

# Precise calculations of the Higgs decay rates in multi-Higgs models

Based on Phys. Lett. B783 (2018) 140;

Comput. Phys. Commun. 233 (2018) 134.

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# Introduction

- Properties of the Higgs bosons have been measured at the LHC:

Masses,  $hVV$  couplings,  $hff$  couplings, ...

Current exp. data are consistent with predictions of the SM.

- Deviations from the SM may be found in the future collider exp.

HL-LHC, ILC, CLIC, FCC-ee, CPEC, HE-LHC, FCC-hh, ...

The Higgs couplings will be measured with a few % accuracy.

➔ Can we clarify the shape the Higgs sector with the precise measurement of the Higgs observables?

In this talk, we discuss whether we can discriminate various extended Higgs modes via 1-loop corrected Higgs decay rates.

# Extended Higgs models

We focus on 4 types of THDMs and the Higgs singlet model (HSM).

THDMs	$\Phi_1$	$\Phi_2$	$u_R$	$d_R$	$e_R$
Type-I	+	-	-	-	-
Type-II	+	-	-	+	+
Type-X	+	-	-	-	+
Type-Y	+	-	-	+	-

[V. D. Barger, J. L. Hewett and R. J. N. Phillips, PRD41 (1990) 3421]

[Y. Grossman, NPB426(1994)355]; [A. G. Akeroyd and W. J. Stirling, Nucl. Phys. B447 (1995)3]

[M. Aoki, S. Kanemura, K. Tsumura and K. Yagyu, PRD80 (2009) 015017]

	THDMs Type I, II, X, Y (Softly broken $Z_2$ sym., CP conserved)	HSM (No global sym. )
Higgs sector	$\Phi_1 + \Phi_2$ $\Phi_i = \begin{pmatrix} w_i \\ 1/\sqrt{2}(v + h_i + iz_i) \end{pmatrix} (i = 1,2)$	$\Phi + S \text{ (Real singlet)}$ $S = v_s + s$
Physical states ( $h$ : 125 GeV Higgs)	$h, H, A, H^\pm$	$h, H$
Free parameters	$m_H, m_A, m_{H^\pm}, \alpha, \beta, M^2$	$m_H, \alpha, m_s^2, \lambda_s \mu_s$
Higgs coup. ( $s_\theta \equiv \sin \theta,$ $c_\theta \equiv \cos \theta$ )	$\Gamma_{hVV}^{1,\text{tree}} = -\frac{2m_V^2}{v} s_{\beta-\alpha},$ $\Gamma_{hff}^{S,\text{tree}} = -\frac{m_f}{v} (s_{\beta-\alpha} + \xi_f c_{\beta-\alpha})$	$\Gamma_{hff}^{1,\text{tree}} = -\frac{2m_V^2}{v} c_\alpha,$ $\Gamma_{hff}^{S,\text{tree}} = -\frac{m_f}{v} c_\alpha$

# H-COUP

[Kanemura, Kikuchi KS, Yagyu, CPC 233 (2018) 134]

H-COUP is a program to evaluate Higgs boson couplings at the 1-loop level.

- Renormalization scheme: improved on-shell scheme

[S. Kanemura, M. Kikuchi, KS, K. Yagyu, PRD96 (2017),035014 ]

Gauge dependence coming from the renormalization for mixing angles are removed by the pinch technique.

[ F. Bojarski, G. Chalons, D. Lopez-Val, T. Robens, JHEP02 (2016) 147.]; [M. Krause, R. Lorenz, M. Muhlleitner, R. Santos, H. Ziesche, JHEP 09 (2016) 143.]

- Model:

Higgs singlet model (HSM)

Two Higgs Doublet Models (THDMs) Type I, II, X, Y

Inert Doublet Model

- Outputs : Renormalized Higgs 3-point vertex functions  $\Gamma_{hXX}$

$$\Gamma_{hVV}^i(2), \Gamma_{hff}^j(8), \Gamma_{hhh}, \Gamma(h \rightarrow \gamma\gamma), \Gamma(h \rightarrow Z\gamma), \Gamma(h \rightarrow gg)$$

Higgs decay rates are evaluated at the 1-loop level by using H-COUP:

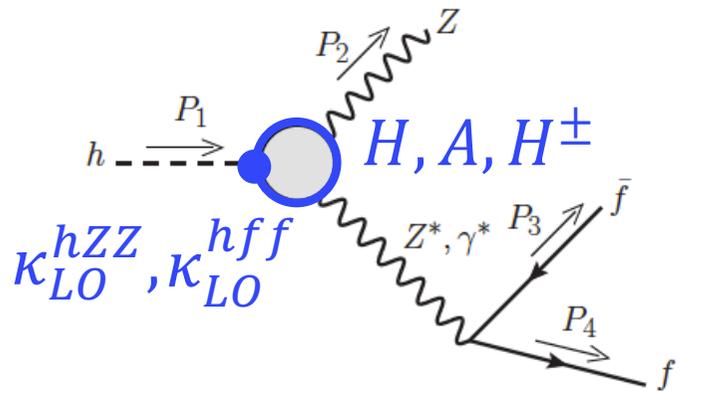
$$\Gamma(h \rightarrow f\bar{f}), \Gamma(h \rightarrow ZZ^* \rightarrow Zf\bar{f}), \Gamma(h \rightarrow \gamma\gamma), \Gamma(h \rightarrow Z\gamma), \Gamma(h \rightarrow gg)$$

# Diagrams for $h \rightarrow ZZ^* \rightarrow Zf\bar{f}$

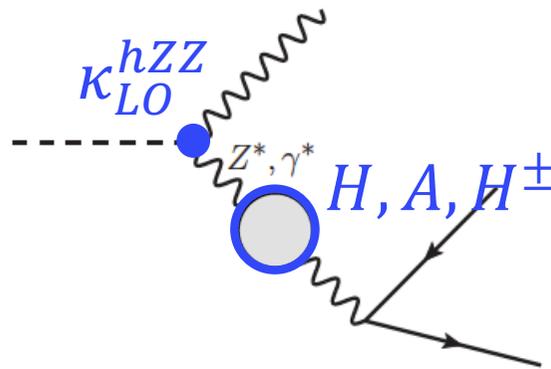
(THDM):  $\kappa_{LO}^{hZZ} = \sin(\beta - \alpha)$

$\kappa_{LO}^{hff} = \sin(\beta - \alpha) + \xi_f \cos(\beta - \alpha)$

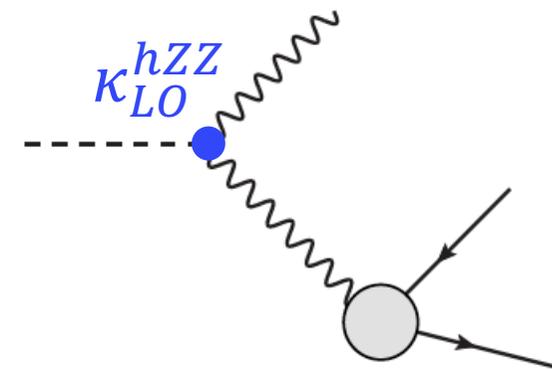
(HSM):  $\kappa_{LO}^{hZZ} = \kappa_{LO}^{hff} = \cos(\alpha)$



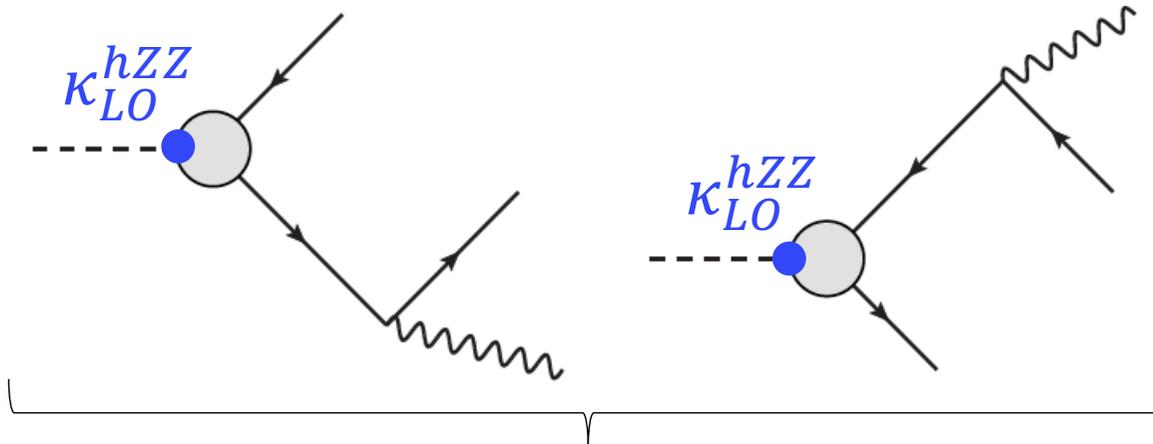
$hZZ$  vertex correction



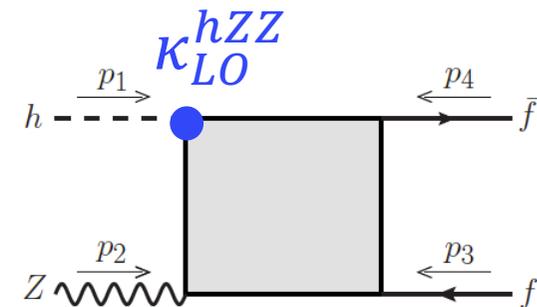
Self-energy correction



$Zff$  vertex correction



$hff$  vertex correction



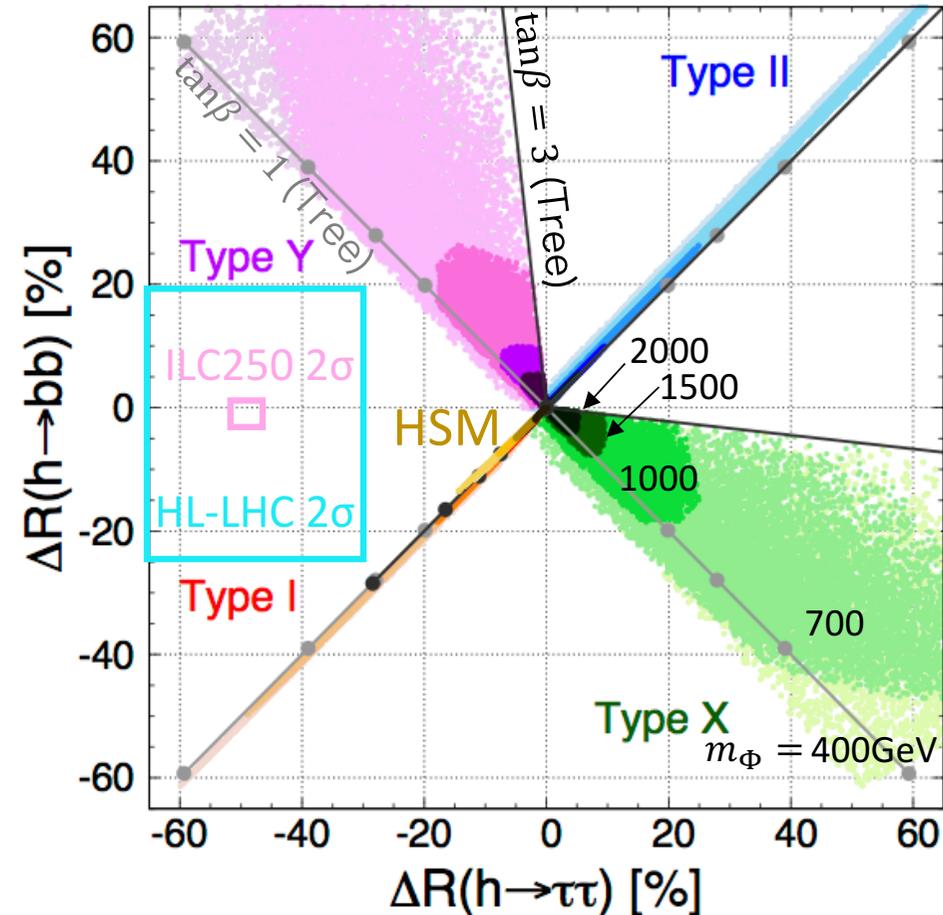
Box diagram correction

# $\Delta R(h \rightarrow b\bar{b})$ vs $\Delta R(h \rightarrow \tau\bar{\tau})$

$$\Delta R(h \rightarrow XX) = \frac{\Gamma(h \rightarrow XX)_{EX}}{\Gamma(h \rightarrow XX)_{SM}} - 1$$

[S. Kanemura, M. Kikuchi, K. Mawatari, KS, K. Yagyu,]  $\cos(\beta-\alpha) < 0$

- Color plots : predictions at the 1-loop level for each model
- A contrast of color : values of mass of extra Higgs bosons
- Black line : predictions at the tree level ( $\tan\beta = 1, 3$ ).

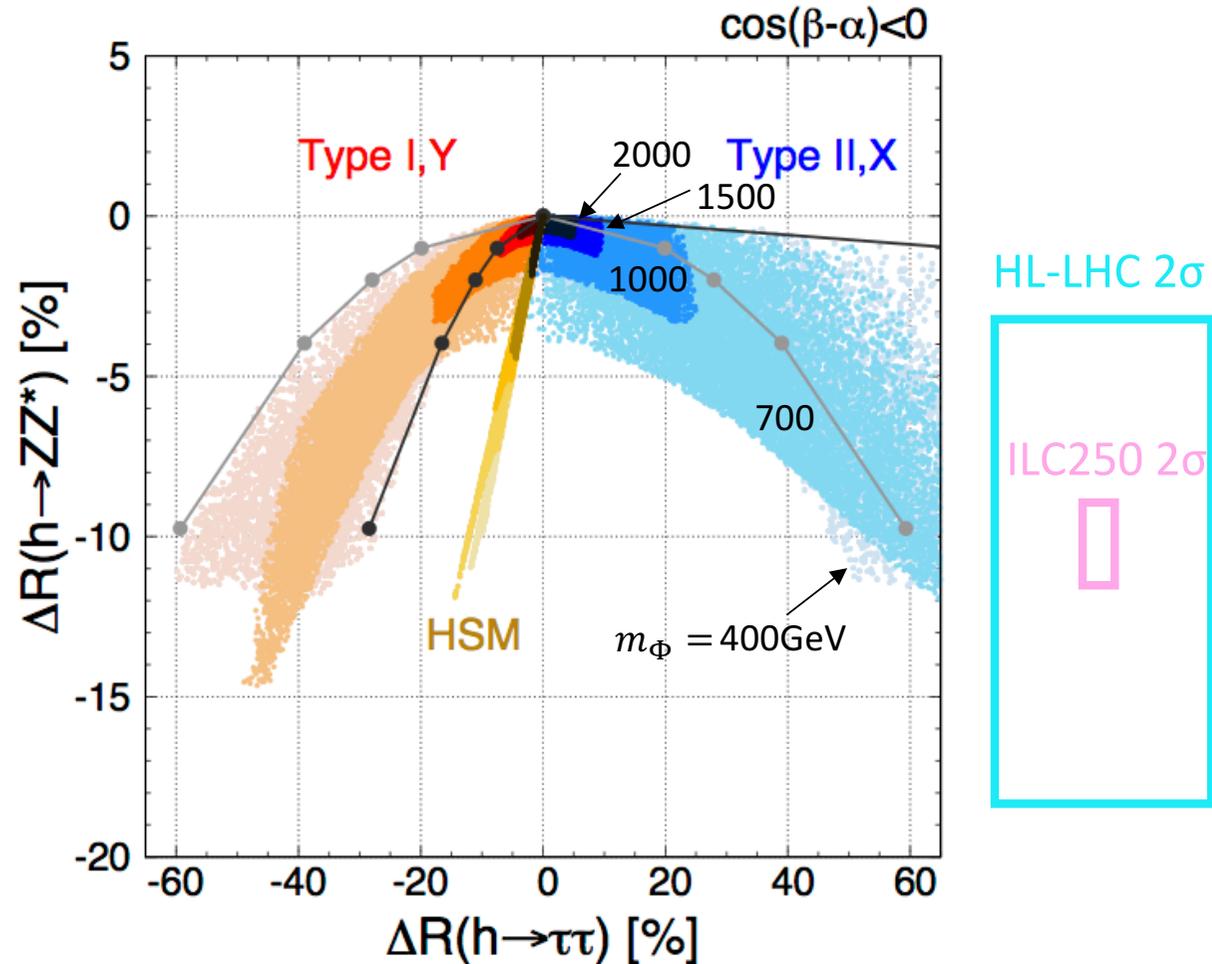


- 4 types of THDMs are discriminated by pattern of deviations.
- The upper bounds of mass of extra Higgs can be obtained from magnitude of deviations due to theoretical constraints.

# $\Delta R(h \rightarrow ZZ^*)$ vs $\Delta R(h \rightarrow \tau\bar{\tau})$

[S. Kanemura, M. Kikuchi, K. Mawatari, KS, K. Yagyu]

$$\Delta R(h \rightarrow XX) = \frac{\Gamma(h \rightarrow XX)_{EX}}{\Gamma(h \rightarrow XX)_{SM}} - 1$$



→ HSM can be separated from the THDMs.

→ Type I and Type II can be discriminated in this plane.

# Summary

- We discussed a possibility of discrimination among the HSM and 4 types of THDMs with precise measurement of Higgs decay rates.
- We evaluated the Higgs decay rates at the 1-loop level by utilizing H-COUP.
  - Pattern of deviations : HSM and 4 types of THDMs can be discriminated.
  - Magnitude of deviations : Information of the mass of extra Higgs bosons can be obtained.

**We need the ILC!**

**Back up slides**

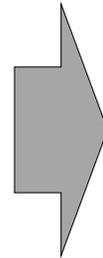
# Accuracy of the Higgs couplings measurements

Current data (LHC Run I)

scaling factor:  $\kappa_X = g_{hXX}^{exp.} / g_{hXX}^{SM}$

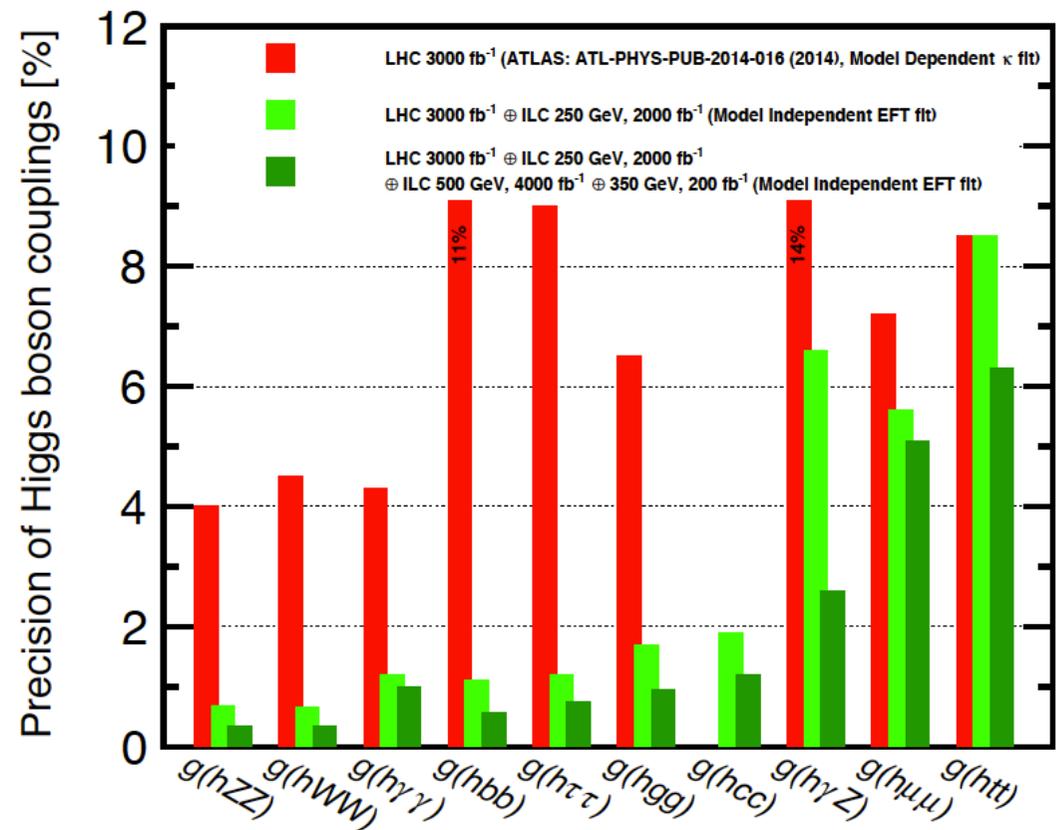
[ATLAS and CMS, JHEP08(2016)045]

$\kappa_Z$	-0.98 [-1.08, -0.88] $\cup$ [0.94, 1.13]
$\kappa_W$	0.87 [0.78, 1.00]
$\kappa_t$	$1.40^{+0.24}_{-0.21}$
$ \kappa_\tau $	$0.84^{+0.15}_{-0.11}$
$ \kappa_b $	$0.49^{+0.27}_{-0.15}$
$ \kappa_g $	$0.78^{+0.13}_{-0.10}$
$ \kappa_\gamma $	$0.87^{+0.14}_{-0.09}$



Future prospect (HL-LHC, ILC)

[K. Fujii, et al., arXiv:1710.07621]



→ We should evaluate the theoretical predictions including radiative corrections.

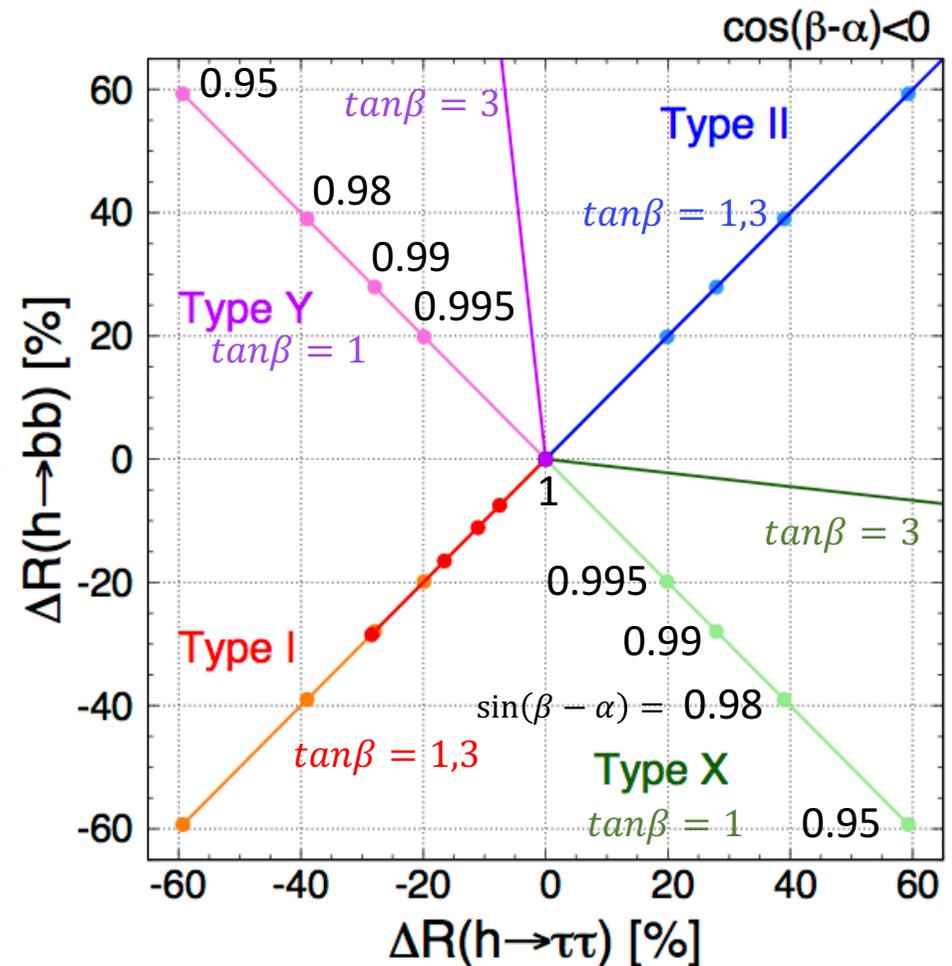
# $\Delta R(h \rightarrow b\bar{b})$ vs $\Delta R(h \rightarrow \tau\bar{\tau})$ [Tree]

$$\Delta R(h \rightarrow XX) = \frac{\Gamma(h \rightarrow XX)_{EX}}{\Gamma(h \rightarrow XX)_{SM}} - 1$$

For THDMs :

$$\Delta R(h \rightarrow f\bar{f})^{LO} = (\sin(\beta - \alpha) - \xi_f \cos(\beta - \alpha))^2 - 1$$

	$\xi_u$	$\xi_d$	$\xi_e$
Type-I	$\cot \beta$	$\cot \beta$	$\cot \beta$
Type-II	$\cot \beta$	$-\tan \beta$	$-\tan \beta$
Type-X	$\cot \beta$	$\cot \beta$	$-\tan \beta$
Type-Y	$\cot \beta$	$-\tan \beta$	$\cot \beta$



[S. Kanemura, K. Tsumura, K. Yagyu, H. Yokoya, PRD90 (2014) 075001.]

In this plane, 4 types of THDMs can be distinguished.

# Numerical calculations (1-loop) <sup>13/18</sup>

We discuss whether or not THDMs and the HSM can be distinguished by deviations from the SM in the decay widths.

- Scan region of input parameters in the THDMs :

$$0.95 < \sin(\beta - \alpha) < 1, \quad 1 < \tan\beta < 3,$$

$$m_\Phi = m_H = m_A = m_{H^\pm},$$

$$m_\Phi = 400, 700, 1000, 1500, 2000 \text{ GeV},$$

$$0 < M < m_\Phi$$

- Scan region for the HSM :

$$0.95 < \cos\alpha < 1,$$

$$m_\Phi = 500, 1000, 2000, 3000, 5000 \text{ GeV},$$

$$0 < m_s < m_\Phi, \quad \lambda_s = \mu_s = 0$$

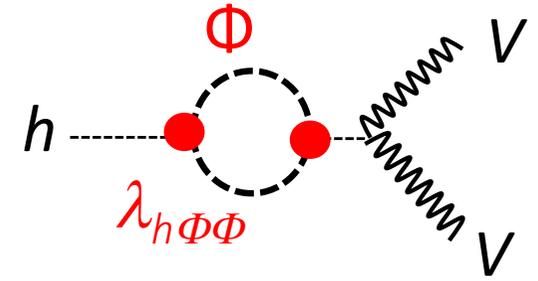
- Constraints :

Perturbative unitarity, Vacuum stability, Wrong vacuum condition (for HSM),

S, T parameters

# Impact of Loop correction of extra Higgs in $hZZ$ vertex

Approximate formula ( $m_A, m_{H^\pm}, m_H \gg m_h$ )  
 $\sin(\beta - \alpha) = 1, \tan\beta = 1$

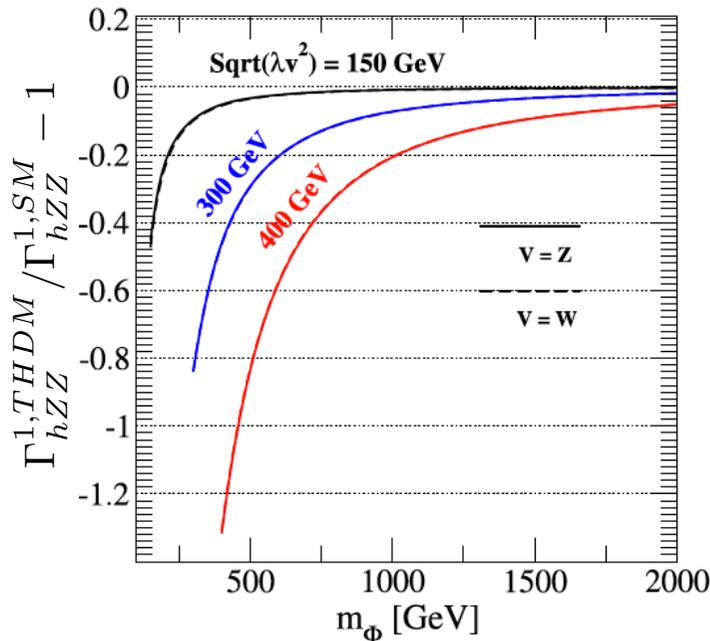


$$\Gamma_{hZZ}^1 = \frac{2m_Z^2}{v} \left\{ 1 - \sum_{\Phi=A,H,H^\pm} c_\Phi \frac{1}{6} \frac{m_\Phi^2}{v^2} \left( 1 - \frac{M^2}{m_\Phi^2} \right)^2 \right\}$$

1)  $M^2 \gg v^2$  Decoupling case

$$m_\Phi^2 \simeq M^2 + \lambda_i v^2 \sim M^2$$

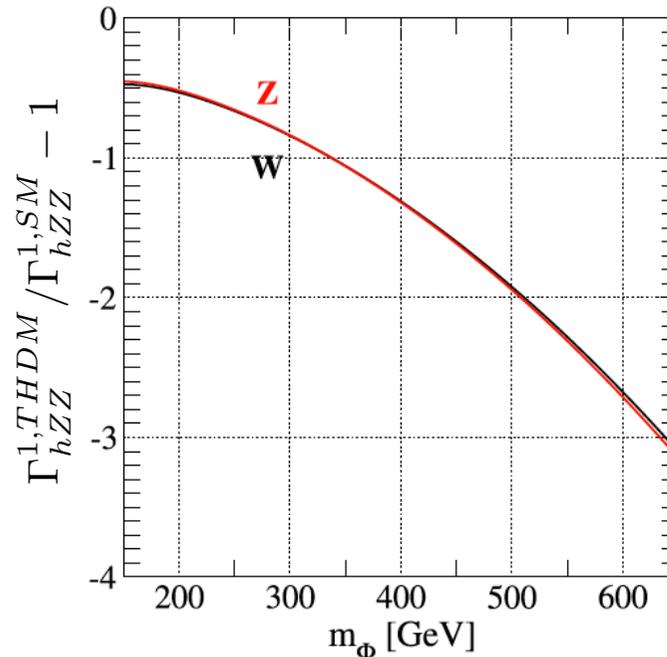
[S. Kanemura, M. Kikuchi, K. Yagyu Nucl.Phys. B896,80.]



2)  $M^2 \lesssim v^2$  Non-decoupling case

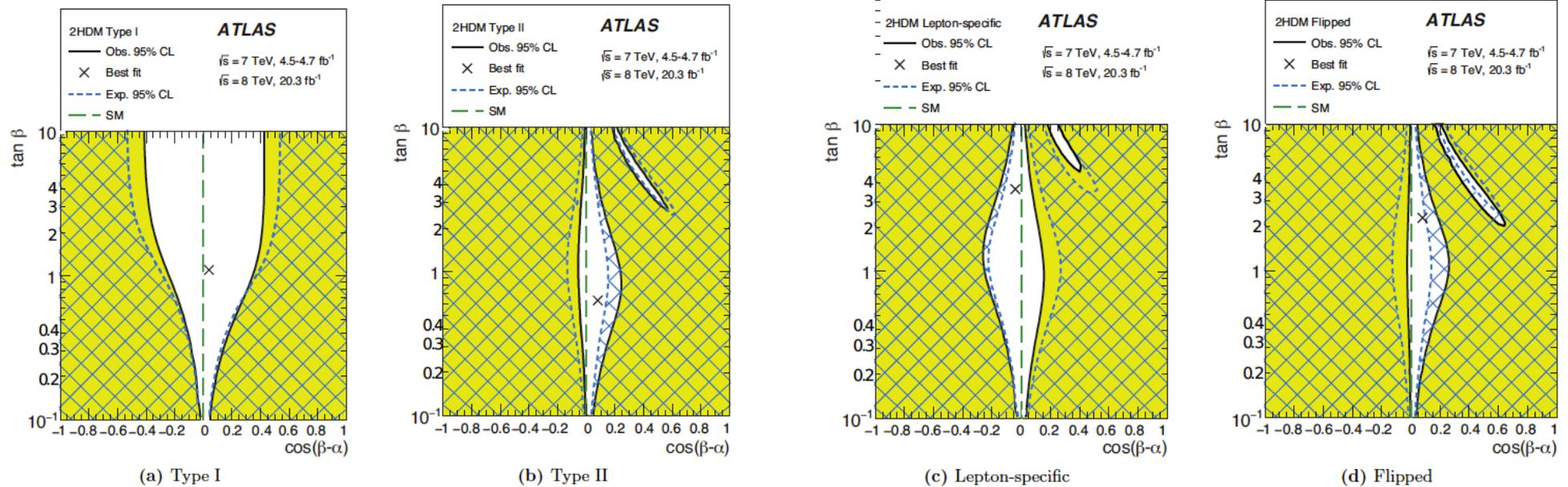
$$m_\Phi^2 \simeq M^2 + \lambda_i v^2 \sim \lambda_i v^2$$

[S. Kanemura, M. Kikuchi, K. Yagyu Nucl.Phys. B896,80.]



# constraint for THDMs (Higgs signal strength)

[ATLAS, JHEP1511(2015)206]



$$c_{\beta-\alpha} = 0.1 \rightarrow s_{\beta-\alpha} = 0.99$$

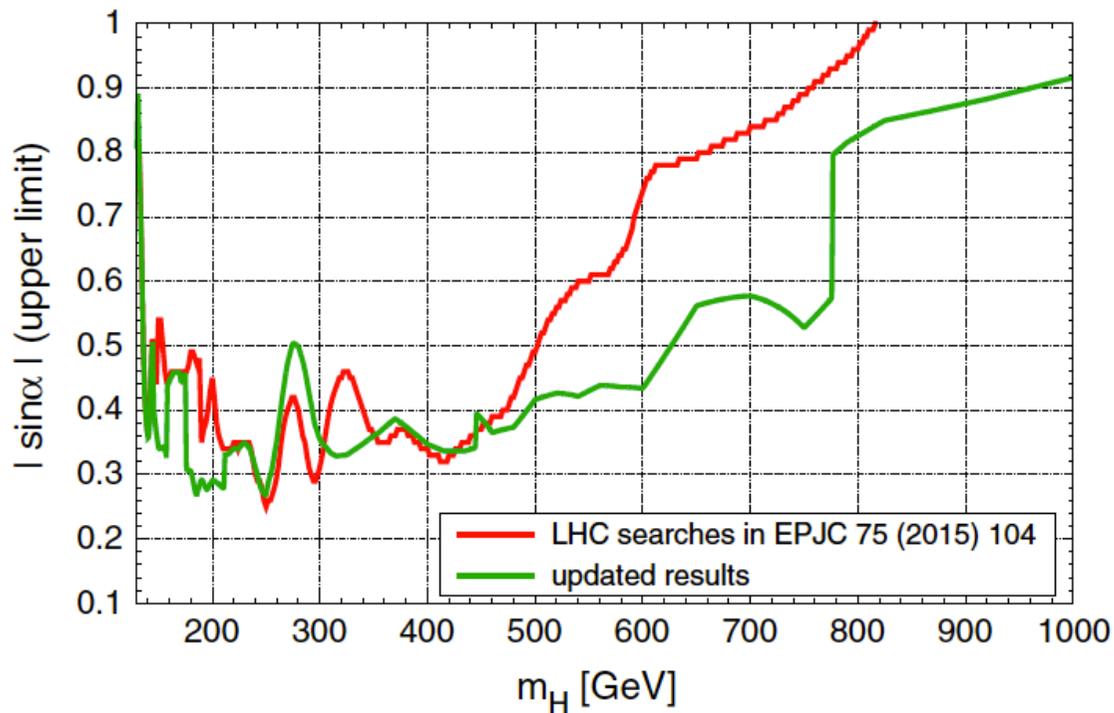
$$c_{\beta-\alpha} = 0.2 \rightarrow s_{\beta-\alpha} = 0.98$$

$$c_{\beta-\alpha} = 0.3 \rightarrow s_{\beta-\alpha} = 0.95$$

# Constraint of direct search (HSM)

[T. Robens, T. Stefaniak, Eur. Phys. J. C (2016) 76,268]

LHC Run II



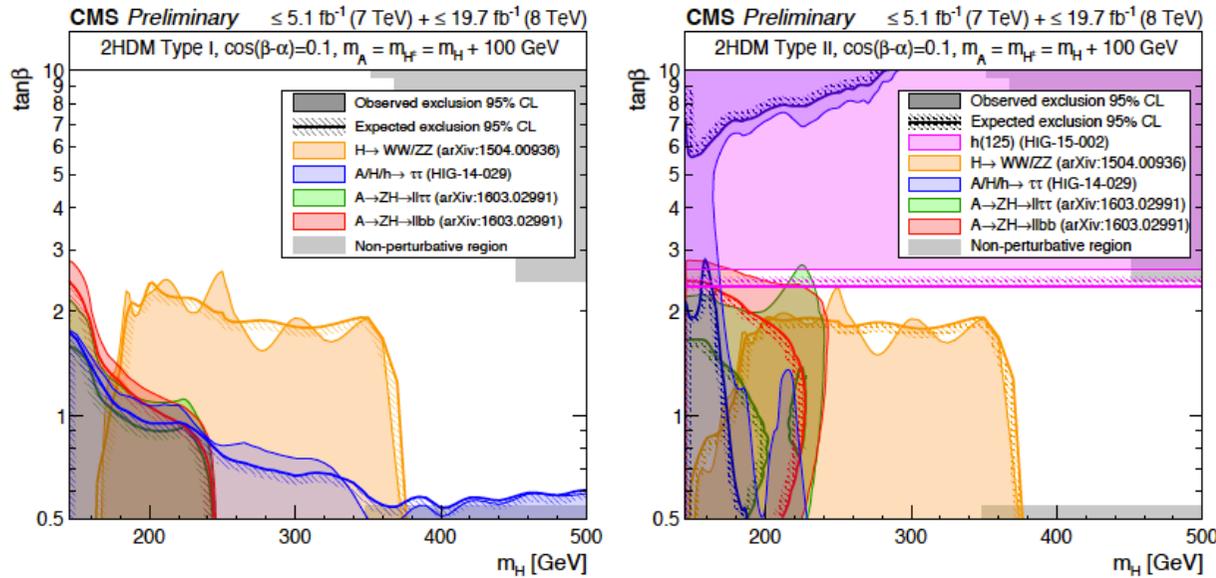
**Table 1** List of LHC Higgs search channels that are applied by HiggsBounds in the high-mass region, yielding the upper limit on  $|\sin\alpha|$  shown in Figs. 1 and 2

Range of $m_H$ [GeV]	Search channel	Reference
130–145	$H \rightarrow ZZ \rightarrow 4l$	[94] (CMS)
145–158	$H \rightarrow VV$ ( $V=W,Z$ )	[66] (CMS)
158–163	SM comb.	[95] (CMS)
163–170	$H \rightarrow WW$	[96] (CMS)
170–176	SM comb.	[95] (CMS)
176–211	$H \rightarrow VV$ ( $V=W,Z$ )	[66] (CMS)
211–225	$H \rightarrow ZZ \rightarrow 4l$	[94] (CMS)
225–445	$H \rightarrow VV$ ( $V=W,Z$ )	[66] (CMS)
445–776	$H \rightarrow ZZ$	[70] (ATLAS)
776–1000	$H \rightarrow VV$ ( $V=W,Z$ )	[66] (CMS)

# Status of direct search of extra Higgs

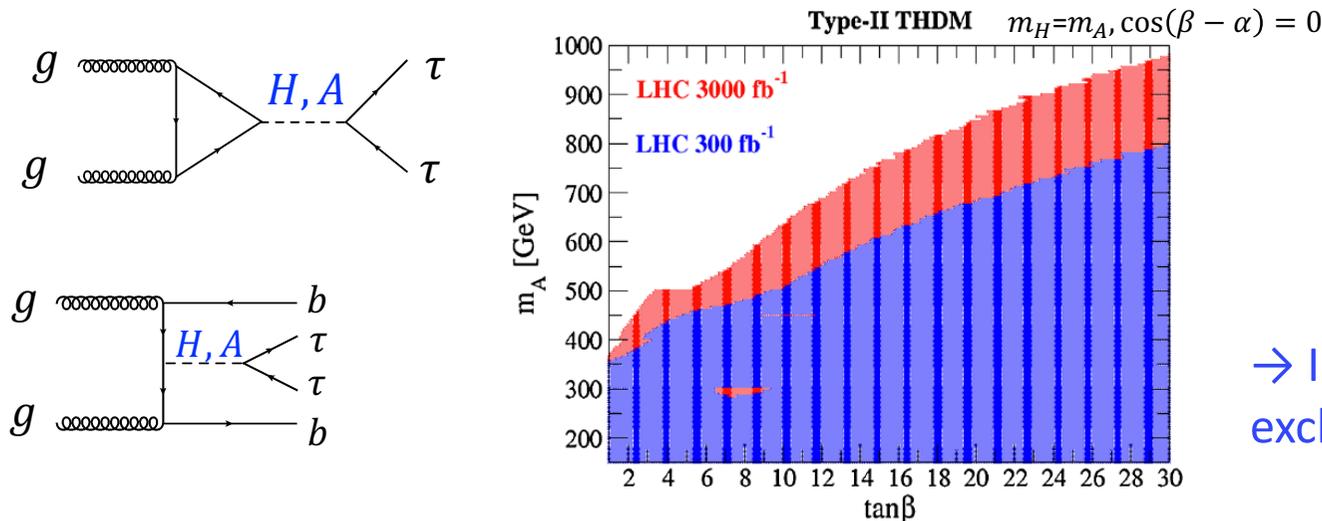
- constraint for THDMs (LHC Run I, Summary plots by CMS)

[CMS PAS HIG-16-007]



→ Basically, for Type II  $\tan\beta < 2$ ,  $m_H < 380 \text{ GeV}$  are excluded.

- Future prospect of excluded regions [ Kanemura, Tsumura, Yagyu, Yokoya, PRD90(2014)075001]



→ In the future exp. excluded regions are spread.

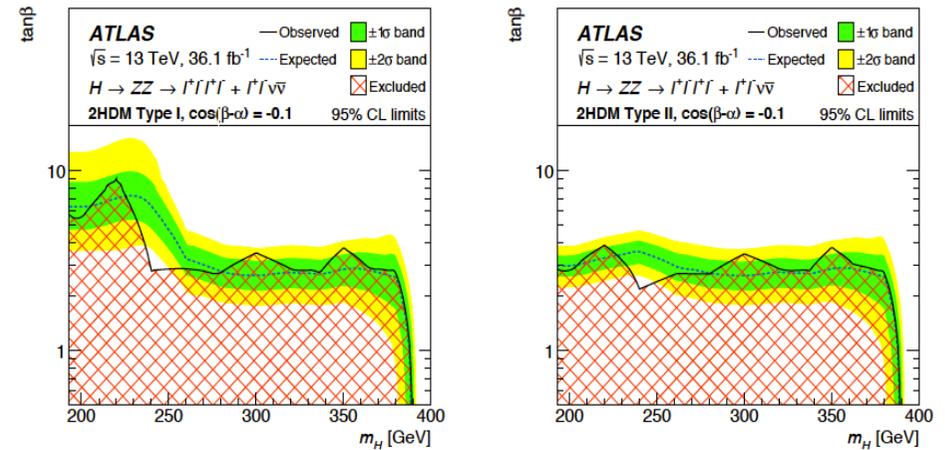
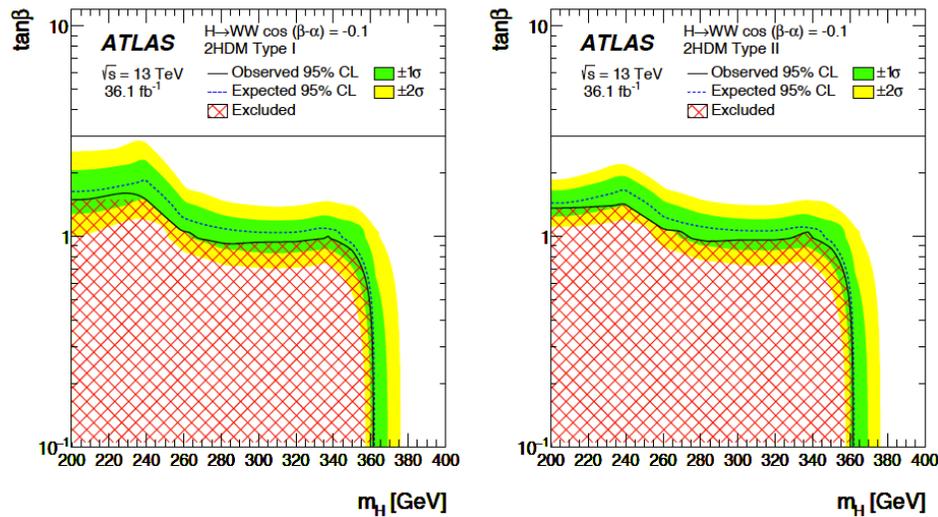
# Constraint of direct search (THDM)

At LHC Run II

[ATLAS, Eur.Phys.J. C78 (2018) 24]

[ATLAS, arXiv:1712.06386]

$\ell: e, \mu$



final state:  $e\nu\mu\mu$

# Signal strength( current data)

JHEP08,045

Definition of  $\mu^f$

$$\mu^f = \frac{\text{BR}_{EX}}{\text{BR}_{SM}}$$

Decay channel	ATLAS+CMS	ATLAS	CMS
$\mu^{\gamma\gamma}$	1.14 $^{+0.19}_{-0.18}$ $\left( \begin{smallmatrix} +0.18 \\ -0.17 \end{smallmatrix} \right)$	1.14 $^{+0.27}_{-0.25}$ $\left( \begin{smallmatrix} +0.26 \\ -0.24 \end{smallmatrix} \right)$	1.11 $^{+0.25}_{-0.23}$ $\left( \begin{smallmatrix} +0.23 \\ -0.21 \end{smallmatrix} \right)$
$\mu^{ZZ}$	1.29 $^{+0.26}_{-0.23}$ $\left( \begin{smallmatrix} +0.23 \\ -0.20 \end{smallmatrix} \right)$	1.52 $^{+0.40}_{-0.34}$ $\left( \begin{smallmatrix} +0.32 \\ -0.27 \end{smallmatrix} \right)$	1.04 $^{+0.32}_{-0.26}$ $\left( \begin{smallmatrix} +0.30 \\ -0.25 \end{smallmatrix} \right)$
$\mu^{WW}$	1.09 $^{+0.18}_{-0.16}$ $\left( \begin{smallmatrix} +0.16 \\ -0.15 \end{smallmatrix} \right)$	1.22 $^{+0.23}_{-0.21}$ $\left( \begin{smallmatrix} +0.21 \\ -0.20 \end{smallmatrix} \right)$	0.90 $^{+0.23}_{-0.21}$ $\left( \begin{smallmatrix} +0.23 \\ -0.20 \end{smallmatrix} \right)$
$\mu^{\tau\tau}$	1.11 $^{+0.24}_{-0.22}$ $\left( \begin{smallmatrix} +0.24 \\ -0.22 \end{smallmatrix} \right)$	1.41 $^{+0.40}_{-0.36}$ $\left( \begin{smallmatrix} +0.37 \\ -0.33 \end{smallmatrix} \right)$	0.88 $^{+0.30}_{-0.28}$ $\left( \begin{smallmatrix} +0.31 \\ -0.29 \end{smallmatrix} \right)$
$\mu^{bb}$	0.70 $^{+0.29}_{-0.27}$ $\left( \begin{smallmatrix} +0.29 \\ -0.28 \end{smallmatrix} \right)$	0.62 $^{+0.37}_{-0.37}$ $\left( \begin{smallmatrix} +0.39 \\ -0.37 \end{smallmatrix} \right)$	0.81 $^{+0.45}_{-0.43}$ $\left( \begin{smallmatrix} +0.45 \\ -0.43 \end{smallmatrix} \right)$
$\mu^{\mu\mu}$	0.1 $^{+2.5}_{-2.5}$ $\left( \begin{smallmatrix} +2.4 \\ -2.3 \end{smallmatrix} \right)$	-0.6 $^{+3.6}_{-3.6}$ $\left( \begin{smallmatrix} +3.6 \\ -3.6 \end{smallmatrix} \right)$	0.9 $^{+3.6}_{-3.5}$ $\left( \begin{smallmatrix} +3.3 \\ -3.2 \end{smallmatrix} \right)$

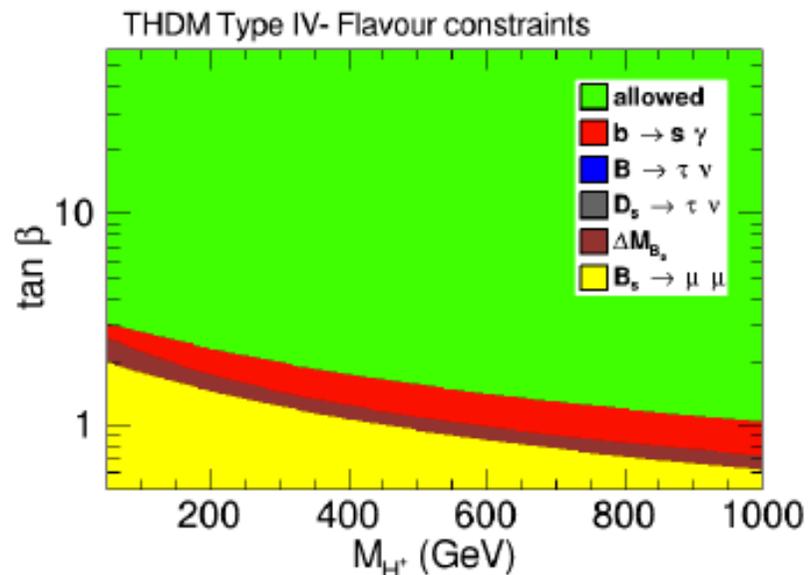
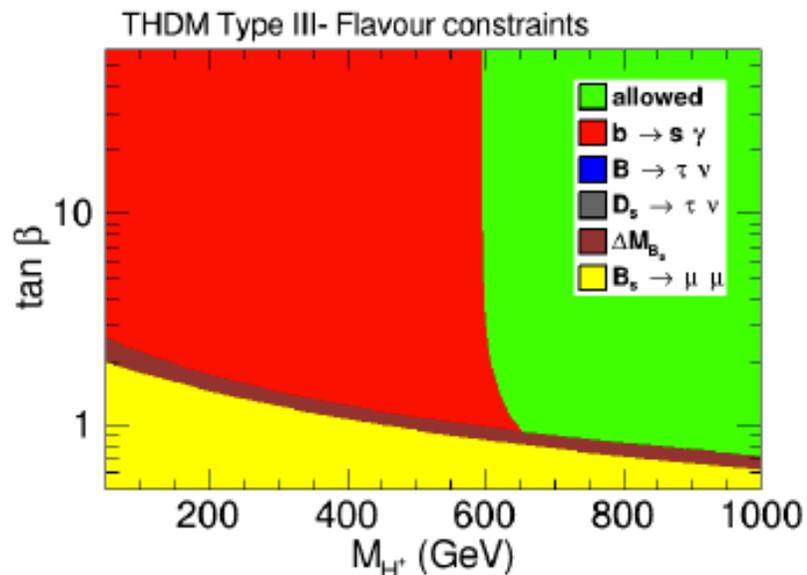
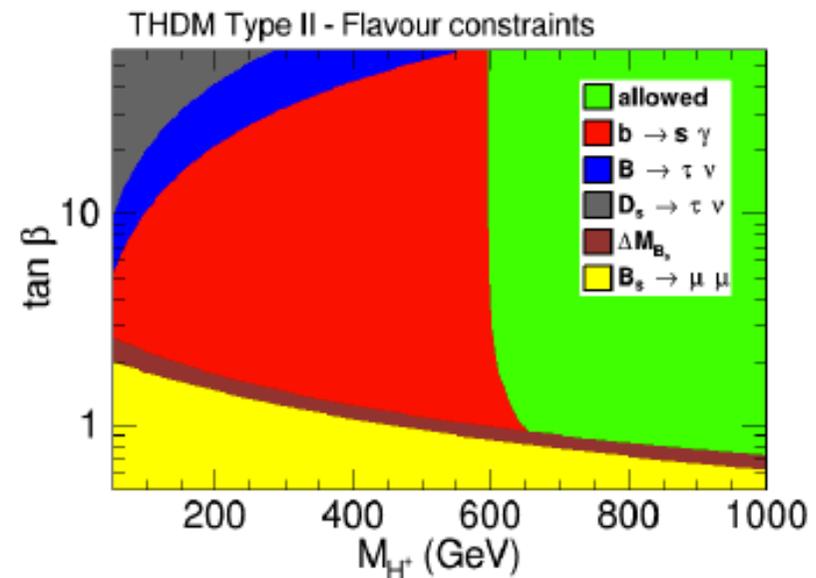
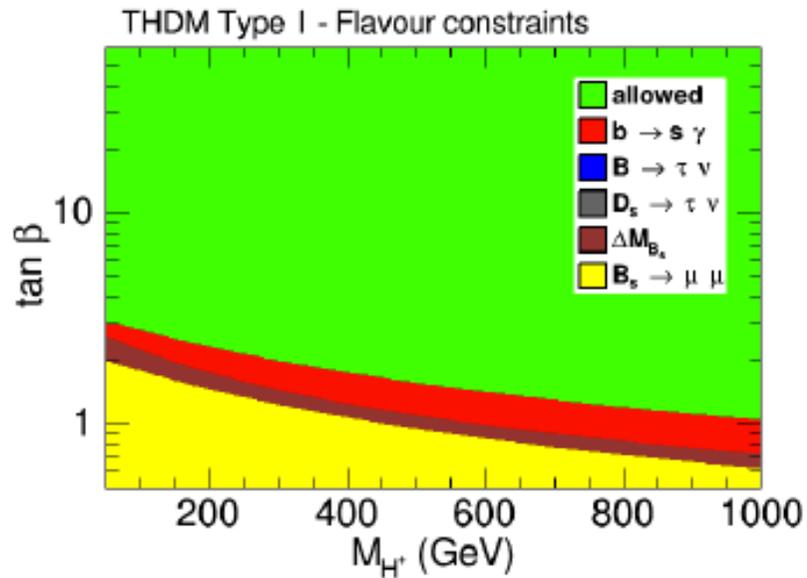
# Signal strength by ILC (prospect)

ArXiv: 1310.8361

	ILC		ILC LumiUp <sup>‡</sup>	
	250/500/1000 GeV		250/500/1000 GeV	
	$ZH$	$\nu\bar{\nu}H$	$ZH$	$\nu\bar{\nu}H$
Inclusive	2.6/3.0/–%	–	1.2/1.7/–%	–
$H \rightarrow \gamma\gamma$	29-38%	–/20-26/7-10%	16/19/–%	–/13/5.4%
$H \rightarrow gg$	7/11/–%	–/4.1/2.3%	3.3/6.0/–%	–/2.3/1.4%
$H \rightarrow ZZ^*$	19/25/–%	–/8.2/4.1%	8.8/14/–%	–/4.6/2.6%
$H \rightarrow WW^*$	6.4/9.2/–%	–/2.4/1.6%	3.0/5.1/–%	–/1.3/1.0%
$H \rightarrow \tau\tau$	4.2/5.4/–%	–/9.0/3.1%	2.0/3.0/–%	–/5.0/2.0%
$H \rightarrow b\bar{b}$	1.2/1.8/–%	11/0.66/0.30%	0.56/1.0/–%	4.9/0.37/0.30%
$H \rightarrow c\bar{c}$	8.3/13/–%	–/6.2/3.1%	3.9/7.2/–%	–/3.5/2.0%
$H \rightarrow \mu\mu$	–	–/–/31%	–	–/–/20%
	$t\bar{t}H$		$t\bar{t}H$	
$H \rightarrow b\bar{b}$	–/28/6.0%		–/16/3.8%	

# Constraint from flavor experiments

A. Arbey, F. Mahmoudi, O. Stal T. Stefaniak [arXiv:1706.07414v1](https://arxiv.org/abs/1706.07414v1)



# Higgs singlet model(HSM)

• Higgs potential  $\Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + \phi + iG^0) \end{pmatrix}, \quad S = v_S + s.$

$$V(\Phi, S) = m_\Phi^2 |\Phi|^2 + \lambda |\Phi|^4 + \mu_{\Phi S} |\Phi|^2 S + \lambda_{\Phi S} |\Phi|^2 S^2 + t_S S + m_S^2 S^2 + \mu_S S^3 + \lambda_S S^4$$

- Mass eigenstates

$$\begin{pmatrix} s \\ \phi \end{pmatrix} = R(\alpha) \begin{pmatrix} H \\ h \end{pmatrix} \quad \text{with} \quad R(\alpha) = \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix}$$

Physical state :  $h, H$

- Physical parameters

$$v, m_h, m_H, \alpha, m_S^2, \lambda_S, \mu_{\Phi S}$$

# Two Higgs doublet model (THDM)

- Higgs potential

$$\Phi_i = \begin{pmatrix} w_i^+ \\ \frac{1}{\sqrt{2}}(v_i + h_i + iz_i) \end{pmatrix} \quad \text{with } i = 1, 2$$

$$V = m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 - m_3^2 (\Phi_1^\dagger \Phi_2 + \text{h.c.}) \\ + \frac{1}{2} \lambda_1 |\Phi_1|^4 + \frac{1}{2} \lambda_2 |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \frac{1}{2} \lambda_5 [(\Phi_1^\dagger \Phi_2)^2 + \text{h.c.}]$$

- Mass eigenstates

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = R(\alpha) \begin{pmatrix} H \\ h \end{pmatrix}, \quad \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = R(\beta) \begin{pmatrix} z \\ A \end{pmatrix}, \quad \begin{pmatrix} w_1^+ \\ w_2^+ \end{pmatrix} = R(\beta) \begin{pmatrix} w^+ \\ H^+ \end{pmatrix}$$

Physical state :  $h, H, A, H^\pm$

- Physical parameters

$$v, m_h, m_H, m_A, m_{H^\pm}, \alpha, \beta, M^2$$

Scaling factor

$$\kappa_X = \Gamma_{hXX}^{EX} / \Gamma_{hXX}^{SM}$$

THDM :

$$\Gamma(h \rightarrow \gamma\gamma) \simeq \frac{G_F \alpha_{\text{em}}^2 m_h^3}{128 \sqrt{2} \pi^3} \left| -\frac{1}{3} \left( 1 - \frac{M^2}{m_{H^\pm}^2} \right) + \sum_f Q_f N_c^f \left( 1 + \xi_f x - \frac{x^2}{2} \right) I_F + \left( 1 - \frac{x^2}{2} \right) I_W \right|^2$$

HSM :

$$\kappa_\gamma = \cos \alpha^2$$

# Definition of form factors for $hVV$ and $hff$

$$\hat{\Gamma}_{hVV}^{\mu\nu} = \hat{\Gamma}_{hVV}^1 g^{\mu\nu} + \hat{\Gamma}_{hVV}^2 \frac{p_1^\mu p_2^\nu}{m_V^2} + i \hat{\Gamma}_{hVV}^3 \epsilon^{\mu\nu\rho\sigma} \frac{p_{1\rho} p_{2\sigma}}{m_V^2},$$

$$\begin{aligned} \hat{\Gamma}_{hff} = & \hat{\Gamma}_{hff}^S + \gamma_5 \hat{\Gamma}_{hff}^P + \not{p}_1 \hat{\Gamma}_{hff}^{V1} + \not{p}_2 \hat{\Gamma}_{hff}^{V2} \\ & + \not{p}_1 \gamma_5 \hat{\Gamma}_{hff}^{A1} + \not{p}_2 \gamma_5 \hat{\Gamma}_{hff}^{A2} + \not{p}_1 \not{p}_2 \hat{\Gamma}_{hff}^T + \not{p}_1 \not{p}_2 \gamma_5 \hat{\Gamma}_{hff}^{PT}, \end{aligned}$$

$hVV$ : 7 form factors

$hff$ : 3 form factors

# Another tools

## Prophecy4f :

- Model: THDMs, HSM
- $h \rightarrow WW/ZZ \rightarrow 4$  fermions

[arXiv:1710.07598](https://arxiv.org/abs/1710.07598)

## RECOLA2 :

- Model: THDMs, HSM
- Calculation to NLO amplitude

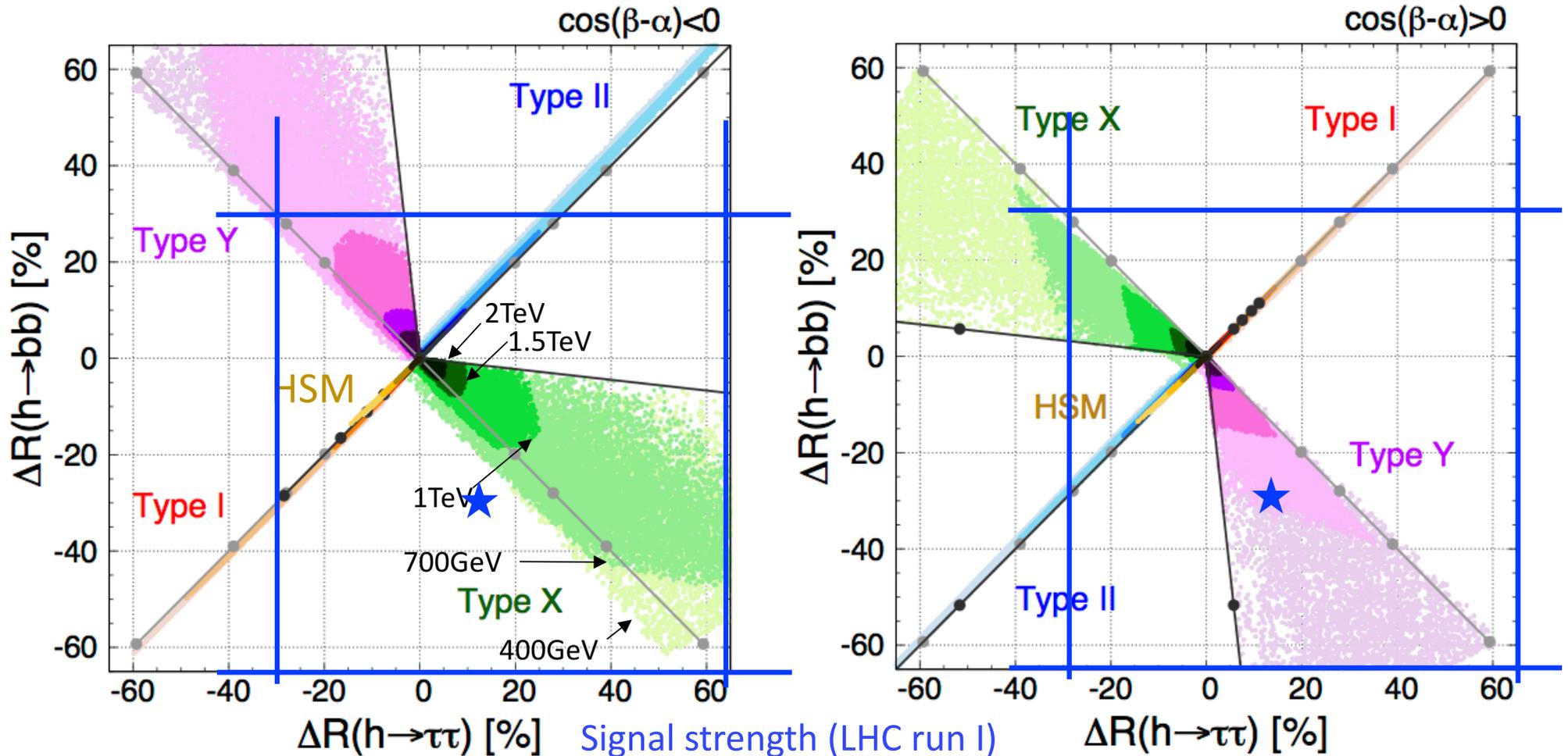
[arXiv:1711.07388](https://arxiv.org/abs/1711.07388)

**Other plot**

# $\Delta R(h \rightarrow b\bar{b})$ vs $\Delta R(h \rightarrow \tau\bar{\tau})$

[S. Kanemura, M. Kikuchi, K. Mawatari, KS, K. Yagyu]

$$\Delta R(h \rightarrow XX) = \frac{\Gamma(h \rightarrow XX)_{EX}}{\Gamma(h \rightarrow XX)_{SM}} - 1$$



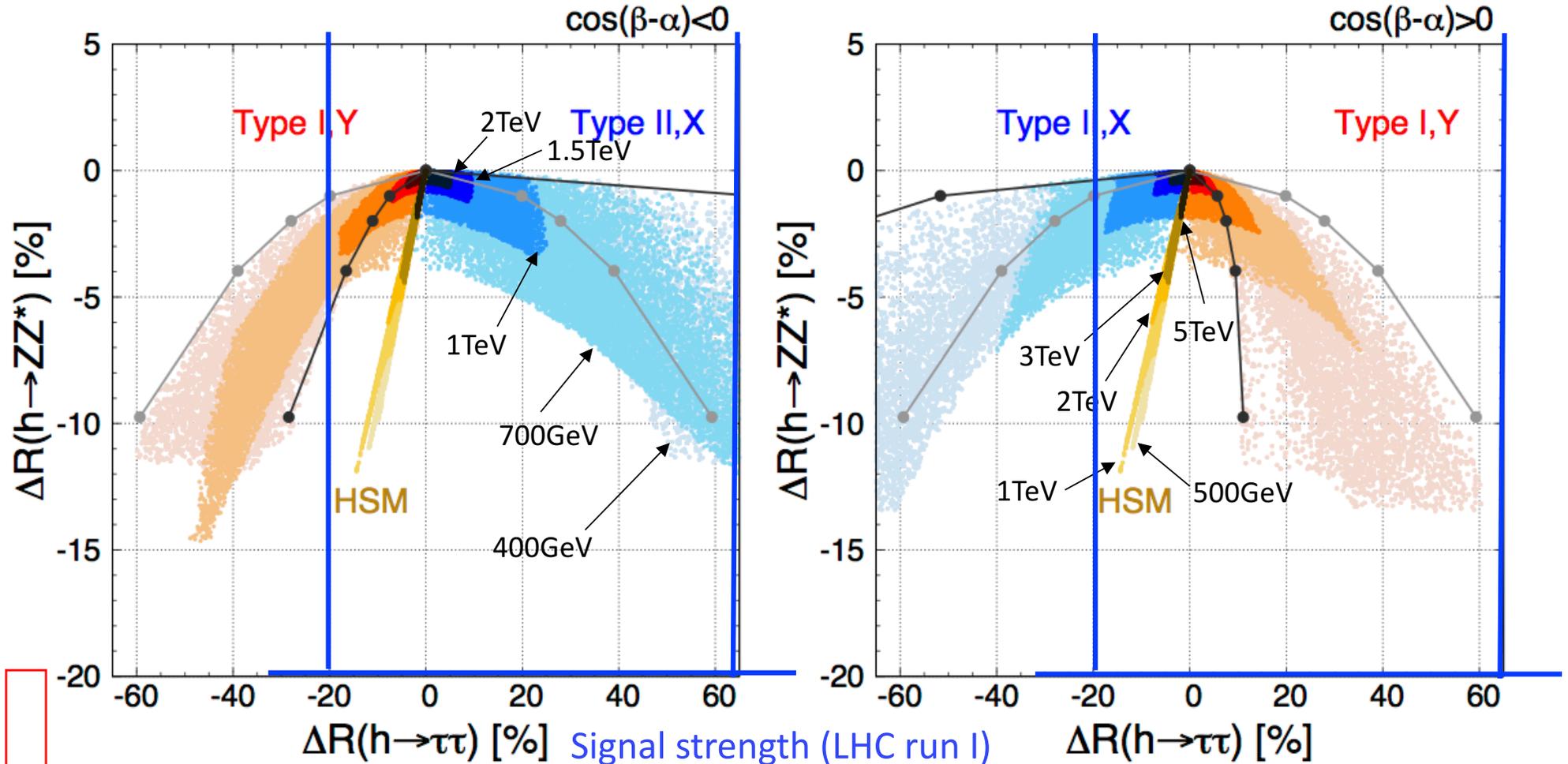
ILC250 ( $2\sigma$ ) [K. Fujii, et al., arXiv:1710.07621]

→ Type X and Y can be discriminated from other models.

# $\Delta R(h \rightarrow ZZ^*)$ vs $\Delta R(h \rightarrow \tau\bar{\tau})$

[S. Kanemura, M. Kikuchi, K. Mawatari, KS, K. Yagyu]

$$\Delta R(h \rightarrow XX) = \frac{\Gamma(h \rightarrow XX)_{EX}}{\Gamma(h \rightarrow XX)_{SM}} - 1$$



ILC250 ( $2\sigma$ ) [K. Fujii, et al., arXiv:1710.07621]

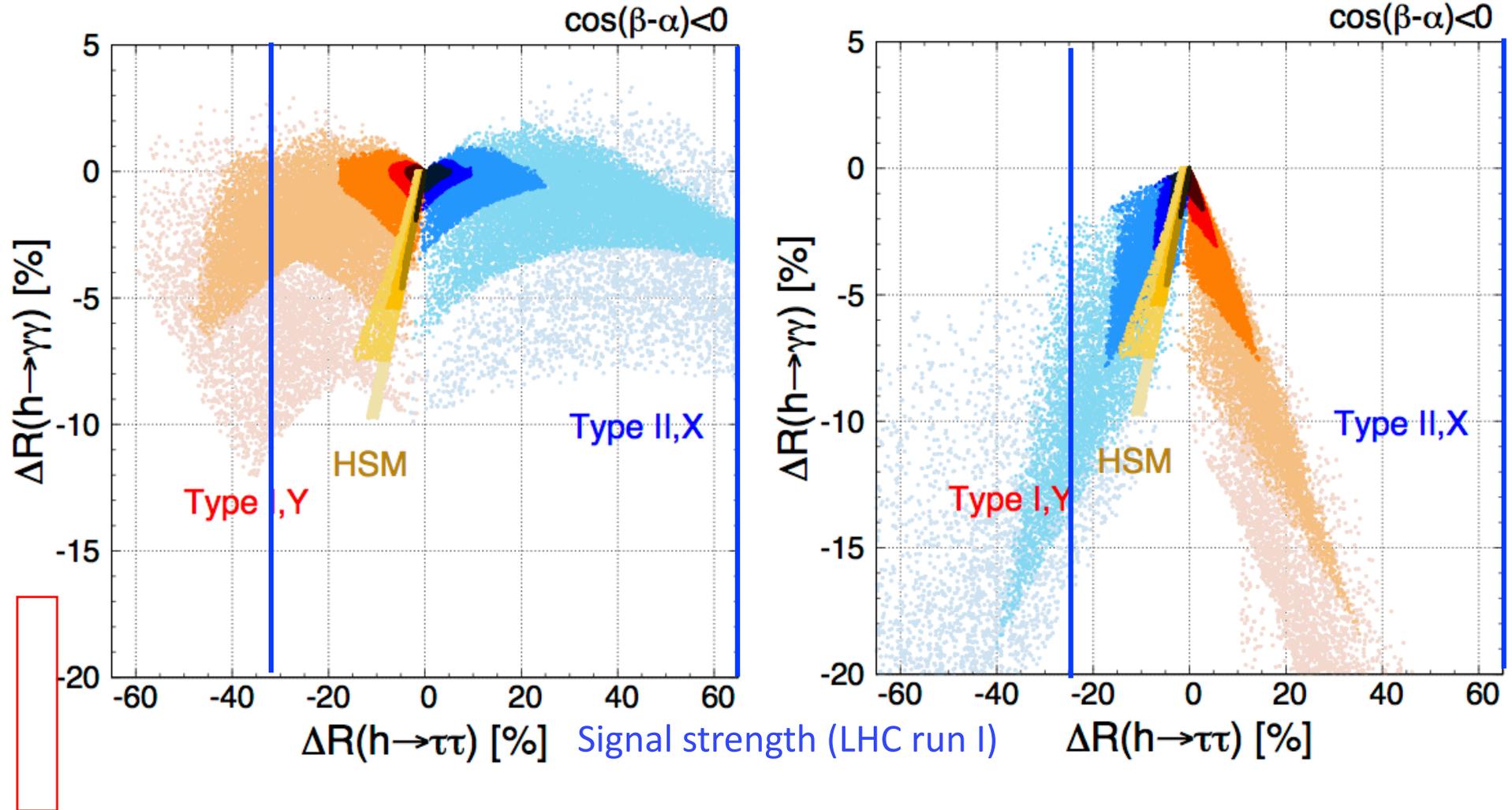
→ HSM can be separated from the THDMs.

→ Type I and Type II can be discriminated in this plane.

# $\Delta R(h \rightarrow \gamma\gamma)$ vs $\Delta R(h \rightarrow \tau\bar{\tau})$

[S. Kanemura, M. Kikuchi, K. Mawatari, KS, K. Yagyu]

$$\Delta R(h \rightarrow XX) = \frac{\Gamma(h \rightarrow XX)_{EX}}{\Gamma(h \rightarrow XX)_{SM}} - 1$$



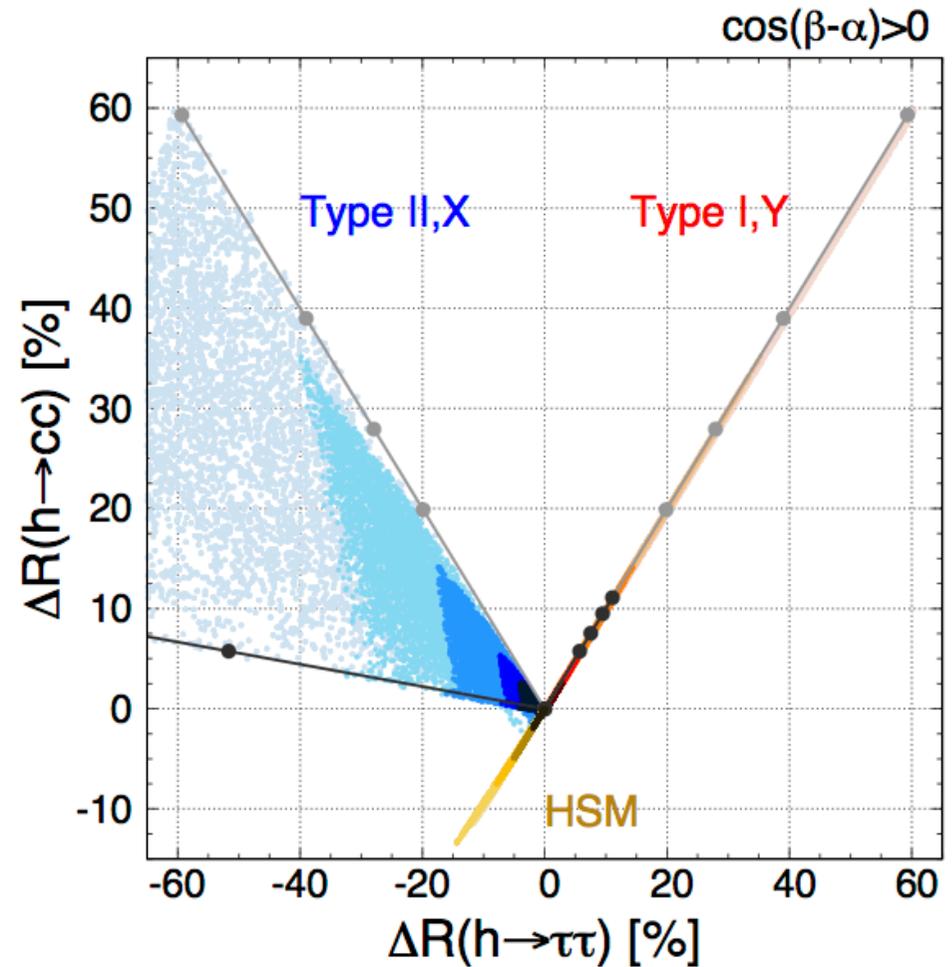
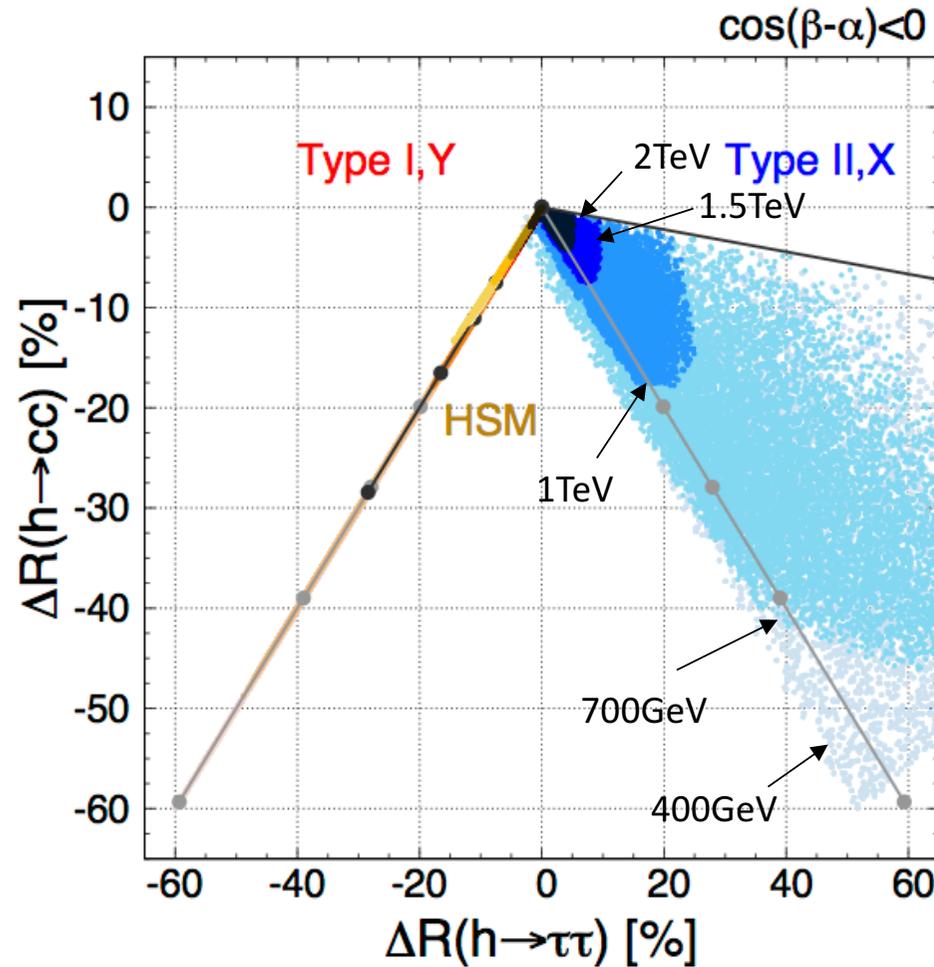
ILC250 ( $2\sigma$ ) [K. Fujii, et al., arXiv:1710.07621]

→ Type X and Y can be discriminated from other models.

# $\Delta R(h \rightarrow c\bar{c})$ vs $\Delta R(h \rightarrow \tau\bar{\tau})$

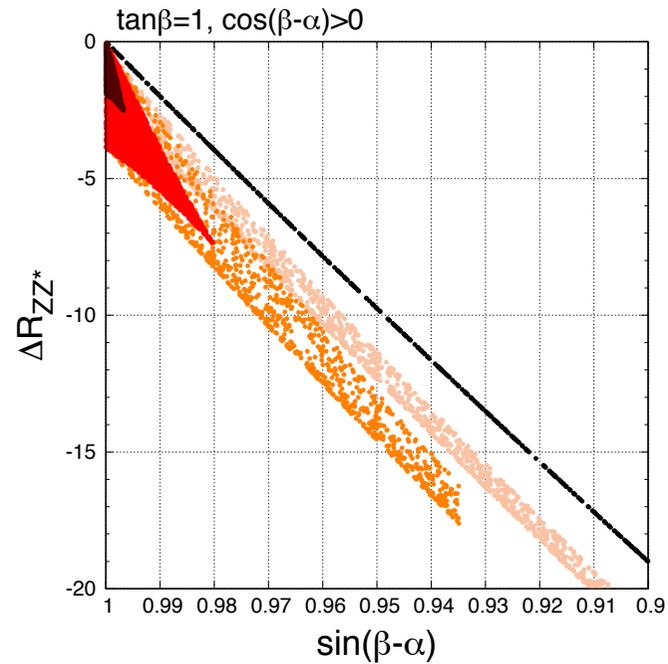
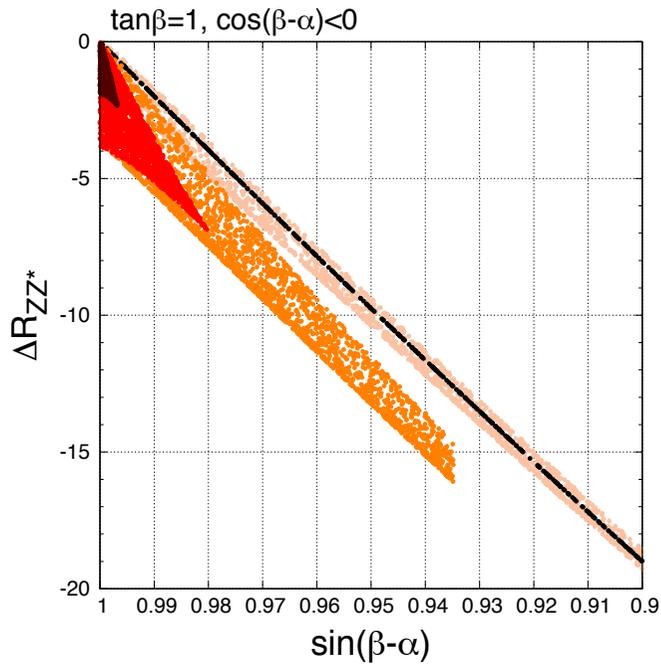
[S. Kanemura, M. Kikuchi, K. Mawatari, KS, K. Yagyu]

$$\Delta R(h \rightarrow XX) = \frac{\Gamma(h \rightarrow XX)_{EX}}{\Gamma(h \rightarrow XX)_{SM}} - 1$$

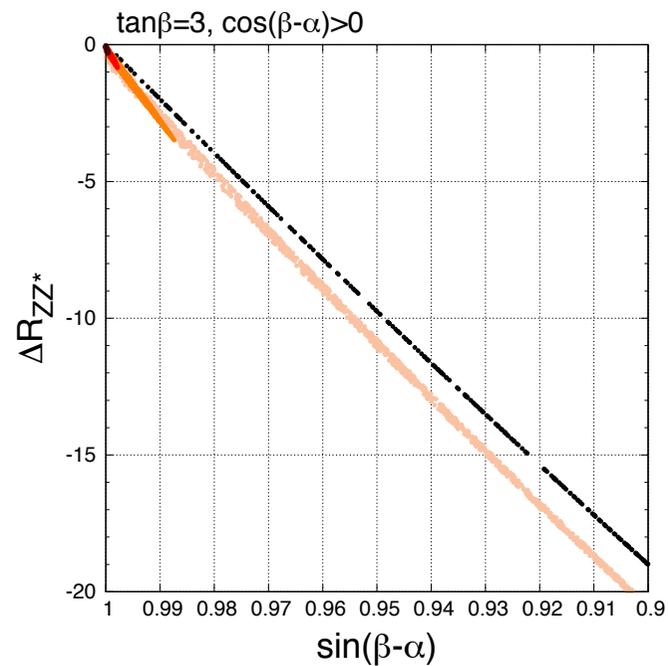
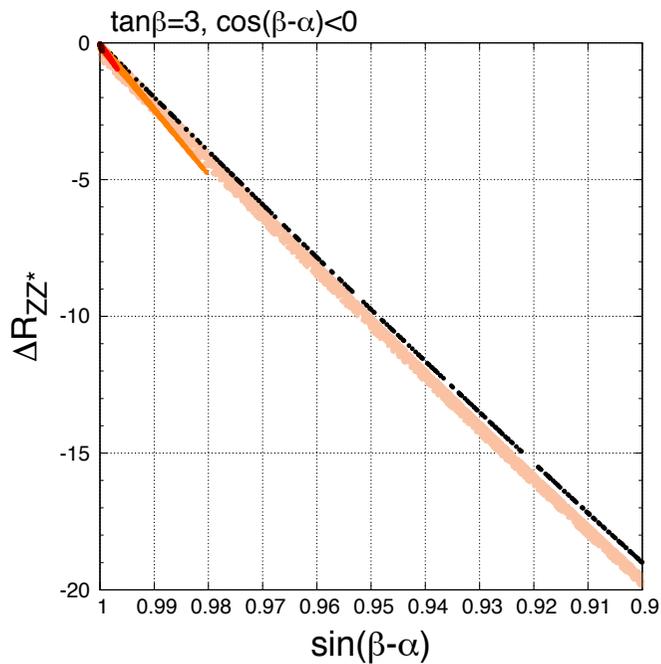


→ Type I and Type II can be discriminated in this plane.

# $\Delta R(h \rightarrow ZZ^*)$ vs $\sin(\beta - \alpha)$



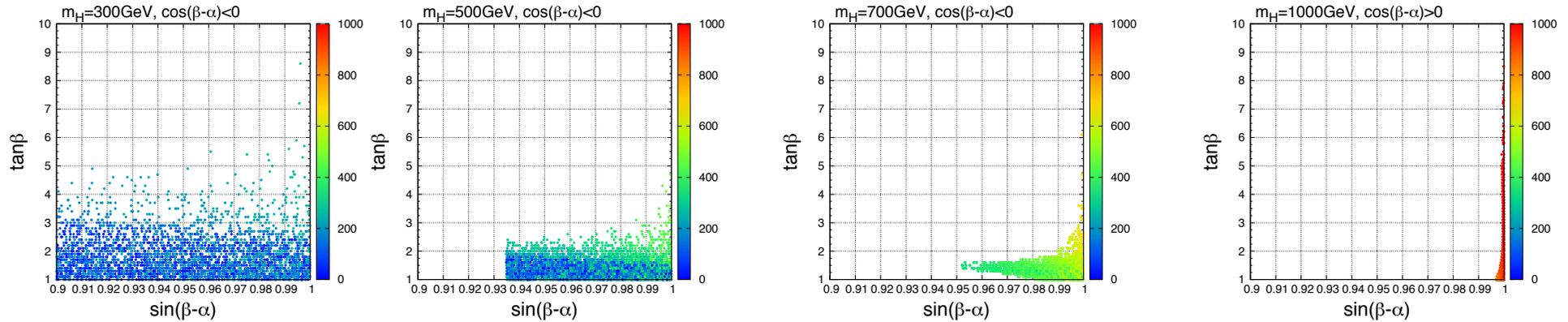
300GeV,  
500GeV,  
700GeV,  
1TeV



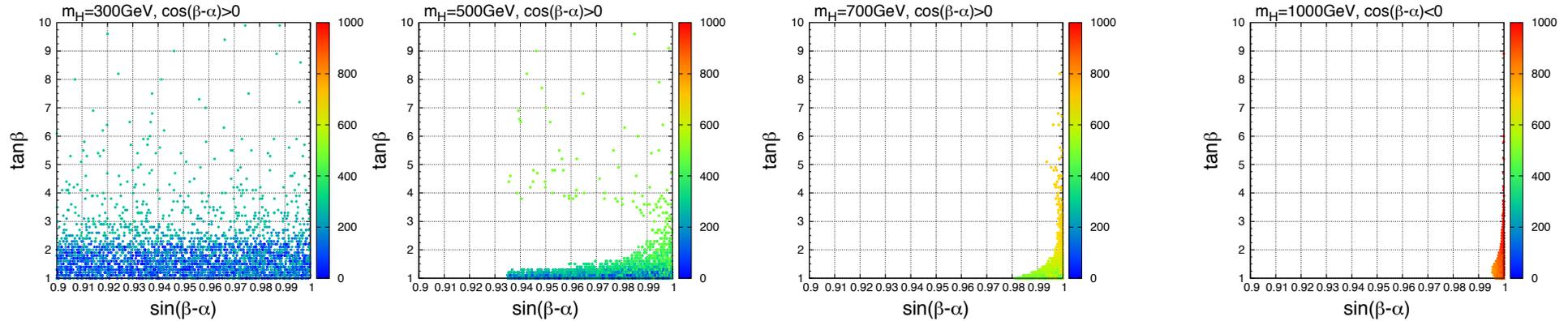
# $\tan\beta$ vs $\sin(\beta - \alpha)$

constraint:  
Perturbative unitarity  
vacuum stability  
S, T parameters

M



M



# Renormalization of scalar sector

# Renormalization of Higgs sector

We have used **improved on-shell scheme renormalization** in calculations for Higgs boson couplings at the 1 loop level. [S. Kanemura, M. Kikuchi, KS, K. Yagyu, PRD96,035014]

- We introduce counter terms

- Parameters in Higgs potential : 8

$$v, m_h, m_H, m_A, m_{H^\pm}, M^2, \alpha, \beta$$

- Fields of Higgs sector : 6

$$h, H^\pm, H, A, G^0, G^\pm$$

- Shift of parameters : ( $\Phi = h, H^\pm, H, A$ )

$$m_\Phi \rightarrow m_\Phi + \delta m_\Phi, M \rightarrow M + \delta M, \alpha \rightarrow \alpha + \delta\alpha, \beta \rightarrow \beta + \delta\beta,$$

$$\begin{pmatrix} H \\ h \end{pmatrix} \rightarrow \begin{pmatrix} 1 + \frac{1}{2}\delta Z_H & \delta C_{Hh} + \delta\alpha \\ \delta C_{hH} - \delta\alpha & 1 + \frac{1}{2}\delta Z_h \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}$$

→ We obtain 20 counter terms :

$$v, \delta m_\Phi^2, \delta M^2, \delta\alpha, \delta\beta, \delta Z_\Phi, \delta C_{\Phi_1\Phi_2} \quad (\Phi = h, H, A, H^\pm, G^0, G^\pm)$$

$$\hat{\Pi}_{ij}(q^2) \equiv i \text{---} \textcircled{1\text{PI}} \text{---} j + \begin{array}{c} \textcircled{1\text{PI}} \\ | \\ i \text{---} \text{---} h, H \text{---} j \end{array} + \text{---} \textcircled{\times} \text{---}$$

- We set following on shell conditions :

$$\underline{\delta m}_\Phi : \hat{\Pi}_{\varphi\varphi}(m_\varphi^2) = 0 \quad (\Phi = h, H, A, H^\pm)$$

$$\underline{\delta Z}_\Phi : \frac{d}{dp^2} \hat{\Pi}_{\varphi\varphi}(p^2) \Big|_{p^2=m_\varphi^2} = 0 \quad (\Phi = h, H, A, H^\pm, G^\pm, G^0)$$

$$\delta\alpha, \delta\beta, \delta C_{\Phi_1\Phi_2} : \hat{\Pi}_{\phi_1\phi_2}(m_{\phi_1}^2) = \hat{\Pi}_{Hh}(m_{\phi_2}^2) = 0 \quad (\Phi_1, \Phi_2 = h, H, A, H^\pm, G^\pm, G^0)$$

$\delta v$  : This counter term is determined in gauge sector.

$\delta M$  : This counter term is determined by minimal subtraction

In this way, with above 20 renormalization conditions, all counter terms are determined.

# Gauge dependence on mixing parameters

- for renormalization of scalar mixing parameters  $\delta\alpha, \delta\beta$  originally, We had used ordinary on-shell scheme.
- Renormalized Higgs observables in ordinal on-shell scheme possess a gauge dependence through  $\delta\alpha, \delta\beta$ .  
[ M. Krause, R. Lorenz, M. Muhlleitner, R. Santos, H. Ziesche, JHEP 09 (2016) 143 ]
- We have removed this gauge dependence by using the pinch technique.  
[J. Papavassiliou, PRD50, 5958]

# Gauge dependence on the counter terms

$$\Pi_{ij}(q^2) \equiv i \text{ --- } \textcircled{1\text{PI}} \text{ --- } j + i \text{ --- } \textcircled{1\text{PI}} \text{ --- } \textcircled{h, H} \text{ --- } j$$

Nielsen identity : [N. K. Nielsen, NPB101 (1975) 173, Y. Yamada, PRD64(2001)036008]

$$\partial_{\xi} \Pi_{ij}(q^2) = (q^2 - m_i^2) \Lambda_i(q^2) + (q^2 - m_j^2) \Lambda_j(q^2)$$

$\Lambda_i(q^2), \Lambda_j(q^2)$ : sum of loop function

ex.1)  $\delta m_h^2$  (the counter term of the Higgs boson mass  $m_h^2$ )

$$\delta m_h^2 = \Pi_{hh}(m_h^2)$$

From the Nielsen identity  $q^2 = m_h^2, m_i^2 = m_j^2 = m_h^2$

$$\Rightarrow \partial_{\xi}(\delta m_h^2) = 0$$

# Gauge dependence on the counter terms

$$\Pi_{ij}(q^2) \equiv i \text{ --- } \textcircled{1\text{PI}} \text{ --- } j + \textcircled{1\text{PI}} \text{ --- } \textcircled{h, H} \text{ --- } j$$

Nielsen identity : [N. K. Nielsen, NPB101 (1975) 173, Y. Yamada, PRD64(2001)036008]

$$\partial_\xi \Pi_{ij}(q^2) = (q^2 - m_i^2) \Lambda_i(q^2) + (q^2 - m_j^2) \Lambda_j(q^2)$$

$\Lambda_i(q^2), \Lambda_j(q^2)$ : sum of loop function

ex.2)  $\delta\beta$  (the counter term of the mixing angle  $\beta$  )

$$\delta\beta = -\frac{1}{2m_A^2} [\Pi_{AG^0}(m_A^2) + \Pi_{AG^0}(0)]$$

Applying to the Nielsen identity

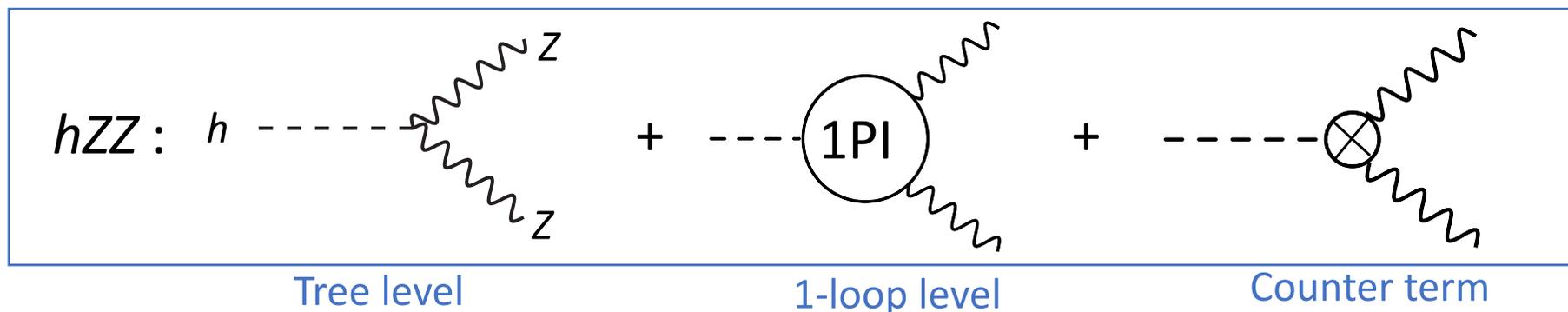
$$\partial_\xi(\delta\beta) = -\frac{1}{2m_A^2} [(m_A^2 - 0)\Lambda_G(0) + (0 - m_A^2)\Lambda_A(0)]$$

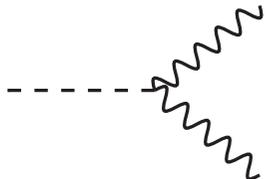
➡  $\partial_\xi(\delta\beta) \neq 0$

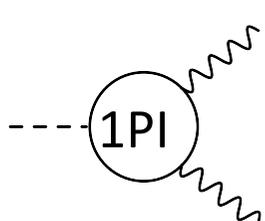
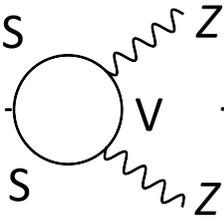
With the same argument, we can also find  $\partial_\xi(\delta\alpha) \neq 0$

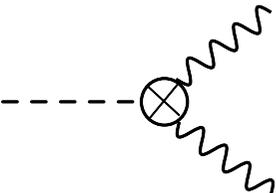
# Gauge dep. for Higgs 3point vertex

EX.) renormalized  $hZZ$  vertex



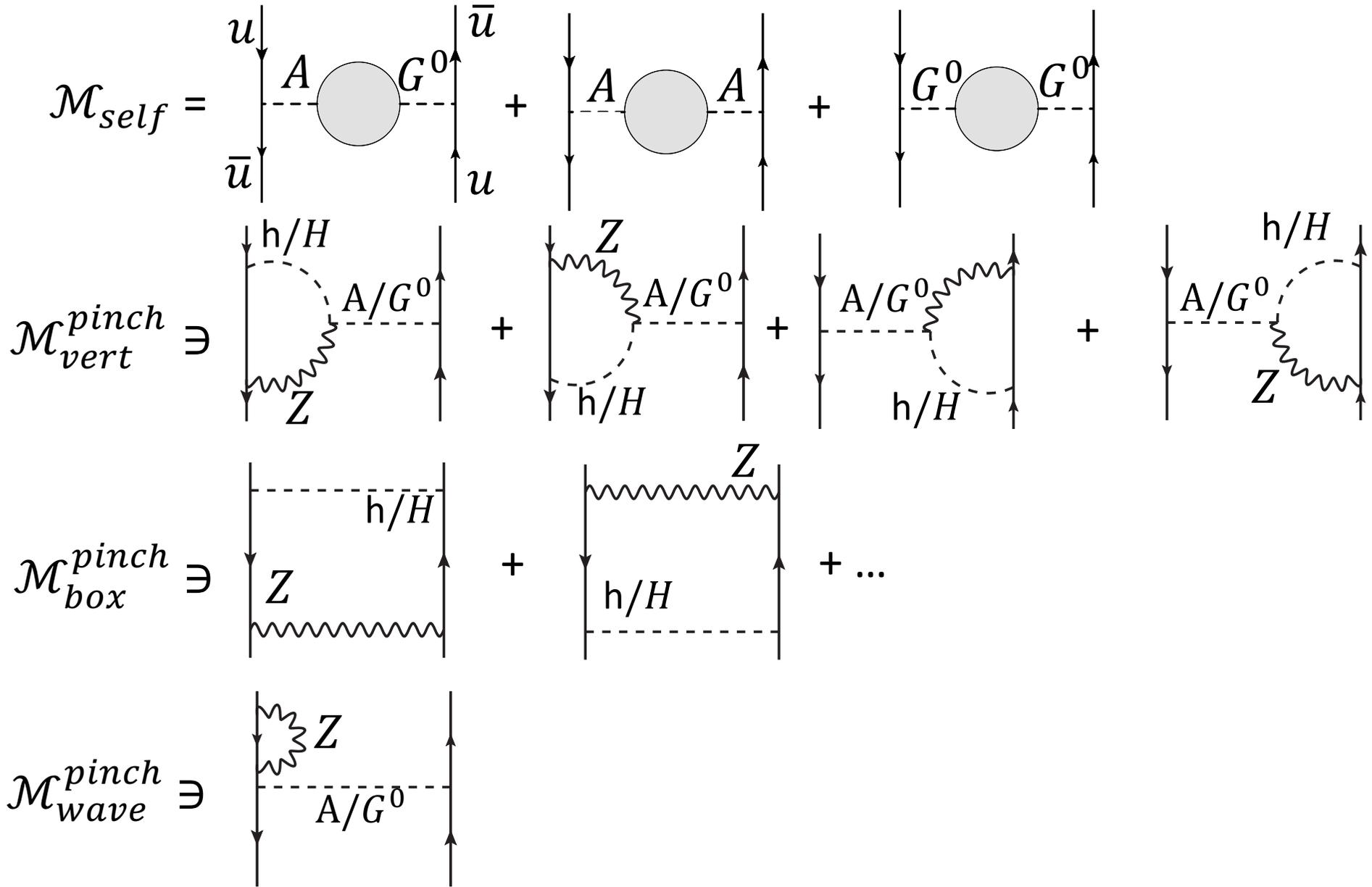
tree level   $= -2 \frac{m_Z^2}{v} \sin(\beta - \alpha)$

1-loop level   $= h$    $+ \dots$

counter term   $= -2 \frac{m_Z^2}{v} \left[ s_{\beta-\alpha} \left( \frac{\delta m_Z^2}{m_Z^2} + \delta Z_Z + \frac{1}{2} \delta Z_h - \frac{\delta v}{v} \right) + c_{\beta-\alpha} (\delta\beta + \delta C_h) \right]$

# Pinch technique

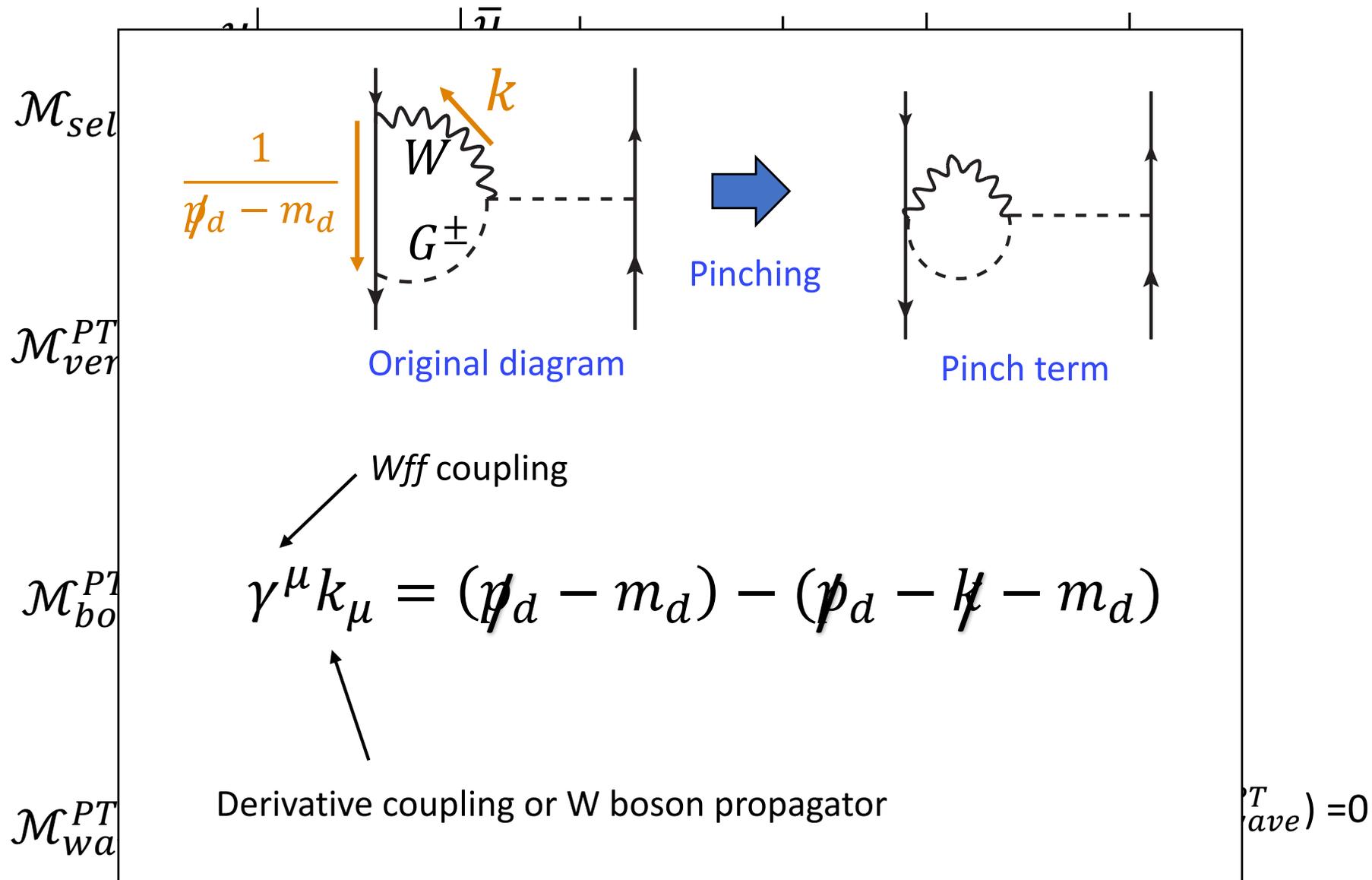
I demonstrate that  $\xi_Z$  dependence for  $\Pi_{AG}(q^2)$  are removed.



# Pinch technique

Toy process :  $u\bar{u} \rightarrow u\bar{u}$

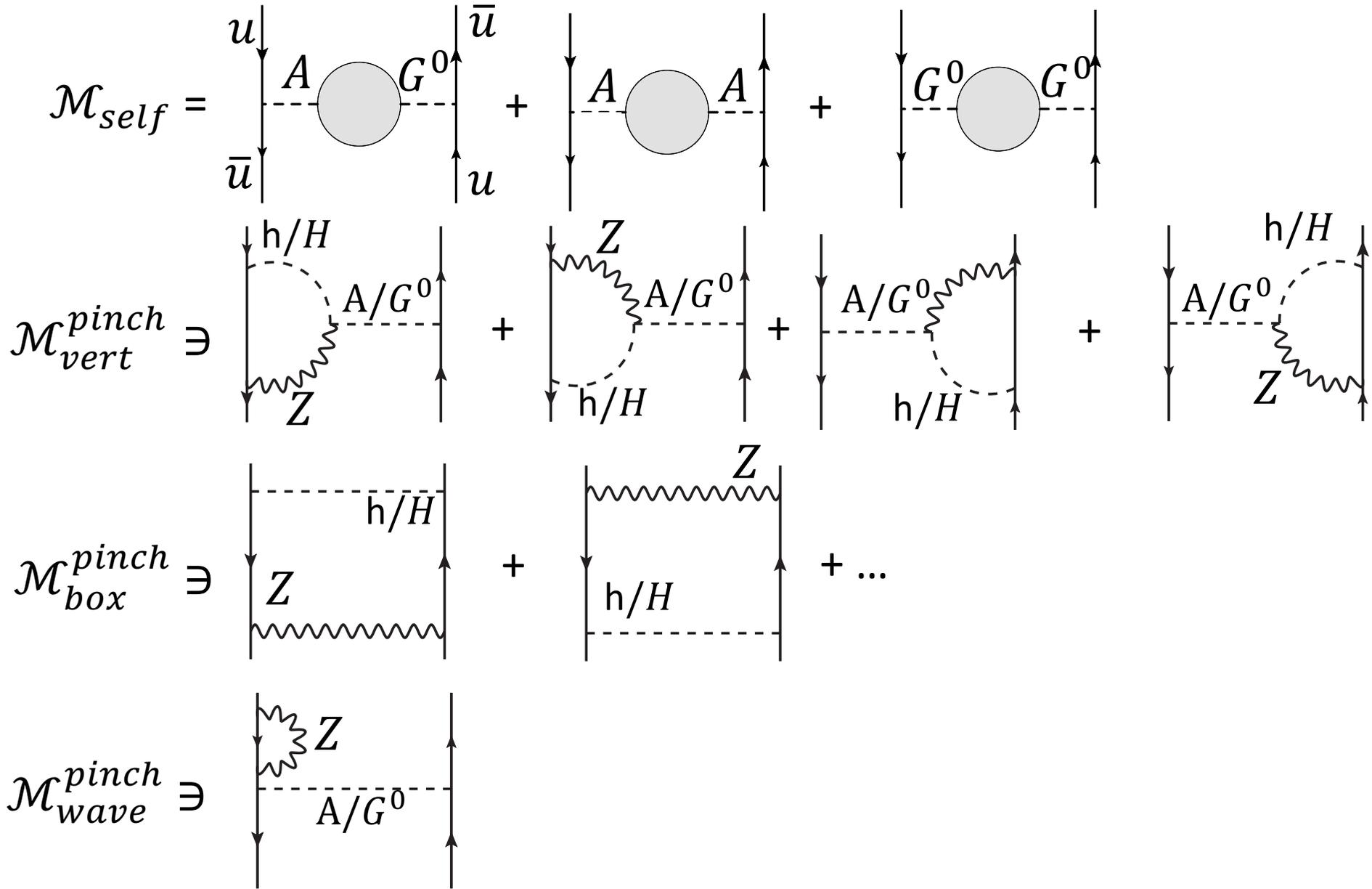
$\mathcal{M}_{vert}^{PT}, \mathcal{M}_{box}^{PT}, \mathcal{M}_{wave}^{PT}$  : Pinch term



We obtained the gauge invariant  $\Pi_{hH}$ ,  
adding the pinch terms

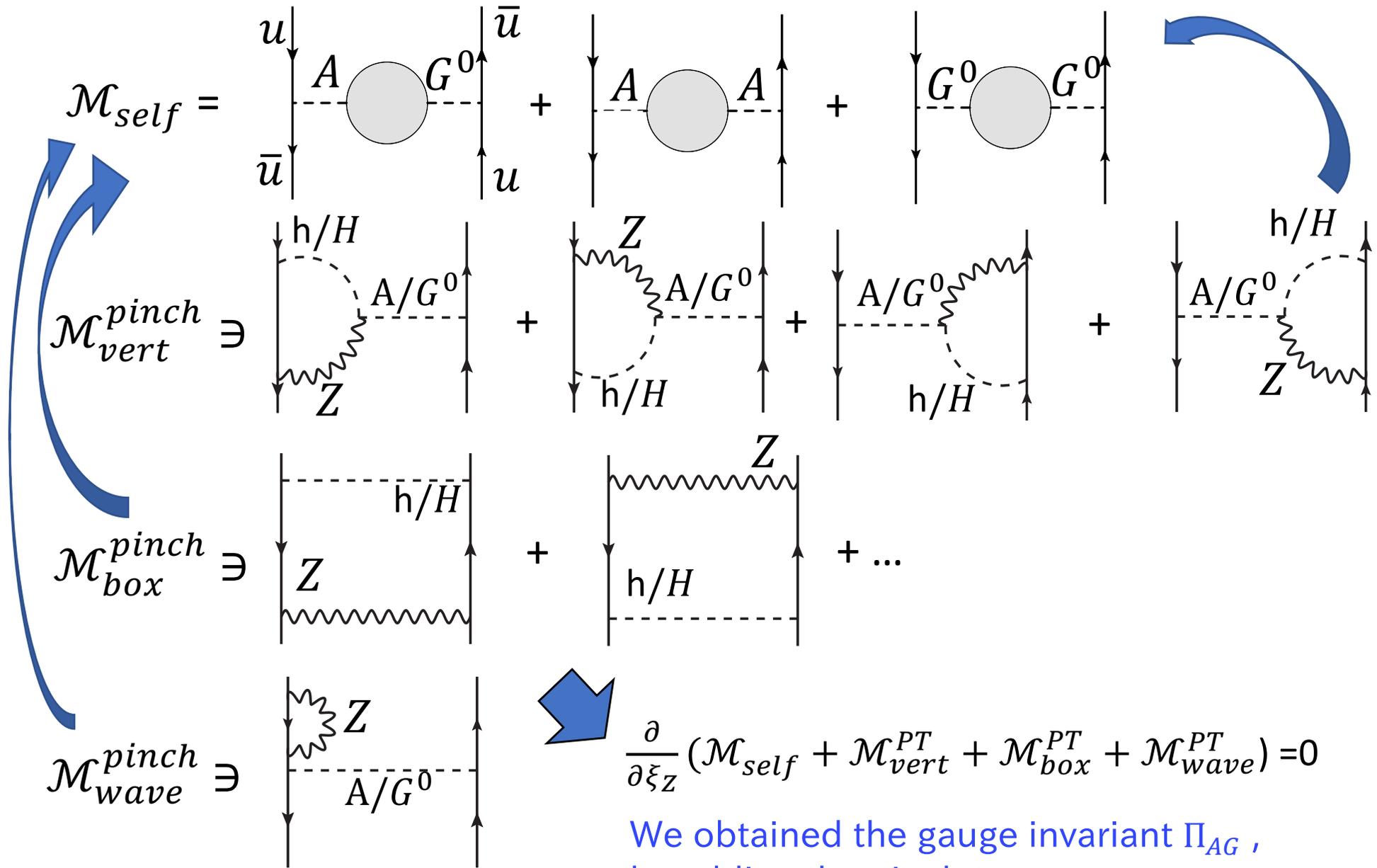
# Pinch technique

I demonstrate that  $\xi_Z$  dependence for  $\Pi_{AG}(q^2)$  are removed.



# Pinch technique

I demonstrate that  $\xi_Z$  dependence for  $\Pi_{AG}(q^2)$  are removed.



We obtained the gauge invariant  $\Pi_{AG}$ ,  
by adding the pinch terms