High scale boundary conditions in multi-Higgs theories

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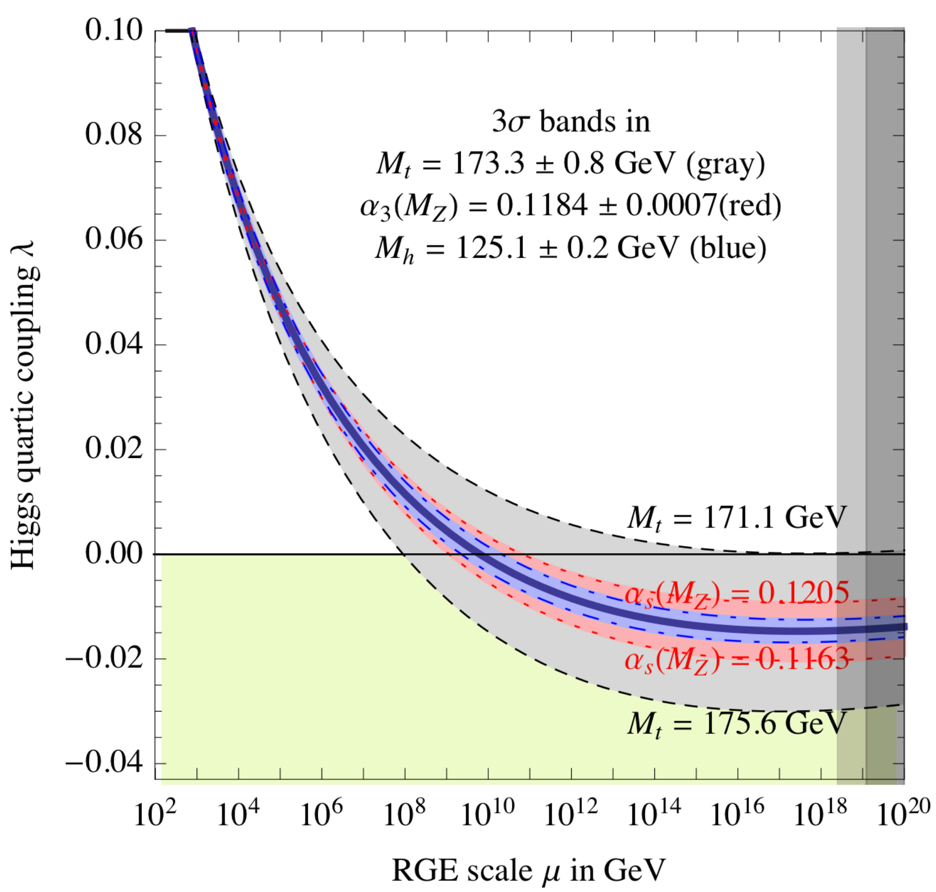
The Higgs potential at High energies

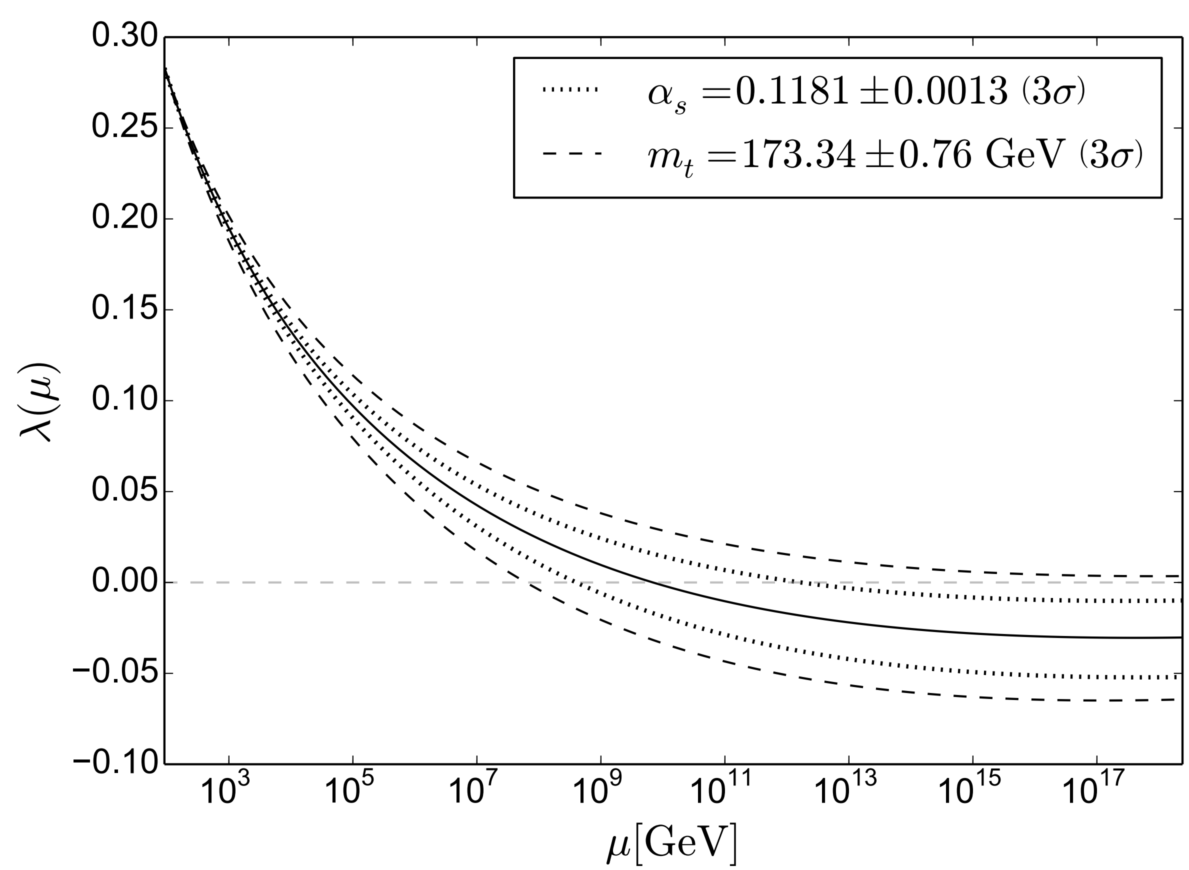
The accurate measurement of the Higgs boson mass $m_H = 125.18 \pm 0.16\,\mathrm{GeV}$ allows us to extrapolate the Higgs potential's behaviour to the Planck scale.

$$V = -\mu^2|\Phi|^2 + \lambda|\Phi|^4$$

With SM radiative corrections, the quartic coupling decreases at high energies and becomes almost zero.

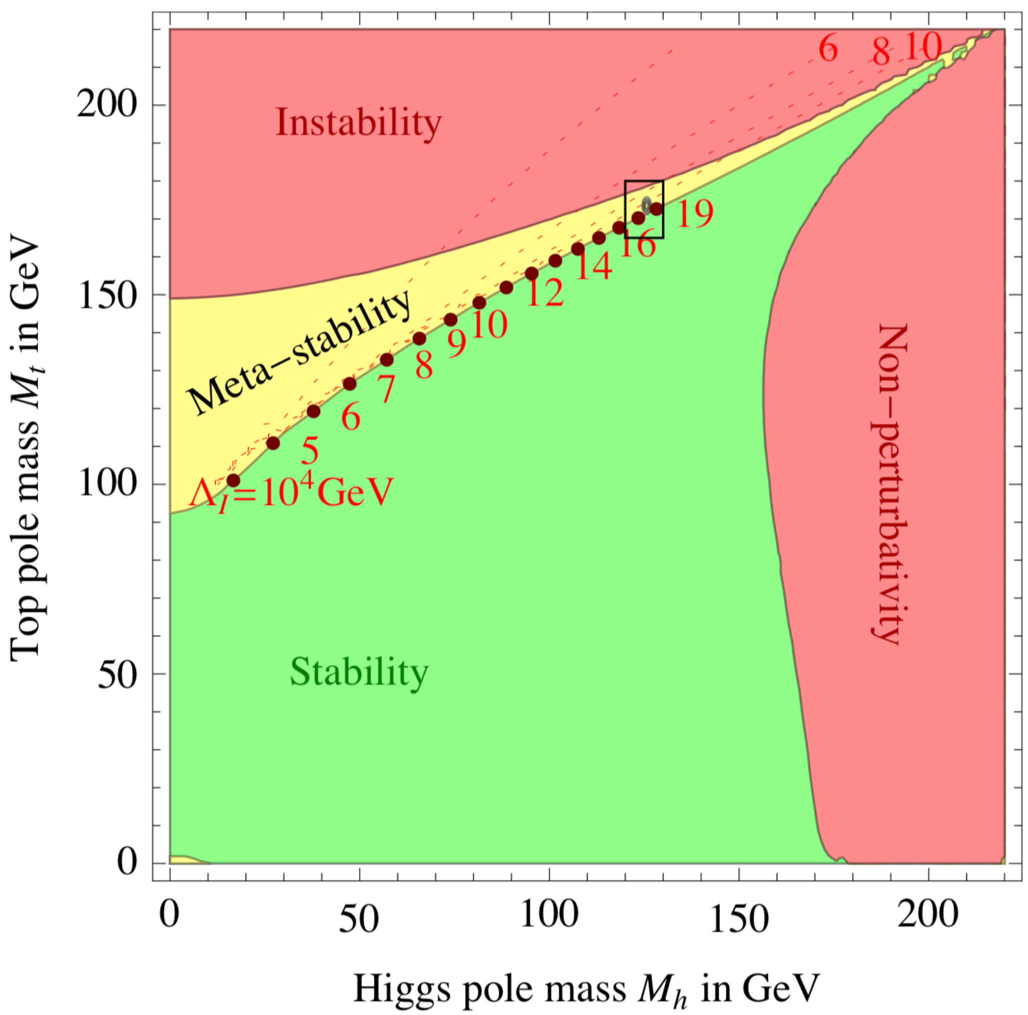


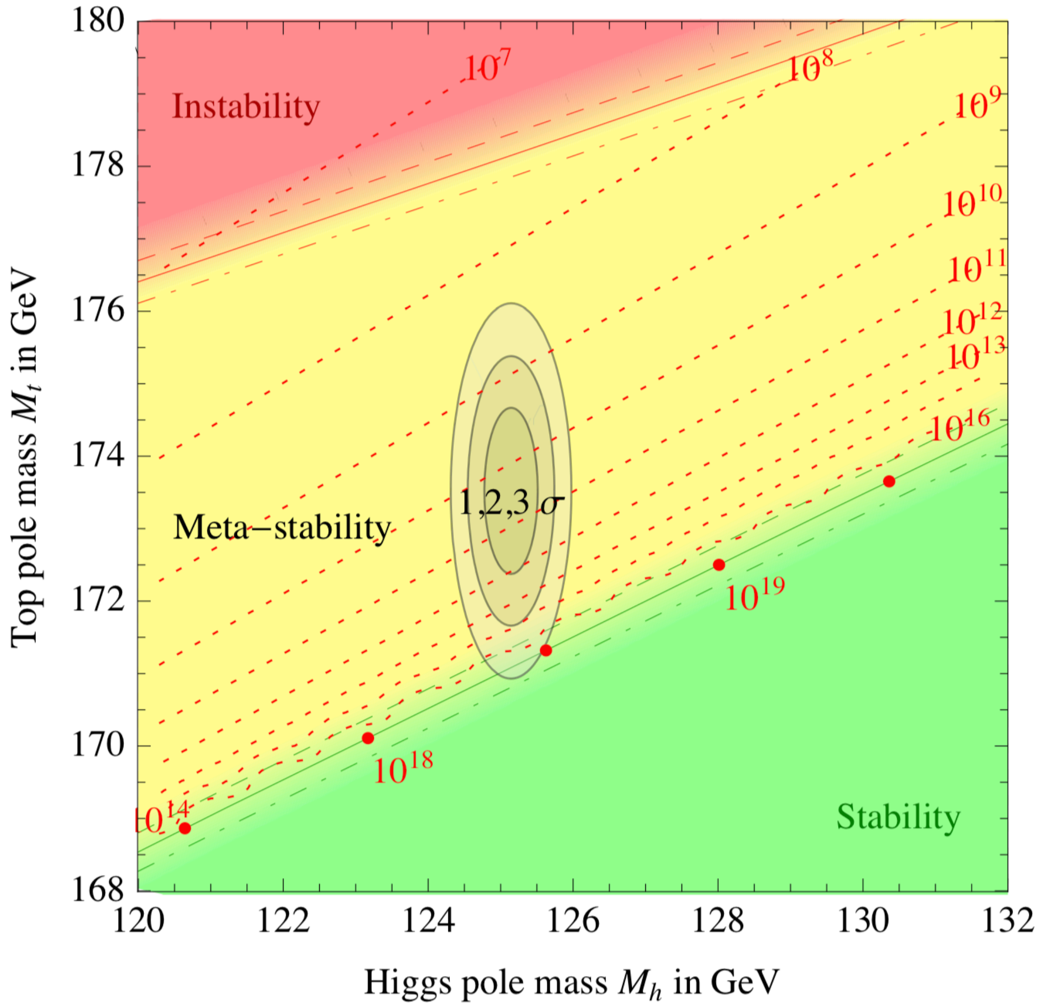
Degrassi et al (2013); Buzatto et al (2014)



DJM, McDowall, arXiv:1802.02391

Degrassi et al (2013)

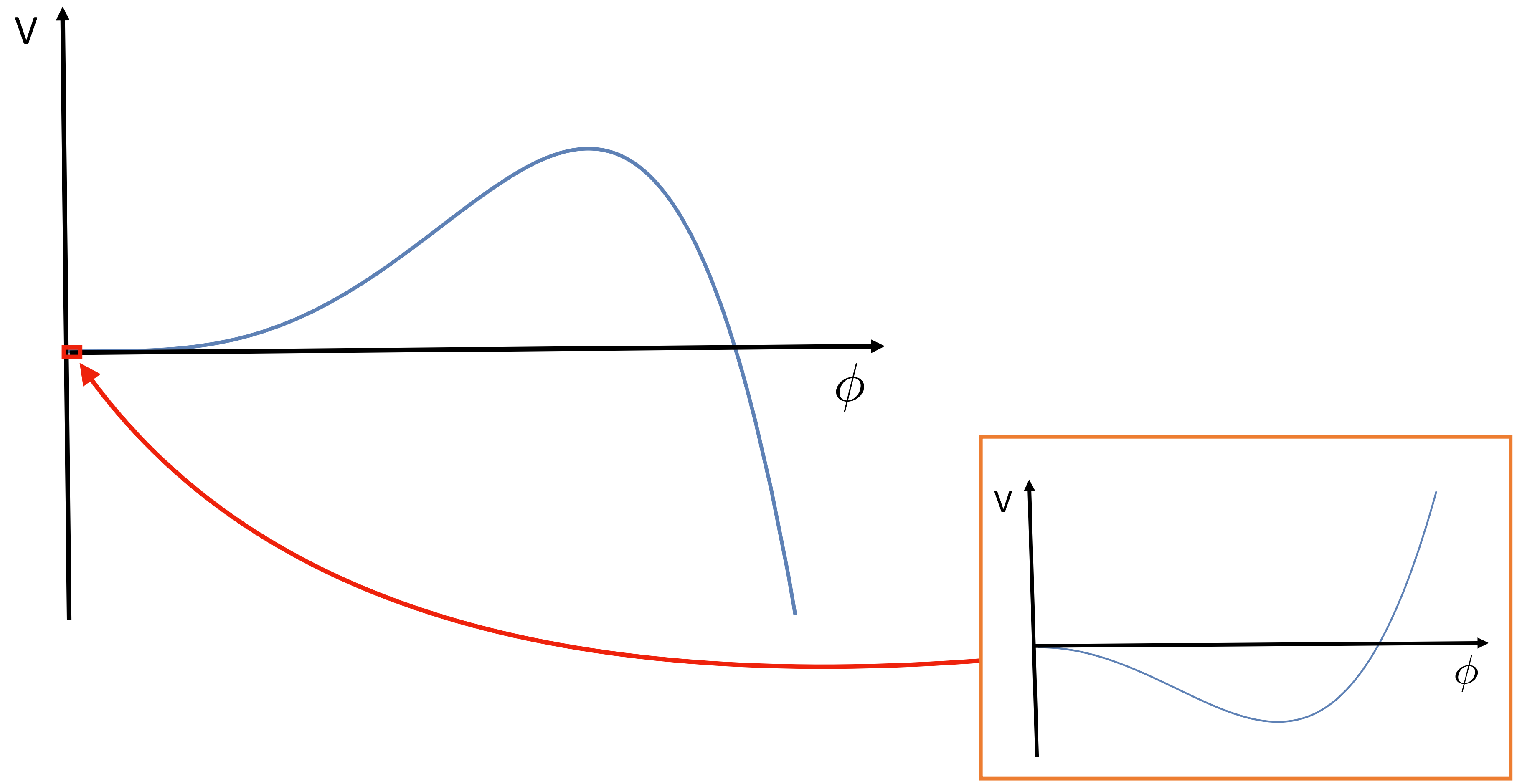




The Higgs mass appears to be just right to allow for metastability of the Higgs vacuum.

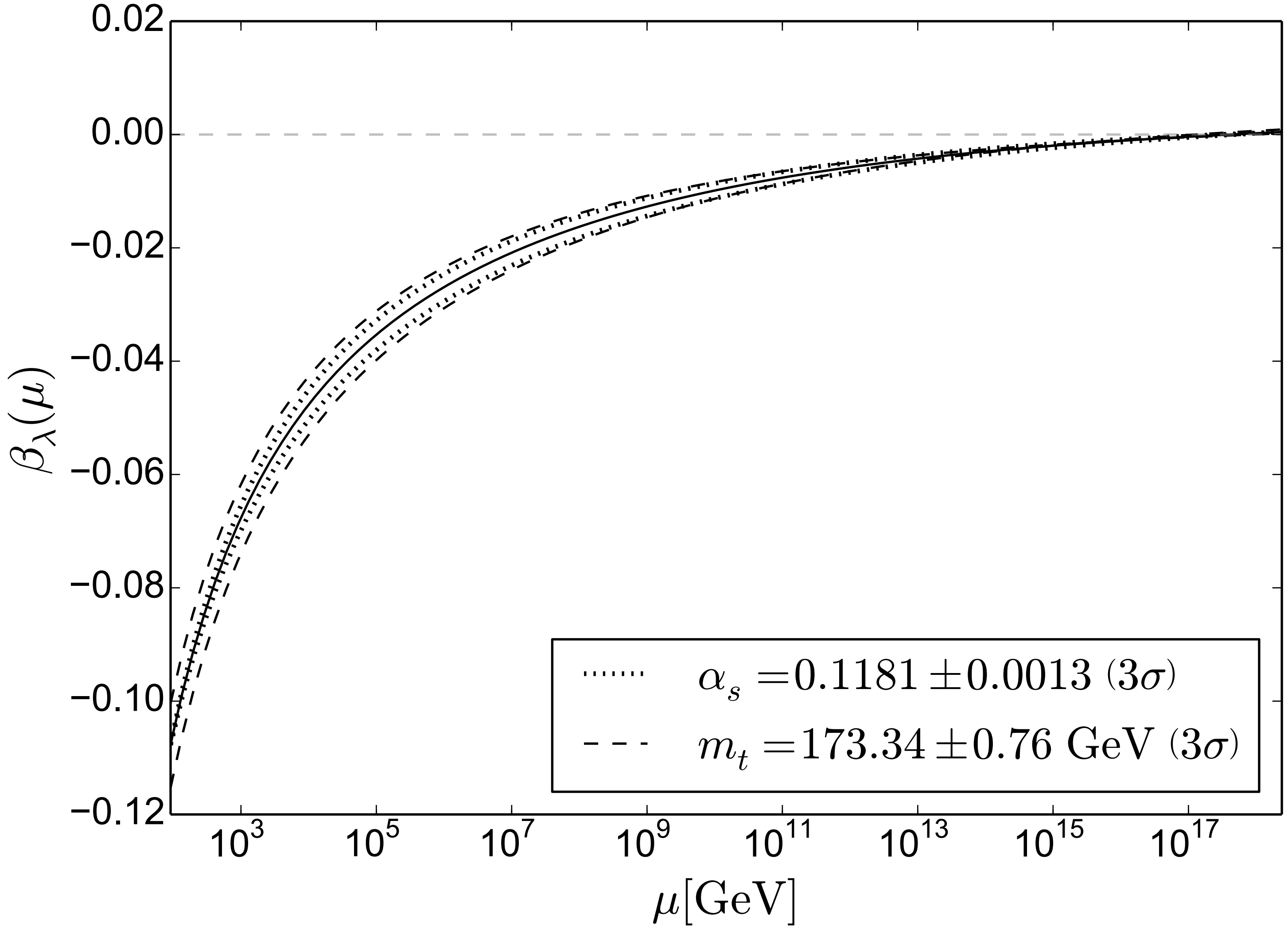
Is this a coincidence or a hint of something deeper?

The behaviour is due to logarithms like $-\Phi^4 \log\left(\frac{\Phi^2}{Q^2}\right)$ in the Coleman-Weinberg potential.



This is not to scale!

The slope of λ also appears to (almost!) vanish at the Planck scale $\beta_\lambda(M_{\rm Pl}) \approx 0$.



This is **not** a fixed point!

The Multiple Point Principle

Way back in 1995, Froggatt and Neilsen predicted the Higgs and top quark masses:

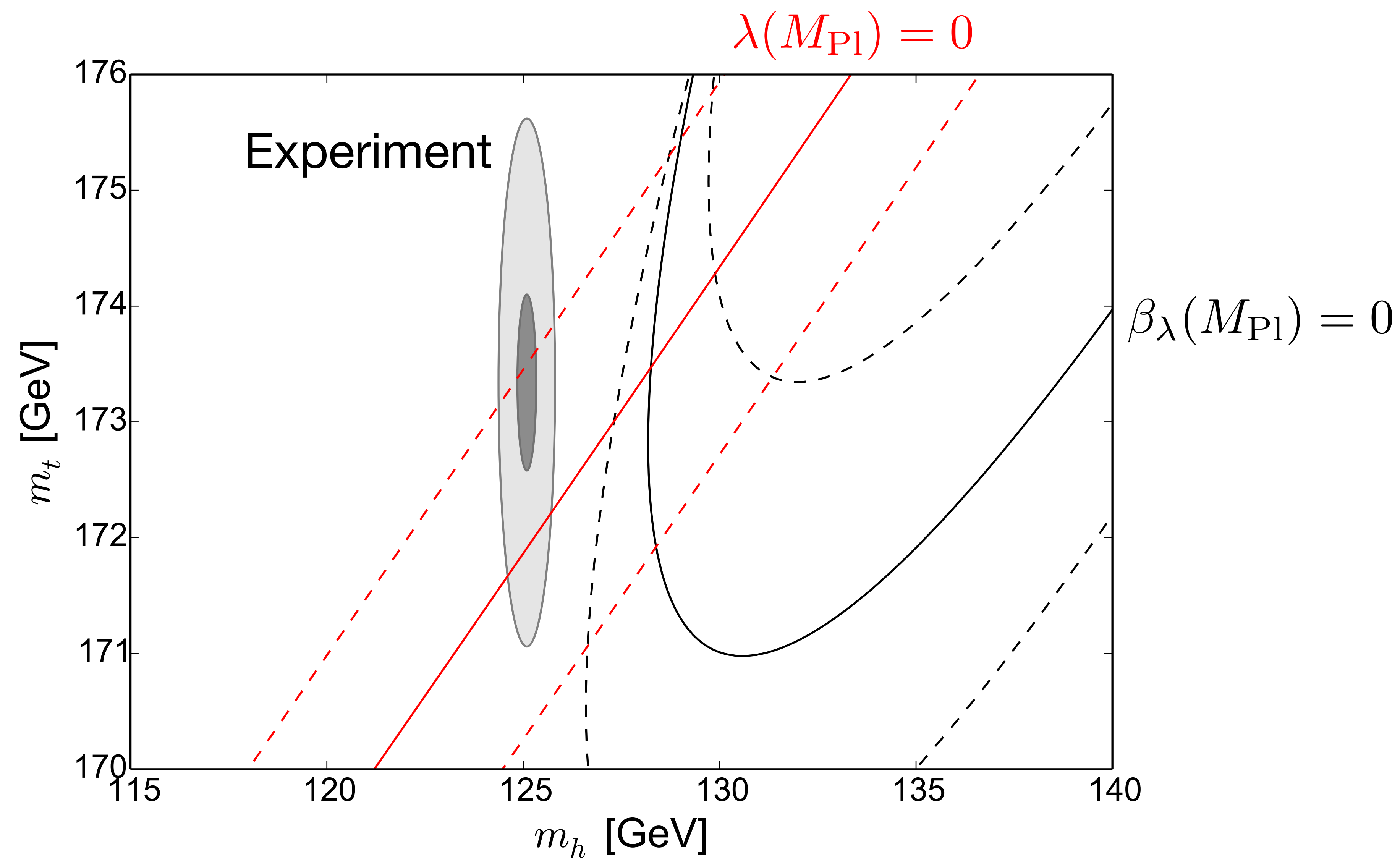
$$m_H = 135 \pm 9\,\mathrm{GeV} \qquad\qquad m_t = 175 \pm 5\,\mathrm{GeV}$$

To do this they applied the "**Multiple Point Principle**" which insists that there is another vacuum at the Planck scale **degenerate** with the Electroweak vacuum.

The condition for this in the SM is $\lambda = \beta_\lambda = 0$.

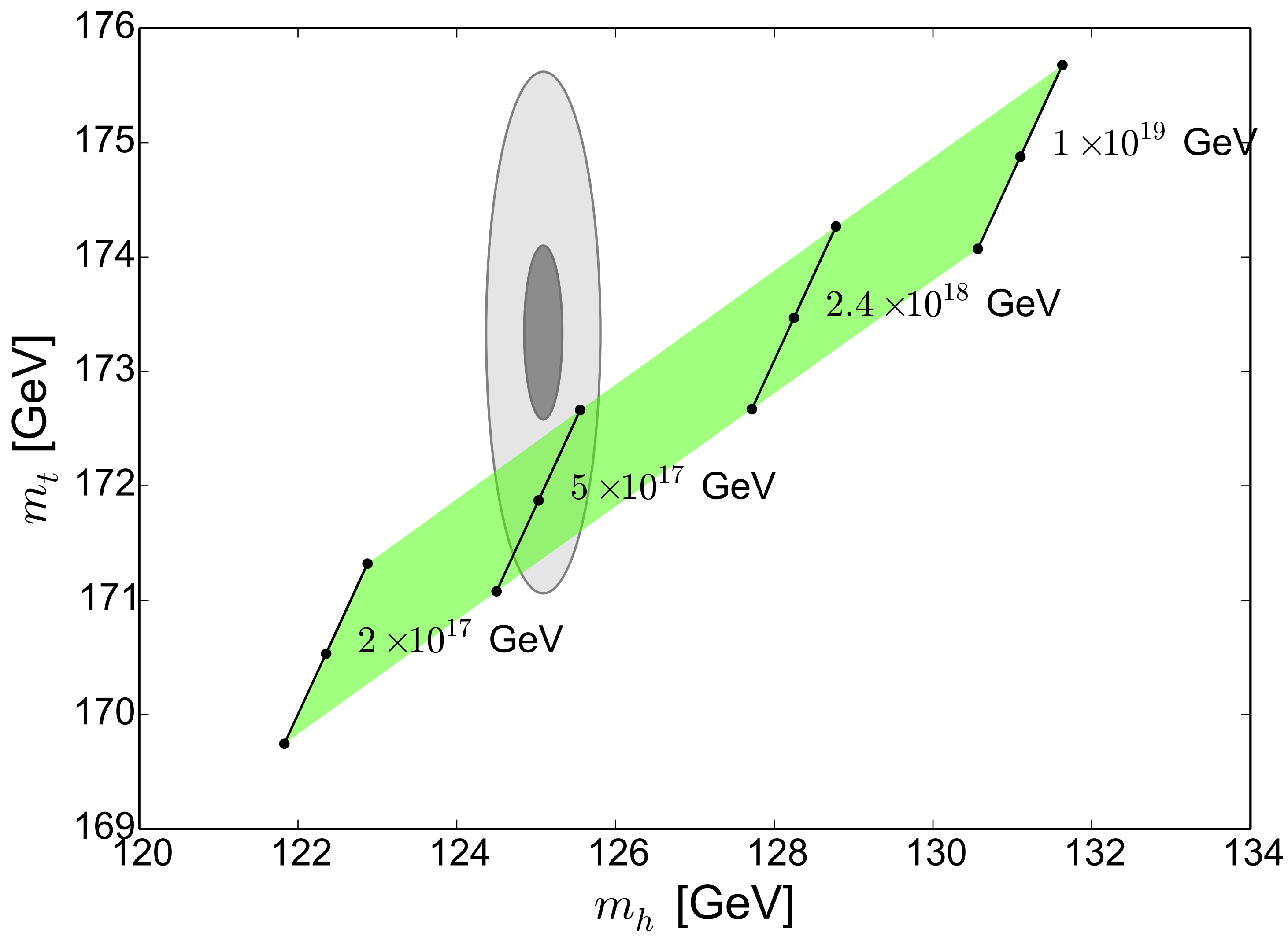
This result was only at one-loop and doesn't use the measured top mass (so the top mass above is a ***prediction***).

With more loops and recent top mass constraints this principle or asymptotic safety all predict a Higgs mass that is a bit too high.



The multiple point principle does not work in the SM but comes surprisingly close.

This is of course also rather sensitive to where the boundary conditions are enforced.



Setting $\lambda = \beta_\lambda = 0$ at 5×10^{17} GeV does rather well at predicting the Higgs mass.

Asymptotic Safety

In 2010, Wetterich and Shaposhnikov predicted a Higgs boson $m_H = 126\,\mathrm{GeV}$ with “a few GeV uncertainty”.

They did this by imposing “asymptotic safety” $\beta_\lambda(M_{\rm Pl}) = 0$ and saw that this also gave $\lambda \approx 0$ (as we have already seen).

$\lambda = \beta_\lambda = 0$ is **not** a fixed point. Gauge interactions will move one away from $\lambda = \beta_\lambda = 0$ at higher scales. Wetterich and Shaposhnikov argued that quantum gravity could tame the gauge couplings, reducing their beta functions to zero, so that $\beta_\lambda = 0$ remains stable.

However, unfortunately the corrections to the beta functions that they used appear to be gauge dependent and are in dispute.

See also Giudice, Isidori, Salvio, Strumia (2014) for models with Total Asymptotic Freedom using “softened gravity”.

An additional complex scalar

$$V = \frac{\mu^2}{2} H^\dagger H + \frac{\lambda}{4} \left(H^\dagger H\right)^2 + \frac{\delta}{2} \left(H^\dagger H\right) |\mathbb{S}|^2 + \frac{b_2}{2} |\mathbb{S}|^2 + \frac{d_2}{4} |\mathbb{S}|^4 + \left(\frac{b_1}{4}\mathbb{S}^2 + a_1 \mathbb{S} + c.c\right)$$

U(1) breaking terms

Choose to study two scenarios:

- "**Broken phase**" where both real and imaginary parts of $\mathbb{S}$ get Vevs.

  The EWSB conditions replace b_1 and b_2 so the remaining parameters are

  $$\lambda, \quad d_2, \quad \delta, \quad v_{s_1}, \quad v_{s_2}, \quad a_1.$$

- "**Dark Matter phase**" where only the real part of $\mathbb{S}$ gets a Vev.

  Now only one parameter is replaced and we are left with

  $$\lambda, \quad d_2, \quad \delta, \quad v_{s_1}, \quad b_-, \quad a_1.$$

  $$b_- = (b_2 - b_1)/2$$

Theoretical and Experimental Constraints

We use SARAH 4.9.3 [Staub 2014] for the beta functions at two loops, and FlexibleSUSY 1.6.1 [Athron et al 2015] to generate the spectrum.

To ensure stability of the potential during running we insist that

$$\begin{aligned} \lambda &\geq 0, \\ d_2 &\geq 0, \\ \delta + \sqrt{\lambda d_2} &\geq 0, \end{aligned}$$

and keep perturbativity via $\lambda, \delta, d_2 < \sqrt{4\pi}$.

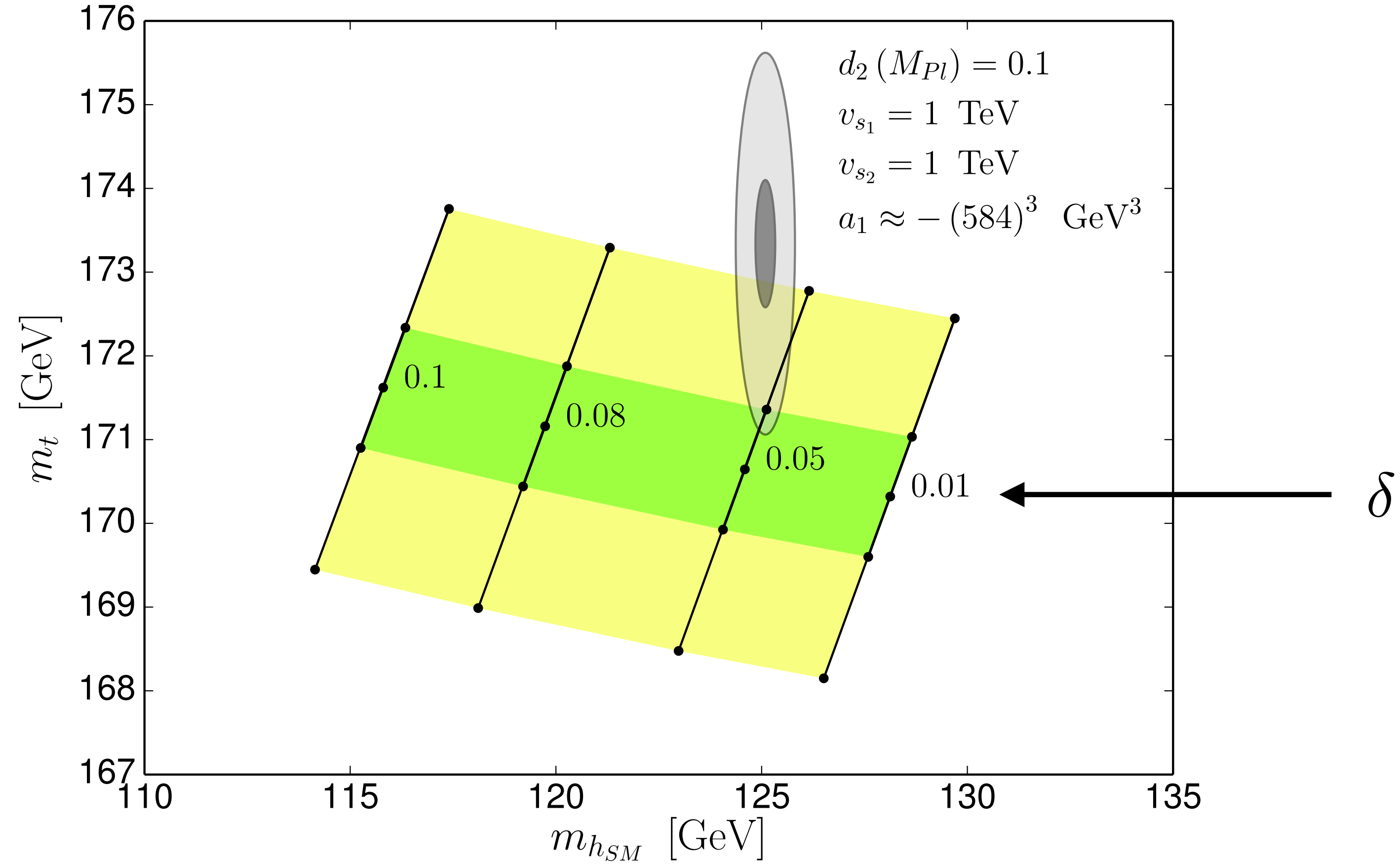
We also check that we have a global minimum using Vevacious [Camargo-Molina at al 2013]

We apply experimental constraints using HiggsBounds 4.3.1 [Bechtle at al 2014] and HiggsSignals 1.4.0 [Bechtle et al 2014] and use sHDecay [Costa et al 2016] to calculate branching ratios and widths.

[Note that the version of HiggsBounds we used doesn't include $h_i \to h_j h_j$.]

In the DM case, we also apply a constraint on the DM Relic density from Planck (2016) and also DM direct detection from LUX (2017), both calculated using micrOMEGAS [Bélanger 2015].

Broken Phase



These are 1σ and 3σ contours.

All three Higgs bosons mix.

It is relatively straightforward to keep $\lambda = \beta_\lambda = 0$ at the Planck scale while simultaneously getting the correct Higgs and top mass.

But this is **not** the Multiple Point Principle since the new quartics are not zero.

If we set $\delta = 0$ then our new scalar decouples and we return to the SM.

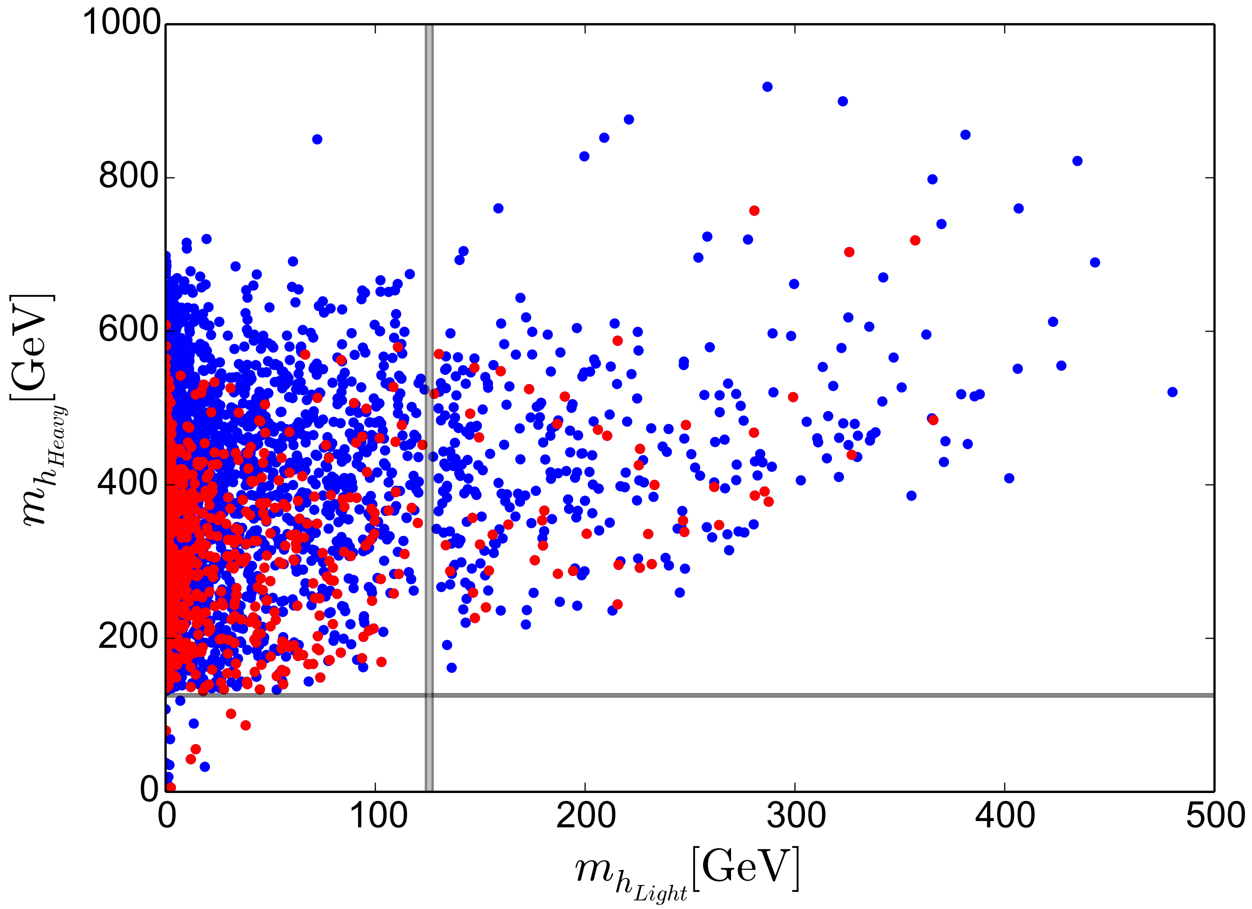
With $\delta \neq 0$ and $d_2 = 0$, d_2 runs negative right away and destabilises the vacuum.

What about Asymptotic Safety instead?

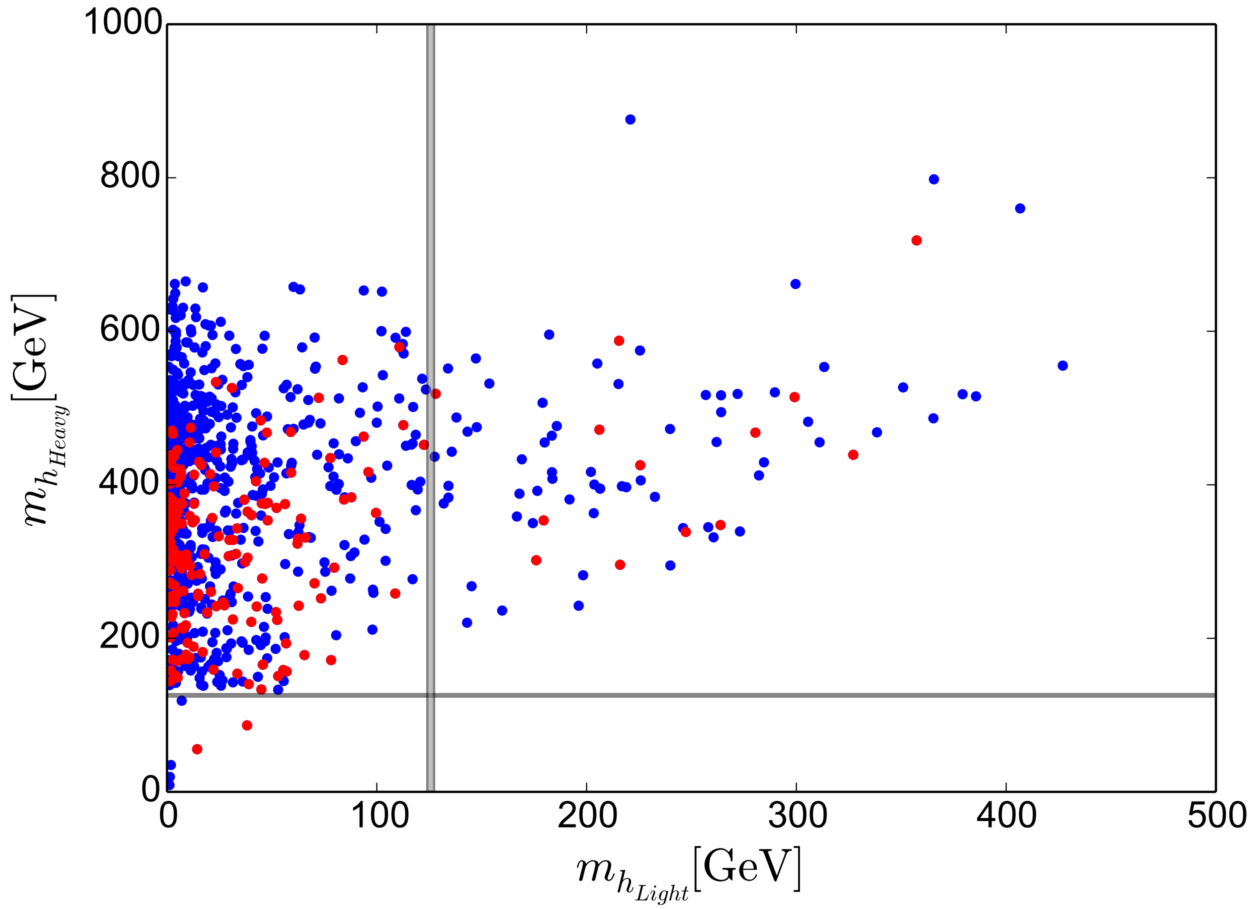
Setting $\beta_\delta = \beta_{d_2} = 0$ at the Planck scale, no points survive.

However, they can get very small. In the plots below blue points have $\beta_{\lambda,\delta,d_2} < 0.05$ while red points have $\beta_\lambda < 0.0005,\ \beta_\delta < 0.0025$ and $\beta_{d_2} < 0.01$

All three neutral scalars mix h_{Heavy} and h_{Light} are the extra mass eigenstates.



Without experimental constraints

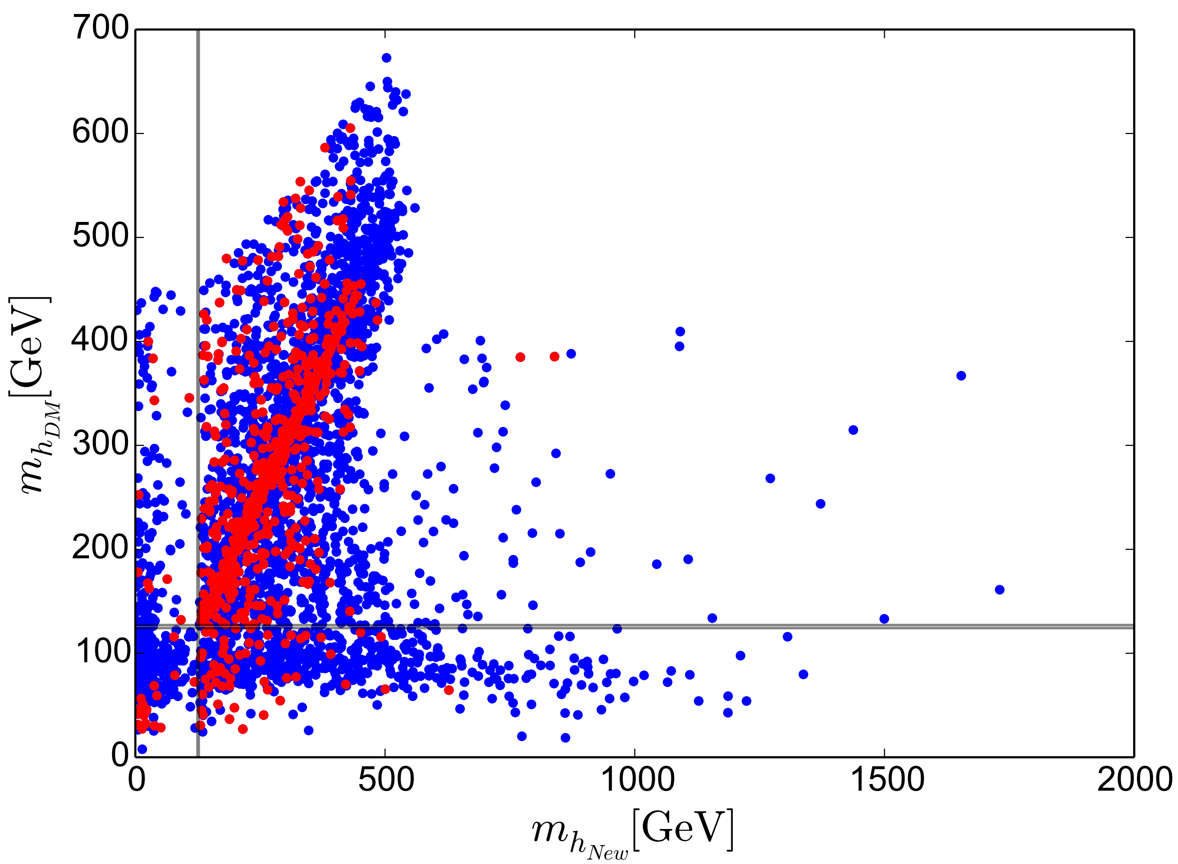


With experimental constraints

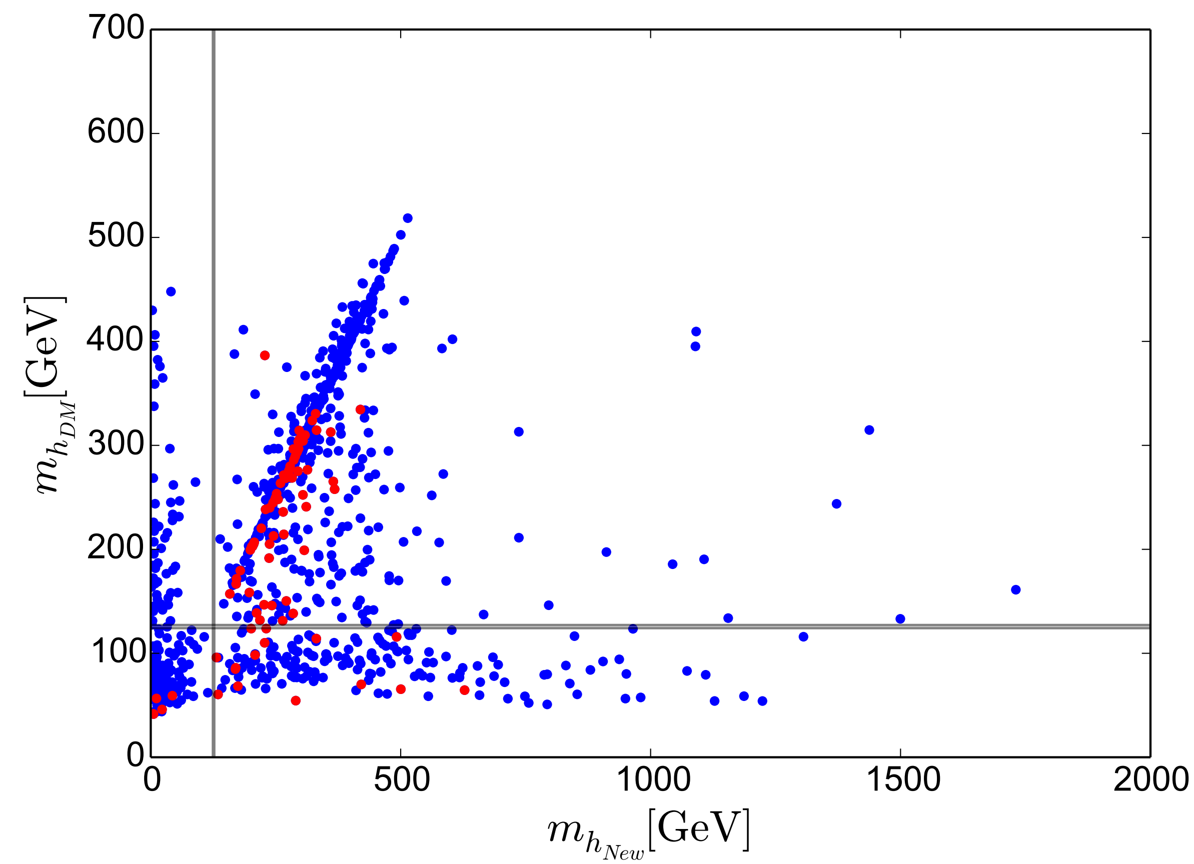
Dark Matter Phase

Now only two neutral scalars mix to give $h_{\rm SM}$ and $h_{\rm New}$, leaving $h_{\rm DM}$ as the Dark Matter candidate.

As for the broken phase, we can find scenarios with very small -functions. Blue points have $\beta_{\lambda,\delta,d_2} < 0.05$ while red points have $\beta_\lambda < 0.0005,\ \beta_\delta < 0.0025$ and $\beta_{d_2} < 0.01$



Without experimental constraints



With experimental constraints

A two Higgs doublet model

$$V(H_1, H_2) = m_{11}^2 H_1^\dagger H_1 + m_{22}^2 H_2^\dagger H_2 - \left(m_{12}^2 H_1^\dagger H_2 + c.c\right) + \lambda_1 \left(H_1^\dagger H_1\right)^2$$
$$+\lambda_2 \left(H_2^\dagger H_2\right)^2 + \lambda_3 \left(H_1^\dagger H_1\right)\left(H_2^\dagger H_2\right) + \lambda_4 \left(H_1^\dagger H_2\right)\left(H_2^\dagger H_1\right)$$
$$+\left(\frac{\lambda_5}{2}\left(H_1^\dagger H_2\right)^2 + \lambda_6 \left(H_1^\dagger H_1\right)\left(H_1^\dagger H_2\right) + \lambda_7 \left(H_2^\dagger H_2\right)\left(H_1^\dagger H_2\right) + c.c\right)$$

Since we are interested in scenarios where the quartic couplings become zero at the Planck scale, we will right away set $\lambda_6 = \lambda_7 = 0$, and these are not regenerated by RGE running.

Here we will show only results for a Type-II 2HDM, but this choice is not very significant, since only the top coupling to the Higgs doublets is important.

As before, we use (newer versions of) SARAH 4.12.2 and FlexibleSUSY 2.0.1, and require vacuum stability and perturbativity, as well as checking the vacuum using Vevacious.

$$\lambda_1 > 0,$$
$$\lambda_2 > 0,$$
$$\lambda_3 > -2\sqrt{\lambda_1 \lambda_2}$$
$$\lambda_3 + \lambda_4 - |\lambda_5| > -2\sqrt{\lambda_1 \lambda_2}$$

We use 2HDMC 1.7.0 [Eriksson et al 2010] to calculate the branching ratios, and apply experimental constraints using HiggsBounds 4.3.1 and HiggsSignals 1.4.0.

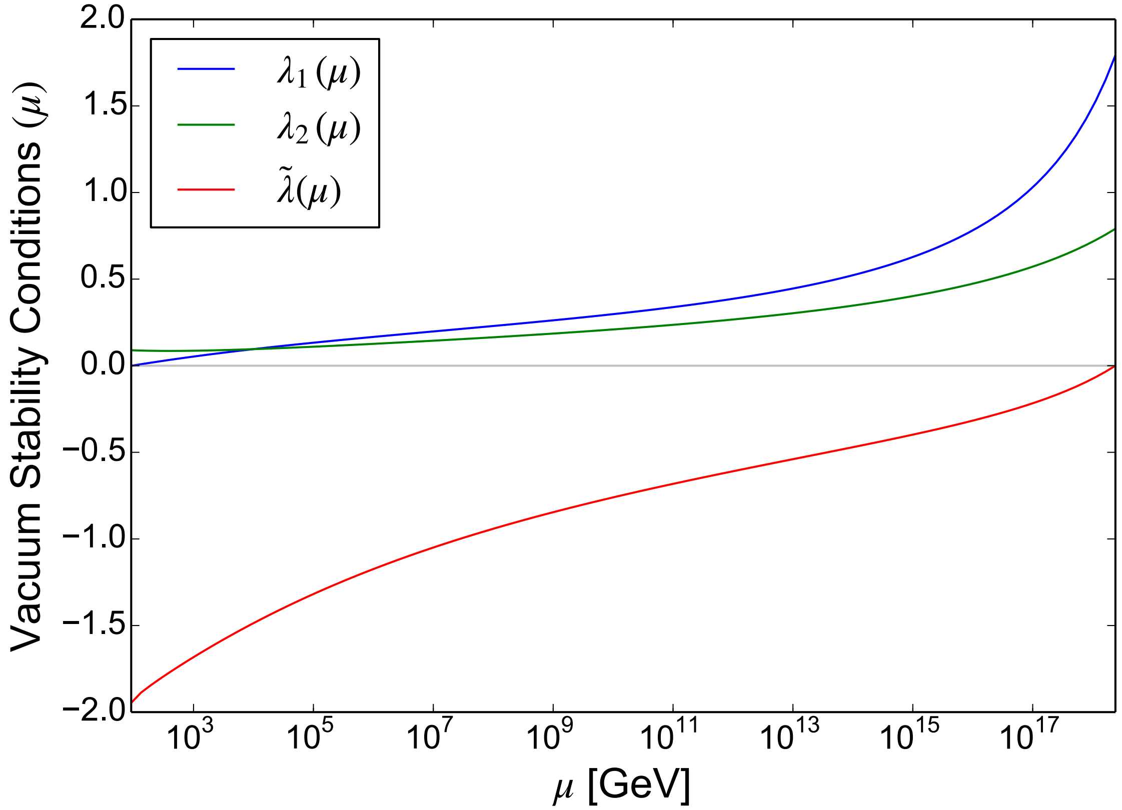
We also use 2HDMC to test precision observables S, T and U, as well as SuperIso [Mahmoudi, 2008] to test lots of flavour observables that could be affected by the charged Higgs.

A naive implementation of the **Multiple Point Principle** for the 2HDM would be to set $\lambda_i = 0, i = 1\ldots5$ but this automatically fails due to vacuum stability [Froggatt et al, 2006]

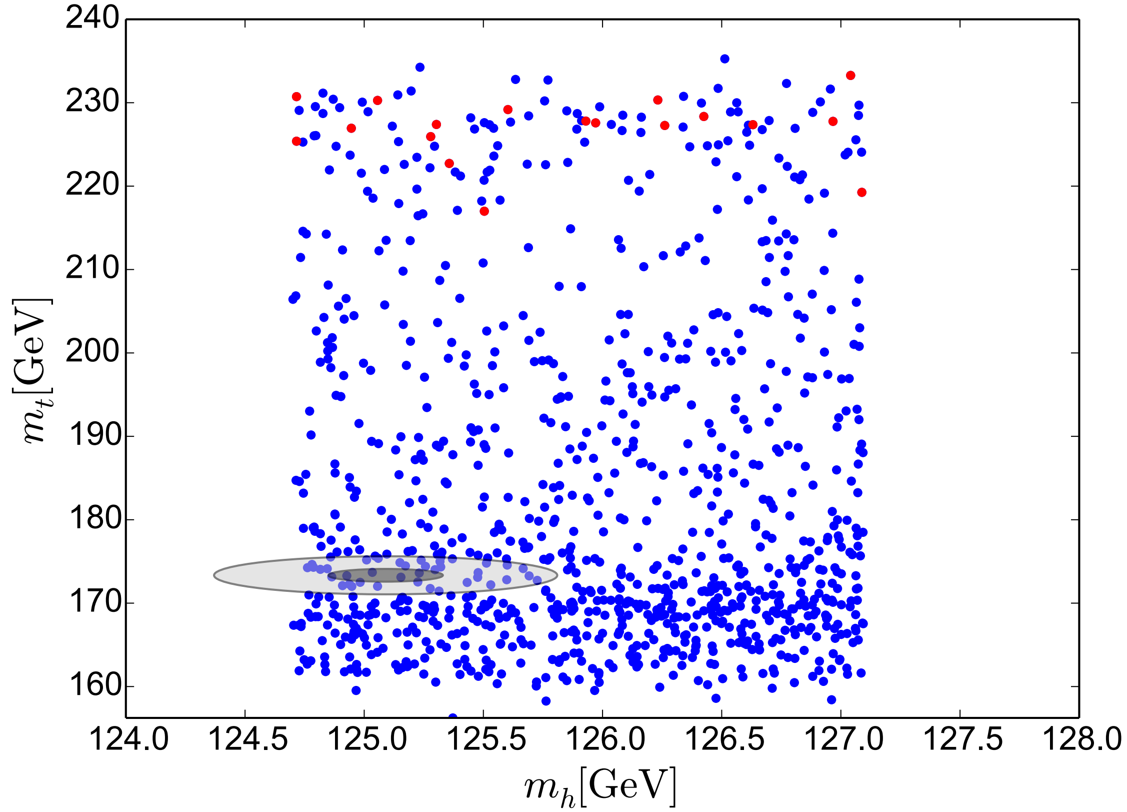
However there is a second solution for a degenerate vacuum:

$$\begin{aligned}
\lambda_5 &= 0 \\
\lambda_4 &< 0 \\
\tilde{\lambda} = \sqrt{\lambda_1\lambda_2} + \lambda_3 + \min\{0, \lambda_4\} &= 0 \\
\beta_{\tilde{\lambda}} &= 0
\end{aligned}$$

Unfortunately, we were completely unable to find scenarios with vacuum stability and the correct SM Higgs mass.



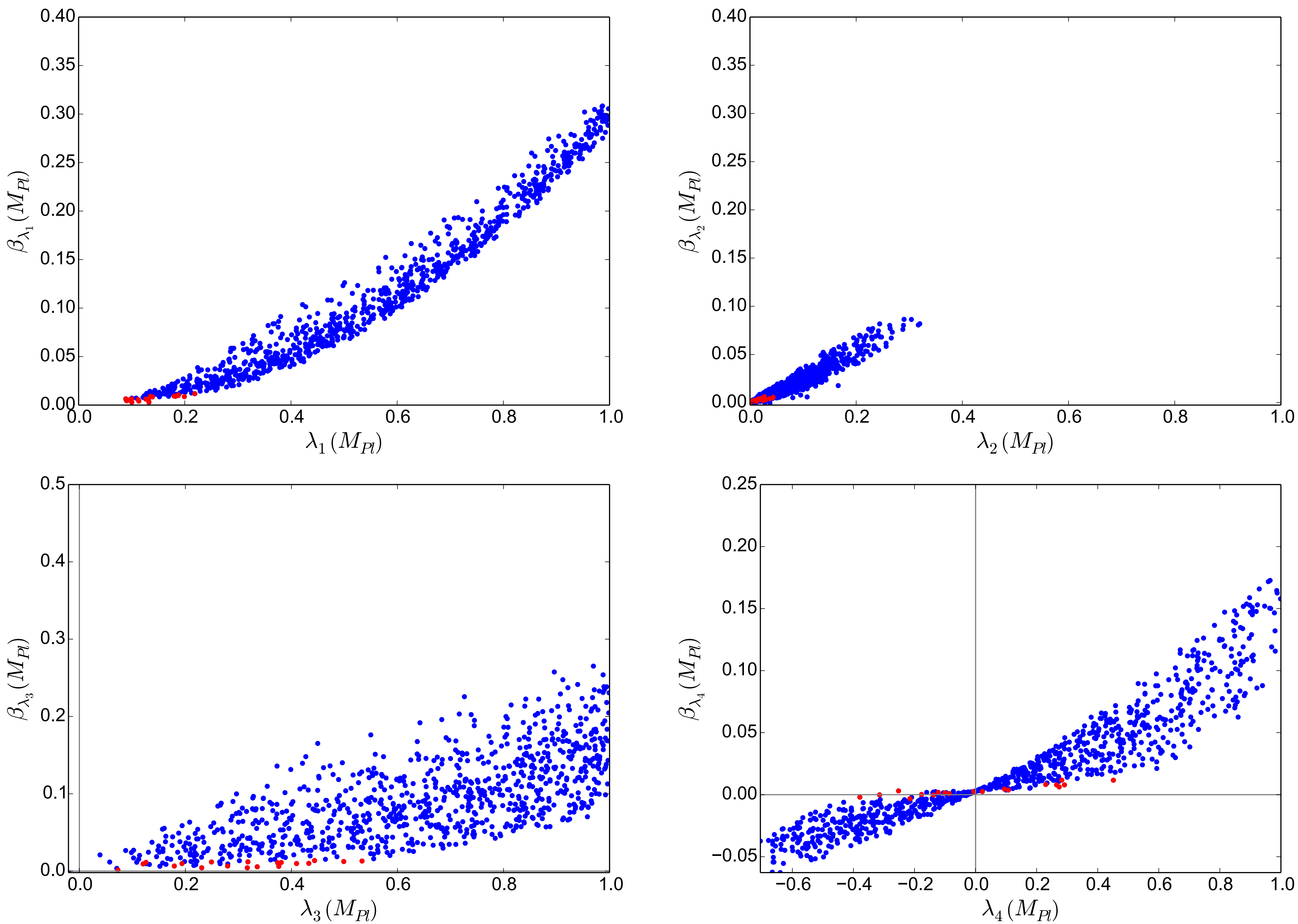
For stability, all of these should remain above zero. For most parameter choices $\tilde{\lambda}$ is instantly driven negative!



This is only avoided if the top Yukawa y_t is very large, positively contributing to the running, but this then requires a very heavy top.

We do much better with asymptotic safety as long as we allow very small β-functions.

Blue points only have $\beta_{\lambda_i} < 1$, but red points have $\beta_{\lambda_1} < 0.0127$, $\beta_{\lambda_2} < 0.0064$, $\beta_{\lambda_3} < 0.0139$ and $\beta_{\lambda_4} < 0.003$



The inert doublet model

We also considered a model with an **inert doublet**. This extra doublet only couples to the SM via the Higgs quartic interactions, and doesn't get a Vev.

$$\begin{aligned} V(H,\Phi) &= m_{11}^2 H^\dagger H + m_{22}^2 \Phi^\dagger \Phi + \lambda_1 \left(H^\dagger H\right)^2 + \lambda_2 \left(\Phi^\dagger \Phi\right)^2 \\ &+\lambda_3 \left(H^\dagger H\right)\left(\Phi^\dagger \Phi\right) + \lambda_4 \left(H^\dagger \Phi\right)\left(\Phi^\dagger H\right) + \left(\frac{\lambda_5}{2}\left(H^\dagger \Phi\right)^2 + c.c\right) \end{aligned}$$

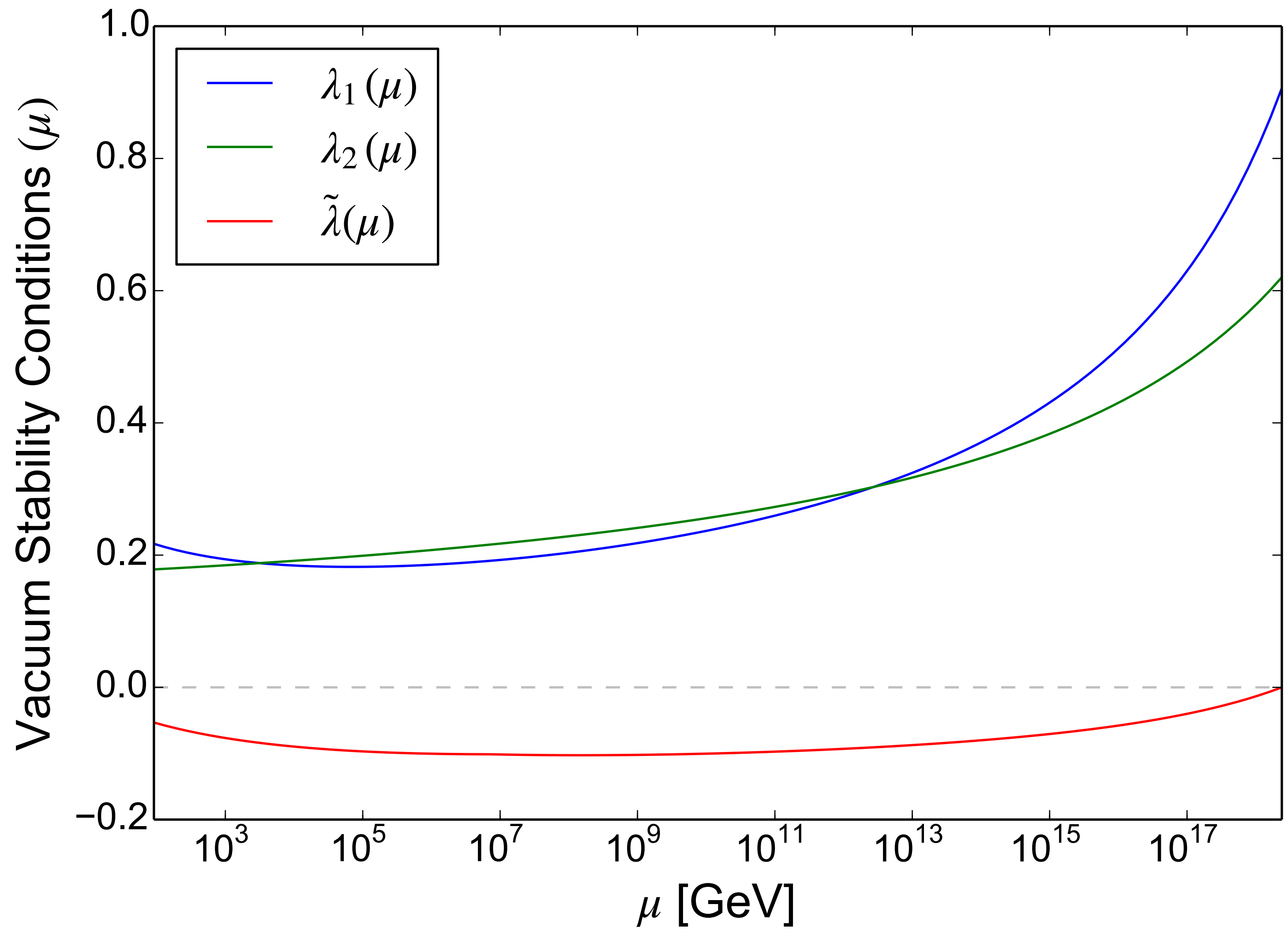
The lightest inert scalar is stable so a Dark Matter candidate. Therefore we need to apply Dark Matter constraints on the DM Relic Density and Direct Detection.

We also need some constraints on the additional scalar masses to prevent violating LEP limits,

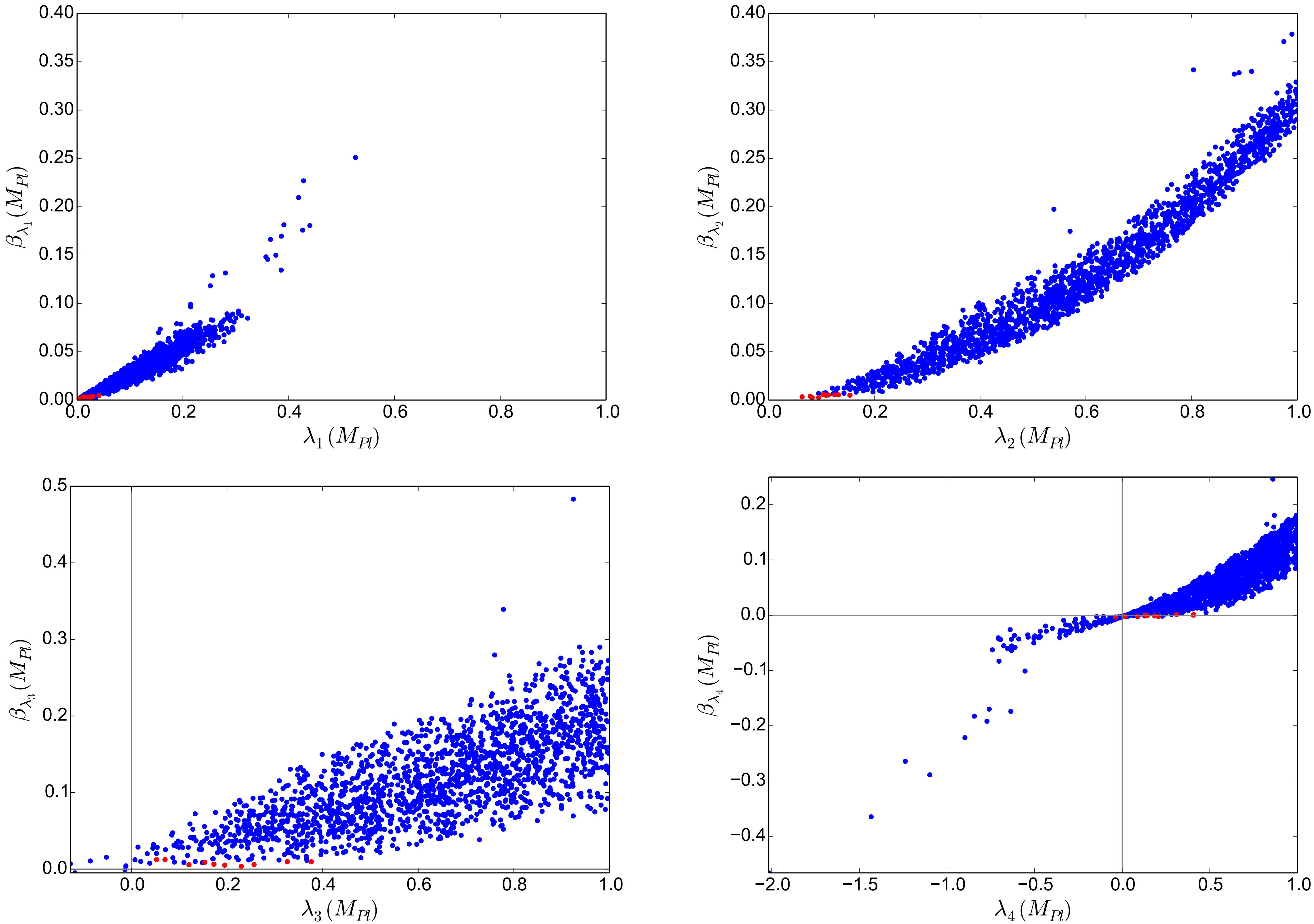
i.e. $M_H + M_A > M_Z$, $M_{H^\pm} > M_W$ and $M_{H^\pm} > \min\{M_H, M_A\}$

The **Multiple Point Principle** for the inert doublet model falls foul of exactly the same issue we had for the 2HDM. We can't keep the vacuum stable due to $\tilde{\lambda}$ running negative.

Here is a fairly typical example of the running:



For asymptotic safety, the inert doublet model again parallels the 2HDM, allowing really very small values of the quartic couplings' β-functions while giving the correct Higgs mass and satisfying all experimental constraints.



Conclusions

The Standard Model Higgs Boson mass measurement implies the Higgs quartic coupling and its β-function are very close to zero at the Planck Scale.

It has been suggested that this could be due to the **Multiple Point Principle** which postulates a second degenerate vacuum at the Planck scale, or alternatively, the requirement for **asymptotic safety** where all the quartic β -functions must vanish.

Neither of these work in the SM with the measured Higgs mass (though they come quite close), so we have asked if they can be made to work in models with extended Higgs sectors.

We examined a model with **an extra singlet** (in the broken phase and the Dark Matter phases); as well as a type-II 2HDM and an inert doublet model.

We applied theoretical constraints such as vacuum stability, as well as experimental constraints including the LHC and Dark Matter experiments.

We could find no scenarios where the Multiple Point Principle gave the correct Higgs mass, but were able to find scenarios in all cases that satisfied asymptotic safety as long as we were willing to admit very small β -functions.