Theory and phenomenology of multi-Higgs effective low-energy field theories of Grand-unified Quartification: status report and the road ahead

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“Top-bottom” approach in BSM building

Despite of remarkable consistency of the SM with to-date observations, it remains remarkably unsatisfactory. We have to

(i) to explain why/where its features as they are/originate from,

(ii) to better understand certain features that cannot be explained by the SM (neutrino sector, Dark Matter, Higgs “generations” and properties etc etc)

One possible way:

- **Postulate: a consistent SUSY GUT valid at high energy scales**
- **Explore: the larger the symmetry, the fewer free parameters, the higher degree of unification**
- **Study: to what extent the SM emerges as an EFT at low energies?**
- **Hope: to explain seemingly arbitrary features of the SM (number of families, mass/mixing hierarchies, neutrino status etc)**

Any good candidates?

Old story of Trinification

- The trinification gauge group (Glashow, '84)

$$[\mathrm{SU}(3)_\mathrm{L}\times\mathrm{SU}(3)_\mathrm{R}\times\mathrm{SU}(3)_\mathrm{C}]\rtimes\mathbb{Z}_3^{(\mathrm{LRC})}$$
$$\downarrow$$
$$\mathrm{SU}(3)_\mathrm{C}\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{SU}(2)_\mathrm{R}\times\mathrm{U}(1)_{\mathrm{L+R}}$$
$$\downarrow$$
$$\mathrm{SU}(3)_\mathrm{C}\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_\mathrm{Y}$$

- Subgroup of $\mathrm{E}_6\supset[\mathrm{SU}(3)]^3$

- SM fields sit in chiral superfields that are bi-fundamental representatic of the gauge group: $\mathbf{L}\sim(\mathbf{3},\bar{\mathbf{3}},\mathbf{1})$, $\mathbf{Q}_\mathrm{L}\sim(\bar{\mathbf{3}},\mathbf{1},\mathbf{3})$, and $\mathbf{Q}_\mathrm{R}\sim(\mathbf{1},\mathbf{3},\bar{\mathbf{3}})$:

$$(\mathbf{L}^i)^l{}_r=\begin{pmatrix}\mathbf{H}_{11} & \mathbf{H}_{12} & \nu_\mathrm{L}\\ \mathbf{H}_{21} & \mathbf{H}_{22} & \mathbf{e}_\mathrm{L}\\ \nu^c_\mathrm{R} & \mathbf{e}^c_\mathrm{R} & \phi\end{pmatrix}^i,\quad \begin{array}{l}(\mathbf{Q}^i_\mathrm{L})^x{}_l=\begin{pmatrix}\mathbf{u}^x_\mathrm{L} & \mathbf{d}^x_\mathrm{L} & \mathbf{D}^x_\mathrm{L}\end{pmatrix}^i,\\ (\mathbf{Q}^i_\mathrm{R})^r{}_x=\begin{pmatrix}\mathbf{u}^c_{\mathrm{R}x} & \mathbf{d}^c_{\mathrm{R}x} & \mathbf{D}^c_{\mathrm{R}x}\end{pmatrix}^{\top\ i},\end{array}$$

- Each family can be arranged into an E_6 **27**-plet:

$$\mathbf{27}^i=(\mathbf{3},\bar{\mathbf{3}},\mathbf{1})^i\otimes(\mathbf{1},\mathbf{3},\bar{\mathbf{3}})^i\otimes(\bar{\mathbf{3}},\mathbf{1},\mathbf{3})^i$$

Why Trinification?

Positives:

- The model accommodates any quark and lepton masses and mixings (Sayre et al 2016)
- Naturally light neutrinos via radiative see-saw with split-SUSY (Cauet et al 2011)
- Gauge symmetry preserve baryon number, i.e. no gauge-mediated proton decay (Achiman&Stech'78; Glashow&Kang'84)
- Motivated as a low-energy version of E8xE8 heterotic string theory (Gross et al 1985)
- GUT scale fermion masses through $L \cdot L' \cdot L''$ type operators
  - Higher dimensional operators needed **(Cauet et al. 2011)**

Negatives:

- Considerable amount of particles and many couplings involved
  - Realistic calculations cumbersome
- Unmotivated Hierarchy between the trinification and the EW breaking scales (common to most GUTs)

Trinification-based models were left as the least developed GUT scenarios

Quartification: SUSY Trinification with family symmetry

Our proposal: **Extend the SUSY trinification model (Georgi, Glashow and De Rujula 1984) with a local family $\mathrm{SU}(3)_\mathrm{F}$ symmetry**

$$[\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(3)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{R}] \rtimes \mathbb{Z}_3 \times \mathrm{SU}(3)_\mathrm{F}$$

We refer to the model as **Supersymmetric Higgs-Unified Trinification**

- Use the minimal field content:

$$(1, \mathbf{3}, \overline{\mathbf{3}}, \mathbf{3}) = (L^i)^l{}_r = \begin{pmatrix} H_\mathrm{u}^0 & H_\mathrm{d}^- & e_\mathrm{L} \\ H_\mathrm{u}^+ & H_\mathrm{d}^0 & \nu_\mathrm{L} \\ e_\mathrm{R} & \nu_\mathrm{R} & \phi \end{pmatrix}^i, \quad \begin{aligned} (\mathbf{3}, \overline{\mathbf{3}}, 1, \mathbf{3}) &= (Q_\mathrm{L}^i)^x{}_l = \begin{pmatrix} u_\mathrm{L}^x & d_\mathrm{L}^x & D_\mathrm{L}^x \end{pmatrix}^i, \\ (\overline{\mathbf{3}}, 1, \mathbf{3}, \mathbf{3}) &= (Q_\mathrm{R}^i)^r{}_x = \begin{pmatrix} u_{\mathrm{R}x}^c & d_{\mathrm{R}x}^c & D_{\mathrm{R}x}^c \end{pmatrix}^{\top\, i}. \end{aligned}$$

- Higgs and leptons unified in L due to SUSY

full unification of Yukawa couplings

- $W_1 = \lambda_{27} \varepsilon_{ijk} (Q_\mathrm{L}^i)^x{}_l (Q_\mathrm{R}^j)^r{}_x (L^k)^l{}_r$
  - > **One** family of quarks and **all** leptons massless at tree-level $\rightarrow$ **radiatively generated**,
  - > Exact Yukawa unification for all three families.

- $\mathrm{SU}(3)_\mathrm{F}$ also fits neatly into an $\mathrm{E}_8 \subset \mathrm{E}_6 \times \mathrm{SU}(3)_\mathrm{F}$ embedding

$\mathbf{SU(3)_F} \times \mathbf{E_6}$ $\quad \mathbf{248} = (\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{78}) \oplus (\mathbf{3}, \mathbf{27}) \oplus (\overline{\mathbf{3}}, \overline{\mathbf{27}})$ $\quad \Rightarrow$ **No gauge anomalies.**

Yukawa and Higgs-lepton unification: the SHUT model

A. Morais, A. Ordell, J.-E. Camargo-Molina, RP, J. Wessen, PRD95 (2017) 075031

$$[\mathrm{SU}(3)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{R} \times \mathrm{SU}(3)_\mathrm{C}] \ltimes \mathbb{Z}_3 \times \{\mathrm{SU}(3)_\mathrm{F}\}\,,$$

Chiral Supermultiplet Fields

Superfield		$\mathrm{SU}(3)_\mathrm{C}$	$\mathrm{SU}(3)_\mathrm{L}$	$\mathrm{SU}(3)_\mathrm{R}$	$\{\mathrm{SU}(3)_\mathrm{F}\}$
Lepton	$(L^i)^l{}_r$	$\mathbf{1}$	$\mathbf{3}^l$	$\bar{\mathbf{3}}_r$	$\mathbf{3}^i$
Right-Quark	$(Q_R^i)^r{}_x$	$\bar{\mathbf{3}}_x$	$\mathbf{1}$	$\mathbf{3}^r$	$\mathbf{3}^i$
Left-Quark	$(Q_L^i)^x{}_l$	$\mathbf{3}^x$	$\bar{\mathbf{3}}_l$	$\mathbf{1}$	$\mathbf{3}^i$
Colour-adjoint	Δ_C^a	$\mathbf{8}^a$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Left-adjoint	Δ_L^a	$\mathbf{1}$	$\mathbf{8}^a$	$\mathbf{1}$	$\mathbf{1}$
Right-adjoint	Δ_R^a	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{8}^a$	$\mathbf{1}$
Flavour-adjoint	Δ_F^a	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{8}^a$

Gauge Supermultiplet Fields

Superfield		$\mathrm{SU}(3)_\mathrm{C}$	$\mathrm{SU}(3)_\mathrm{L}$	$\mathrm{SU}(3)_\mathrm{R}$	$\{\mathrm{SU}(3)_\mathrm{F}\}$
Gluon	$G_C^{\mu a},\ \lambda_C^a$	$\mathbf{8}^a$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Left-Gluon	$G_L^{\mu a},\ \lambda_L^a$	$\mathbf{1}$	$\mathbf{8}^a$	$\mathbf{1}$	$\mathbf{1}$
Right-Gluon	$G_R^{\mu a},\ \lambda_R^a$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{8}^a$	$\mathbf{1}$

$$W = \sum_{A=L,R,C} \left(\lambda_{78} d_{abc} \Delta_A^a \Delta_A^b \Delta_A^c + \mu_{78} \delta_{ab} \Delta_A^a \Delta_A^b\right) + \left(\lambda_1 d_{abc} \Delta_F^a \Delta_F^b \Delta_F^c + \mu_1 \delta_{ab} \Delta_F^a \Delta_F^b\right)$$

$$+ \lambda_{27} \varepsilon_{ijk} \left(Q_L^i\right)^x{}_l \left(Q_R^j\right)^r{}_x \left(L^k\right)^l{}_r\,, \quad \text{with} \;\; d_{abc} = 2\mathrm{Tr}\left[\{T_a, T_b\} T_c\right]$$

A set of accidental symmetries $\mathrm{U}(1)_\mathrm{A} \times \mathrm{U}(1)_\mathrm{B}$, where $\mathrm{U}(1)_\mathrm{B}$ gives a conserved baryon number

The chiral SUSY EFT and GUT-scale SSB

We motivate our field content in the Katsuki et. al. $\mathbb{Z}_3$-orbifold for the breaking $\mathrm{E}_8 \to \mathrm{E}_6 \times \mathrm{SU}(3)_\mathrm{F}$ (Prog.Theor.Phys. 82 (1989) 171)

> massless physical $(\mathbf{27}, \mathbf{3})$,
> massive adjoint $(\mathbf{78}, 1) \oplus (1, \mathbf{8})$,
> $(\overline{\mathbf{27}}, \overline{\mathbf{3}})$ removed by orbifolding.
> No dangerous $(\mathbf{27}, \mathbf{3}) \cdot (\overline{\mathbf{27}}, \overline{\mathbf{3}})$ terms

Fundamental tri-triplets:

$$(L^i)^l{}_r = \begin{pmatrix} H_\mathrm{u}^0 & H_\mathrm{d}^- & e_\mathrm{L} \\ H_\mathrm{u}^+ & H_\mathrm{d}^0 & \nu_\mathrm{L} \\ e_\mathrm{R} & \nu_\mathrm{R} & \phi \end{pmatrix}^i, \; (Q^i_R)^r{}_x = \begin{pmatrix} u_\mathrm{R}^{\bar 1} & u_\mathrm{R}^{\bar 2} & u_\mathrm{R}^{\bar 3} \\ d_\mathrm{R}^{\bar 1} & d_\mathrm{R}^{\bar 2} & d_\mathrm{R}^{\bar 3} \\ D_\mathrm{R}^{\bar 1} & D_\mathrm{R}^{\bar 2} & D_\mathrm{R}^{\bar 3} \end{pmatrix}^i, \; (Q^i_L)^x{}_l = \begin{pmatrix} u_\mathrm{L}^1 & d_\mathrm{L}^1 & D_\mathrm{L}^1 \\ u_\mathrm{L}^2 & d_\mathrm{L}^2 & D_\mathrm{L}^2 \\ u_\mathrm{L}^3 & d_\mathrm{L}^3 & D_\mathrm{L}^3 \end{pmatrix}^i$$

Adjoint octets:

$$\Delta_\mathrm{A} = \begin{pmatrix} \Delta^1 \\ \Delta^2 \\ \Delta^3 \\ \hline \Delta^4 \\ \Delta^5 \\ \Delta^6 \\ \Delta^7 \\ \hline \Delta^8 \end{pmatrix}_{\mathrm{A=L,R,C,F}}$$

No VEV in fundamental reps are generated in the exact SUSY theory (as long as the color SU(3) is preserved)!

The GUT-scale SSB

$$\langle \tilde{\Delta}^8_\mathrm{L} \rangle = \langle \tilde{\Delta}^8_\mathrm{R} \rangle \neq 0$$

$$\mathrm{SU}(3)_{\mathrm{L,R}} \to \mathrm{SU}(2)_{\mathrm{L,R}} \times \mathrm{U}(1)_{\mathrm{L,R}}$$

All components of $\mathbf{\Delta}_{\mathrm{L,R,C}}$ receive $\mathcal{O}$(GUT scale) masses and are integrated out. Neutral components still relevant for neutrino sector (See-Saw mechanism)

On the top-bottom path down to a SM-like theory

Q-GUT:
$[\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(3)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{R}] \times \mathrm{SU}(3)_\mathrm{F}$

non-SUSY multi-scalar and multi-fermion models

Region - I $\left\langle \widetilde{\Delta}^8_{\mathrm{L,R,F}} \right\rangle \sim M_{\mathrm{GUT}}$

SUSY stabilizes the $M_{\mathrm{GUT}} \gg M^{(1)}_{\mathrm{soft}}$ hierarchy.

SUSY-LRF:
$\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \times \mathrm{SU}(2)_\mathrm{F} \times \mathrm{U}(1)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{R} \times \mathrm{U}(1)_\mathrm{F}$

Region - II $\left\langle \widetilde{\phi}^3 \right\rangle \sim M^{(1)}_{\mathrm{soft}} \ll M_{\mathrm{GUT}}$

$T_{\mathrm{L+R}} = T^8_\mathrm{L} + T^8_\mathrm{R}\,, \quad T_\mathrm{S} = T^8_\mathrm{L} - T^8_\mathrm{R} - 2T_\mathrm{F}$

non-SUSY-LRF:
$\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \times \mathrm{SU}(2)_\mathrm{F} \times \mathrm{U}(1)_{\mathrm{L+R}} \times \mathrm{U}(1)_\mathrm{S}$

Natural hierarchy between GUT and EW scales!

Region - III $\left\langle \widetilde{\phi}^2 \right\rangle \sim M^{(2)}_{\mathrm{soft}} \lesssim M^{(1)}_{\mathrm{soft}}$

$T_\mathrm{V} = T^3_\mathrm{F} - \frac{1}{2\sqrt{2}} T_\mathrm{S}$

non-SUSY-LR:
$\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \times \mathrm{U}(1)_{\mathrm{L+R}} \times \mathrm{U}(1)_\mathrm{V}$

Region - IV $\langle \widetilde{\nu}^1_\mathrm{R} \rangle \sim M^{(3)}_{\mathrm{soft}} \lesssim M^{(2)}_{\mathrm{soft}}$

$T_\mathrm{Y} = \sqrt{3} T^3_\mathrm{R} + T_{\mathrm{L+R}}\,, \quad T_\mathrm{T} = T_\mathrm{V} - \frac{3}{2} T^3_\mathrm{R}$

3HDM EFTs:
$\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y} \times \mathrm{U}(1)_\mathrm{T}$

Region - V $\langle S \rangle \sim m_{z'} \lesssim M^{(3)}_{\mathrm{soft}}$

3HDM EFTs:
$\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$

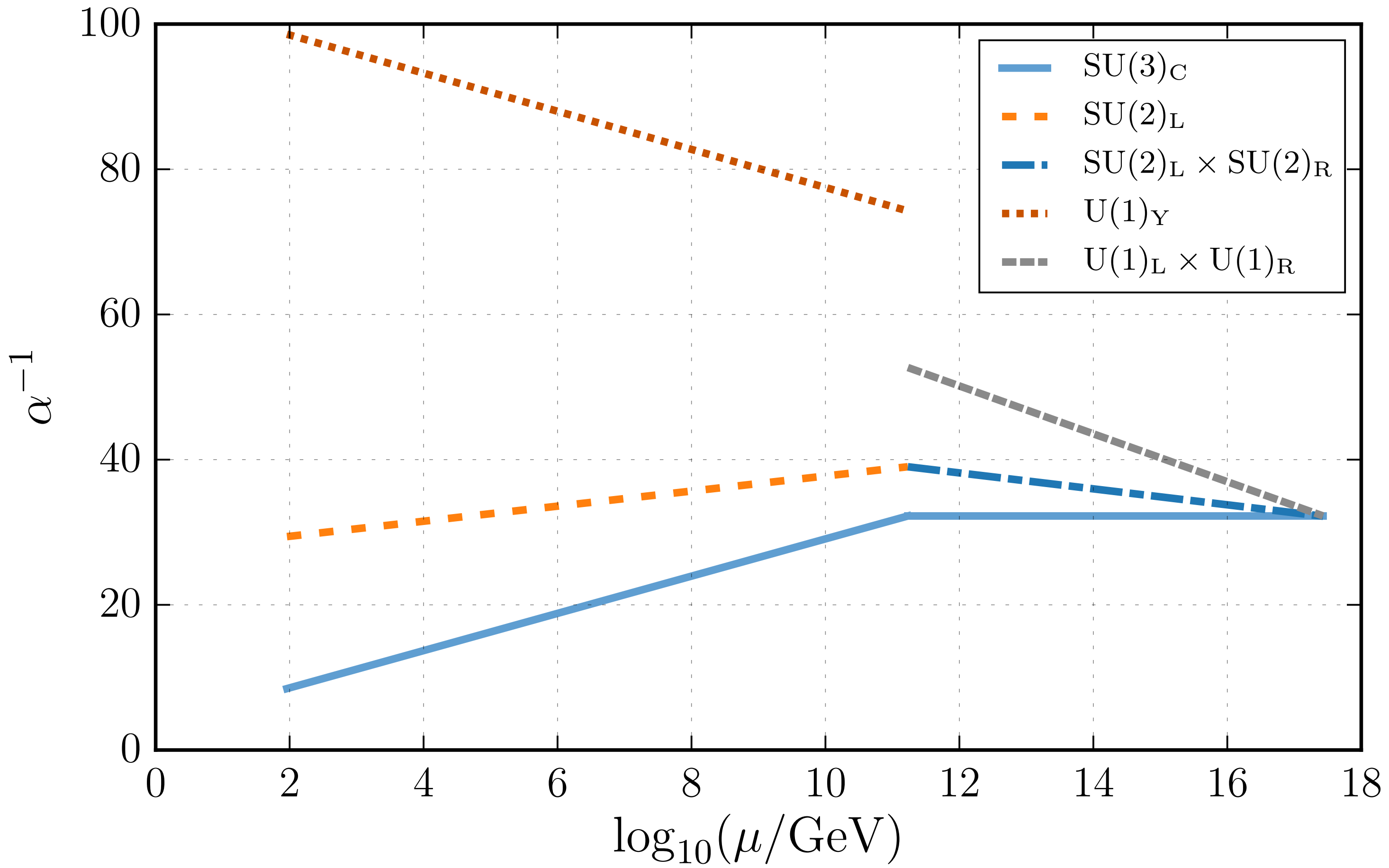
All symmetry breaking scales (including the elecro-weak) except v_{GUT} are controlled by SSB parameters $\Rightarrow$ No μ-problem!

Soft scale hierarchies

A. Morais, A. Ordell, J.-E. Camargo-Molina, RP, J. Wessen, arXiv:1711.05199

- Consider for simplicity $\omega \simeq f \simeq p = M_{\rm soft}$
- Unification condition: $\alpha^{-1}_{\tilde{g}_{\rm L,R}}(M_{\rm GUT}) = \alpha^{-1}_{\tilde{g}_{\rm L,R,C}}(M_{\rm GUT}) = \alpha^{-1}_{\rm U}$

$$\alpha_i^{-1}(\mu_2) = \alpha_i^{-1}(\mu_1) - \frac{b_i}{2\pi} \ln\left(\frac{\mu_2}{\mu_1}\right)$$



- Threshold conditions:

$$\alpha^{-1}_{\tilde{g}_{\rm L+R}}(p) = \alpha^{-1}_{\tilde{g}_{\rm L}}(p) + \alpha^{-1}_{\tilde{g}_{\rm R}}(p)$$

$$\alpha^{-1}_{\tilde{g}_{\rm Y}}(\omega) = \alpha^{-1}_{\tilde{g}_{\rm R}}(\omega) + \frac{1}{3}\alpha^{-1}_{\tilde{g}_{\rm L+R}}(\omega)$$

$$\alpha^{-1}_{\tilde{g}_{\rm Y}}(m_z) = \cos^2\theta_{\rm W}\,\alpha^{-1}_{\rm EM}$$

$$\alpha^{-1}_{\tilde{g}_{L}}(m_z) = \sin^2\theta_{\rm W}\,\alpha^{-1}_{\rm EM},$$

- Solution:

$$M_{\rm soft} \sim 8.8 \cdot 10^{10}\ \mathrm{GeV},$$

$$M_{\rm GUT} \sim 4.9 \cdot 10^{17}\ \mathrm{GeV},$$

$$\alpha^{-1}_{\rm U} \sim 31.5,$$

Too large?

The role of E6 dim-5 operators

- Consider unification at E_6 level relaxing $\mathbb{Z}_3$ in the original $[SU(3)]^3 \rtimes \mathbb{Z}_3$ unification.
- Consider correction to gauge-kinetic terms from dim-5 operators (Chakrabortty, Raychaudhuri 0812.2783 [hep-ph])

$$\mathcal{L}_{d5} = -\frac{\eta}{M_{\rm Pl}}\left[\frac{1}{4c}Tr\left(F_{\mu\nu}\Phi_{\boldsymbol{R}}F^{\mu\nu}\right)\right]$$

  - $\Phi_{\boldsymbol{R}}$ sits in $(\mathbf{78}\otimes\mathbf{78})_{\rm sym} = \mathbf{1}\oplus\mathbf{650}\oplus\mathbf{2430}$
  - $\mathbf{650}$ contains two $[SU(3)]^3$ singlets which provide linearly independent contributions whose VEVs break $E_6 \to [SU(3)]^3$.
  - In general we consider $\Phi_{\boldsymbol{R}} = \kappa_1\Phi_{\mathbf{1}} + \kappa_{650}\Phi_{\mathbf{650}} + \kappa_{650'}\Phi_{\mathbf{650'}} + \kappa_{2430}\Phi_{\mathbf{2430}}$ with $\kappa_1^2 + \kappa_{650}^2 + \kappa_{650'}^2 + \kappa_{2430}^2 = 1$

A possible (non-unique) solution:

$$\epsilon = 0.66 \qquad M_{3333} = 10^{17.5}\ \mathrm{GeV} \qquad p = 10^6\ \mathrm{GeV}$$

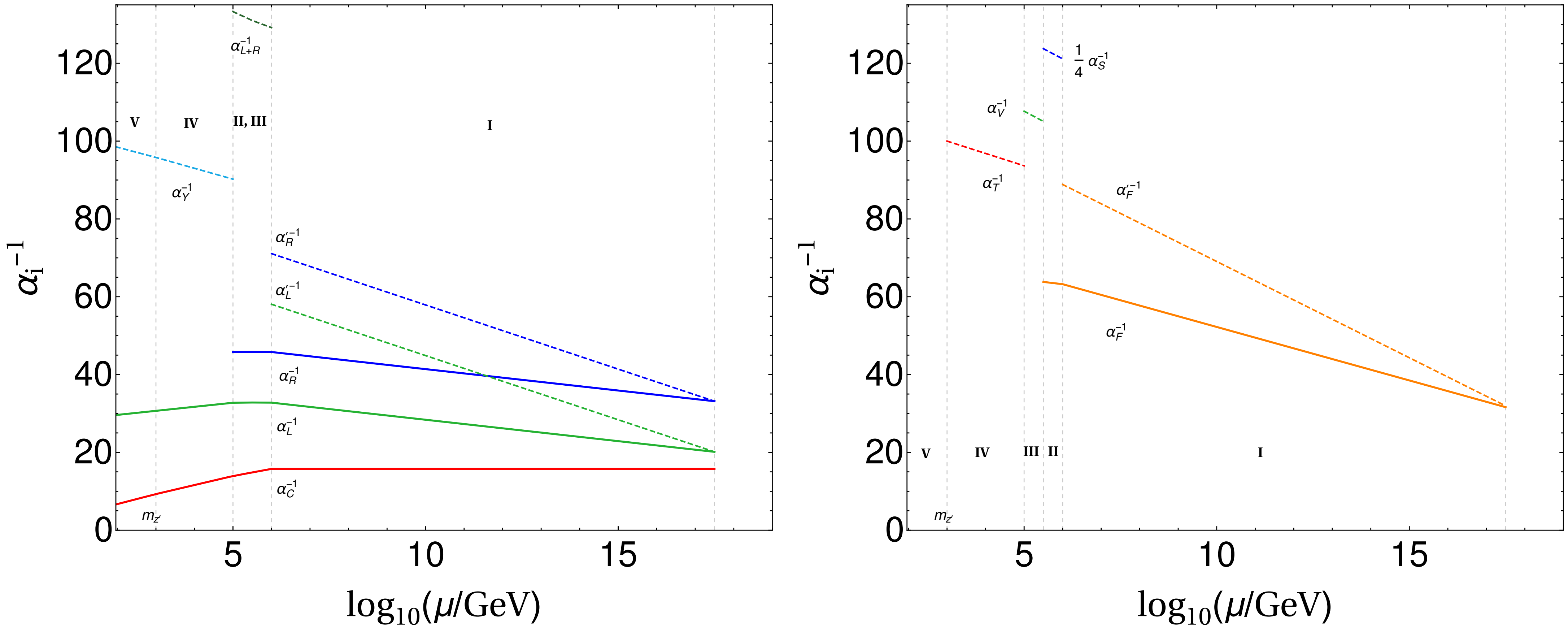
$$f = 10^{5.5}\ \mathrm{GeV} \qquad \omega = 10^5\ \mathrm{GeV} \qquad m_{z'} = 10^3\ \mathrm{GeV}$$

$$\Phi = -0.61\Phi_{\mathbf{650}} + 0.75\Phi_{\mathbf{650'}} + 0.27\Phi_{\mathbf{2430}}$$

$$\alpha_C^{-1} = 1.15\alpha_L^{-1} = 1.89\alpha_R^{-1} \qquad \alpha_{L+R}^{-1} = \alpha_L'^{-1} + \alpha_R'^{-1} \qquad \alpha_S^{-1} = \alpha_L'^{-1} + \alpha_R'^{-1} + 4\alpha_F'^{-1}$$

$$\alpha_V^{-1} = \alpha_F^{-1} + \tfrac{1}{12}\alpha_S^{-1} \qquad \alpha_T^{-1} = \tfrac{4}{9}\alpha_V^{-1} + \alpha_R^{-1} \qquad \alpha_Y^{-1} = \tfrac{1}{3}\alpha_{L+R}^{-1} + \alpha_R^{-1}$$

Possible low-scale phenomenologies



- **Region I:** Keep all states in $\mathbf{27} = (\boldsymbol{L}, \boldsymbol{Q}_{\mathrm{L}}, \boldsymbol{Q}_{\mathrm{R}})$
- **Region II:** Keep all fermions in $\mathbf{27}$ and remove all squarks and $\tilde{\phi}^3$
- **Region III:** Keep all fermions in $\mathbf{27}$, all Higgs doublets and the $\mathrm{SU}(2)_{\mathrm{R}}$ doublet $\begin{pmatrix} e_{\mathrm{R}}^1 & \nu_{\mathrm{R}}^1 \end{pmatrix}$
- **Regions IV and V contain:** (low-scale phenomenology)
  - All chiral (SM-like) quarks and leptons with family non-universal $\mathrm{U}(1)_{\mathrm{T}}$-symmetry,
  - One generation of VLQs,
  - Three generations of VLLs,
  - Three Higgs doublets
  - One $\mathrm{U}(1)_{\mathrm{T}}$-charged complex singlet $\to$ new Z' after $\mathrm{U}(1)_{\mathrm{T}}$-breaking

Why 3HDMs?

After Q-GUT breaking the SUSY-LRF theory reads

$$W = \varepsilon_{ijk} \{ y_{1-3} \phi^i \boldsymbol{D}_{\mathrm{L}}{}^j \boldsymbol{D}_{\mathrm{R}}{}^k + y_{4-6} (\boldsymbol{H}^i)^L{}_R (\boldsymbol{q}_{\mathrm{L}}{}^j)_L (\boldsymbol{q}_{\mathrm{R}}{}^k)^R + y_{7-9} (\boldsymbol{E}_{\mathrm{L}}{}^i)^L (\boldsymbol{q}_{\mathrm{L}}{}^j)_L \boldsymbol{D}_{\mathrm{R}}{}^k + y_{10-12} (\boldsymbol{E}_{\mathrm{R}}{}^i)_R \boldsymbol{D}_{\mathrm{L}}{}^j (\boldsymbol{q}_{\mathrm{R}}{}^k)^R \} \,.$$

Introduce soft SUSY breaking sector!

The simplest scenario that automatically provides CKM mixing with Cabibbo form.

$$\langle L^1 \rangle \sim \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \omega & 0 \end{pmatrix} \qquad \langle L^2 \rangle \sim \frac{1}{\sqrt{2}} \begin{pmatrix} v_2 & 0 & 0 \\ 0 & v_3 & 0 \\ 0 & 0 & f \end{pmatrix} \qquad \langle L^3 \rangle \sim \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & p \end{pmatrix} .$$

$$M_{\mathrm{EW}} \sim v_{1,2,3} \ll \omega \lesssim f \lesssim p \lll M_{\mathrm{GUT}}, \qquad (p, f, \omega) \sim M_{\mathrm{soft}}^{(1,2,3)}$$

Classical approach: (y_{1-12} matched to λ_{27}) Cabibbo form readily at tree-level

$$m_{\mathrm{c,t}}^2 = \frac{1}{2}\lambda_{27}^2 ({v_1}^2 + {v_2}^2), \quad m_{\mathrm{b}}^2 = 3 m_{\mathrm{s}}^2 = \frac{1}{2}\lambda_{27}^2 v_3^2, \quad m_{\mathrm{u,d}}^2 = 0, \quad \tan\theta_{\mathrm{C}} = \frac{v_1}{v_2}$$

$$m_{\mathrm{B}}^2 = \frac{1}{2}\lambda_{27}^2 (2p^2 + f^2 + \omega^2), \quad m_{\mathrm{S}}^2 = \frac{1}{2}\lambda_{27}^2 (p^2 + f^2), \quad m_{\mathrm{D}}^2 = \frac{1}{2}\lambda_{27}^2 \omega^2.$$

Minimal low-scale 3HDMs

- Build a low scale *phenomenological* model *inspired* by the SHUT framework: **pSHUT**
- A new scale set by soft-interactions: Choose $M_{\rm soft} \simeq \left\langle \tilde{\phi}^3 \right\rangle \sim 10^{3-4}$ TeV
- Consider that the VEVs $\left\langle \tilde{\phi}^3 \right\rangle$, $\left\langle \tilde{\phi}^2 \right\rangle$ and $\langle \tilde{\nu}^1_{\rm R} \rangle$ can occur up to two orders of magnitude separation

$$\left\langle \tilde{\phi}^3 \right\rangle \equiv p\,, \qquad \underbrace{\left\langle \tilde{\phi}^2 \right\rangle \equiv f = \xi_1 p\,,}_{\rm SU(2)_F-breaking} \qquad \underbrace{\langle \tilde{\nu}^1_{\rm R} \rangle \equiv \omega = \xi_2 p\,,}_{\rm SU(2)_R-breaking} \qquad 10^{-2} \leqslant \xi_1,\, \xi_2 \leqslant 1\,.$$

- 9 $\mathrm{SU}(2)_{\rm L}$ doublets at our disposal from $\tilde{L}$ tri-triplet
- Study the case of a 3HDM low-scale limit with $H_{\rm u}^{1,2}$ and $H_{\rm d}^3$.

Example:

Boson	$\mathrm{SU}(3)_{\rm C}$	$\mathrm{SU}(2)_{\rm L}$	$\mathrm{U}(1)_{\rm Y}$	$\{\mathrm{U}(1)_{\rm T}\}$
$H_{\rm u}^{1\,l}\,,\ H_{\rm d}^{3\,*\,l}$	$\mathbf{1}$	$\mathbf{2}^l$	1	5
$H_{\rm u}^{2\,l}$	$\mathbf{1}$	$\mathbf{2}^l$	1	1

- The choice of the Higgs sector has an impact in the fermion masses:
  - If e.g. $H_{\rm u}^{1,2,3}$ bottom quark mass would be unacceptably light.

Cabibbo mixing at tree-level

$$\mathcal{L}_{\rm q} = \sum_{i=2}^{3} Y^u_{1i} Q^1_{\rm L} H^2_{\rm u} u^i_{\rm R} + Y^d_{12} Q^1_{\rm L} H^{2\,\dagger}_{\rm u} d^2_{\rm R} + \sum_{i=2}^{3} Y^u_{i1} Q^i_{\rm L} H^2_{\rm u} u^1_{\rm R} + \sum_{i=2}^{3} Y^d_{i3} Q^i_{\rm L} H^{2\,\dagger}_{\rm u} d^3_{\rm R}$$
$$+ Y^d_{13} Q^1_{\rm L} H^{1\,\dagger}_{\rm u} d^3_{\rm R} + \sum_{i,j=2}^{3} Y^u_{ij} Q^i_{\rm L} H^1_{\rm u} u^j_{\rm R} + \sum_{i=2}^{3} Y^d_{i1} Q^i_{\rm L} H^{1\,\dagger}_{\rm u} d^1_{R} + Y^d_{14} Q^1_{\rm L} H^{1\,\dagger}_{\rm u} \mathcal{D}_{\rm R}$$
$$+ \sum_{i=2}^{3} Y^d_{i4} Q^i_{\rm L} H^{2\,\dagger}_{\rm u} \mathcal{D}_{\rm R} + m_D \mathcal{D}_{\rm L} \mathcal{D}_{\rm R} + \left(H^2_{\rm u} \to H^{3\,\dagger}_{\rm d}\right) + h.c.$$

- A subset of quark Yulawa couplings are generated at tree-level $\left(W \supset \varepsilon_{ijk} \boldsymbol{L}^i \boldsymbol{Q}^j_{\rm L} \boldsymbol{Q}^k_{\rm R}\right)$
- Up and down quark sectors with tree-level contributions

$$d^3_{\rm R} = a_1 d^3_{\rm R} + a_2 d^2_{\rm R} + a_3 D^1_{\rm R} \qquad \mathcal{D}_{\rm R} = b_1 d^3_{\rm R} + b_2 d^2_{\rm R} + b_3 D^1_{\rm R}$$

take $a_2 \sim 1 \Rightarrow d^1_{\rm L} d^3_{\rm R} H^3_{\rm d} \simeq d^1_{\rm L} d^2_{\rm R} H^3_{\rm d}$ tree-level

$$\mathcal{M}^u = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 & 0 & k_2 Y^u_{13} \\ 0 & 0 & k_1 Y^u_{23} \\ k_1 Y^u_{31} & k_2 Y^u_{32} & 0 \end{pmatrix} \qquad \mathcal{M}^d = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 & 0 & Y^d_{13} k_d & 0 \\ Y^d_{21} k_d & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{2} m_D / v \end{pmatrix}$$

Mass eigenstates: $\tilde{u}^i_{\rm L,R} = (u_{\rm L,R},\, c_{\rm L,R},\, t_{\rm L,R}) \quad \tilde{d}^i_{\rm L,R} = (d_{\rm L,R},\, s_{\rm L,R},\, b_{\rm L,R}\,, D_{\rm L,R})$

Bi-unitary transformations: $u^i_{\rm L,R} = \left(V^{\rm u}_{\rm L,R}\right)^{ij} \tilde{u}^j_{\rm L,R}\,,\; d^I_{\rm L,R} = \left(V^{\rm d}_{\rm L,R}\right)^{IJ} \tilde{d}^J_{\rm L,R}$

Tree-level CKM mixing:

$$V'_{\rm CKM} = V^{{\rm d}\dagger}_{\rm L} E V^{\rm u}_{\rm L} = \left(\begin{array}{ccc} \cos\theta_C & \sin\theta_C & 0 \\ -\sin\theta_C & \cos\theta_C & 0 \\ 0 & 0 & 1 \\ \hline 0 & 0 & 0 \end{array}\right) \qquad \tan\theta_C = \frac{k_1 Y^u_{13}}{k_2 Y^u_{13}} \qquad E = \left(\mathbb{1}_{3\times 3}\ 0\right)^\top$$

The impact of quantum effects on fermion spectra

A schematic illustration:

$$\frac{m_t^2}{m_c^2} = \frac{{y'^{(1)}_u}^2 {v_1}^2 + {y'^{(2)}_u}^2 {v_2}^2}{{y^{(1)}_u}^2 {v_1}^2 + {y^{(2)}_u}^2 {v_2}^2}, \qquad \frac{m_b^2}{m_s^2} = \frac{{y'^{(2)}_d}^2}{a_{11}^2 {y_d^{(2)}}^2}, \qquad m_{u,d}^2 = \text{loop-suppressed.}$$

Full CKM mixing radiatively generated

$$\tan\theta_C = \frac{y_u^{(1)} v_1}{y_u^{(2)} v_2},$$

$$y_u^{(2)} \sim \frac{1}{16\pi^2} \frac{p^3 f A_{soft} M_{gau}}{\Lambda^6} y' y \lambda' \lambda$$

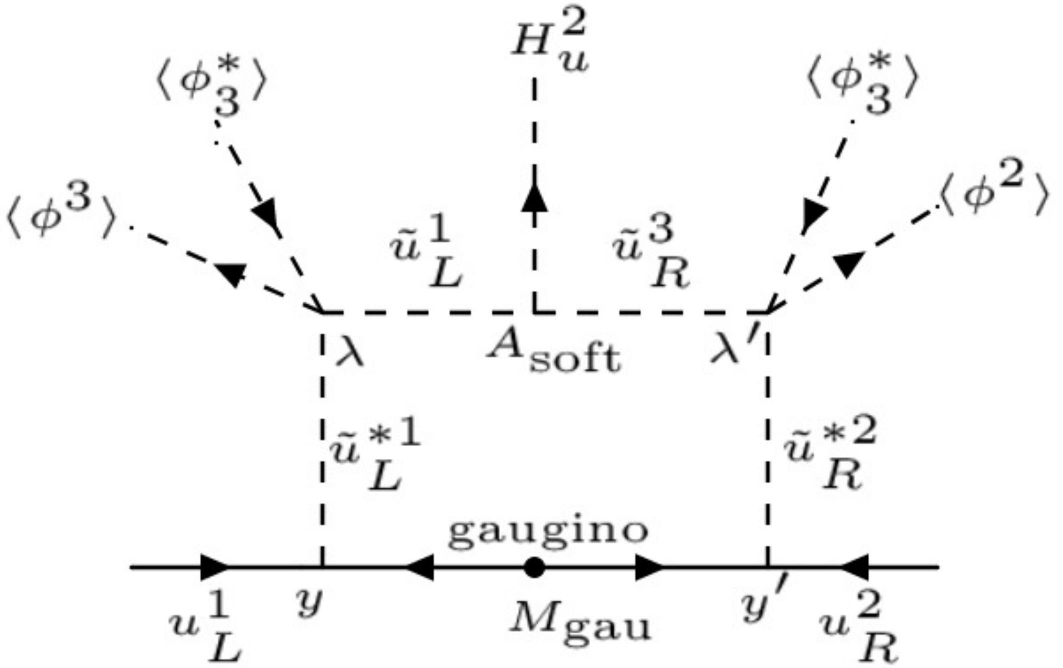


Figure: Example of radiative generation of Yukawa interactions: $y_u^{(2)} H_u^2 u_L^1 u_R^2$

- It is possible to get one generation of VLQ close to TeV scale $m_D \propto y_D \omega$
- Charged lepton masses all loop-generated: 3 chiral, $m_{chi} \propto v_{1,2,3}$, and 3 vector-like, $m_{vec} \propto \omega, f, p$.
- Neutrino masses contain both loop and See-Saw effects.

Quark masses and CKM mixing: one-loop example

$$d_R^3 = 0.71 d_R^3 + 0.06 d_R^2 + 0.7 D_R^1 \qquad \mathcal{D}_R = -0.11 d_R^3 + 0.99 d_R^2 + 0.04 D_R^1$$

$$\mathcal{M}^u = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 & \beta_{12}^{\mathrm{u}} & k_2 Y_{13}^u \\ \beta_{12}^{\mathrm{u}} & \alpha_{22}^{\mathrm{u}} & k_1 Y_{23}^u \\ k_1 Y_{31}^u & k_2 Y_{32}^u & \alpha_{33}^{\mathrm{u}} \end{pmatrix} \qquad \mathcal{M}^d = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 & \beta_{12}^{\mathrm{d}} & 0.06 Y_{13}^d k_d & 0.99 Y_{13}^d k_d \\ k_d Y_{21}^d & 0 & \beta_{23}^{\mathrm{d}} & \beta_{24}^{\mathrm{d}} \\ \alpha_{31}^{\mathrm{d}} & 0 & \beta_{33}^{\mathrm{d}} & \beta_{34}^{\mathrm{d}} \\ 0 & 0 & 0 & \sqrt{2} m_D / v \end{pmatrix}$$

Spectrum and CKM mixing:

$m_t = 173.2$ **GeV** $\quad m_c = 1338$ **MeV** $\quad m_u = 3.552$ **MeV**

$m_D = 2.369$ **TeV** $\quad m_b = 4022$ **MeV** $\quad m_s = 105.4$ **MeV** $\quad m_d = 2.595$ **MeV**

VLQ!

$$V'_{\mathrm{CKM}} \simeq \begin{pmatrix} 0.972436 & 0.232173 & 0.00438568 \\ 0.232023 & 0.972665 & 0.0200507 \\ 0.0092939 & 0.0216599 & 0.999756 \\ \hline 0.0015349 & 0.00786742 & 0.000153942 \end{pmatrix}$$

- Tree-level entries on the last row have a suppression factor $\propto 1/m_D$
- VLQ mass depends on the $\mathrm{SU}(2)_R$-breaking scale ($\xi_2 = 0.0326$)

Loop-generated lepton effective Yukawas and mass terms

- Due to $\mathrm{SU}(3)_{\mathrm{F}}$ there are no tree-level lepton masses $\left(W \supset \varepsilon_{ijk} \boldsymbol{L}^i \boldsymbol{Q}_{\mathrm{L}}^j \boldsymbol{Q}_{\mathrm{R}}^k\right)$
- **ALL Yukawa couplings and bilinears generated at quantum level**

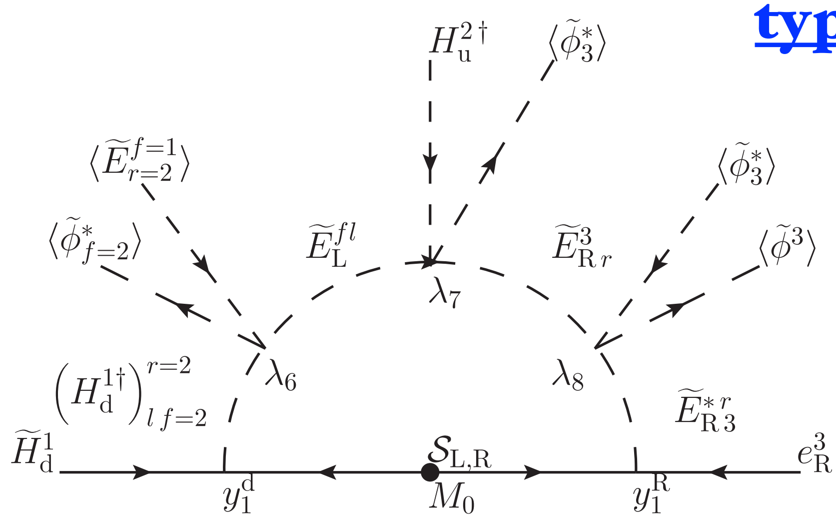
$$\mathcal{L}_{\mathrm{C}} = \left(H_{\mathrm{u}}^{1\dagger} + H_{\mathrm{d}}^{3}\right) \sum_{i=2}^{3} \left(Y_{11i}^{\mathrm{e}} E_{\mathrm{L}}^{1} e_{\mathrm{R}}^{i} + Y_{1i1}^{\mathrm{e}} E_{\mathrm{L}}^{i} e_{\mathrm{R}}^{1}\right) + \sum_{j=2}^{3} \sum_{k=2}^{3} Y_{ijk}^{\mathrm{e}} H_{\mathrm{u}}^{2\dagger} E_{\mathrm{L}}^{j} e_{\mathrm{R}}^{k}$$

$$+ Y_{111}^{\mathrm{eh}} \left(H_{\mathrm{u}}^{1\dagger} + H_{\mathrm{d}}^{3}\right) \widetilde{H}_{\mathrm{d}}^{1} e_{\mathrm{R}}^{1} + \sum_{j=2}^{3} \sum_{k=2}^{3} H_{\mathrm{u}}^{2\dagger} \left(Y_{i1j}^{\mathrm{eh}} \widetilde{H}_{\mathrm{d}}^{1} e_{\mathrm{R}}^{j} + Y_{ik1}^{\mathrm{eh}} \widetilde{H}_{\mathrm{d}}^{k} e_{\mathrm{R}}^{1}\right)$$

$$+ \sum_{i=2}^{3} \sum_{j=2}^{3} \sum_{k=2}^{3} M_{ij}^{\mathrm{eh}} e_{\mathrm{L}}^{i} \widetilde{H}_{\mathrm{d}}^{j} + \sum_{i=2}^{3} M_{1i}^{\mathrm{h}} \widetilde{H}_{\mathrm{d}}^{1} \widetilde{H}_{\mathrm{u}}^{i} + \sum_{i=2}^{3} M_{i1}^{\mathrm{h}} \widetilde{H}_{\mathrm{d}}^{i} \widetilde{H}_{\mathrm{u}}^{1}$$

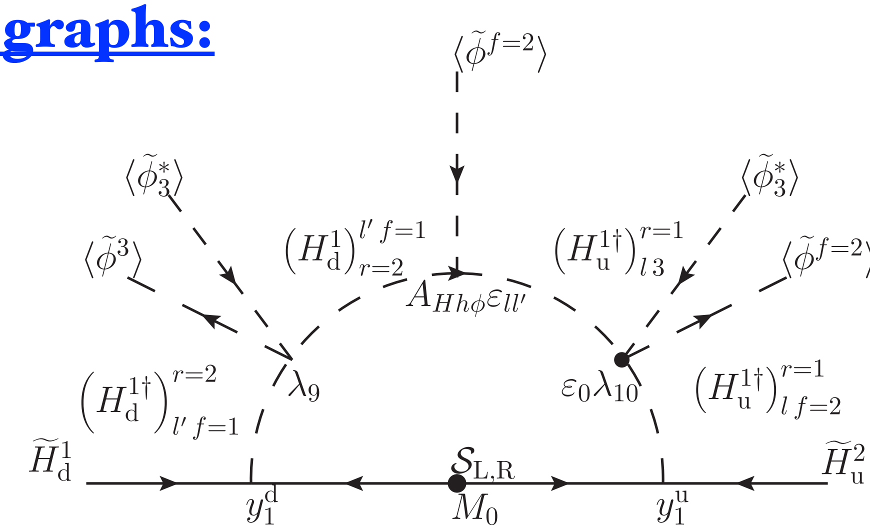
typical graphs:

One-loop Yukawa



$Y_{213}^{\mathrm{eh}} \propto \xi_1 \xi_2 \varepsilon_0 \lambda_6 \lambda_7 \lambda_8 y_1^{\mathrm{d}} y_1^{\mathrm{R}} \equiv \delta_{213}^{\mathrm{eh}} \xi_1 \xi_2 \varepsilon_0$

Effective two-loop mass-term



$M_{12}^{\mathrm{h}} \propto \xi_1^2 \varepsilon_0^2 \lambda_9 \lambda_{10} y_1^{\mathrm{d}} y_1^{\mathrm{u}} \kappa p \equiv \delta_{12}^{\mathrm{h}} \xi_1 \varepsilon_0^2 \kappa p$

Sources of suppression:

$\varepsilon_0 = \frac{1}{16\pi^2}$ $\qquad \kappa = \frac{A_{Hh\phi}}{p}$ $\qquad \xi_1 = \frac{f}{p}$ $\qquad \xi_2 = \frac{\omega}{p}$ $\qquad \delta_{ijk}^{\mathrm{x}},\ \delta_{ij}^{\mathrm{x}} = \lambda_m \cdots \lambda_q y_n y_s$

Loop-generated charged lepton spectrum: numerical examples

- Quantum effects also present in δ^{x}_{ijk} and δ^{x}_{ij} due to RG-flow
- Once EW symmetry is broken, consider $v = \sqrt{v_1^2 + v_2^2 + v_3^2} = 246$ GeV

$$\mathcal{L}_C = \begin{pmatrix} e^1_{\mathrm{L}} & e^2_{\mathrm{L}} & e^3_{\mathrm{L}} & \widetilde{H}^{1-}_{\mathrm{d}} & \widetilde{H}^{2-}_{\mathrm{d}} & \widetilde{H}^{3-}_{\mathrm{d}} \end{pmatrix} \mathcal{M}^C \begin{pmatrix} e^1_{\mathrm{R}} & e^2_{\mathrm{R}} & e^3_{\mathrm{R}} & \widetilde{H}^{1+}_{\mathrm{u}} & \widetilde{H}^{2+}_{\mathrm{u}} & \widetilde{H}^{3+}_{\mathrm{u}} \end{pmatrix}^{\top} + h.c..$$

$$\mathcal{M}^C = \varepsilon_0 \left(\begin{array}{ccc|ccc} 0 & a^{\mathrm{e}}_{12}v & a^{\mathrm{e}}_{13}v & 0 & 0 & 0 \\ a^{\mathrm{e}}_{21}v & b^{\mathrm{e}}_{22}v & b^{\mathrm{e}}_{23}v & 0 & c^{\mathrm{eh}}_{25}p & c^{\mathrm{eh}}_{26}p \\ a^{\mathrm{e}}_{31}v & b^{\mathrm{e}}_{32}v & \varepsilon_0 b^{\mathrm{e}}_{33}v & 0 & c^{\mathrm{eh}}_{35}p & c^{\mathrm{eh}}_{36}p \\ \hline a^{\mathrm{eh}}_{41}v & b^{\mathrm{eh}}_{42}v & b^{\mathrm{eh}}_{43}v & 0 & c^{\mathrm{h}}_{45}p & c^{\mathrm{h}}_{46}p \\ b^{\mathrm{eh}}_{51}v & 0 & 0 & c^{\mathrm{h}}_{54}p & 0 & 0 \\ b^{\mathrm{eh}}_{61}v & 0 & 0 & c^{\mathrm{h}}_{64}p & 0 & 0 \end{array}\right),$$

VLLs!

Example of working points:

Physical lepton	Point 1	Point 2
m_e (keV)	510.7	511.4
m_μ (MeV)	105.2	105.8
m_τ (MeV)	1778	1779
m_E (TeV)	0.3331	0.4258
m_M (TeV)	4.249	0.4258
$m_{\mathcal{T}}$ (TeV)	4.254	1.534

$p \simeq 1.06 \times 10^3$ TeV $\quad \xi_1 \simeq 0.924 \quad \xi_2 \simeq 0.0926 \quad \kappa_2 \simeq 1.67 \quad \kappa_3 \simeq 1.1$

$k_1 \simeq 0.428 \quad k_2 \simeq 0.849 \quad k_d \simeq 0.310$

- δ^{x}_{ijk} and δ^{x}_{ij} preferably from $\mathcal{O}(10^{-6})$ to $\mathcal{O}(1)$
- Fitted values put constraints on the high scale theory parameters

See talk by Antonio this afternoon for more details on low-energy phenomenology!

Summary and the future goals

- Developed a **phenomenological** model *inspired* buy a T-GUT framework
- 3HDM low scale limit reproduces the observed fermion masses in the SM
  - **interplay between $\mathrm{SU}(3)_\mathrm{F}$, quantum effects and ratios between breaking scales (ξ_1, ξ_2).**
- Extra exotic fermions $\longrightarrow$ collider signatures? (Future studies)
  - 1 generation of VLQ
  - Heavy electrons
- Cabibbo mixing emerges at tree-level
- CKM structure from quantum effects

Future research paths:

- Matching and RG running analysis
- Search for all viable regions (flavour, EW-precision),
- New physics: exotic states (VLF, flavour non-universal Z', scalars), DM, GW, EWBG.