Symmetry and Geometry of Generalized Higgs Sectors

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in collaboration with
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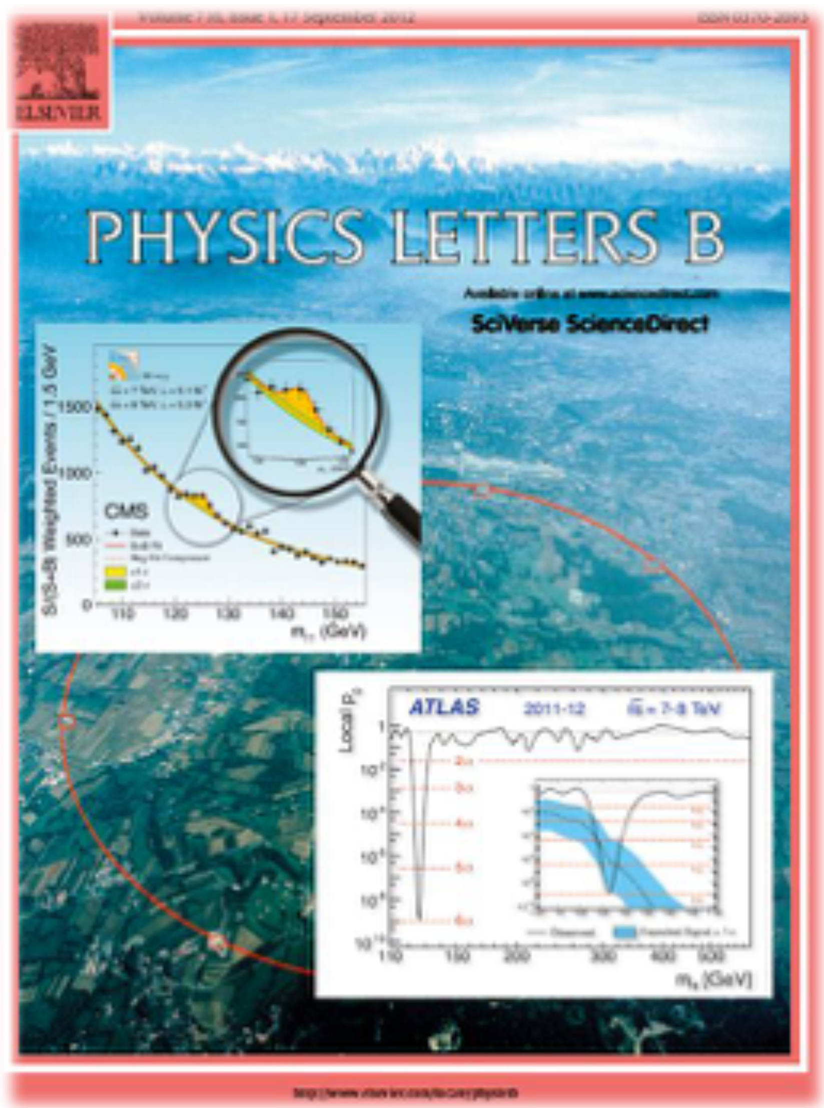
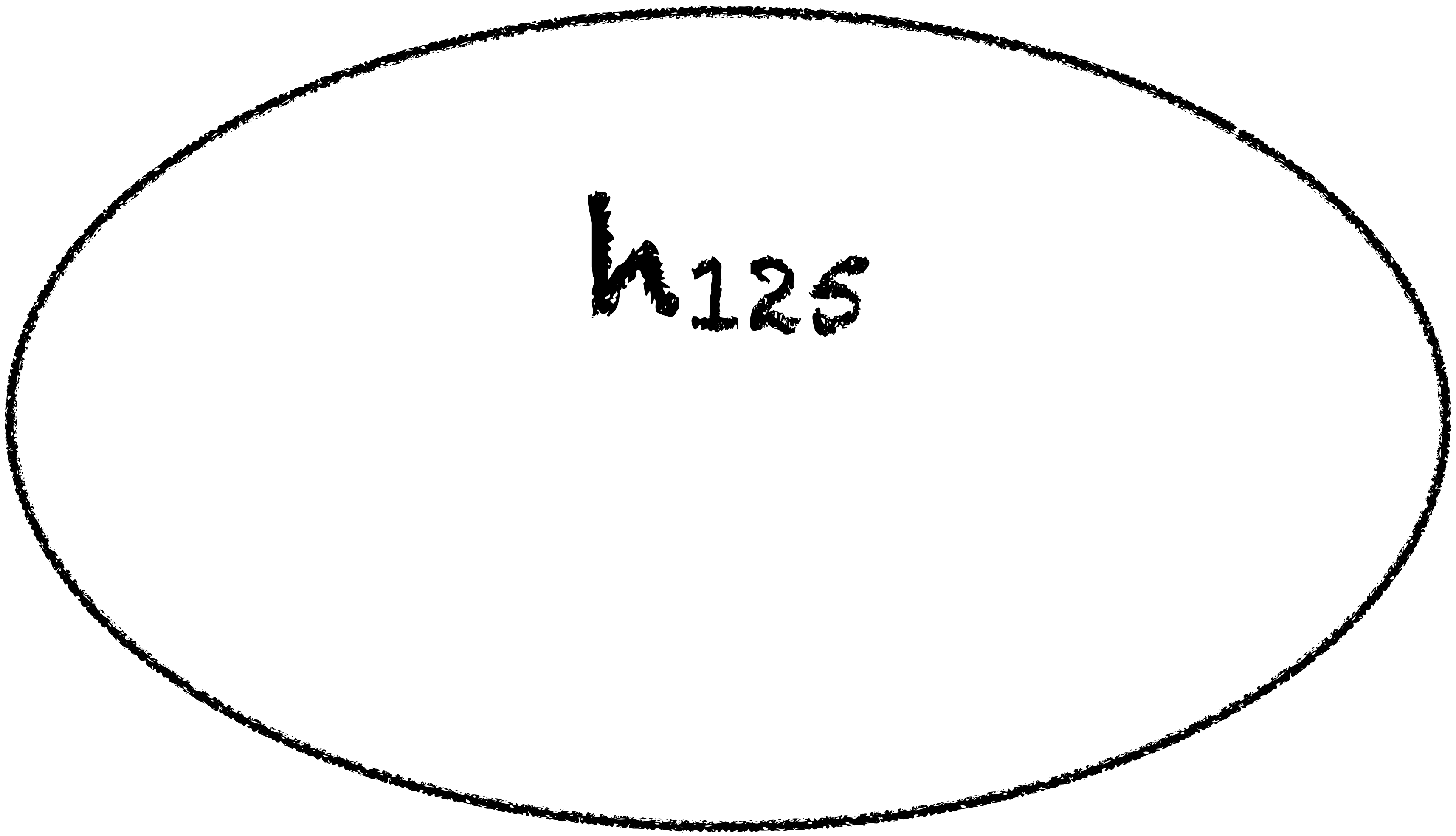
We found a h(125) !

- The 125GeV Higgs boson was discovered at the LHC.



h(125) = h(SM) ?

Higgs Sector



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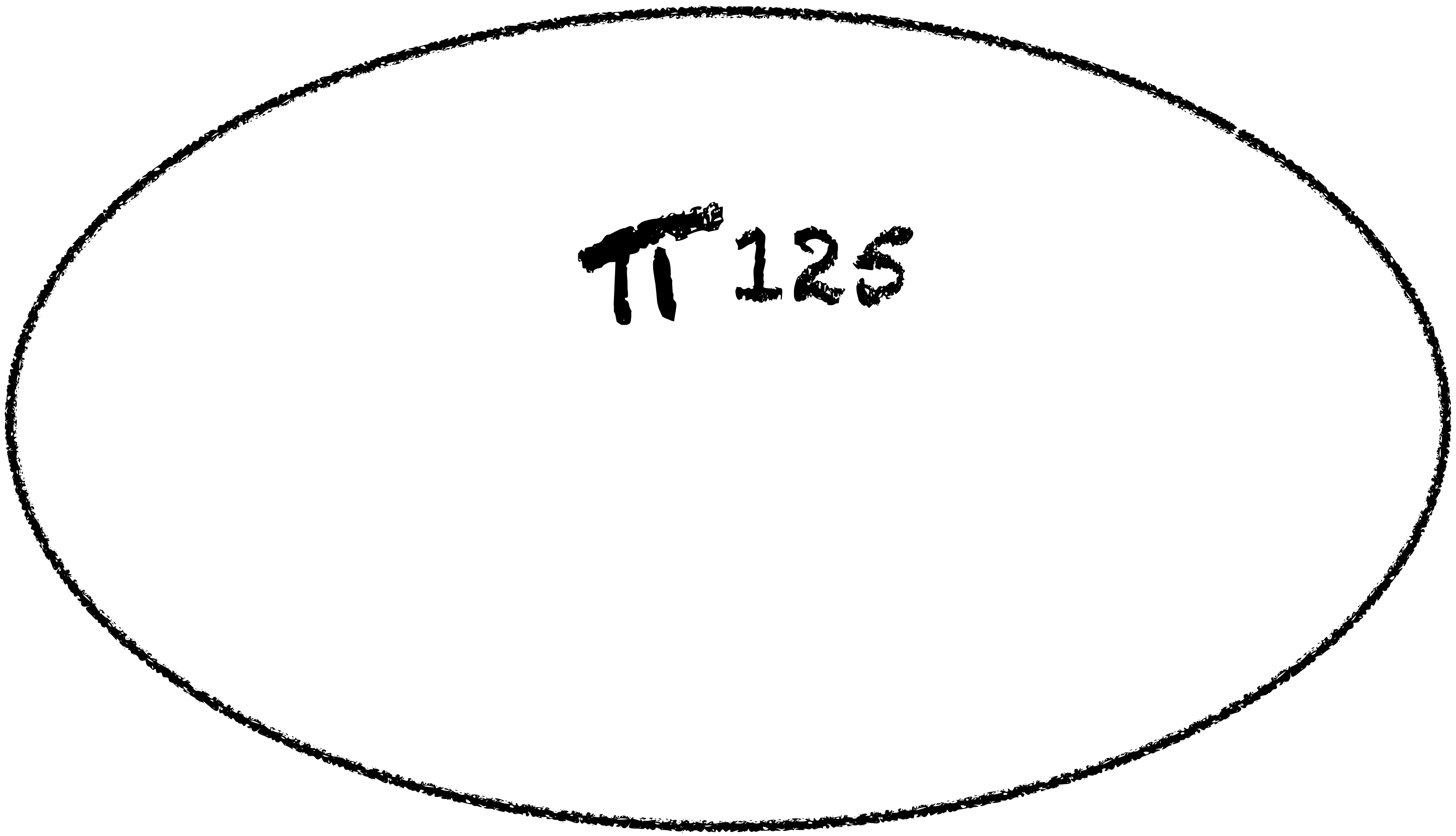
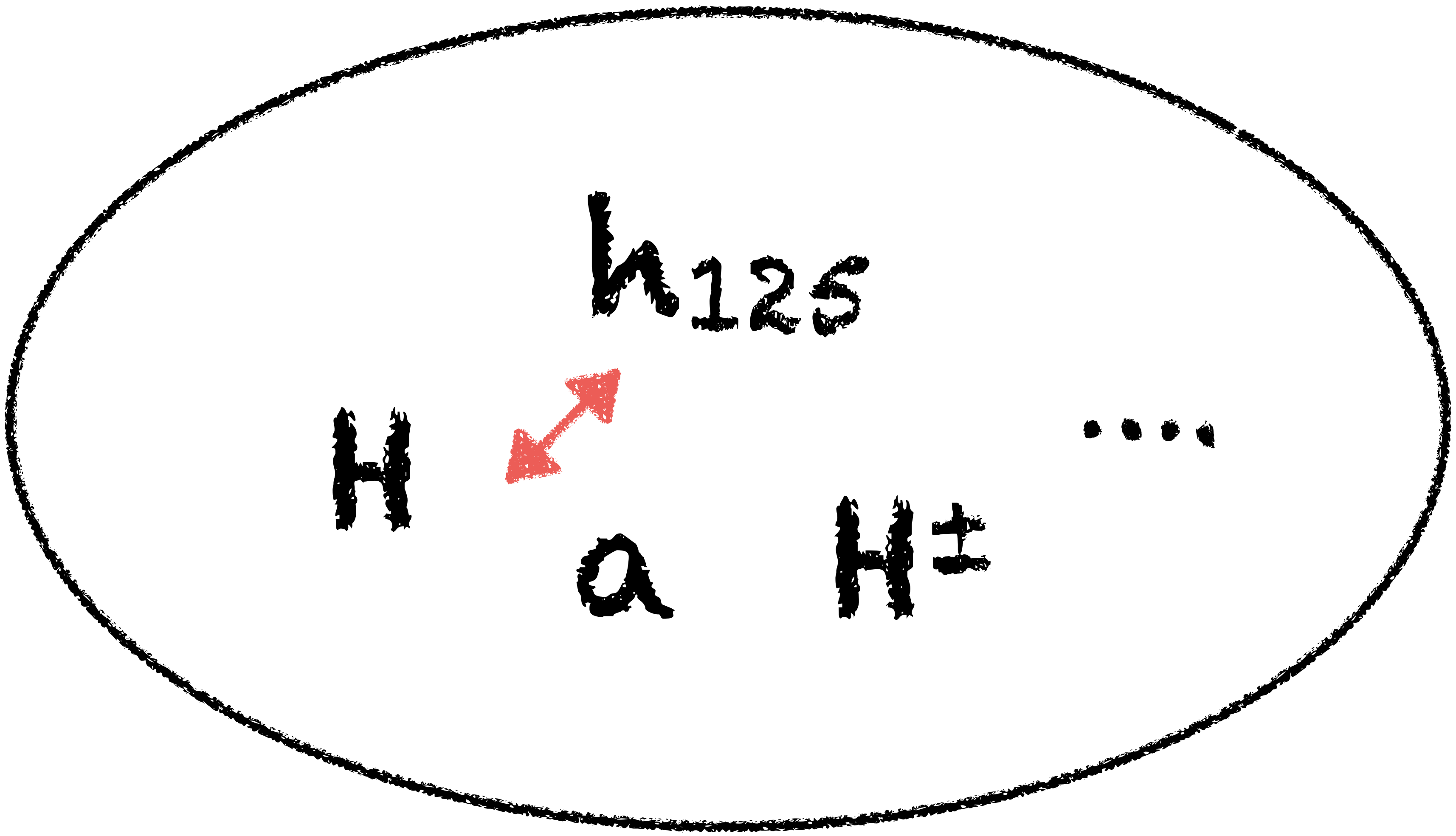


Composite ?

h(125) = h(SM) ?

Higgs Sector



Elementary. But mixed state ?

Outline

- Introduction
- Higgs Effective Field Theory
- Perturbative Unitarity vs. EWPTs
- Summary and Outlook

Higgs EFT

Q. How can we formulate the Higgs sector ?

Higgs EFT

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- $SU(2)_W$ doublet filed : 4 = 3(NGBs) + 1(higgs)

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} i(\phi^1 + i\phi^2) \\ \phi^0 - i\phi^3 \end{pmatrix}$$

Higgs EFT

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$$\Phi = \frac{1}{\sqrt{2}} \exp\left(\frac{i}{v}\pi^a \tau^a\right) \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

NGBs   125GeV higgs

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- Examples :

$$\mathcal{L}_{\rm SM} = (D_\mu \Phi)^\dagger (D^\mu \Phi) + \cdots$$

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$$\mathcal{L}_{\mathrm{SM}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) + \cdots$$

$$\mathcal{L}_{\mathrm{SMEFT}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) + \frac{C}{\Lambda^2} \Phi^\dagger \Phi \, (D_\mu \Phi)^\dagger (D^\mu \Phi) + \cdots$$

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- What if $h(125) \not\in \Phi$? e.g. higgs = Composite state, Dilaton, Radion, ….

CCWZ method

- Systematic way for describing physics associated with SSB

Coleman-Wess-Zumino (1969)
Coleman-Callen-Wess-Zumino (1969)

CCWZ method

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Coleman-Wess-Zumino (1969)
Coleman-Callen-Wess-Zumino (1969)

- General Lagrangian for NGBs arising through EWSB :

$$\mathcal{L}_\pi = \frac{v^2}{4}\mathrm{Tr}[D_\mu U^\dagger D^\mu U]$$

* We here assume "custodial symmetry" for simplicity.

- NGB fields : $U = e^{\frac{i}{v}\pi^a \tau^a}$
- Gauge bosons : $D_\mu U = \partial_\mu U - igU\mathbf{W}_\mu + ig_Y \mathbf{B}_\mu U$

CCWZ method

- General Lagrangian for NGBs and singlet scalar(s)

$$\mathcal{L} = \frac{v^2}{4}\mathrm{Tr}[D_\mu U^\dagger D^\mu U]\left(1 + 2\sum_h \kappa^h_{WW}\frac{h}{v} + \sum_{h,h'}\kappa^{hh'}_{WW}\frac{hh'}{v^2} + \cdots\right)$$

Nagai-Tanabashi-Tsumura (2014)

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- Examples:

$$\mathcal{L}_{\mathrm{SM}} = \frac{v^2}{4}\mathrm{Tr}[D_\mu U^\dagger D^\mu U]\left(1 + 2\frac{h}{v} + \frac{h^2}{v^2}\right)$$

$$\mathcal{L}_{\mathrm{SM+S}} = \frac{v^2}{4}\mathrm{Tr}[D_\mu U^\dagger D^\mu U]\left(1 + 2c\frac{h}{v} + 2s\frac{H}{v} + c^2\frac{h^2}{v^2} + s^2\frac{H^2}{v^2} + 2cs\frac{hH}{v^2}\right)$$

$$\mathcal{L}_{\mathrm{MCSM}} = \frac{v^2}{4}\mathrm{Tr}[D_\mu U^\dagger D^\mu U]\left(1 + 2\left(1 - \frac{v^2}{2f^2}\right)\frac{h}{v} + \left(1 - \frac{v^2}{f^2}\right)\frac{h^2}{v^2} + \cdots\right)$$

CCWZ method

- General Lagrangian for NGBs and singlet scalar(s)

$$\mathcal{L} = \frac{v^2}{4}\mathrm{Tr}[\partial_\mu U^\dagger \partial^\mu U]\left(1 + 2\sum_h \kappa_{WW}^h \frac{h}{v} + \sum_{h,h'} \kappa_{WW}^{hh'} \frac{hh'}{v^2} + \cdots\right)$$

$$= \frac{1}{2}\,\partial_\mu(\pi^a\ h\ h'\ \cdots)\begin{pmatrix} \left(1+2\sum\kappa_{WW}^h \frac{h}{v} + \sum \kappa_{WW}^{hh'}\frac{hh'}{v^2}\right)(\delta_{ab} + \mathcal{O}(\pi^2)) & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} \partial^\mu \begin{pmatrix} \pi^a \\ h \\ h' \\ \vdots \end{pmatrix}$$

$$= \frac{1}{2} g_{ij}(\phi)\,\partial_\mu \phi^i\,\partial^\mu \phi^j$$

- NGBs and Higgs field(s) define coordinates on a **scalar manifold**.

Generalized Higgs sector

- Example : SM Higgs sector

$\Phi = (\pi, h)$

$$
g_{ij}(\phi) = \begin{array}{cc} \begin{array}{cc} \pi & h \end{array} & \\ \begin{pmatrix} g_{ab}(\phi) & \\ & \mathbf{1} \end{pmatrix} & \begin{array}{c} \pi \\ h \end{array} \end{array}
$$

Flat

$$
g_{ab}(\phi) = \left(1 + 2\frac{h}{v} + \frac{h^2}{v^2}\right)\left(\delta_{ab} + \mathcal{O}(\pi^2)\right)
$$

Generalized Higgs sector

- Example : Minimal Composite Higgs Model $\Phi = (\pi, h)$

Agashe-Contino-Pomarol (2005)

$$g_{ij}(\phi) = \begin{array}{cc} \begin{array}{cc} \pi & h \end{array} & \\ \begin{pmatrix} g_{ab}(\phi) & \\ & \ \ \end{pmatrix} & \begin{array}{c} \pi \\ h \end{array} \end{array}$$

S^4

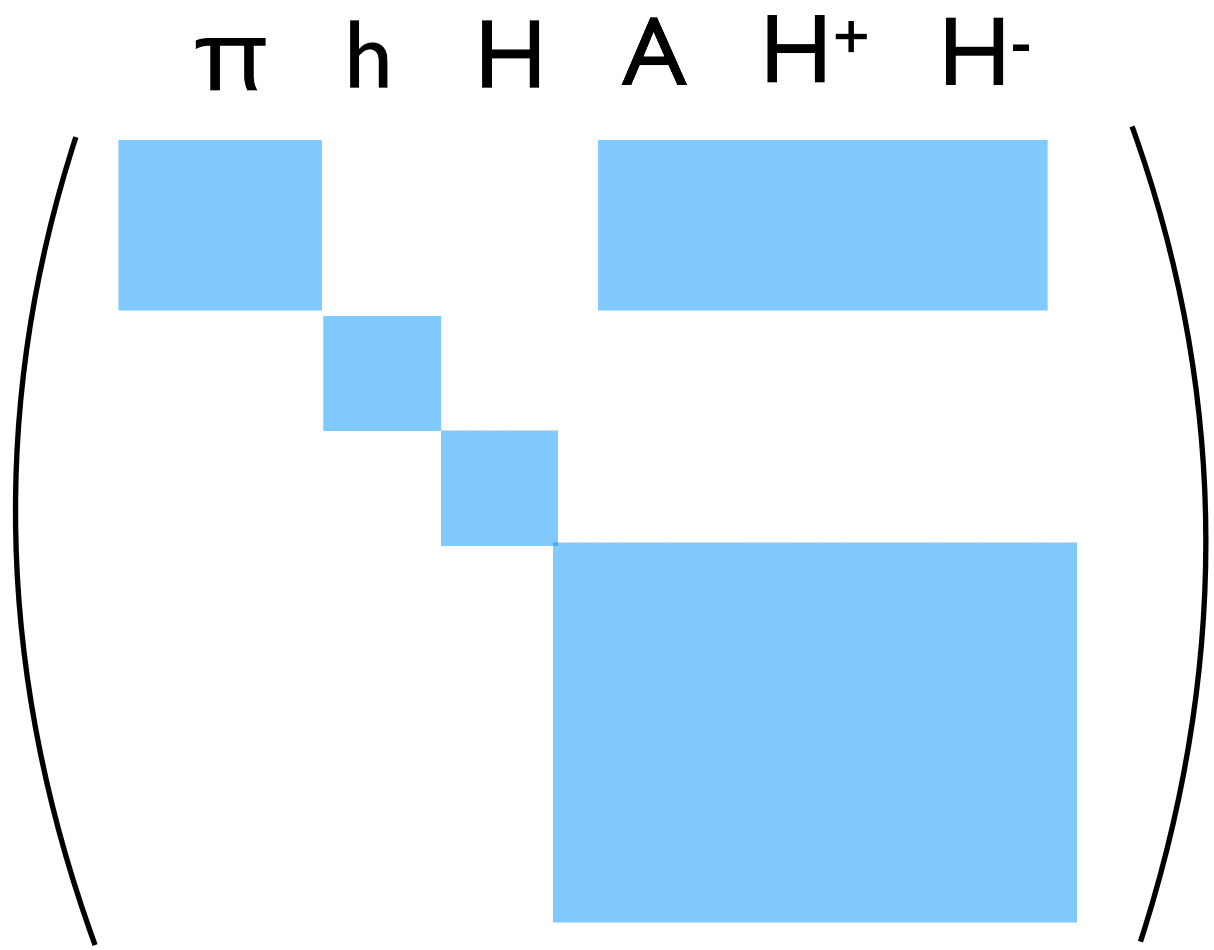
(curvature ~ v/f)

$$g_{ab}(\phi) = \left(1 + 2\left(1 - \frac{v^2}{2f^2}\right)\frac{h}{v} + \left(1 - \frac{2v^2}{f^2}\right)\frac{h^2}{v^2} + \cdots\right)\left(\delta_{ab} + \mathcal{O}(\pi^2)\right)$$

Generalized Higgs sector

- Example : Two Higgs Doublet Model $\Phi = (\pi, h, H, A, H^+, H^-)$

$$g_{ij}(\phi) =$$



Flat

Generalized Higgs sector

- The EW sector in general extended Higgs scenarios can be expressed by the following form:

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi)\, \partial_\mu \phi^i\, \partial^\mu \phi^j$$

$$\phi^i = (\underbrace{\pi^a}_{\text{NGBs}}, \underbrace{h, h', \cdots}_{\text{Higgses}})$$

- The interaction between π (~ W_L/Z_L) and Higgs bosons is determined by the structure of $g_{ij}\,(\Phi)$

Alonso-Jenkins-Manohar (2016)

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Perturbative Unitarity

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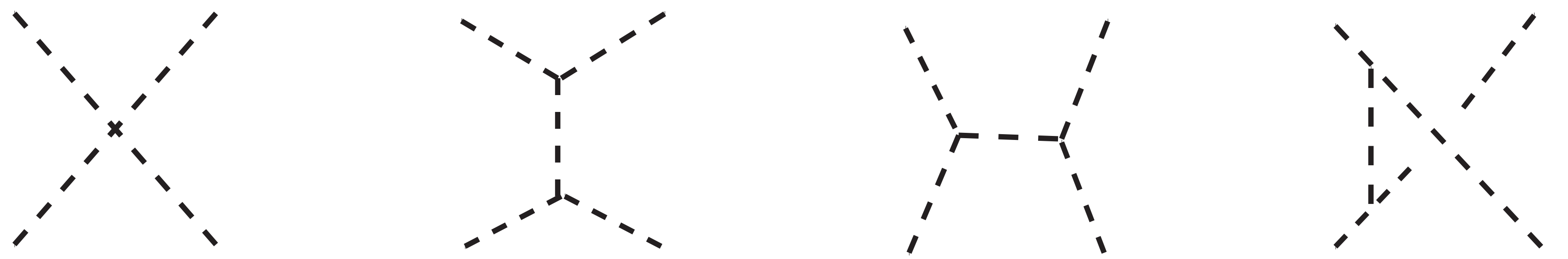
Alonso-Jenkins-Manohar (2016)

- **Perturbative Unitarity** / EWPTs → g_{ij} (Φ) ??

Perturbative Unitarity

$\underline{\Phi_i\,\Phi_j \rightarrow \Phi_k\,\Phi_l}$



$$\phi^i = (\underbrace{\pi^a}_{\text{NGBs}}, \underbrace{h, h', \cdots}_{\text{Higgses}})$$

Perturbative Unitarity

$\underline{\phi_i \, \phi_j \rightarrow \phi_k \, \phi_l}$

- Tree-level scattering amplitude:

$$\mathcal{M}_{\phi_i\phi_j\to\phi_k\phi_l} \sim \frac{s}{3}\left(\overline{R}_{iklj}+\overline{R}_{ilkj}\right)+\frac{t}{3}\left(\overline{R}_{ijlk}+\overline{R}_{iljk}\right)+\frac{u}{3}\left(\overline{R}_{ijkl}+\overline{R}_{ikjl}\right)$$

Riemann curvature tensor

Perturbative Unitarity

$\Phi_i \Phi_j \rightarrow \Phi_k \Phi_l$

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- Perturbative unitarity condition: $\boxed{\overline{R}_{ijkl}=0}$

EWPTs

- The EW sector in general extended Higgs scenarios can be expressed by the following form:

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi)\, \partial_\mu \phi^i\, \partial^\mu \phi^j$$

$$\phi^i = (\underbrace{\pi^a}_{\text{NGBs}}, \underbrace{h, h', \cdots}_{\text{Higgses}})$$

- The interaction between π (~ W_L/Z_L) and Higgs bosons is determined by the structure of g_{ij} (Φ) Alonso-Jenkins-Manohar (2016)

- Perturbative Unitarity / **EWPTs** → g_{ij} (Φ) ??

EW corrections

- EW oblique corrections depend on not only geometry but also symmetry of the scalar manifold.
- The scalar manifold respects $SU(2)_W \times U(1)_Y$ gauge sym.

<u>4 Killing vectors</u>

w_a^i (a=1,2,3) y^i

$SU(2)_W$ symmetry $U(1)_Y$ symmetry

For examples:

$$w_a^b = \frac{v}{2}\delta_{ab} - \frac{1}{2}\epsilon_{abc}\pi^c + \mathcal{O}(\pi^2)$$

$$y^b = -\frac{v}{2}\delta_{3b} - \frac{1}{2}\epsilon_{3bc}\pi^c + \mathcal{O}(\pi^2)$$

EW corrections

S-parameter at one-loop

- UV divergence:

$$S_{\log} \propto (\bar{w}_3^j)_{;i} (\bar{y}^i)_{;j}$$

Alonso-Jenkins-Manohar (2016)

$$(w_a^i)_{;j} = \frac{\partial}{\partial \phi^j} w_a^i + \Gamma^i_{kj} w_a^k$$

$$(y^i)_{;j} = \frac{\partial}{\partial \phi^j} y^i + \Gamma^i_{kj} y^k$$

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Alonso-Jenkins-Manohar (2016)

SU(2)$_W$×U(1)$_Y$ sym.

$$[w_a, w_b] = -\epsilon_{abc} w_c$$

$$[w_a, y] = 0$$

$$v_{i;j;k} = R^l{}_{kji} v_l \quad (v = w_a, y)$$

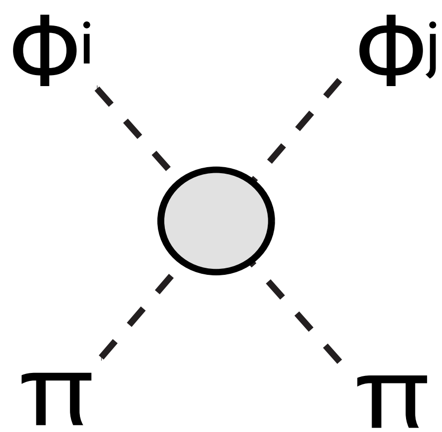
$$\boxed{S_{\log} \propto \epsilon_{3bc} \overline{R}^i{}_{jc3} (\bar{w}_b^j)_{;i} + \epsilon_{3bc} \overline{R}^i{}_{jbc} (\bar{y}^j)_{;i}}$$

EW corrections

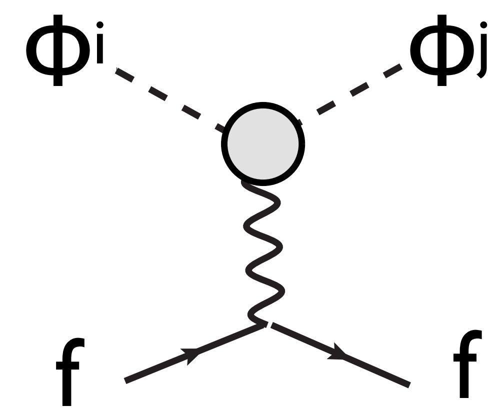
S-parameter at one-loop

- UV divergence:

$\pi\pi \to \phi^i\phi^j$



$f\bar{f} \to \phi^i\phi^j$



$$S_{\log} \propto \epsilon_{3bc}\underline{\overline{R}^i{}_{jc3}}\,\underline{(\bar{w}^j_b)_{;i}} + \epsilon_{3bc}\underline{\overline{R}^i{}_{jbc}}\,\underline{(\bar{y}^j)_{;i}}$$

EW corrections

<u>U-parameter at one-loop</u>

- UV divergence:

$$U_{\log} \propto (\bar{w}_1^j)_{;i}(\bar{w}_1^i)_{;j} - (\bar{w}_3^j)_{;i}(\bar{w}_3^i)_{;j}$$

<u>SU(2)$_W$×U(1)$_Y$ sym.</u>

$$[w_a, w_b] = -\epsilon_{abc} w_c$$

$$[w_a, y] = 0$$

$$v_{i;j;k} = R^l{}_{kji} v_l \quad (v = w_a, y)$$

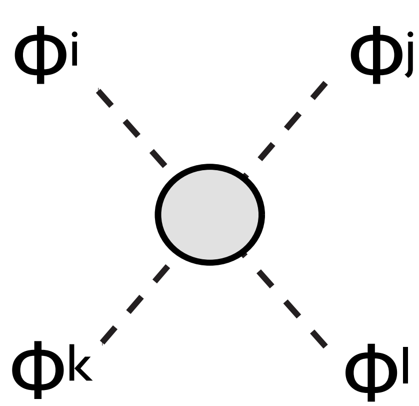
$$\boxed{U_{\log} \propto \epsilon_{1bc} \overline{R}^i{}_{jbc} (\bar{w}_1^j)_{;i} - \epsilon_{3bc} \overline{R}^i{}_{jbc} (\bar{w}_3^j)_{;i}}$$

PU vs. EWPTs

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi) D_\mu \phi^i D^\mu \phi^j \qquad \phi^i = (\pi^a, h, h', \cdots)$$

Perturbative Unitarity

$$* \ \mathcal{M}_{\phi^i \phi^j \to \phi^k \phi^l}\Big|_{E^2} \sim \overline{R}_{ijkl}$$



EW oblique corrections

$$* \quad S_{\log} \sim \epsilon_{3bc} \overline{R}^{i}{}_{jc3} (\bar{w}^j_b)_{;i} + \epsilon_{3bc} \overline{R}^{i}{}_{jbc} (\bar{y}^j)_{;i}$$

$$* \quad U_{\log} \sim \epsilon_{1bc} \overline{R}^{i}{}_{jbc} (\bar{w}^j_1)_{;i} - \epsilon_{3bc} \overline{R}^{i}{}_{jbc} (\bar{w}^j_3)_{;i}$$

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Summary and Outlook

- Physics in extended Higgs scenarios can be understood in terms of geometry and symmetry of "the scalar manifold".

Flat → Perturbative

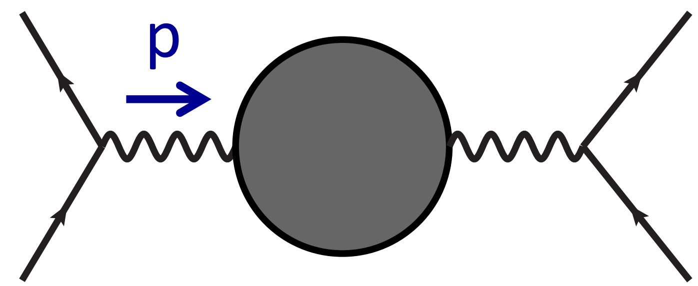
Curved → Non-Perturbative

- Consistency with EWPTs does not imply the complete flatness of the scalar manifold.

- Other constraints on "the scalar manifold" ?

Back-up slides

EW corrections



Vacuum polarization functions

Charged SU(2) current	$\Pi_{11}(p^2) = \Pi_{11}(0) + p^2\Pi'_{11}(0)$
Neutral SU(2) current	$\Pi_{33}(p^2) = \Pi_{33}(0) + p^2\Pi'_{33}(0)$
EM U(1) current	$\Pi_{QQ}(p^2) = p^2\Pi'_{QQ}(0)$
Neutral SU(2) - EM U(1) mixing	$\Pi_{3Q}(p^2) = p^2\Pi'_{3Q}(0)$

- Some linear combinations among Πs can be renormalized by redefinitions of Lagrangian parameters (g, g_Z, v, v_Z).

EW corrections

- In order to make the fermion scattering amps. UV-finite, the following combinations are required to be cutoff-independent.

$$S \sim \quad \Pi'_{33}(0) - \Pi'_{3Q}(0)$$

$$U \sim \quad \Pi'_{11}(0) - \Pi'_{33}(0)$$

$$\Pi_{33}(0) + \frac{v_Z^2}{4}$$

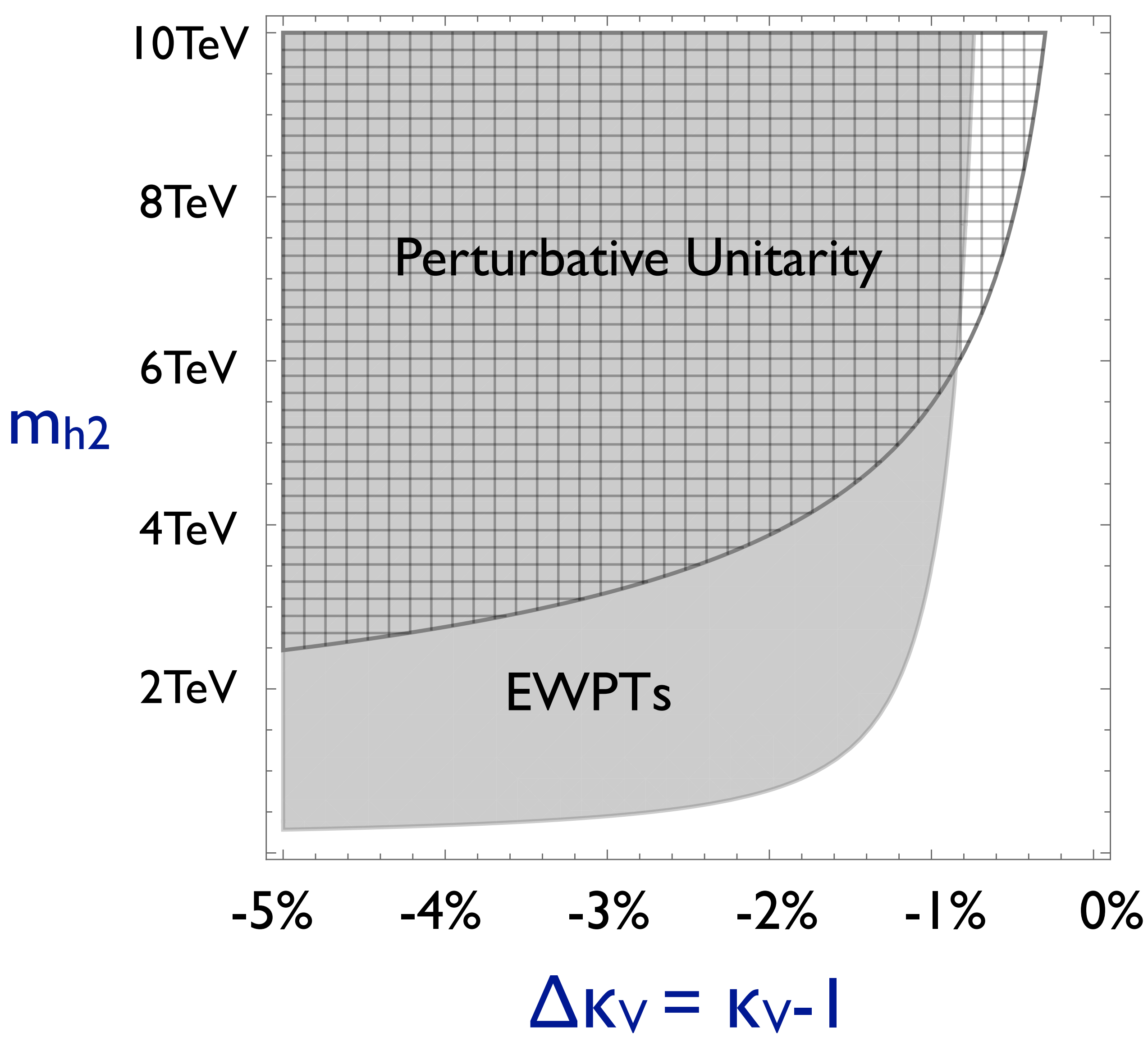
$$\Pi_{11}(0) + \frac{v^2}{4}$$

Kennedy-Lynn (1989)
Peskin-Takeuchi (1992)

- In the model with $v^2 \not\propto v_Z^2$, we can always tune the values of Π_{11} and Π_{33}. (e.g. Georgi-Machacek Model)

Constraints on heavy Higgses

SM + (h_2, h_3, ...)



- Only from the unitarity sum rules, we derive upper mass bound on extra scalars as a functions of the deviation of hVV coupling ($\Delta\kappa_V$).
- If $|\Delta\kappa_V| > 0.008$, the constraints from EWPTs are stronger than perturbative unitarity (PU) limit.
- Precise measurements of κ_V is important for investigating BSM.