A hard/soft split of the effective potential and its relation to gauge invariance

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Spontaneous symmetry breaking in QFT

$$V(\phi) = V_0(\phi) + \hbar V_1(\phi) + \hbar^2 V_2(\phi) + \ldots$$

In Higgs theories, we minimize the effective potential $V(\phi)$ to find the quantum corrected VEV ϕ_{min}.

Frustration with the effective potential

The static energy density $V_{\text{min}} \equiv V(\phi_{\text{min}})$ is in principle measurable, but it is hard to find physical values for it.

Particular problems:

1. Gauge dependence
2. IR divergences due to massless Goldstone bosons

Gauge dependence

The $\hbar$-expansion

In perturbation theory we must be careful.

$$\phi_{\text{min}} = \phi_0 + \hbar\phi_1 + \hbar^2\phi_2 + \ldots$$

Such that

$$V_{\text{min}} = V_0(\phi_0) + \hbar V_1(\phi_0) + \hbar^2 \left[V_2 - \frac{1}{2}\phi_1^2 \partial^2 V_0 \right]_{\phi_0} + \ldots$$

Then V_{min} is gauge invariant order by order in $\hbar$.

REFERENCE: [Patel & Ramsey-Musolf, Arxiv:1101.4665]

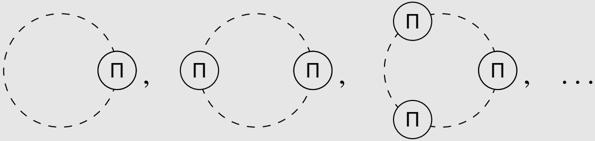
The $\hbar$-expansion

The longitudinal unphysical modes can only cancel consistently if we take the perturbative expansion seriously. This is manifested in the non-zero masses of Goldstone bosons:

$$m_G^2\Big|_{\phi=\phi_{\min}} = \cancel{m_G^2\Big|_{\phi=\phi_0}} + \hbar\phi_1(\xi)\ \partial m_G^2\Big|_{\phi=\phi_0} + \mathcal{O}(\hbar^2)$$

IR divergences due to Goldstone bosons

IR divergent Daisy diagrams



A Daisy with n petals:

$$\int_k \left(\frac{\Pi(k^2)}{k^2 - m_G^2} \right)^n \sim \left(m_G^2\right)^{2-n} \log\left[m_G^2\right]$$

Solution: these diagrams can be resummed!

REFERENCES: [Elias-Miró, Espinosa & Konstandin, Arxiv:1406.2652], and [Martin, Arxiv:1406.2355]

A hard/soft split of the effective potential

- If $\phi \approx \phi_0$ such that $m_G^2 \approx 0$:

$$V(\phi) = V^{\mathrm{H}}(\phi) + V^{\mathrm{S}}(\phi),$$
$$V^{\mathrm{H}}(\phi) \leftarrow \textit{hard} \text{ physics}, k^2 \gg m_G^2$$
$$V^{\mathrm{S}}(\phi) \leftarrow \textit{soft} \text{ physics}, k^2 \sim m_G^2$$

- All IR divergences reside in $V^{\mathrm{S}}(\phi)$
- Goldstone mass is shifted according to $V^{\mathrm{H}}(\phi)$:

$$\overline{m_G^2} = m_G^2 + \Delta = \frac{1}{\phi} \partial_\phi V^{\mathrm{H}}$$

REFERENCE: [Espinosa & Konstandin, Arxiv:1712.08068]

Can resummation help us understand the gauge dependence?

The hard/soft split and gauge invariance

- Again the Goldstone boson masses are relevant. What if this can help with the gauge dependence?
- We calculated the 2 loop effective potential in Abelian Higgs in Fermi gauge.
- There are problematic gauge-dependent contributions that cannot be resummed!

SEE ALSO: [Espinosa, Garny & Konstandin, Arxiv:1607.08432], [Andreassen, Frost & Schwartz, Arxiv:1408.0292]

The powers of $\hbar$

Using the hard/soft split **and** the $\hbar$-expansion, we find that

$$V_{\text{min}} = V^{\text{H}}(\phi^{\text{H}}_{\text{min}}) + \cancel{\left\{\text{terms from } V^{\text{S}}(\phi)\right\}}$$

We have shown this up to four loops in Abelian Higgs. We conjecture it holds to all loops.

Outlook:

- Why is the $\hbar$-expansion finite?
- Is it possible to modify or extend the resummation to deal with the gauge-dependent singularities?
- Further applications of the hard/soft split?