Phenomenology of a 3HDM+complex singlet model with a gauged U(1)-family symmetry

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Workshop on Multi-Higgs Models – IST-Lisboa

Outline

1. High scale motivation
2. Q-GUT breaking steps
3. The Q-GUT inspired 3HDSM
4. First results
5. Conclusions and outlook

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Overview of the Quartification model (Q-GUT)

Extension the SUSY trinification model (Georgi, Glashow and De Rujula 1984) with a local family $\mathrm{SU}(3)_{\mathrm{F}}$ symmetry (recap Roman's talk this morning)

$$[\mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{SU}(3)_{\mathrm{L}} \times \mathrm{SU}(3)_{\mathrm{R}}] \rtimes \mathbb{Z}_3 \times \mathrm{SU}(3)_{\mathrm{F}}$$

- Use the minimal field content:

$$(1,\mathbf{3},\overline{\mathbf{3}},\mathbf{3}) = (L^i)^l{}_r = \begin{pmatrix} H_{\mathrm{u}}^0 & H_{\mathrm{d}}^- & e_{\mathrm{L}} \\ H_{\mathrm{u}}^+ & H_{\mathrm{d}}^0 & \nu_{\mathrm{L}} \\ e_{\mathrm{R}} & \nu_{\mathrm{R}} & \phi \end{pmatrix}^i , \quad \begin{array}{l} (\mathbf{3},\overline{\mathbf{3}},1,\mathbf{3}) = (Q_{\mathrm{L}}^i)^x{}_l = \begin{pmatrix} u_{\mathrm{L}}^x & d_{\mathrm{L}}^x & D_{\mathrm{L}}^x \end{pmatrix}^i , \\ (\overline{\mathbf{3}},1,\mathbf{3},\mathbf{3}) = (Q_{\mathrm{R}}^i)^r{}_x = \begin{pmatrix} u_{\mathrm{R}x}^c & d_{\mathrm{R}x}^c & D_{\mathrm{R}x}^c \end{pmatrix}^{\top\, i} . \end{array}$$

- Higgs and leptons unified in L due to SUSY
- $W_1 = \lambda_{27}\varepsilon_{ijk}\,(Q_{\mathrm{L}}^i)^x{}_l\,(Q_{\mathrm{R}}^j)^r{}_x\,(L^k)^l{}_r$
  - > **One** family of quarks and **all** leptons massless at tree-level → **radiatively generated**,
  - > Exact Yukawa unification for all three families.

Mass scale introduced via adjoint superfields inspired by the embedding of $[\mathrm{SU}(3)]^4$ into E_8.

$$(\mathbf{8},1,1,1) = \boldsymbol{\Delta}_C^a\,, \quad (1,\mathbf{8},1,1) = \boldsymbol{\Delta}_L^a\,, \quad (1,1,\mathbf{8},1) = \boldsymbol{\Delta}_R^a\,, \quad (1,1,1,\mathbf{8}) = \boldsymbol{\Delta}_F^a$$

$$W_2 = \sum_{A=\mathrm{L,R,C}} \left(\lambda_{\mathbf{78}} d_{abc} \Delta_A^a \Delta_A^b \Delta_A^c + \mu_{\mathbf{78}} \delta_{ab} \Delta_A^a \Delta_A^b\right) + \left(\lambda_{\mathbf{1}} d_{abc} \Delta_{\mathrm{F}}^a \Delta_{\mathrm{F}}^b \Delta_{\mathrm{F}}^c + \mu_{\mathbf{1}} \delta_{ab} \Delta_{\mathrm{F}}^a \Delta_{\mathrm{F}}^b\right)$$

- $\mu_{78} \sim \mu_1 \sim M_{\mathrm{GUT}}$
- No μ-like problem as Higgs sits in fundamental sector.

- Embedding of $[\mathrm{SU}(3)]^4$ into $\mathrm{E}_8 \Rightarrow$ **No gauge anomalies.**

$$[\mathrm{SU}(3)]^3 \times \mathrm{SU}(3)_\mathrm{F} \subset \mathrm{E}_6 \times \mathrm{SU}(3)_\mathrm{F} \subset \mathrm{E}_8$$

- We motivate our field content in the Katsuki et. al. $\mathbb{Z}_3$-orbifold for the breaking $\mathrm{E}_8 \to \mathrm{E}_6 \times \mathrm{SU}(3)_\mathrm{F}$ (Prog.Theor.Phys. 82 (1989) 171)
  - > massless physical $(\mathbf{27}, \mathbf{3})$,
  - > massive adjoint $(\mathbf{78}, 1) \oplus (1, \mathbf{8})$,
  - > $(\overline{\mathbf{27}}, \overline{\mathbf{3}})$ removed by orbifolding.
  - > No dangerous $(\mathbf{27}, \mathbf{3}) \cdot (\overline{\mathbf{27}}, \overline{\mathbf{3}})$ terms and low energy limit is chiral.

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Q-GUT:
$[\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(3)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{R}] \times \mathrm{SU}(3)_\mathrm{F}$

Region - I $\left\langle \widetilde{\Delta}^8_{\mathrm{L,R,F}} \right\rangle \sim M_\mathrm{GUT}$

SUSY-LRF:
$\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \times \mathrm{SU}(2)_\mathrm{F} \times \mathrm{U}(1)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{R} \times \mathrm{U}(1)_\mathrm{F}$

Region - II $\left\langle \widetilde{\phi}^3 \right\rangle \sim M^{(1)}_\mathrm{soft} \ll M_\mathrm{GUT}$ $\qquad T_{\mathrm{L+R}} = T^8_\mathrm{L} + T^8_\mathrm{R}\,, \quad T_\mathrm{S} = T^8_\mathrm{L} - T^8_\mathrm{R} - 2T_\mathrm{F}$

non-SUSY-LRF:
$\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \times \mathrm{SU}(2)_\mathrm{F} \times \mathrm{U}(1)_{\mathrm{L+R}} \times \mathrm{U}(1)_\mathrm{S}$

Region - III $\left\langle \widetilde{\phi}^2 \right\rangle \sim M^{(2)}_\mathrm{soft} \lesssim M^{(1)}_\mathrm{soft}$ $\qquad T_\mathrm{V} = T^3_\mathrm{F} - \frac{1}{2\sqrt{2}} T_\mathrm{S}$

non-SUSY-LR:
$\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \times \mathrm{U}(1)_{\mathrm{L+R}} \times \mathrm{U}(1)_\mathrm{V}$

Region - IV $\langle \widetilde{\nu}^1_\mathrm{R} \rangle \sim M^{(3)}_\mathrm{soft} \lesssim M^{(2)}_\mathrm{soft}$ $\qquad T_\mathrm{Y} = \sqrt{3} T^3_\mathrm{R} + T_{\mathrm{L+R}}\,, \quad T_\mathrm{T} = T_\mathrm{V} - \frac{3}{2} T^3_\mathrm{R}$

3HDM EFTs:
$\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y} \times \mathrm{U}(1)_\mathrm{T}$

Region - V $\langle S \rangle \sim m_{z'} \lesssim M^{(3)}_\mathrm{soft}$

3HDM EFTs:
$\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$

- $\mathrm{U}(1)_{\mathrm{T}}$ provides family non-universal Z' boson.
- Mass scale in the fundamental sector emerges solely from soft-SUSY breaking interactions.
- SUSY stabilizes the $M_{\mathrm{GUT}} \gg M_{\mathrm{soft}}^{(1)}$ hierarchy and "disappears" well above TeV-scale.

Why 3HDM EFTs?

The simplest scenario that automatically provides CKM mixing with Cabibbo form.

$$V_{\mathrm{CKM}} \sim \begin{pmatrix} \cos\theta_c & \sin\theta_c & 0 \\ -\sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{pmatrix} + \text{Perturbations}\,.$$

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$$\langle L^1 \rangle \sim \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \omega & 0 \end{pmatrix} \qquad \langle L^2 \rangle \sim \frac{1}{\sqrt{2}} \begin{pmatrix} v_2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & f \end{pmatrix} \qquad \langle L^3 \rangle \sim \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & v_3 \\ 0 & 0 & p \end{pmatrix}\,.$$

$$\tan\theta_C = \frac{v_1}{v_2} \qquad M_{\mathrm{EW}} \sim v_{1,2,3} \ll \omega \lesssim f \lesssim p \lll M_{\mathrm{GUT}}, \qquad (p, f, \omega) \sim M^{(1,2,3)}_{\mathrm{soft}}$$

Consider the simplest realistic low-scale limit

- All chiral (SM-like) quarks and leptons,
- Three Higgs doublets
- One $\mathrm{U}(1)_{\mathrm{T}}$-charged complex singlet $\rightarrow$ new Z' after $\mathrm{U}(1)_{\mathrm{T}}$-breaking

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The Q-GUT inspired 3HDSM

3HDSM	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_T$
q_{Li}	$\mathbf{3}$	$\mathbf{2}$	$1/3$	$(3,-1,-1)$
u_{Ri}	$\mathbf{3}$	$\mathbf{1}$	$4/3$	$(0,4,4)$
d_{Ri}	$\mathbf{3}$	$\mathbf{1}$	$-2/3$	$(-6,2,-2)$
ℓ_{Li}	$\mathbf{1}$	$\mathbf{2}$	-1	$(3,-1,-1)$
e_{Ri}	$\mathbf{1}$	$\mathbf{1}$	-2	$(-6,-2,-2)$
H_i	$\mathbf{1}$	$\mathbf{2}$	1	$(5,1,-3)$
S	$\mathbf{1}$	$\mathbf{1}$	0	-4

- **New scalars and $U(1)_T$ offer rich phenomenology/cosmology:**

  EW-precision, flavour, collider, baryogenesis, primordial GW.

$$H_i = \frac{e^{i\gamma_i}}{\sqrt{2}} \begin{pmatrix} \varphi_i^+ \\ v_i + \rho_i + i\eta_i \end{pmatrix} \qquad S = \frac{e^{i\gamma_S}}{\sqrt{2}} \left(v_S + s + i\sigma \right)$$

$$V = \sum_i m_i^2 \left|H_i\right|^2 + \sum_{j \geqslant i} \lambda_{ij} \left|H_i\right|^2 \left|H_j\right|^2 + \sum_{i>j} \lambda'_{ij} \left|H_i^\dagger H_j\right|^2 + m_S^2 \left|S\right|^2 + \lambda_S \left|S\right|^4 + \sum_i \lambda'_{Si} \left|H_i\right|^2 \left|S\right|^2$$
$$+ \delta \left(H_1^\dagger H_2\right) \left(H_3^\dagger H_2\right) + \delta_S \left(H_3^\dagger H_1\right) S^2 + S \left[A_1 \left(H_2^\dagger H_1\right) + A_2 \left(H_2^\dagger H_3\right)\right] + \text{c.c.}\,,$$

- Our first analysis will consider no CP-phases and purely real parameters
- 6 neutral and 4 charged physical scalars
  - H_{125}, H_{2-6}, $h^{\pm}$, $H^{\pm}$
- Study the mass spectrum and its viability

$$q_{\mathrm{L}i} = \begin{pmatrix} u_{\mathrm{L}i} \\ d_{\mathrm{L}i} \end{pmatrix} \qquad \ell_{\mathrm{L}i} = \begin{pmatrix} \nu_{\mathrm{L}i} \\ e_{\mathrm{L}i} \end{pmatrix}$$

$$-\mathcal{L}_{\mathrm{Y}} = \Gamma^a_{ij}\overline{q_{\mathrm{L}i}}H_a d_{\mathrm{R}j} + \Delta^a_{ij}\overline{q_{\mathrm{L}i}}\widetilde{H}_a u_{\mathrm{R}j} + \Pi^a_{ij}\overline{\ell_{\mathrm{L}i}}H_a e_{\mathrm{R}j}$$

$$M_d = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & v_2\Gamma^2_{12} & v_1\Gamma^1_{13} \\ v_1\Gamma^1_{21} & v_3\Gamma^3_{22} & v_2\Gamma^2_{23} \\ v_1\Gamma^1_{31} & v_3\Gamma^3_{32} & v_2\Gamma^2_{33} \end{pmatrix} \qquad M_u = \frac{1}{\sqrt{2}}\begin{pmatrix} v_3\Delta^3_{11} & v_2\Delta^2_{12} & v_2\Delta^2_{13} \\ v_2\Delta^2_{21} & v_1\Delta^1_{22} & v_1\Delta^1_{23} \\ v_2\Delta^2_{31} & v_1\Delta^1_{32} & v_1\Delta^1_{33} \end{pmatrix}$$

$$M_e = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & v_1\Pi^1_{12} & v_1\Pi^1_{13} \\ v_1\Pi^1_{21} & v_2\Pi^2_{22} & v_2\Pi^2_{23} \\ v_1\Pi^1_{31} & v_2\Pi^2_{32} & v_2\Pi^2_{33} \end{pmatrix}$$

- Recover SM-like fermion masses and CKM structure
- FCNC studies (work in progress not fully addressed in this talk)
- **Sizes of the Yukawa couplings motivated by Q-GUT**

Quantum effects - a schematic view:

$$\frac{m_t^2}{m_c^2} = \frac{{y'}_u^{(1)\,2} {v_1}^2 + {y'}_u^{(2)\,2} {v_2}^2}{{y_u^{(1)}}^2 {v_1}^2 + {y_u^{(2)}}^2 {v_2}^2}, \qquad \frac{m_b^2}{m_s^2} = \frac{{y'}_d^{(2)\,2}}{a_{11}^2 {y_d^{(2)}}^2}, \qquad m_{u,d}^2 = \text{loop-suppressed.}$$

Full CKM mixing radiatively generated

$$\tan\theta_C = \frac{y_u^{(1)} v_1}{y_u^{(2)} v_2},$$

$$y_u^{(2)} \sim \frac{1}{16\pi^2} \frac{p^3 f A_{\rm soft} M_{\rm gau}}{\Lambda^6} y' y \lambda' \lambda$$

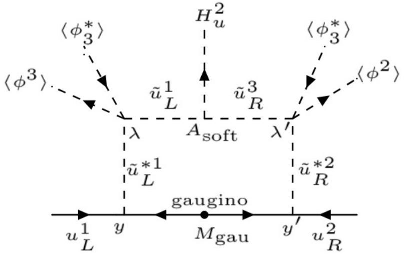

Figure: Example of radiative generation of Yukawa interactions: $y_u^{(2)} H_u^2 u_L^1 u_R^2$

- Charged lepton masses all loop-generated

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1. Implement model in **SARAH** and find a point that provides SM fermion masses and CKM mixing

$$M_d = \begin{pmatrix} 0 & -0.12 & 4.1 \\ 0.0041 & 0. & -0.15 \\ 0.11 & 0. & 0.005 \end{pmatrix} \qquad M_u = \begin{pmatrix} 0. & 0.0001 & 170 \\ -0.00005 & 0.006 & 33.8 \\ -0.58 & -1.0 & -0.002 \end{pmatrix}$$

$$V_{\text{CKM}} \simeq \begin{pmatrix} 0.972 & 0.232 & 0.002 \\ 0.232 & 0.971 & 0.043 \\ 0.008 & 0.042 & 0.999 \end{pmatrix} \quad \begin{array}{l} \boldsymbol{m_t = 173\ \text{GeV},\ m_c = 1.2\ \text{MeV},\ m_u = 2.9\ \text{MeV}} \\ \boldsymbol{m_b = 4.1\ \text{GeV},\ m_s = 105.4\ \text{MeV},\ m_d = 4.06\ \text{MeV}} \end{array}$$

$$M_e = \begin{pmatrix} 0 & 0.008 & 0.004 \\ 0.006 & 0.105 & 0. \\ 0.189 & 0. & 1.777 \end{pmatrix} \quad \boldsymbol{m_\tau = 1.777\ \text{GeV},\ m_\mu = 105\ \text{MeV},\ m_e = 510\ \text{keV}}$$

2. $Y^a_{ij} = M_{ij}\dfrac{\sqrt{2}}{v_a} \longrightarrow$ Different VEVs imply different Yukawas (M_{ij} is fixed)
3. Study the Higgs sector with this input

Scan on the parameter space

Quartic couplings:

$$\lambda_{ij},\ \lambda'_{ij},\ \lambda_S,\ \lambda'_{Si},\ \delta,\ \delta_S \to \pm\left[10^{-6}, 6\right]$$

Cubic couplings:

$$A_1, A_2 \to \pm\,[0,6]\ \text{TeV}$$

VEVs:

$$\tan\beta_{12}, \tan\beta_{23} \to [0,20] \qquad \begin{cases} v_1 = v_{\text{SM}}\cos\beta_{12} \\ v_2 = v_{\text{SM}}\sin\beta_{12}\cos\beta_{23} \\ v_3 = v_{\text{SM}}\sin\beta_{12}\sin\beta_{23} \end{cases} \qquad \begin{cases} v_s \to [1,8]\ \text{TeV} \\ g_{\text{T}} = 0.9,\, g_{\text{mix}} = 0 \Rightarrow M_{Z'} \gtrsim 630\text{GeV} \end{cases}$$

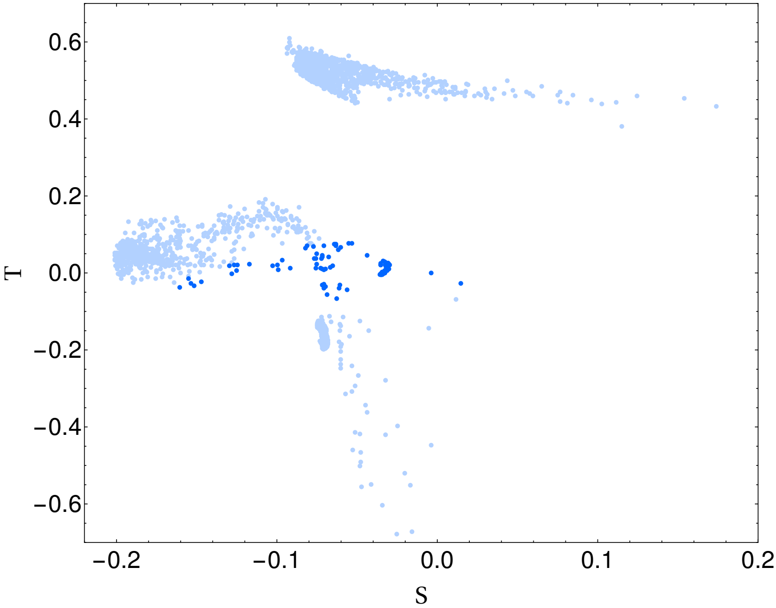
- Used **`SARAH`** to generate a sample with one 125 GeV scalar (challenging... large parameter space and only 5000 input points),
- Feed data to **`SPheno`** to compute observables,
- Used **`HiggsBounds`** and **`HiggsSignals`** for collider constraints.

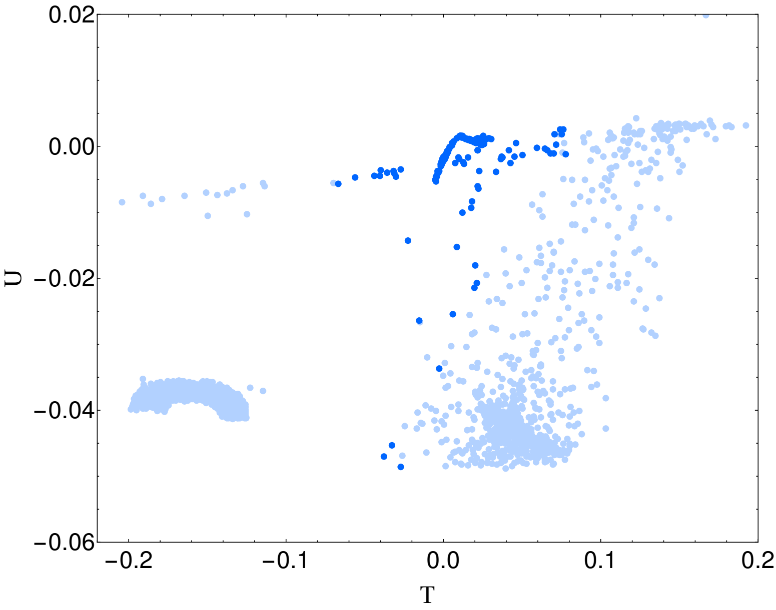
Oblique parameters

$$\Delta S^{(0)} = 0.05 \pm 0.10 \qquad \Delta T^{(0)} = 0.08 \pm 0.12 \qquad \Delta U^{(0)} = 0.02 \pm 0.10$$

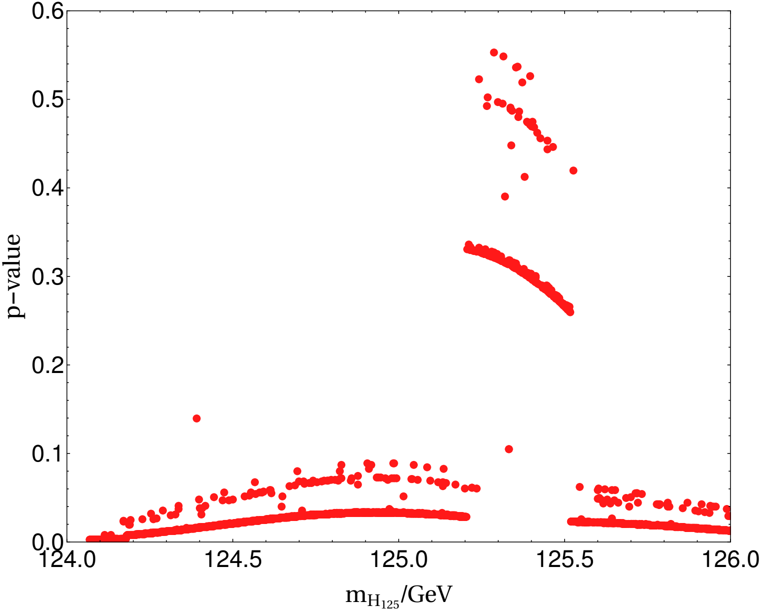
$$\rho_{ij} = \begin{pmatrix} 1 & 0.91 & -0.61 \\ 0.91 & 1 & -0.82 \\ -0.61 & -0.82 & 1 \end{pmatrix}$$

$$\Delta\chi^2(99\% \text{ C.L.}) = \sum_{ij} \left(\Delta O_i - \Delta O_i^{(0)}\right) [\sigma_i \rho_{ij} \sigma_j]^{-1} \left(\Delta O_i - \Delta O_i^{(0)}\right) < 11.345$$

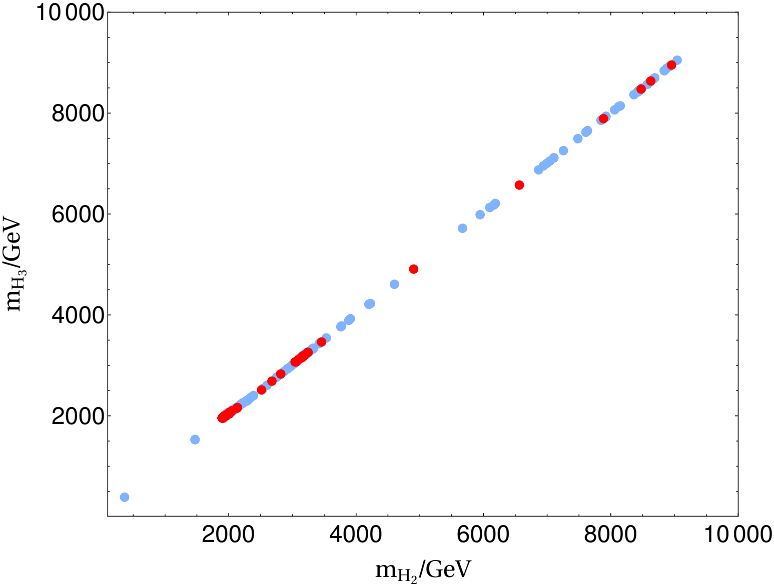

0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
T
−0.2
−0.1
0.0
0.1
0.2
S


0.02
0.00
−0.02
−0.04
−0.06
U
−0.2
−0.1
0.0
0.1
0.2
T

HS and HB (points allowed by STU)



Heavy scalars: Red points for p-value > 0.2



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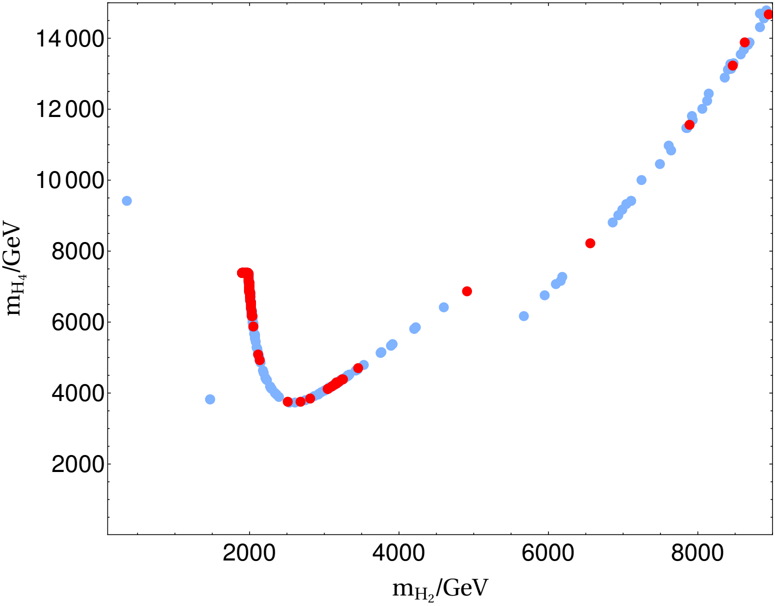
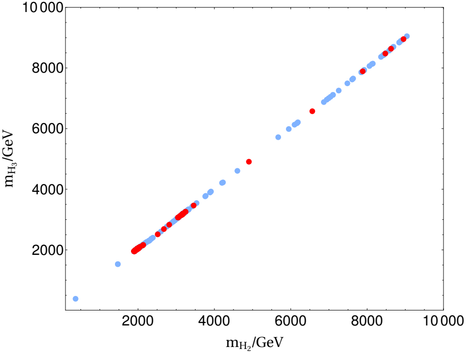
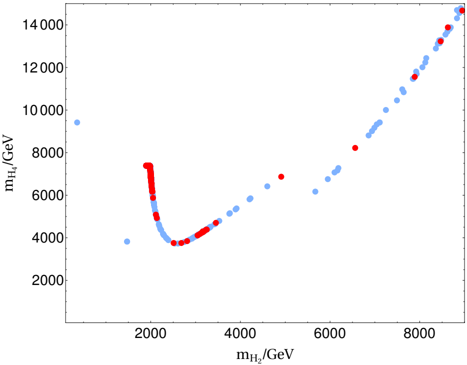
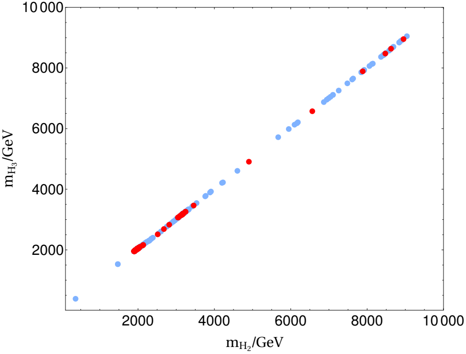
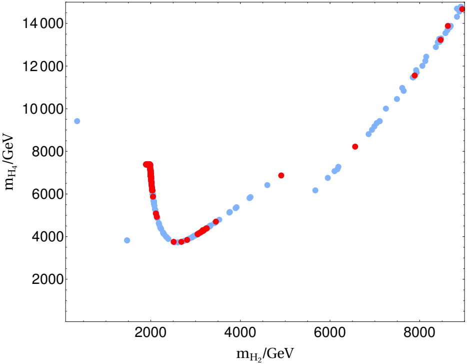




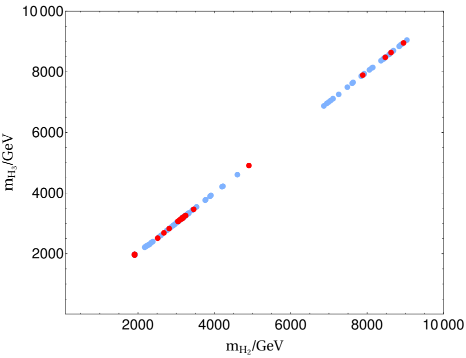


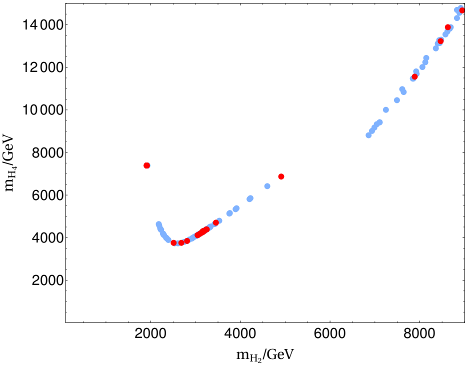
- Electroweak precision observables
- Tree-level unitarity
- Collider constraints
- **No FCNC studies yet... work ongoing**





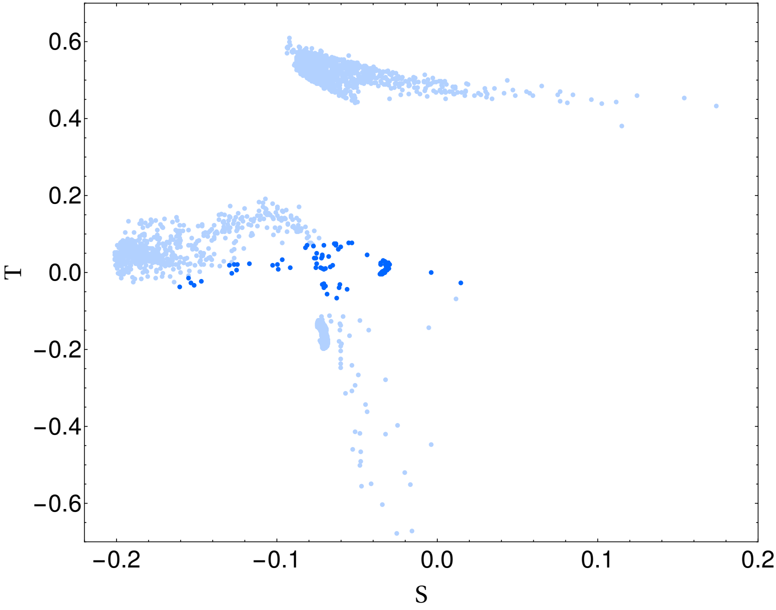
- Electroweak precision observables
- Tree-level unitarity
- Collider constraints
- **No FCNC studies yet... work ongoing**
- **But let just try $b \to s\gamma$**



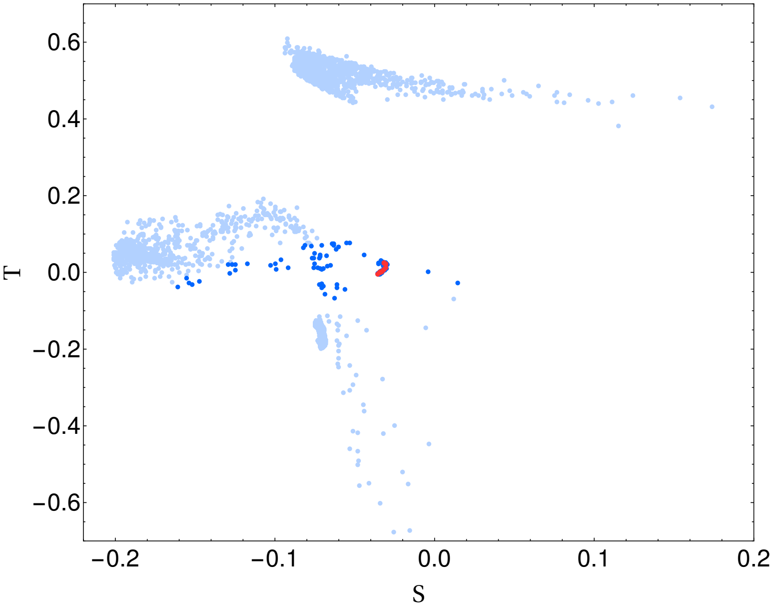


- Electroweak precision observables
- Tree-level unitarity
- Collider constraints
- **No FCNC studies yet... work ongoing**
- $b \to s\gamma$, **brings further constrains and eliminates light scalars**

Surviving points



Surviving points



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Conclusions and outlook

- Constructed a 3HDSM framework based on the quartification group
- Performed a first scan on the parameter space
- Studied EW-precision, tree-level unitarity and collider constraints
- About 1% of the points survived with $m_{H_2} \gtrsim 2\ \mathrm{TeV}$

Next steps

- Full FCNC studies to validate the model
  - $b \to s\gamma$ already excluded light scalars
  - Heavy scalars may be safe but what about Yukawa sector and Z'?

Further steps

- Physically viable light scalars? (alignment limit etc...)
- Z' phenomenology
- CP violation

Backup slides

Quantum effects - a schematic view:

$$\frac{m_{\rm t}^2}{m_{\rm c}^2} = \frac{{y'}_{\rm u}^{(1)\,2} {v_1}^2 + {y'}_{\rm u}^{(2)\,2} {v_2}^2}{{y_{\rm u}^{(1)}}^2 {v_1}^2 + {y_{\rm u}^{(2)}}^2 {v_2}^2}, \qquad \frac{m_{\rm b}^2}{m_{\rm s}^2} = \frac{{y'}_{\rm d}^{(2)\,2}}{a_{11}^2 {y_{\rm d}^{(2)}}^2}, \qquad m_{\rm u,d}^2 = \text{loop-suppressed.}$$

Full CKM mixing radiatively generated

$$\tan\theta_{\rm C} = \frac{y_{\rm u}^{(1)} v_1}{y_{\rm u}^{(2)} v_2},$$

$$y_u^{(2)} \sim \frac{1}{16\pi^2}\frac{p^3 f A_{\rm soft} M_{\rm gau}}{\Lambda^6} y' y \lambda' \lambda$$

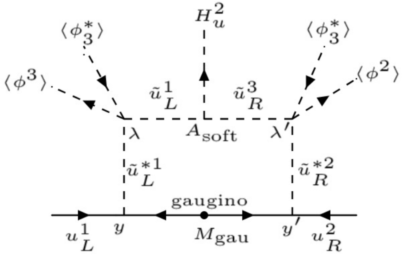


Figure: Example of radiative generation of Yukawa interactions: $y_u^{(2)} H_u^2 u_{\rm L}^1 u_{\rm R}^2$

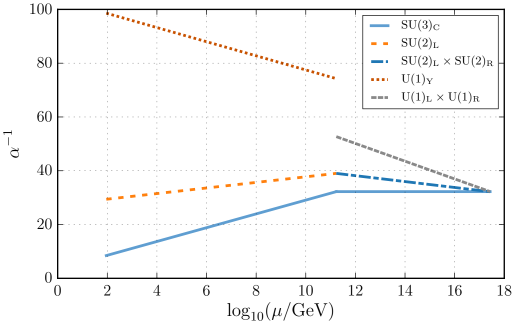
- It is possible to get one generation of VLQ close to TeV scale $m_{\rm D} \propto y_{\rm D}\omega$
- Charged lepton masses all loop-generated: 3 chiral, $m_{\rm chi} \propto v_{1,2,3}$, and 3 vector-like, $m_{\rm vec} \propto \omega, f, p$.
- Neutrino masses contain both loop and See-Saw effects.

Estimation of $\omega = \langle \tilde{\nu}^1_R \rangle$, $f = \langle \tilde{\phi}^2 \rangle$, $p = \langle \tilde{\phi}^3 \rangle$ scales

Use 1-loop running and three-level matching to **estimate** scale hierarchies.

- Consider for simplicity $\omega \simeq f \simeq p = M_{\rm soft}$
- Unification condition: $\alpha^{-1}_{\tilde{g}_{\rm L,R}}(M_{\rm GUT}) = \alpha^{-1}_{g_{\rm L,R,C}}(M_{\rm GUT}) = \alpha_{\rm U}^{-1}$

$$\alpha_i^{-1}(\mu_2) = \alpha_i^{-1}(\mu_1) - \frac{b_i}{2\pi} \ln\left(\frac{\mu_2}{\mu_1}\right)$$



- Threshold conditions:

$$\alpha^{-1}_{\tilde{g}_{\rm L+R}}(p) = \alpha^{-1}_{\tilde{g}_{\rm L}}(p) + \alpha^{-1}_{\tilde{g}_{\rm R}}(p)$$
$$\alpha^{-1}_{\tilde{g}_{\rm Y}}(\omega) = \alpha^{-1}_{g_{\rm R}}(\omega) + \frac{1}{3}\alpha^{-1}_{\tilde{g}_{\rm L+R}}(\omega)$$
$$\alpha^{-1}_{\tilde{g}_{\rm Y}}(m_z) = \cos^2\theta_{\rm W}\alpha^{-1}_{\rm EM}$$
$$\alpha^{-1}_{\tilde{g}_{L}}(m_z) = \sin^2\theta_{\rm W}\alpha^{-1}_{\rm EM},$$

- Solution:

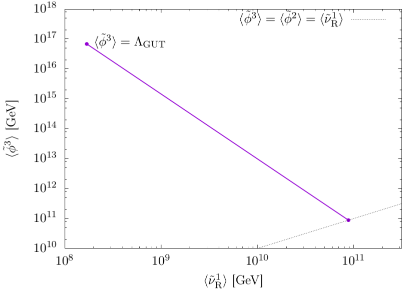
$$M_{\rm soft} \sim 8.8\cdot 10^{10}\ \mathrm{GeV},$$
$$M_{\rm GUT} \sim 4.9\cdot 10^{17}\ \mathrm{GeV},$$
$$\alpha_{\rm U}^{-1} \sim 31.5,$$

Effect of decoupling scales

- **Solve RGEs for ω**

$$\omega = m_Z \exp\left\{20.69 - \frac{1}{19}\ln\left(\frac{p}{f}\right)\left[4b^{\text{II}}_{g_C} - 9b^{\text{II}}_{g_L} + 3b^{\text{II}}_{g_R} + b^{\text{II}}_{\tilde{g}_{L+R}}\right]\right.$$
$$\left. - \frac{1}{19}\ln\left(\frac{p}{f}\right)\left[4b^{\text{III}}_{g_C} - 9b^{\text{III}}_{g_L} + 3b^{\text{III}}_{g_R} + b^{\text{III}}_{\tilde{g}_{L+R}}\right]\right\}.$$

- To minimize ω need to maximise $b^{\text{II,III}}_{g_C}$, $b^{\text{II,III}}_{g_R}$, $b^{\text{II,III}}_{\tilde{g}_{L+R}}$ and minimise $b^{\text{II,III}}_{g_L}$.
  - > Scalar content in regions II and III without $\left(\tilde{u}_L, \tilde{d}_L\right)^{1,2,3}$, $\left(\tilde{e}_L, \tilde{\nu}_L\right)^{1,2,3}$ and $H^3_{u,d}$
  - > $b^{\text{II,III}}_{g_C} = -\frac{13}{3}, \quad b^{\text{II,III}}_{g_L} = -\frac{2}{3}, \quad b^{\text{II,III}}_{g_R} = \frac{4}{3}, \quad b^{\text{II,III}}_{\tilde{g}_{L+R}} = \frac{40}{3}.$



- purple line: run to measured values
- ω decreases with increasing p
- **Optimal scenario when $p = f = \omega$**

How to lower the ω, f and p scales?

- Consider unification at E_6 level relaxing $\mathbb{Z}_3$ in the original $[\mathrm{SU}(3)]^3 \rtimes \mathbb{Z}_3$ unification.
- Consider correction to gauge-kinetic terms from dim-5 operators (Chakrabortty, Raychaudhuri 0812.2783 [hep-ph])

$$\mathcal{L}_{d5} = -\frac{\eta}{M_{\mathrm{Pl}}}\left[\frac{1}{4c} Tr\left(F_{\mu\nu}\Phi_{\boldsymbol{R}} F^{\mu\nu}\right)\right]$$

  - $\Phi_{\boldsymbol{R}}$ sits in $(\mathbf{78}\otimes\mathbf{78})_{\mathrm{sym}} = \mathbf{1}\oplus\mathbf{650}\oplus\mathbf{2430}$
  - $\mathbf{650}$ contains two $[\mathrm{SU}(3)]^3$ singlets which provide linearly independent contributions whose VEVs break $\mathrm{E}_6 \to [\mathrm{SU}(3)]^3$.
  - In general we consider $\Phi_{\boldsymbol{R}} = \kappa_1\Phi_{\mathbf{1}} + \kappa_{650}\Phi_{\mathbf{650}} + \kappa_{650'}\Phi_{\mathbf{650'}} + \kappa_{2430}\Phi_{\mathbf{2430}}$ with $\kappa_1^2 + \kappa_{650}^2 + \kappa_{650'}^2 + \kappa_{2430}^2 = 1$
- Unification condition modified

$$\alpha_{\mathrm{C}}^{-1}\left(1+\epsilon\delta_{\mathrm{C}}\right)^{-1} = \alpha_{\mathrm{L}}^{-1}\left(1+\epsilon\delta_{\mathrm{L}}\right)^{-1} = \alpha_{\mathrm{R}}^{-1}\left(1+\epsilon\delta_{\mathrm{R}}\right)^{-1} \qquad \epsilon \sim \frac{M_{\mathrm{E}_6}}{M_{\mathrm{Pl}}}$$

$$\delta_{\mathrm{C}} = -\frac{1}{\sqrt{2}}\kappa_{650} - \frac{1}{\sqrt{26}}\kappa_{2430}$$

$$\delta_{\mathrm{L,R}} = \frac{1}{2\sqrt{2}}\kappa_{650} \pm \frac{3}{2\sqrt{2}}\kappa_{650'} - \frac{1}{\sqrt{26}}\kappa_{2430}$$

A possible solution: (not unique)

$$\epsilon = 0.66 \qquad M_{3333} = 10^{17.5}\ \text{GeV} \qquad p = 10^{6}\ \text{GeV}$$

$$f = 10^{5.5}\ \text{GeV} \qquad \omega = 10^{5}\ \text{GeV} \qquad m_{z'} = 10^{3}\ \text{GeV}$$

$$\Phi_{\boldsymbol{R}} = -0.61\Phi_{\mathbf{650}} + 0.75\Phi_{\mathbf{650'}} + 0.27\Phi_{\mathbf{2430}}$$

$$\alpha_{\mathrm{C}}^{-1} = 1.15\alpha_{\mathrm{L}}^{-1} = 1.89\alpha_{\mathrm{R}}^{-1} \qquad \alpha_{\mathrm{L+R}}^{-1} = \alpha_{\mathrm{L}}'^{-1} + \alpha_{\mathrm{R}}'^{-1} \qquad \alpha_{\mathrm{S}}^{-1} = \alpha_{\mathrm{L}}'^{-1} + \alpha_{\mathrm{R}}'^{-1} + 4\alpha_{\mathrm{F}}'^{-1}$$

$$\alpha_{\mathrm{V}}^{-1} = \alpha_{\mathrm{F}}^{-1} + \frac{1}{12}\alpha_{\mathrm{S}}^{-1} \qquad \alpha_{\mathrm{T}}^{-1} = \frac{4}{9}\alpha_{\mathrm{V}}^{-1} + \alpha_{\mathrm{R}}^{-1} \qquad \alpha_{\mathrm{Y}}^{-1} = \frac{1}{3}\alpha_{\mathrm{L+R}}^{-1} + \alpha_{\mathrm{R}}^{-1}$$