop on Multi-Higgs Models”,
Lisbon, September 4, 2018



Odd Magne Ogreid
Based on work with B. Grzadkowski and P. Osland

Outline of talk

- Traditional parametrization of 2HDM
- VEVs and basis changes
- Counting of parameters
- Choosing the Higgs basis
- Inpendent couplings, and the introduction of the physical parameter set $\mathcal{P}$
- Translation from standard parameters to the parameter set $\mathcal{P}$

Applications:

- Scalar couplings of the 2HDM
- CP violation
- Spontaneous CP violation
- Alignment
- Z_2-symmetric potential
- Unitarity at tree level
- Oblique parameters

More to do and Summary

Traditional parametrization(s) of the 2HDM potential

$$
\begin{aligned}
V(\Phi_1,\Phi_2) &= -\frac{1}{2}\left\{m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + \left[m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right]\right\} \\
&+\frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\
&+\frac{1}{2}\left[\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right] + \left\{\left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right](\Phi_1^\dagger\Phi_2) + \text{h.c.}\right\} \\
&\equiv Y_{ab}\Phi_a^\dagger\Phi_b + \frac{1}{2}Z_{abcd}(\Phi_a^\dagger\Phi_b)(\Phi_c^\dagger\Phi_d)
\end{aligned}
$$

$$
Y_{11} = -\frac{m_{11}^2}{2}, \quad Y_{12} = -\frac{m_{12}^2}{2},
$$
$$
Y_{21} = -\frac{(m_{12}^2)^*}{2}, \quad Y_{22} = -\frac{m_{22}^2}{2},
$$

$$
\begin{aligned}
&Z_{1111} = \lambda_1, \quad Z_{2222} = \lambda_2, \quad Z_{1122} = Z_{2211} = \lambda_3, \\
&Z_{1221} = Z_{2112} = \lambda_4, \quad Z_{1212} = \lambda_5, \quad Z_{2121} = (\lambda_5)^*, \\
&Z_{1112} = Z_{1211} = \lambda_6, \quad Z_{1121} = Z_{2111} = (\lambda_6)^*, \\
&Z_{1222} = Z_{2212} = \lambda_7, \quad Z_{2122} = Z_{2221} = (\lambda_7)^*.
\end{aligned}
$$

Second form useful in the study of invariants.

Vacuum expectation values (VEVs) and choice of basis

Most general form that conserves electric charge:

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 e^{i\xi_1}\end{pmatrix}, \qquad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\xi_2}\end{pmatrix}$$

$$v_1^2 + v_2^2 = v^2 = (246\,\mathrm{GeV})^2$$

- We demand that the VEVs should represent a minimum of the potential
- Electroweak Symmetry Breaking:
  Work out stationary-point equations by differentiating the potential with respect to the fields and put these to zero.
  [Ref: Grzadkowski, Ogreid, Osland,
  JHEP11 (2014) 084].
- Minimum enforced by demanding all physical scalars have positive squared masses (later).

- Initial expression of potential is defined with respect to doublets Φ_1 and Φ_2 .
- We may rotate to a new basis by

  $$\bar{\Phi}_i = U_{ij}\Phi_j$$

  where *U* is any U(2) matrix.
- Potential parameters change under change of basis.
- Physics is the same regardless of our choice of basis.
- Observables (constructed from masses and couplings) cannot depend on choice of basis – they are **invariant** under a change of basis.

Counting parameters and choosing the Higgs basis

- Potential has initially 14 parameters
- Exploit the freedom to change basis and reduce to 11 independent parameters.
- Traditional approach:
  Work out masses and couplings expressed in terms of the initial parameters of the potential.
- Our approach:
  Work the other way around. Pick a set of 11 independent physical masses and couplings (all invariants) and express the initial parameters in terms of these.

- In the Higgs-basis only one doublet has non-zero VEV.

$$\langle\Phi_1\rangle_{\mathrm{HB}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle\Phi_2\rangle_{\mathrm{HB}} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

- Not unique, as one may still perform a U(1) transform on Φ_2 without giving Φ_2 a non-zero VEV.
- Algebra much simpler in the Higgs-basis than in a general basis.
- Stationary-point equations:

$$\begin{aligned} m_{11}^2 &= v^2\lambda_1, \\ \mathrm{Re}\, m_{12}^2 &= v^2 \mathrm{Re}\,\lambda_6, \\ \mathrm{Im}\, m_{12}^2 &= v^2 \mathrm{Im}\,\lambda_6, \end{aligned}$$

Parametrization of the doublets and the physical masses in the Higgs basis

- Doublets are parametrized as:

$$\Phi_1 = \begin{pmatrix} G^+ \\ (v + \eta_1 + iG^0)/\sqrt{2} \end{pmatrix}$$

$$\Phi_2 = \begin{pmatrix} H^+ \\ (\eta_2 + i\eta_3)/\sqrt{2} \end{pmatrix}$$

We work out the mass of the charged scalars:

$$M_{H^\pm}^2 = -\frac{m_{22}^2}{2} + \frac{v^2}{2}\lambda_3$$

Neutral sector mass terms given by

$$\frac{1}{2}\begin{pmatrix}\eta_1 & \eta_2 & \eta_3\end{pmatrix}\mathcal{M}^2\begin{pmatrix}\eta_1 \\ \eta_2 \\ \eta_3\end{pmatrix}.$$

- With the neutral sector mass matrix

$$\mathcal{M}^2 = v^2\begin{pmatrix} \lambda_1 & \mathrm{Re}\,\lambda_6 & -\mathrm{Im}\,\lambda_6 \\ \mathrm{Re}\,\lambda_6 & \frac{1}{2}(\lambda_3+\lambda_4+\mathrm{Re}\,\lambda_5-\frac{m_{22}^2}{v^2}) & -\frac{1}{2}\mathrm{Im}\,\lambda_5 \\ -\mathrm{Im}\,\lambda_6 & -\frac{1}{2}\mathrm{Im}\,\lambda_5 & \frac{1}{2}(\lambda_3+\lambda_4-\mathrm{Re}\,\lambda_5-\frac{m_{22}^2}{v^2}) \end{pmatrix}$$

- Is diagonalied by an orthogonal 3x3-matrix *R*

$$R = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix}$$

as

$$R\mathcal{M}^2R^{\mathrm{T}} = \mathrm{diag}(M_1^2, M_2^2, M_3^2)$$

- Physical neutral fields are now given as

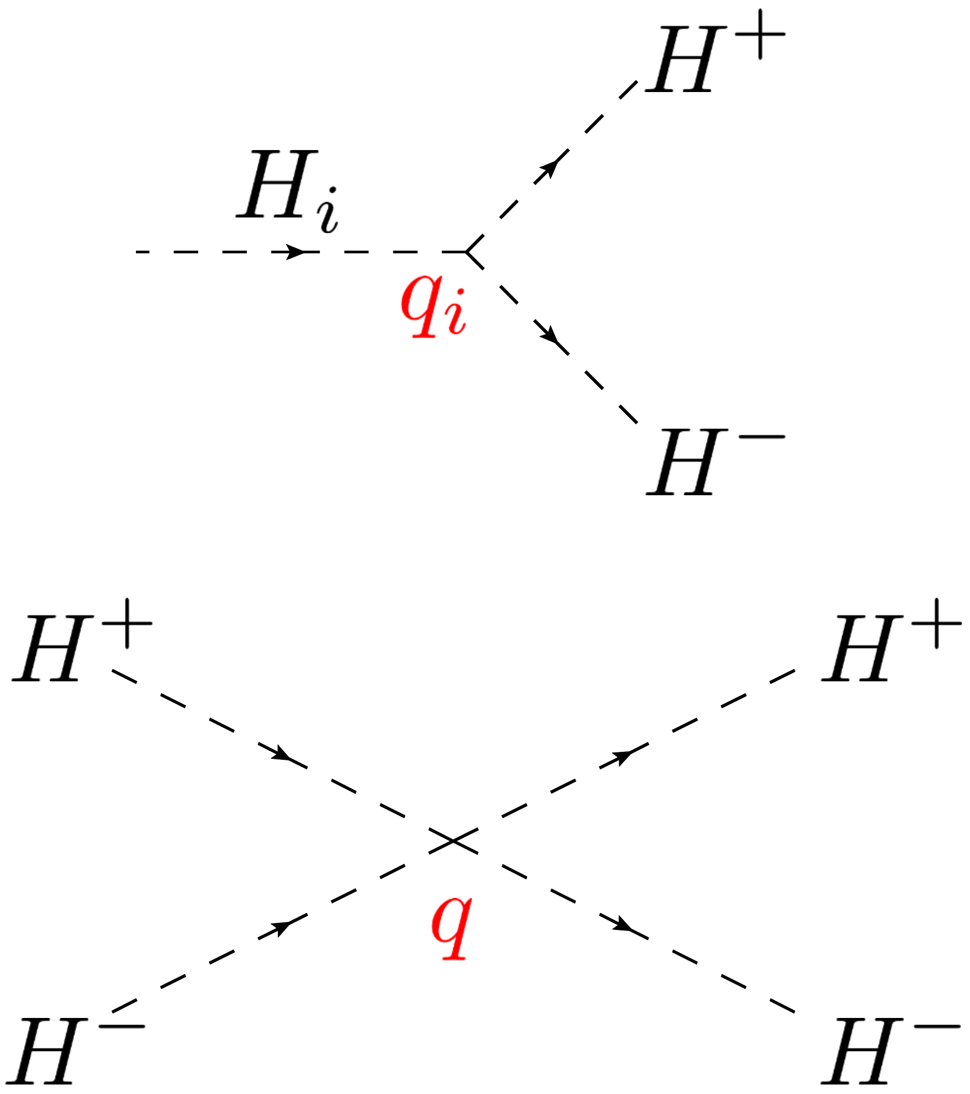
$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = R\begin{pmatrix}\eta_1 \\ \eta_2 \\ \eta_3\end{pmatrix}$$

Some tree-level scalar couplings

- Some important scalar couplings expressed in the Higgs-basis

$$
\begin{aligned}
q_i &\equiv \text{Coefficient}(V, H_i H^- H^+) \\
&= v(R_{i1}\lambda_3 + R_{i2}\text{Re}\,\lambda_7 - R_{i3}\lambda_7), \\
q &\equiv \text{Coefficient}(V, H^- H^- H^+ H^+) \\
&= \frac{1}{2}\lambda_2.
\end{aligned}
$$

- If calculated in a general basis, we can explicitly verify that these couplings are basis invariant, hence observables.



Some tree-level gauge couplings

- Gauge couplings from the Lagrangian

$$(H_i \overleftrightarrow{\partial}_\mu H_j) Z^\mu : \quad -\frac{g}{2v\cos\theta_W}\epsilon_{ijk} e_k ,$$

$$H_i Z_\mu Z^\mu : \quad \frac{g^2}{4\cos^2\theta_W} e_i ,$$

$$H_i W_\mu^+ W^{-\mu} : \quad \frac{g^2}{2} e_i ,$$

$$(H^+ \overleftrightarrow{\partial}_\mu H_i) W^{-\mu} : \quad i\frac{g}{2v} f_i ,$$

$$(H^- \overleftrightarrow{\partial}_\mu H_i) W^{+\mu} : \quad -i\frac{g}{2v} f_i^* .$$

where $e_i \equiv v_1 R_{i1} + v_2 R_{i2}$

$$f_i \equiv v_1 R_{i2} - v_2 R_{i1} - i v R_{i3}$$

- Satisfies

$$e_1^2 + e_2^2 + e_3^2 = v^2 = (246\,\mathrm{GeV})^2$$
$$f_i f_j^* = v^2 \delta_{ij} - e_i e_j + i v \epsilon_{ijk} e_k$$

- In a general basis we can show that e_i is invariant under a change of basis, hence an observable, whereas f_i is a pseudo-observable (it's absolute value is invariant).
- Simpler form in the Higgs-basis:

$$e_i = v R_{i1}$$
$$f_i = v(R_{i2} - i R_{i3})$$

The physical parameter set $\mathcal{P}$

- We now choose our set of 11 independent parameters to consist of
  - Four squared masses
  - Three tree-level gauge couplings
  - Four tree-level scalar couplings

$$\mathcal{P} \equiv \{M_{H^\pm}^2, M_1^2, M_2^2, M_3^2, e_1, e_2, e_3, q_1, q_2, q_3, q\}$$

- All observables (invariants) expressible through these.

We need an algorithm to translate from the traditional parameters to our new parameter set $\mathcal{P}$

All 14 parameters of the traditional parametrization of the potential shall be expressed through $\mathcal{P}$ (and the auxiliary quantities f_i .)

In observables, the f_i will cancel in a way such that they are expressible through the e_i .

$$f_i f_j^* = v^2 \delta_{ij} - e_i e_j + i v \epsilon_{ijk} e_k$$

Expressing the parameters of the potential in terms of $\mathcal{P}$

- Treat $M_{H^\pm}^2 = -\frac{m_{22}^2}{2} + \frac{v^2}{2}\lambda_3$

$$\mathcal{M}^2 = R^{\mathrm{T}} \mathrm{diag}(M_1^2, M_2^2, M_3^2) R$$

$$\begin{aligned} q_i &= v(R_{i1}\lambda_3 + R_{i2}\mathrm{Re}\,\lambda_7 - R_{i3}\lambda_7), \\ q &= \frac{1}{2}\lambda_2. \end{aligned}$$

as 11 independent equations, and invert them.
Also using the stationary-point equations,

$$\begin{aligned} m_{11}^2 &= v^2\lambda_1, \\ \mathrm{Re}\, m_{12}^2 &= v^2 \mathrm{Re}\,\lambda_6, \\ \mathrm{Im}\, m_{12}^2 &= v^2 \mathrm{Im}\,\lambda_6, \end{aligned}$$

- In the Higgs-basis we obtain

$$\begin{aligned} m_{11}^2 &= \frac{e_1^2 M_1^2 + e_2^2 M_2^2 + e_3^2 M_3^2}{v^2}, \\ m_{12}^2 &= \frac{e_1 f_1 M_1^2 + e_2 f_2 M_2^2 + e_3 f_3 M_3^2}{v^2}, \\ m_{22}^2 &= -2M_{H^\pm}^2 + e_1 q_1 + e_2 q_2 + e_3 q_3, \\ \lambda_1 &= \frac{e_1^2 M_1^2 + e_2^2 M_2^2 + e_3^2 M_3^2}{v^4}, \\ \lambda_2 &= 2q, \\ \lambda_3 &= \frac{e_1 q_1 + e_2 q_2 + e_3 q_3}{v^2}, \\ \lambda_4 &= \frac{M_1^2 + M_2^2 + M_3^2 - 2M_{H^\pm}^2}{v^2} - \frac{e_1^2 M_1^2 + e_2^2 M_2^2 + e_3^2 M_3^2}{v^4}, \\ \lambda_5 &= \frac{f_1^2 M_1^2 + f_2^2 M_2^2 + f_3^2 M_3^2}{v^4}, \\ \lambda_6 &= \frac{e_1 f_1 M_1^2 + e_2 f_2 M_2^2 + e_3 f_3 M_3^2}{v^4}, \\ \lambda_7 &= \frac{f_1 q_1 + f_2 q_2 + f_3 q_3}{v^2}. \end{aligned}$$

Expressing non-zero scalar couplings in terms of $\mathcal{P}$

$$
\begin{aligned}
H_iH_iH_i: \quad & \frac{v^2-e_i^2}{2v^2}q_i - \frac{(v^2-e_i^2)e_i}{v^4}M_{H^\pm}^2 \\
& + \frac{(2v^2-e_i^2)e_i}{2v^4}M_i^2 \\
H_iH_iH_j: \quad & -\frac{e_ie_j}{v^2}q_i + \frac{v^2-e_i^2}{2v^2}q_j \\
& + \frac{(3e_i^2-v^2)e_j}{v^4}M_{H^\pm}^2 \\
& + \frac{(v^2-e_i^2)e_j}{v^4}M_i^2 - \frac{e_i^2e_j}{2v^4}M_j^2 \\
H_1H_2H_3: \quad & -\frac{e_2e_3}{v^2}q_1 - \frac{e_1e_3}{v^2}q_2 \\
& - \frac{e_1e_2}{v^2}q_3 + \frac{6e_1e_2e_3}{v^4}M_{H^\pm}^2 \\
& - \frac{e_1e_2e_3}{v^4}(M_1^2+M_2^2+M_3^2) \\
H_iH^+H^-: \quad & q_i
\end{aligned}
$$

$$
\begin{aligned}
H_iH_iH_iH_i: \quad & \frac{(v^2-e_i^2)^2}{4v^4}q + \frac{(v^2-e_i^2)e_i}{2v^4}q_i + \frac{e_i^4}{8v^8}(e_1^2M_1^2+e_2^2M_2^2+e_3^2M_3^2) \\
& -\frac{(v^2-e_i^2)e_i^2}{4v^6}(e_1q_1+e_2q_2+e_3q_3+2M_{H^\pm}^2-2M_i^2) \\
H_iH_iH_iH_j: \quad & \frac{(e_i^2-v^2)e_ie_j}{v^4}q + \frac{(v^2-3e_i^2)e_j}{2v^4}q_i + \frac{(v^2-e_i^2)e_i}{2v^4}q_j \\
& + \frac{(2e_i^2-v^2)e_ie_j}{2v^6}(e_1q_1+e_2q_2+e_3q_3+2M_{H^\pm}^2) \\
& + \frac{(2v^2-3e_i^2)e_ie_j}{2v^6}M_i^2 - \frac{e_i^3e_j}{2v^6}M_j^2 + \frac{e_i^3e_j}{2v^8}(e_1^2M_1^2+e_2^2M_2^2+e_3^2M_3^2) \\
H_iH_iH_jH_j: \quad & \frac{v^4-(e_i^2+e_j^2)v^2+3e_i^2e_j^2}{2v^4}q + \frac{v^2-3e_j^2}{2v^4}e_iq_i + \frac{v^2-3e_i^2}{2v^4}e_jq_j \\
& + \frac{6e_i^2e_j^2-(e_i^2+e_j^2)v^2}{4v^6}(e_1q_1+e_2q_2+e_3q_3+2M_{H^\pm}^2) + \frac{(v^2-3e_i^2)e_j^2}{2v^6}M_i^2 \\
& + \frac{(v^2-3e_j^2)e_i^2}{2v^6}M_j^2 + \frac{3e_i^2e_j^2}{4v^8}(e_1^2M_1^2+e_2^2M_2^2+e_3^2M_3^2) \\
H_iH_iH_jH_k: \quad & \frac{(3e_i^2-v^2)e_je_k}{v^4}q + \frac{(v^2-3e_i^2+6e_j^2)e_k}{2v^4}q_j + \frac{(v^2-3e_i^2+6e_k^2)e_j}{2v^4}q_k \\
& + \frac{(6e_i^2-7v^2)e_je_k}{2v^6}(e_1q_1+e_2q_2+e_3q_3) + \frac{(6e_i^2-v^2)e_je_k}{v^6}M_{H^\pm}^2 \\
& + \frac{(2v^2-3e_i^2)e_je_k}{2v^6}M_i^2 - \frac{3e_i^2e_je_k}{2v^6}(M_1^2+M_2^2+M_3^2) \\
& + \frac{3e_i^2e_je_k}{2v^8}(e_1^2M_1^2+e_2^2M_2^2+e_3^2M_3^2) \\
H_iH_iH^+H^-: \quad & \frac{v^2-e_i^2}{v^2}q + \frac{e_i}{v^2}q_i - \frac{e_i^2}{2v^4}(e_1q_1+e_2q_2+e_3q_3) \\
H_iH_jH^+H^-: \quad & -\frac{2e_ie_j}{v^2}q + \frac{1}{v^2}(e_jq_i+e_iq_j) - \frac{e_ie_j}{v^4}(e_1q_1+e_2q_2+e_3q_3) \\
H^+H^+H^-H^-: \quad & q
\end{aligned}
$$

[Ref: Grzadkowski, Haber, Ogreid, Osland, arXiv:1808.01472]

Application: CP-violation

- CP-properties determined by three CP-odd invariants, first discovered by Lavoura and Silva. Re-expressed by Gunion and Haber as:

$$\mathrm{Im}\,J_1 = -\frac{2}{v^2}\mathrm{Im}\left[V_{da}Y_{ab}Z_{bccd}\right],$$
$$\mathrm{Im}\,J_2 = \frac{4}{v^4}\mathrm{Im}\left[V_{ab}V_{dc}Y_{be}Y_{cf}Z_{eafd}\right],$$
$$\mathrm{Im}\,J_3 = \mathrm{Im}\left[V_{ab}V_{dc}Z_{bgge}Z_{chhf}Z_{eafd}\right].$$

- Here, $V_{ab} = \frac{v_a v_b^*}{v^2}$

- In Higgs-basis: $V_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

- We translate these to $\mathcal{P}$ to find

$$\begin{aligned}
\mathrm{Im}\,J_1 &= \frac{1}{v^5}[e_1e_3q_2(M_1^2-M_3^2)+e_2e_1q_3(M_2^2-M_1^2) \\
&\qquad +e_3e_2q_1(M_3^2-M_2^2)], \\
\mathrm{Im}\,J_2 &= 2\frac{e_1e_2e_3}{v^9}(M_1^2-M_2^2)(M_2^2-M_3^2)(M_3^2-M_1^2), \\
\mathrm{Im}\,J_3 &= \frac{2}{v^4}\left[(e_1^2M_1^2+e_2^2M_2^2+e_3^2M_3^2)\right. \\
&\qquad \left. +v^2(e_1q_1+e_2q_2+e_3q_3)+2v^2M_{H^\pm}^2\right]\mathrm{Im}\,J_1 \\
&\quad +\mathrm{Im}\,J_2+\frac{2}{v^7}\sum_{i,j,k}\epsilon_{ijk}(v^2q_i+2e_iM_i^2)M_i^2e_jq_k
\end{aligned}$$

- Put Im J_1 = Im J_2 = Im J_3 = 0 and solve

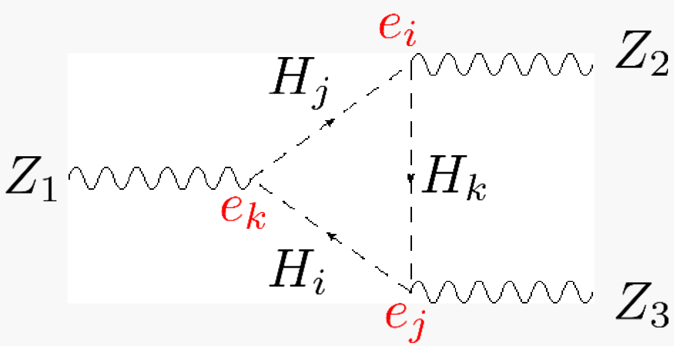
6 cases of CP-conservation:

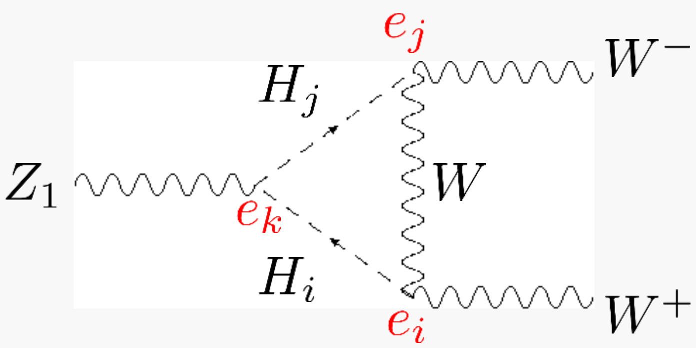
- Case 1: $M_1=M_2=M_3$.
- Case 2: $M_1=M_2$ and $e_1q_2 = e_2q_1$
- Case 3: $M_2=M_3$ and $e_2q_3 = e_3q_2$
- Case 4: $e_1=0$ and $q_1=0$
- Case 5: $e_2=0$ and $q_2=0$
- Case 6: $e_3=0$ and $q_3=0$

[Ref: Grzadkowski, Ogreid, Osland, JHEP11 (2014) 084]

Processes containing Im J_2:

ZZZ vertex and ZWW vertex





› Summing over all possible combinations of *i,j,k*, we find

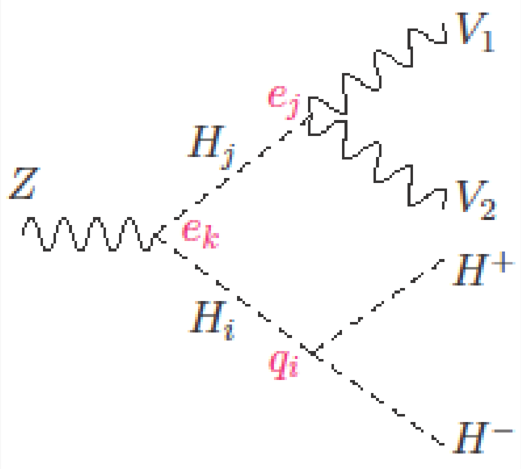
$$\mathcal{M} \;\propto\; \mathrm{Im} J_2$$

[Ref: Grzadkowski, Ogreid, Osland, JHEP05 (2016) 025]

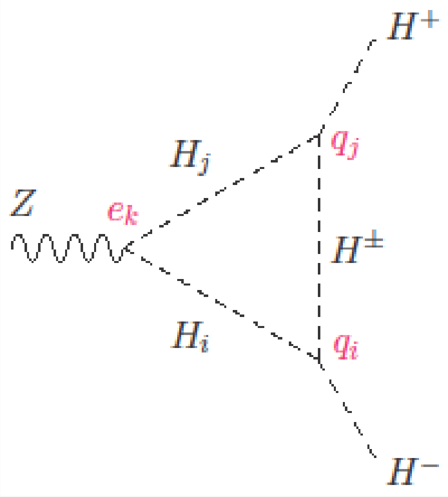
Processes containing Im J_1 and Im J_3 :

$Z \to VVH^+H^-$ or $Z \to H^+H^-$

Summing over all possible combinations of *i,j,k*, we find



$\mathcal{M}$ contains $\mathrm{Im} J_1$



$\mathcal{M}$ contains $\mathrm{Im} J_3$

Application: Spontaneous CP-violation

- Nature of CP-violation determined by four invariants presented by Gunion and Haber:

$$I_{Y3Z} = \mathrm{Im}\left[Z^{(1)}_{a\bar{c}} Z^{(1)}_{e\bar{b}} Z_{b\bar{e}c\bar{d}} Y_{d\bar{a}}\right],$$

$$I_{2Y2Z} = \mathrm{Im}\left[Y_{a\bar{b}} Y_{c\bar{d}} Z_{b\bar{a}d\bar{f}} Z^{(1)}_{f\bar{c}}\right],$$

$$I_{3Y3Z} = \mathrm{Im}\left[Z_{a\bar{c}b\bar{d}} Z_{c\bar{e}d\bar{g}} Z_{e\bar{h}f\bar{q}} Y_{g\bar{a}} Y_{h\bar{b}} Y_{q\bar{f}}\right],$$

$$I_{6Z} = \mathrm{Im}\left[Z_{a\bar{b}c\bar{d}} Z^{(1)}_{b\bar{f}} Z^{(1)}_{d\bar{h}} Z_{f\bar{a}j\bar{k}} Z_{k\bar{j}m\bar{n}} Z_{n\bar{m}h\bar{c}}\right].$$

- We translate all these to $\mathcal{P}$, demand that they should all vanish, and obtain:

Theorem. Let us assume that the quantity

$$D = e_1^2 M_2^2 M_3^2 + e_2^2 M_3^2 M_1^2 + e_3^2 M_1^2 M_2^2$$

is non-zero. Then, in a charge-conserving general 2HDM, CP is violated spontaneously if and only if the following three statements are satisfied simultaneously:

- At least one of the three invariants $\mathrm{Im}\, J_1$, $\mathrm{Im}\, J_2$, $\mathrm{Im}\, J_3$ is nonzero.
- $M_{H^\pm}^2 = \frac{v^2}{2D}[e_1 q_1 M_2^2 M_3^2 + e_2 q_2 M_3^2 M_1^2 + e_3 q_3 M_1^2 M_2^2 - M_1^2 M_2^2 M_3^2],$
- $q = \frac{1}{2D}[(e_2 q_3 - e_3 q_2)^2 M_1^2 + (e_3 q_1 - e_1 q_3)^2 M_2^2 + (e_1 q_2 - e_2 q_1)^2 M_3^2 + M_1^2 M_2^2 M_3^2].$

[Ref: Grzadkowski, Ogreid, Osland, Phys. Rev. D 94, 115002 (2016)]

Application: Alignment Limit (AL)

- 2HDM is aligned if H_1 couples to the gauge-bosons in the same way as the Higgs of the Standard Model.
- Alignment expressed in terms of $\mathcal{P}$ simply become

$$e_1 = v, \qquad e_2 = e_3 = 0$$

- Also possible to study "near-alignment" by expanding in the small parameters

e_2 and e_3 .

CPV in AL:

$$\begin{aligned} \operatorname{Im} J_1 &= 0, \\ \operatorname{Im} J_2 &= 0, \\ \operatorname{Im} J_3 &= \frac{2q_2q_3}{v^2}(M_3^2 - M_2^2). \end{aligned}$$

SCPV in AL:

$$\begin{aligned} M_{H^\pm}^2 &= \frac{vq_1 - M_1^2}{2}, \\ q &= \frac{1}{2}\left(\frac{q_2^2}{M_2^2} + \frac{q_3^2}{M_3^2} + \frac{M_1^2}{v^2}\right). \end{aligned}$$

Non-zero scalar couplings in AL:

$$\begin{aligned} H_1H_1H_1 &: \frac{M_1^2}{2v}, \\ H_jH_kH_k &: \frac{q_j}{2}, \\ H_1H_jH_j &: \frac{q_1}{2} + \frac{M_j^2 - M_{H^\pm}^2}{v}, \\ H_iH^+H^- &: q_i. \end{aligned}$$

$$j, k = 2, 3$$

$$\begin{aligned} H_1H_1H_1H_1 &: \frac{M_1^2}{8v^2}, \\ H_jH_jH_jH_j &: \frac{q}{4}, \\ H_1H_jH_kH_k &: \frac{q_j}{2v}, \\ H_1H_1H_jH_j &: \frac{q_1}{4v} + \frac{M_j^2 - M_{H^\pm}^2}{2v^2}, \\ H_2H_2H_3H_3 &: \frac{q}{2}, \\ H_1H_1H^+H^- &: \frac{q_1}{2v}, \\ H_jH_jH^+H^- &: q, \\ H_1H_jH^+H^- &: \frac{q_j}{v}, \\ H^+H^+H^-H^- &: q. \end{aligned}$$

Application: Z_2-symmetric potential

- Potential Z_2-symmetric if the three commutators of Davidson and Haber vanish

$$[Z^{(1)}, Y] = 0,$$
$$[Z^{(1)}, Z^{(11)}] = 0,$$
$$[Y^{(1)}, Y] = 0.$$

- Expressing these commutators in terms of $\mathcal{P}$ and find six different cases of Z_2–symmetric potential.

- Note: CP-conservation necessary, but not sufficient for Z_2–symmetric potential, (thus we continue from the six cases of CP-conservation).

Full mass degeneracy: $M_1 = M_2 = M_3$

Two distinct cases that gives Z_2–symmetric potential.

Case 1:

$$M^2_{H^\pm} = \frac{e_1 q_1 + e_2 q_2 + e_3 q_3 - M_1^2}{2}$$

$$q = \frac{(e_1 q_2 - e_2 q_1)^2 + (e_1 q_3 - e_3 q_1)^2 + (e_2 q_3 - e_3 q_2)^2 + M_1^4}{2 M_1^2 v^2}$$

Case 2:

$$e_1 q_2 = e_2 q_1$$

$$e_1 q_3 = e_3 q_1 \qquad (\text{ implies } \quad e_2 q_3 = e_3 q_2 \text{ })$$

[Ref: Davidson and Haber, Phys.Rev.D72:035004 (2005)]

Application: Z_2-symmetric potential

Partial mass degeneracy: $M_i = M_j$

$$e_i q_j = e_j q_i$$

Two distinct cases that gives Z_2–symmetric potential.

Case 1:

$$M_{H^\pm}^2 = \frac{v^2}{2}\frac{(e_i q_i + e_j q_j)M_k^2 + e_k q_k M_i^2 - M_i^2 M_k^2}{e_k^2 M_i^2 + (e_i^2 + e_j^2)M_k^2}$$

$$q = \frac{1}{2}\frac{(e_i q_k - e_k q_i)^2 + (e_j q_k - e_k q_j)^2 + M_i^2 M_k^2}{e_k^2 M_i^2 + (e_i^2 + e_j^2)M_k^2}$$

Case 2:

$$e_k = q_k = 0$$

No mass degeneracy: $e_k = q_k = 0$

Two distinct cases that gives Z_2–symmetric potential.

Case 1:

$$M_{H^\pm}^2 = \frac{v^2}{2}\frac{e_j q_j M_i^2 + e_i q_i M_j^2 - M_j^2 M_i^2}{e_j^2 M_i^2 + e_i^2 M_j^2}$$

$$q = \frac{1}{2}\frac{(e_j q_i - e_i q_j)^2 + M_i^2 M_j^2}{e_j^2 M_i^2 + e_i^2 M_j^2}$$

Case 2:

$$e_j = q_j = 0$$

Application: Unitarity at tree-level

Determined from four matrices given by Ginzburg and Ivanov:

$$
\begin{aligned}
8\pi S_{Y=2,\sigma=1} &= \begin{pmatrix} \lambda_1 & \lambda_5 & \sqrt{2}\lambda_6 \\ \lambda_5^* & \lambda_2 & \sqrt{2}\lambda_7^* \\ \sqrt{2}\lambda_6^* & \sqrt{2}\lambda_7 & \lambda_3+\lambda_4 \end{pmatrix}, \\
8\pi S_{Y=2,\sigma=0} &= \lambda_3 - \lambda_4, \\
8\pi S_{Y=0,\sigma=1} &= \begin{pmatrix} \lambda_1 & \lambda_4 & \lambda_6 & \lambda_6^* \\ \lambda_4 & \lambda_2 & \lambda_7 & \lambda_7^* \\ \lambda_6^* & \lambda_7^* & \lambda_3 & \lambda_5^* \\ \lambda_6 & \lambda_7 & \lambda_5 & \lambda_3 \end{pmatrix}, \\
8\pi S_{Y=0,\sigma=0} &= \begin{pmatrix} 3\lambda_1 & 2\lambda_3+\lambda_4 & 3\lambda_6 & 3\lambda_6^* \\ 2\lambda_3+\lambda_4 & 3\lambda_2 & 3\lambda_7 & 3\lambda_7^* \\ 3\lambda_6^* & 3\lambda_7^* & \lambda_3+2\lambda_4 & 3\lambda_5^* \\ 3\lambda_6 & 3\lambda_7 & 3\lambda_5 & \lambda_3+2\lambda_4 \end{pmatrix}.
\end{aligned}
$$

All eigenvalues must satisfy $|\Lambda| < \frac{1}{8\pi}$

Eigenvalues determined from characteristic equation for each matrix.

Each coefficient in the characteristic equation is an invariant, translatable to $\mathcal{P}$.

[Ref: Ginzburg, Ivanov, Phys.Rev. D72 (2005) 115010]

Application: Unitarity at tree-level

$$8\pi S_{Y=2,\sigma=0} = 2\frac{M_{H^\pm}^2}{v^2} - \frac{M_1^2 + M_2^2 + M_3^2}{v^2} + \frac{e_1^2 M_1^2 + e_2^2 M_2^2 + e_3^2 M_3^2}{v^4} + \frac{e_1 q_1 + e_2 q_2 + e_3 q_3}{v^2}$$

Alignment limit:

$$8\pi S_{Y=2,\sigma=0} = \frac{2M_{H^\pm}^2 - M_2^2 - M_3^2 + v q_1}{v^2}$$

Application: Unitarity at tree-level

- For $8\pi S_{Y=2,\sigma=1}$, the characteristic equation becomes cubic. $\alpha_0 + \alpha_1\Lambda + \alpha_2\Lambda^2 - \Lambda^3 = 0,$
- Coefficients given by

$$\begin{aligned}
\alpha_2 &= -2m_+ + d_{010} + d_{101} + 2q, \\
\alpha_1 &= q\left(4m_+ - 2d_{010} - 2d_{101}\right) + 2m_+ d_{012} \\
&\quad + d_{010}d_{012} - d_{010}^2 - d_{012}d_{101} + 2d_{020} - 2d_{022} - 2d_{101}^2 + 2d_{200}, \\
\alpha_0 &= m_+\left(4d_{010}d_{012} - 2d_{010}^2 + 2d_{012}^2 + 4d_{020} - 8d_{022}\right) - 4qm_+d_{012} + 2d_{010}d_{012}d_{101} \\
&\quad + q\left(2d_{010}d_{012} + 2d_{012}^2\right) + d_{010}d_{012}^2 + d_{010}d_{020} - 2d_{010}d_{022} + d_{010}^2 d_{101} - 4d_{010}d_{111} \\
&\quad - d_{012}d_{020} - 4d_{012}d_{022} + 2qd_{012}d_{101} + 3d_{012}^2 d_{101} + 2d_{012}d_{101}^2 - 4d_{012}d_{111} - 2d_{012}d_{200} \\
&\quad + d_{012}^3 - 2d_{020}d_{101} - 4d_{022}d_{101} - 4qd_{022} - 2d_{030} + 6d_{032} + 8d_{121}.
\end{aligned}$$

Possible to express eigenvalues, hence unitarity constrains in terms of $\mathcal{P}$, unfortunately not very transparent.

Abbreviations:

$$d_{ijk} = \frac{q_1^i M_1^{2j} e_1^k + q_2^i M_2^{2j} e_2^k + q_3^i M_3^{2j} e_3^k}{v^{i+2j+k}},$$

$$m_+ = \frac{M_{H^\pm}^2}{v^2}.$$

Application: Oblique parameters

$$\Delta\rho = \frac{g^2}{64\pi^2 m_W^2 v^2}$$
$$\Big\{e_1^2\Big[F(M_{H_\pm}^2, M_2^2) + F(M_{H_\pm}^2, M_3^2) - F(M_2^2, M_3^2)\Big]$$
$$+ e_2^2\Big[F(M_{H_\pm}^2, M_1^2) + F(M_{H_\pm}^2, M_3^2) - F(M_1^2, M_3^2)$$
$$+ 3F(M_Z^2, M_2^2) - 3F(M_Z^2, M_1^2)$$
$$-3F(M_W^2, M_2^2) + 3F(M_W^2, M_1^2)]$$
$$+ e_3^2\Big[F(M_{H_\pm}^2, M_1^2) + F(M_{H_\pm}^2, M_2^2) - F(M_1^2, M_2^2)$$
$$+ 3F(M_Z^2, M_3^2) - 3F(M_Z^2, M_1^2)$$
$$-3F(M_W^2, M_3^2) + 3F(M_W^2, M_1^2)]\Big\},$$

where

$$F(I, J) \equiv \begin{cases} \dfrac{I+J}{2} - \dfrac{IJ}{I-J}\ln\dfrac{I}{J} & , \quad I \neq J, \\ 0 & , \quad I = J. \end{cases}$$

Application: Oblique parameters

$$\bar{S} = \frac{g^2}{384\pi^2 c_W^2 v^2}\left\{e_1^2\left[G(M_2^2, M_3^2, M_Z^2)\right]\right.$$
$$+ e_2^2\left[G(M_1^2, M_3^2, M_Z^2) + \hat{G}(M_2^2, M_Z^2) - \hat{G}(M_1^2, M_Z^2)\right]$$
$$+ e_3^2\left[G(M_1^2, M_2^2, M_Z^2) + \hat{G}(M_3^2, M_Z^2) - \hat{G}(M_1^2, M_Z^2)\right]$$
$$\left. + v^2\left[\ln\frac{M_2^2 M_3^2}{M_{H_\pm}^4} + (1-2s_W^2)^2 G(M_{H_\pm}^2, M_{H_\pm}^2, M_Z^2))\right]\right\}$$

where

$$G(I,J,Q) \equiv -\frac{16}{3} + \frac{5(I+J)}{Q} - \frac{2(I-J)^2}{Q^2} + \frac{3}{Q}\left[\frac{I^2+J^2}{I-J} - \frac{I^2-J^2}{Q} + \frac{(I-J)^3}{3Q^2}\right]\ln\frac{I}{J} + \frac{Q^2 - 2Q(I+J) + (I-J)^2}{Q^3} \times f\left(I+J-Q, Q^2 - 2Q(I+J) + (I-J)^2\right),$$

$$\hat{G}(I,Q) \equiv -\frac{79}{3} + 9\frac{I}{Q} - 2\frac{I^2}{Q^2} + \left(-10 + 18\frac{I}{Q} - 6\frac{I^2}{Q^2} + \frac{I^3}{Q^3} - 9\frac{I+Q}{I-Q}\right)\ln\frac{I}{Q} + \left(12 - 4\frac{I}{Q} + \frac{I^2}{Q^2}\right)\frac{f(I, I^2 - 4IQ)}{Q}.$$

$$f(t,r) \equiv \begin{cases} \sqrt{r}\,\ln\left|\dfrac{t-\sqrt{r}}{t+\sqrt{r}}\right| & , \quad r > 0, \\ 0 & , \quad r = 0, \\ 2\sqrt{-r}\,\arctan\dfrac{\sqrt{-r}}{t} & , \quad r < 0. \end{cases}$$

To do list + Summary

To-do list:

- Soft breaking of Z_2 in terms of $\mathcal{P}$
- PQ-symmetry in terms of $\mathcal{P}$
- Dark Matter in tems of $\mathcal{P}$
- Positivity in terms of $\mathcal{P}$

- Summary:
- All couplings of the 2HDM expressible in terms of a subset of masses/tree-level couplings.
- Observables from the scalar sector always expressible in terms of couplings/masses of $\mathcal{P}$.
- Couplings/masses provide direct connection to experiments and tells us what measurements to make in order to test properties and predictions of the 2HDM.