Gauge dependence of tadpole and mass renormalization for a seesaw extended 2HDM

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- We extend general 2HDM with one additional neutral Weyl spinor N that has a Majorana mass term:

$$-\frac{1}{2}M\left(NN+h.c.\right),$$

we have four neutrinos in total $\nu_i^F=\left(\nu_e,\nu_\mu,\nu_\tau,N\right)_i$.
- The seesaw mechanism yields 2 non zero masses at tree level.
- The third mass is generated at loop level.
- Hence 4 mass eigenstates for neutrinos:

$$\nu_i^F\to\nu_i\,,i=1,2,3,4\,.$$

- Loop induced mass for ν_2 is gauge independent.
- We will talk about the gauge dependence of renormalization of the seesaw induced masses of ν_4 and ν_3.

Some remarks on notation

- Bare parameters and fields: p_0 and ϕ_0.
- Renormalized: p and ϕ.
- All counterterms are derived with:

$$p_0 = (1+\delta_p)\,p\,,\ \phi_{0i} = \sum_j \left(1+\delta_\phi\right)_{ij}\phi_j$$

- We start from the Higgs basis, where:

$$H_{01} = \begin{pmatrix} \chi^+_{W0} \\ \frac{1}{\sqrt{2}}\left(v_0 + h_0 + i\chi_{Z0}\right) \end{pmatrix},\ H_{02} = \begin{pmatrix} H^+_0 \\ \frac{1}{\sqrt{2}}\left(H_0 + iA_0\right) \end{pmatrix}.$$

- We work in a CP symmetric potential.
  - A_0 doesn't mix.

Yukawa sector

- The Yukawa interactions for neutrinos together with the Majorana mass term in the Higgs basis:

$$\mathscr{L}_{\text{Yuk}} = -Y_i^1 \nu_{0i}^F N_0 H_{01} - Y_i^2 \nu_{0i}^F N_0 H_{02} - \frac{1}{2} M_0 N_0 N_0 + h.c.$$

- By unitary transformation on the fields, we parametrize the first Yukawa couplings by a single value y_0:

$$\begin{aligned} \mathscr{L}_{\text{Yuk1}} &= -y_0 \, \nu'_{03} N_0 H_{01} - \frac{1}{2} M_0 N_0 N_0 + h.c. \\ &= -\left(\frac{1}{\sqrt{2}} y_0 v_0 \right) \nu'_{03} N_0 - \frac{1}{2} M_0 N_0 N_0 + \ldots \end{aligned}$$

- Seesaw transformation is done between ν'_{03} and N_0.
- Bare masses of new eigenstates m_{03} and m_{04} are:

$$M_0 = m_{04} - m_{03} \,, \quad y_0^2 v_0^2 = 2 m_{03} m_{04}$$

- We use complex mass scheme for mass renormalization to get gauge invariant renormalized masses.
- Bare parameter relations:

$$M_0 = m_{04} - m_{03}\,, \quad y_0^2 v_0^2 = 2 m_{03} m_{04}$$

- Renormalized parameter relations:

$$M = m_4 - m_3\,, \quad y^2 v^2 = 2 m_3 m_4$$

- We relate renormalization constants:

$$\delta_{m3} + \delta_{m4} = 2(\delta_v + \delta_y)\,, \quad m_4 \delta_{m4} - m_3 \delta_{m3} = (m_4 - m_3)\,\delta_M$$

- δ_v is gauge dependent and is expressed with scalar tadpole functions.
- The usual tadpole scheme explicitly includes gauge dependence in mass renormalization constants!

FJ (Fleischer and Jegerlehner) scheme

- v is dynamically defined (not independent) parameter that stands for minimizing the potential at every loop order.
- Tree level: $v = v_0$.
- At loop level v_0 does not give the correct minimum, but v does.
- We define all our bare mass parameters with "proper" VEV v:

$$y_0^2 v^2 = 2m'_{03} m'_{04}$$

- Mass renormalization constants are then:

$$\delta'_{m3} + \delta'_{m4} = 2\delta_y$$

- They do not include VEV shift anymore.

Applied FJ scheme

- Bare masses are now gauge independent and differs from the masses got without the FJ scheme by Δ:

$$m_{0i} = m'_{0i} + \Delta_0\,,\ \Delta_0 = 2\frac{m_{03}m_{04}}{m_{04}+m_{03}}\delta_v\,,\ \text{for } i = 3,4$$

- Δ is the same for both masses.
  - Because we have single value y to couple to H_1.
- This construction is equivalent to attaching tadpoles to the propagator.
- The Seesaw Majorana mass term is unaffected and is gauge independent:

$$M_0 = m_{04} - m_{03} = m'_{04} + \Delta_0 - m'_{03} - \Delta_0 = M'_0$$

- We implemented the model in FeynArts with the help of SARAH package and analytically checked the gauge invariance using FormCalc.
- For more on this, see:
  V. Dūdėnas and T. Gajdosik. Phys. Rev. D 98, 035034, 2018.

Backup slides

Renormalizing masses

- We use definitions:

$$\langle \phi_1 ... \phi_n \rangle_{1PI}^{[loop]} = \frac{\delta^n \hat{\Gamma}^{[loop]}}{\delta \phi_1 .. \delta \phi_n}\Big|_{\phi_i = 0} \equiv \hat{\Gamma}^{[loop]}_{\phi_1 ... \phi_n} \equiv \Gamma^{[loop]}_{\phi_1 ... \phi_n} + \delta \Gamma^{[loop]}_{\phi_1 ... \phi_n} ,$$

$$\hat{\Gamma}_{\nu_i \nu_i} = m_i \hat{\Sigma}_{\nu_i \nu_i} , \; \hat{\Gamma}_{\nu_i^\dagger \nu_i^\dagger} = m_i \hat{\Sigma}_{\nu_i^\dagger \nu_i^\dagger} ,$$
$$\hat{\Gamma}_{\nu_i \nu_j^\dagger} = p\sigma \hat{\Sigma}_{\nu_i \nu_j^\dagger} , \; \hat{\Gamma}_{\nu_i^\dagger \nu_j} = p\bar{\sigma} \hat{\Sigma}_{\nu_i^\dagger \nu_j} . \quad (1)$$

- The complex mass scheme fixed mass counterterm in the usual tadpole scheme is:

$$\delta_{mi} = \frac{1}{2} \left(\Sigma_{\nu_i \nu_i} + \Sigma_{\nu_i^\dagger \nu_i^\dagger} + \Sigma_{\nu_i \nu_i^\dagger} + \Sigma_{\nu_i^\dagger \nu_i} \right) \Big|_{p^2 = m_i^2} . \quad (2)$$

With the FJ scheme:

$$\delta'_{mi} = \frac{1}{2} \left(\Sigma_{\nu_i \nu_i} + \Sigma_{\nu_i^\dagger \nu_i^\dagger} + \Sigma_{\nu_i \nu_i^\dagger} + \Sigma_{\nu_i^\dagger \nu_i} \right) \Big|_{p^2 = m_i^2} - \frac{\Delta}{m_i} , \quad (3)$$

Which is equivalent to adding tadpoles.

Tadpoles

- Counterterms:

$$\delta \hat{T}_h^{[1]} = \frac{1}{2}\lambda_1 v^3 \left(2\delta_{m11} - \delta_{\lambda 1} - 2\delta_v\right) ,$$
$$\delta \hat{T}_H^{[1]} = \frac{1}{2}\lambda_6 v^3 \left(2\delta_{m12} - \delta_{\lambda 6} - 2\delta_v\right) ,$$
$$\delta \hat{T}_A^{[1]} = 0 . \qquad (4)$$

- As one of the $\delta_{m11}, \delta_{\lambda 1}, \delta_v$ is redundant, we pick $2\delta_{m11} - \delta_{\lambda 1} = 0$ so:

$$\delta_v = \frac{1}{\lambda_1 v^3} T_h^{[1]} , \qquad (5)$$

$$\left(\delta_{m12} - \frac{1}{2}\delta_{\lambda 6}\right) = \frac{1}{v^3}\left(\frac{1}{\lambda_1} T_h^{[1]} - \frac{1}{\lambda_6} T_H^{[1]}\right) . \qquad (6)$$

- Higgs basis to mass eigenstate basis:

$$T_h = c_\alpha T_{h_{(m)}} - s_\alpha T_{H_{(m)}} , \; T_H = c_\alpha T_{H_{(m)}} + s_\alpha T_{h_{(m)}} ,$$
$$T_A = T_{A_{(m)}} , \qquad (7)$$