Controlled Scalar FCNC, Vacuum Induced CP Violation and a Complex CKM

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Based on work done in collaboration with:
F.J. Botella & F. Cornet-Gómez (IFIC – Valencia)
J. Alves, G.C. Branco & J.P. Silva (IST – Lisbon)

EPJC77 (2017) 9, 585 [arXiv:1703.03796]
EPJC78 (2018) 8, 630 [arXiv:1803.11199]
arXiv:1808.00493

Controlled Scalar FCNC

Generalities – Notation

- Yukawa Lagrangian ($\tilde{\Phi}_j = i\sigma_2\Phi_j^*$):

$$\mathscr{L}_{\rm Y} = -\bar{Q}_L^0[\Gamma_1\Phi_1 + \Gamma_2\Phi_2]d_R^0 - \bar{Q}_L^0[\Delta_1\tilde{\Phi}_1 + \Delta_2\tilde{\Phi}_2]u_R^0 + \text{H.c.}$$

- EWSSB:

$$\langle\Phi_1\rangle = \begin{pmatrix} 0 \\ e^{i\xi_1}v_1/\sqrt{2} \end{pmatrix}, \quad \langle\Phi_2\rangle = \begin{pmatrix} 0 \\ e^{i\xi_2}v_2/\sqrt{2} \end{pmatrix}.$$

$v^2 \equiv v_1^2 + v_2^2$, $c_\beta = \cos\beta \equiv v_1/v$, $s_\beta = \sin\beta \equiv v_2/v$, $t_\beta \equiv \tan\beta$ and $\xi \equiv \xi_2 - \xi_1$

- "Higgs basis"

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ s_\beta & -c_\beta \end{pmatrix} \begin{pmatrix} e^{-i\xi_1}\Phi_1 \\ e^{-i\xi_2}\Phi_2 \end{pmatrix}, \quad \langle H_1\rangle = \frac{v}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \langle H_2\rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

- Expand fields

$$H_1 = \begin{pmatrix} G^+ \\ (v + \mathrm{h}^0 + iG^0)/\sqrt{2} \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ (\mathrm{R}^0 + i\mathrm{I}^0)/\sqrt{2} \end{pmatrix}$$

Generalities – Notation

- Yukawa couplings

$$-\frac{v}{\sqrt{2}}\mathscr{L}_{\mathrm{Y}} = \bar{Q}_L^0(M_d^0 H_1 + N_d^0 H_2)d_R^0 + \bar{Q}_L^0(M_u^0 \tilde{H}_1 + N_u^0 \tilde{H}_2)u_R^0 + \text{H.c.}$$

- Mass matrices M_d^0, M_u^0, and N_d^0, N_u^0 matrices:

$$M_d^0 = \frac{ve^{i\xi_1}}{\sqrt{2}}(c_\beta\Gamma_1 + e^{i\xi}s_\beta\Gamma_2)\,, \qquad N_d^0 = \frac{ve^{i\xi_1}}{\sqrt{2}}(s_\beta\Gamma_1 - e^{i\xi}c_\beta\Gamma_2)$$

$$M_u^0 = \frac{ve^{-i\xi_1}}{\sqrt{2}}(c_\beta\Delta_1 + e^{-i\xi}s_\beta\Delta_2)\,, \quad N_u^0 = \frac{ve^{-i\xi_1}}{\sqrt{2}}(s_\beta\Delta_1 - e^{-i\xi}c_\beta\Delta_2)$$

- Bidiagonalisation ($\mathcal{U}_{q_X} \in U(3)$, CKM $V = \mathcal{U}_{u_L}^\dagger \mathcal{U}_{d_L}$)

$$\mathcal{U}_{d_L}^\dagger M_d^0 \mathcal{U}_{d_R} = M_d = \text{diag}(m_d, m_s, m_b)$$
$$\mathcal{U}_{u_L}^\dagger M_u^0 \mathcal{U}_{u_R} = M_u = \text{diag}(m_u, m_c, m_t)$$

Generalities – Notation

- N_d, N_u matrices

$$\mathcal{U}_{d_L}^\dagger N_d^0 \mathcal{U}_{d_R} = N_d, \quad \mathcal{U}_{u_L}^\dagger N_u^0 \mathcal{U}_{u_R} = N_u$$

- Yukawa couplings

$$-\frac{v}{\sqrt{2}}\mathscr{L}_{\text{Y}} = (\bar{u}_L V, \bar{d}_L)(M_d H_1 + N_d H_2) d_R \\ + (\bar{u}_L, \bar{d}_L V^\dagger)(M_u \tilde{H}_1 + N_u \tilde{H}_2) u_R + \text{H.c.}$$

- Up to $2 \times 3 \times 3 \times 2 = 36$ new real parameters
- Source of Scalar FCNC (SFCNC)
- Symmetry to limit this inflation of parameters

- Abelian symmetry transformations

$$\Phi_1 \mapsto \Phi_1,\ \Phi_2 \mapsto e^{i\theta}\Phi_2$$
$$Q^0_{Lj} \mapsto e^{i\alpha_j\theta} Q^0_{Lj},\ d^0_{Rj} \mapsto e^{i\beta_j\theta} d^0_{Rj},\ u^0_{Rj} \mapsto e^{i\gamma_j\theta} u^0_{Rj}$$

- All possible realistic implementations analysed:

  Ferreira & Silva, PRD83 (2011) 065026 [arXiv:1012.2874]

- Among them, Branco-Grimus-Lavoura (BGL) models

  Branco, Grimus & Lavoura, PLB380 (1996) 119 [hep-ph/9601383]

  - SFCNC only in one quark sector, proportional to CKM entries
  - Interesting relations among Yukawa matrices

$$\text{BGL models: } \Gamma_1^\dagger\Gamma_2 = 0, \quad \Delta_1^\dagger\Delta_2 = 0, \quad \Gamma_1^\dagger\Delta_2 = 0, \quad \Gamma_2^\dagger\Delta_1 = 0,$$
$$\text{and } \Gamma_1\Gamma_2^\dagger = 0 \text{ (dBGL)} \quad \text{or} \quad \Delta_1\Delta_2^\dagger = 0 \text{ (uBGL)}$$

- "Generalised" BGL (gBGL)

  Alves, Botella, Branco, Cornet-Gómez, & N, EPJC77 (2017) 9, 585 [arXiv:1703.03796]

  - SFCNC in both quark sectors, related through CKM

$$\text{gBGL models: } \Gamma_1^\dagger\Gamma_2 = 0, \quad \Delta_1^\dagger\Delta_2 = 0, \quad \Gamma_1^\dagger\Delta_2 = 0, \quad \Gamma_2^\dagger\Delta_1 = 0$$

Strategy

Since in these models symmetry properties $\leftrightarrows$ matrix relations (not always possible) we focus on

- 2HDMs which obey an abelian symmetry and (a) or (b)

(a) Yukawa coupling matrices required to obey *Left conditions*

$$N_d^0 = \mathrm{L}_\mathrm{d}^0\, M_d^0\,, \quad N_u^0 = \mathrm{L}_\mathrm{u}^0\, M_u^0 \qquad \text{with } \mathrm{L}_\mathrm{q}^0 = \ell_1^{[q]}\mathrm{P}_1 + \ell_2^{[q]}\mathrm{P}_2 + \ell_3^{[q]}\mathrm{P}_3$$

$\ell_j^{[q]}$ are, *a priori*, arbitrary numbers.

(b) Yukawa coupling matrices required to obey *Right conditions*

$$N_d^0 = M_d^0\, \mathrm{R}_\mathrm{d}^0\,, \quad N_u^0 = M_u^0\, \mathrm{R}_\mathrm{u}^0 \qquad \text{with } \mathrm{R}_\mathrm{q}^0 = r_1^{[q]}\mathrm{P}_1 + r_2^{[q]}\mathrm{P}_2 + r_3^{[q]}\mathrm{P}_3$$

$r_j^{[q]}$ are, *a priori*, arbitrary numbers

Projection operators P_i, $[\mathrm{P}_i]_{jk} = \delta_{ij}\delta_{jk}$ (no sum in j)

$$\mathrm{P}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \mathrm{P}_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \mathrm{P}_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Left models

- from *Left conditions*, each left-handed doublet Q^0_{Li} couples exclusively, i.e. to one and only one, of the scalar doublets Φ_k

Right models

- from *Right conditions*, each right-handed singlet d^0_{Ri}, u^0_{Rj}, couples exclusively to one scalar doublet Φ_k

- "Generalisation" of Glashow-Weinberg NFC

  types I & II from L^0_d and L^0_u proportional to the identity $\mathbf{1}$

  S. Glashow & S. Weinberg, PRD15 (1977) 1958

In the mass bases

- Going to the mass bases

$$N_d = \mathrm{L_d}\, M_d\,, \quad N_u = \mathrm{L_u}\, M_u$$

with

$$\mathrm{L_d} = \mathcal{U}_{d_L}^\dagger\, \mathrm{L_d^0}\, \mathcal{U}_{d_L}\,, \quad \mathrm{L_u} = \mathcal{U}_{u_L}^\dagger\, \mathrm{L_u^0}\, \mathcal{U}_{u_L}$$

- Transformed projection operators

$$P_j^{[d_L]} \equiv \mathcal{U}_{d_L}^\dagger\, \mathrm{P}_j\, \mathcal{U}_{d_L}\,, \quad P_j^{[u_L]} \equiv \mathcal{U}_{u_L}^\dagger\, \mathrm{P}_j\, \mathcal{U}_{u_L}$$

- One simply has (same $\ell_i^{[q]}$)

$$\mathrm{L_d} = \ell_1^{[d]} P_1^{[d_L]} + \ell_2^{[d]} P_2^{[d_L]} + \ell_3^{[d]} P_3^{[d_L]}\,, \quad \mathrm{L_u} = \ell_1^{[u]} P_1^{[u_L]} + \ell_2^{[u]} P_2^{[u_L]} + \ell_3^{[u]} P_3^{[u_L]}$$

- Since the CKM matrix is $V = \mathcal{U}_{u_L}^\dagger \mathcal{U}_{d_L}$

$$P_k^{[u_L]} = V\, P_k^{[d_L]}\, V^\dagger$$

Determination of $\ell_j^{[q]}$

(1) Assuming an abelian symmetry

$$(\Gamma_1)_{ia} \neq 0 \;\Rightarrow\; (\Gamma_2)_{ia} = 0 \qquad \text{(the converse } 1 \leftrightarrow 2 \text{ also holds)}$$

(2) Consider $(\Gamma_1)_{ia} \neq 0$, then $(\Gamma_2)_{ia} = 0$, and the *Left conditions* read

$$\left(M_d^0\right)_{ia} = \frac{v e^{i\xi_1}}{\sqrt{2}} c_\beta \, (\Gamma_1)_{ia} \,, \qquad \left(N_d^0\right)_{ia} = \frac{v e^{i\xi_1}}{\sqrt{2}} s_\beta \, (\Gamma_1)_{ia} \,,$$

and thus $\left(N_d^0\right)_{ia} = t_\beta \, \left(M_d^0\right)_{ia}$
That is

$$(\Gamma_1)_{ia} \neq 0 \;\Rightarrow\; \ell_i^{[d]} = t_\beta$$

(3) Consider $(\Gamma_2)_{ib} \neq 0$: similarly,

$$(\Gamma_2)_{ib} \neq 0 \;\Rightarrow\; \ell_i^{[d]} = -t_\beta^{-1}$$

Abelian symmetry & *Left conditions*

- Cannot have simultaneously $(\Gamma_1)_{ia} \neq 0$ and $(\Gamma_2)_{ib} \neq 0$, for *any* choices of a and b

i.e.

- Γ_1 and Γ_2 cannot have nonzero matrix elements in the same row
- Each doublet Q^0_{Li} couples to one and only one doublet Φ_k

Rule book for $\ell_i^{[d]}$

$$\text{if } (\Gamma_1)_{ia} \text{ exists, then } \ell_i^{[d]} = t_\beta$$

$$\text{if } (\Gamma_2)_{ia} \text{ exists, then } \ell_i^{[d]} = -t_\beta^{-1}$$

Similarly for Δ_1, Δ_2 and $\ell_i^{[u]}$

Left models – Generalised BGL (gBGL)

- Transformation

$$\Phi_2 \mapsto -\Phi_2, \quad Q^0_{L3} \mapsto -Q^0_{L3}$$

- Yukawa matrices

$$\Gamma_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}, \Delta_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}$$

- Left conditions

$$N^0_d = (t_\beta \mathrm{P}_1 + t_\beta \mathrm{P}_2 - t_\beta^{-1}\mathrm{P}_3)\, M^0_d\,, \quad N^0_u = (t_\beta \mathrm{P}_1 + t_\beta \mathrm{P}_2 - t_\beta^{-1}\mathrm{P}_3) M^0_u$$

$$N_d = (t_\beta \mathbf{1} - (t_\beta + t_\beta^{-1}) P_3^{[d_L]}) M_d\,, \quad N_u = (t_\beta \mathbf{1} - (t_\beta + t_\beta^{-1}) P_3^{[u_L]}) M_u$$

- Parametrisation: introduce complex unit vectors $\hat{n}_{[\mathrm{d}]}$, $\hat{n}_{[\mathrm{u}]}$

$$\hat{n}_{[\mathrm{d}]j} \equiv [\mathrm{P}_3 \mathcal{U}_{d_L}]_{3j}, \quad \hat{n}_{[\mathrm{u}]j} \equiv [\mathrm{P}_3 \mathcal{U}_{u_L}]_{3j}$$

Left models – Generalised BGL (gBGL)

- Parametrisation: introduce complex unit vectors $\hat{n}_{[\mathrm{d}]}$, $\hat{n}_{[\mathrm{u}]}$

$$\hat{n}_{[\mathrm{d}]j} \equiv [\mathrm{P}_3 \mathcal{U}_{d_L}]_{3j}, \quad \hat{n}_{[\mathrm{u}]j} \equiv [\mathrm{P}_3 \mathcal{U}_{u_L}]_{3j}$$

$$\left(P_3^{[d_L]}\right)_{ij} = \hat{n}^*_{[\mathrm{d}]i} \hat{n}_{[\mathrm{d}]j}, \quad \left(P_3^{[u_L]}\right)_{ij} = \hat{n}^*_{[\mathrm{u}]i} \hat{n}_{[\mathrm{u}]j}$$

- N_d and N_u matrices

$$(N_d)_{ij} = (t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) \hat{n}^*_{[\mathrm{d}]i} \hat{n}_{[\mathrm{d}]j}) m_{d_j}$$
$$(N_u)_{ij} = (t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) \hat{n}^*_{[\mathrm{u}]i} \hat{n}_{[\mathrm{u}]j}) m_{u_j}$$

- $\hat{n}_{[\mathrm{d}]}$ and $\hat{n}_{[\mathrm{u}]}$ are not independent:

$$\hat{n}_{[\mathrm{u}]i} V_{ij} = \hat{n}_{[\mathrm{d}]j}$$

- Overall: 4 new parameters

Left models – BGL (bottom)

- Transformation

$$\Phi_2 \mapsto e^{i\theta}\Phi_2, \quad Q^0_{L3} \mapsto e^{-i\theta}Q^0_{L3}, \quad d^0_{R3} \mapsto e^{-i2\theta}d^0_{R3}, \quad \theta \neq 0, \pi$$

- Yukawa coupling matrices

$$\Gamma_1 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}, \Delta_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}$$

- Left conditions

$$N^0_d = (t_\beta \mathrm{P}_1 + t_\beta \mathrm{P}_2 - t_\beta^{-1}\mathrm{P}_3)\, M^0_d\,, \quad N^0_u = (t_\beta \mathrm{P}_1 + t_\beta \mathrm{P}_2 - t_\beta^{-1}\mathrm{P}_3) M^0_u$$

- N_d and N_u

$$(N_d)_{ij} = \delta_{ij}(t_\beta - (t_\beta + t_\beta^{-1})\delta_{j3}) m_{d_j}$$
$$(N_u)_{ij} = (t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) V_{ib} V^*_{jb}) m_{u_j}$$

Left models – jBGL

[Sort of "flipped" generalised BGL]

- Transformation

$$\Phi_2 \mapsto e^{i\theta}\Phi_2, \quad Q^0_{L3} \mapsto e^{-i\theta}Q^0_{L3}, \quad d^0_{Rj} \mapsto e^{-i\theta}d^0_{Rj},\ j=1,2,3$$

- Yukawa matrices

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}, \Gamma_2 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \Delta_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}$$

- *Left conditions*

$$N^0_d = (-t_\beta^{-1}\mathrm{P}_1 - t_\beta^{-1}\mathrm{P}_2 + t_\beta \mathrm{P}_3)\, M^0_d\,, \quad N^0_u = (t_\beta \mathrm{P}_1 + t_\beta \mathrm{P}_2 - t_\beta^{-1}\mathrm{P}_3) M^0_u$$

- N_d and N_u parametrisation

$$(N_d)_{ij} = (-t_\beta^{-1}\delta_{ij} + (t_\beta + t_\beta^{-1})\hat{n}^*_{[\mathrm{d}]i}\hat{n}_{[\mathrm{d}]j})m_{d_j}$$
$$(N_u)_{ij} = (t_\beta\delta_{ij} - (t_\beta + t_\beta^{-1})\hat{n}^*_{[\mathrm{u}]i}\hat{n}_{[\mathrm{u}]j})m_{u_j}$$

Left models – Summary

Model \ Properties	Sym.	Tree FCNC	Parameters
G-W	$\mathbb{Z}_2$	$(N_u)_{ij} \propto \delta_{ij} m_{u_j}$ $(N_d)_{ij} \propto \delta_{ij} m_{d_j}$	t_β, m_{q_k}
uBGL (t)	$\mathbb{Z}_{n\geq 4}$	$(N_u)_{ij} = \delta_{ij}(t_\beta - (t_\beta + t_\beta^{-1})\delta_{j3}) m_{u_j}$ $(N_d)_{ij} = (t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) V_{ti}^* V_{tj}) m_{d_j}$	V, t_β, m_{q_k}
dBGL (b)	$\mathbb{Z}_{n\geq 4}$	$(N_u)_{ij} = (t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) V_{ib} V_{jb}^*) m_{u_j}$ $(N_d)_{ij} = \delta_{ij}(t_\beta - (t_\beta + t_\beta^{-1})\delta_{j3}) m_{d_j}$	V, t_β, m_{q_k}
gBGL	$\mathbb{Z}_2$	$(N_u)_{ij} = (t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) \hat{n}_{[\mathrm{u}]i}^* \hat{n}_{[\mathrm{u}]j}) m_{u_j}$ $(N_d)_{ij} = (t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) \hat{n}_{[\mathrm{d}]i}^* \hat{n}_{[\mathrm{d}]j}) m_{d_j}$	V, t_β, m_{q_k} $\hat{n}_{[\mathrm{q}]}$ (+4)
jBGL	$\mathbb{Z}_{n\geq 2}$	$(N_u)_{ij} = (t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) \hat{n}_{[\mathrm{u}]i}^* \hat{n}_{[\mathrm{u}]j}) m_{u_j}$ $(N_d)_{ij} = (-t_\beta^{-1} \delta_{ij} + (t_\beta + t_\beta^{-1}) \hat{n}_{[\mathrm{d}]i}^* \hat{n}_{[\mathrm{d}]j}) m_{d_j}$	V, t_β, m_{q_k} $\hat{n}_{[\mathrm{q}]}$ (+4)

Right models

- Follow the same steps
  - Right-handed singlets in up & down Yukawa couplings unrelated ⇒ more freedom
  - No “right” CKM
- Rows for *left conditions* ↦ columns for *right conditions*

$$\hat{r}_{[\mathrm{d}]j} \equiv [\mathrm{P}_3 \mathcal{U}_{d_R}]_{3j}, \quad \hat{r}_{[\mathrm{u}]j} \equiv [\mathrm{P}_3 \mathcal{U}_{u_R}]_{3j}$$

Right models

- Model A

$$\Gamma_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \Delta_1 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix}, \Delta_2 = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}$$

- Model B

$$\Gamma_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \Delta_1 = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}, \Delta_2 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix}$$

- Model C

$$\Gamma_1 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}, \Delta_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Right models

- Model D

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}, \Gamma_2 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix}, \Delta_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- Model E

$$\Gamma_1 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}, \Delta_1 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix}, \Delta_2 = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}$$

- Model F

$$\Gamma_1 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}, \Delta_1 = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}, \Delta_2 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix}$$

Right models – Summary

Model \ Properties	Sym.	Tree FCNC	Parameters
G-W	$\mathbb{Z}_2$	$(N_u)_{ij} \propto m_{u_i}\delta_{ij}$ $(N_d)_{ij} \propto m_{d_i}\delta_{ij}$	t_β, m_{q_k}
Type A	$\mathbb{Z}_{n\geq 2}$	$(N_u)_{ij} = m_{u_i}(t_\beta\delta_{ij} - (t_\beta + t_\beta^{-1})\hat{r}^*_{[u]i}\hat{r}_{[u]j})$ $(N_d)_{ij} = m_{d_i}t_\beta\delta_{ij}$	t_β, m_{q_k} $\hat{r}_{[u]}(+4)$
Type B	$\mathbb{Z}_{n\geq 2}$	$(N_u)_{ij} = m_{u_i}(-t_\beta^{-1}\delta_{ij} + (t_\beta + t_\beta^{-1})\hat{r}^*_{[u]i}\hat{r}_{[u]j})$ $(N_d)_{ij} = m_{d_i}t_\beta\delta_{ij}$	t_β, m_{q_k} $\hat{r}_{[u]}(+4)$
Type C	$\mathbb{Z}_{n\geq 2}$	$(N_u)_{ij} = m_{u_i}t_\beta\delta_{ij}$ $(N_d)_{ij} = m_{d_i}(t_\beta\delta_{ij} - (t_\beta + t_\beta^{-1})\hat{r}^*_{[d]i}\hat{r}_{[d]j})$	t_β, m_{q_k} $\hat{r}_{[d]}(+4)$
Type D	$\mathbb{Z}_{n\geq 2}$	$(N_u)_{ij} = m_{u_i}t_\beta\delta_{ij}$ $(N_d)_{ij} = m_{d_i}(-t_\beta^{-1}\delta_{ij} + (t_\beta + t_\beta^{-1})\hat{r}^*_{[d]i}\hat{r}_{[d]j})$	t_β, m_{q_k} $\hat{r}_{[d]}(+4)$
Type E	$\mathbb{Z}_{n\geq 2}$	$(N_u)_{ij} = m_{u_i}(t_\beta\delta_{ij} - (t_\beta + t_\beta^{-1})\hat{r}^*_{[u]i}\hat{r}_{[u]j})$ $(N_d)_{ij} = m_{d_i}(t_\beta\delta_{ij} - (t_\beta + t_\beta^{-1})\hat{r}^*_{[d]i}\hat{r}_{[d]j})$	t_β, m_{q_k} $\hat{r}_{[u]}, \hat{r}_{[d]}(+8)$
Type F	$\mathbb{Z}_{n\geq 2}$	$(N_u)_{ij} = m_{u_i}(-t_\beta^{-1}\delta_{ij} + (t_\beta + t_\beta^{-1})\hat{r}^*_{[u]i}\hat{r}_{[u]j})$ $(N_d)_{ij} = m_{d_i}(t_\beta\delta_{ij} - (t_\beta + t_\beta^{-1})\hat{r}^*_{[d]i}\hat{r}_{[d]j})$	t_β, m_{q_k} $\hat{r}_{[u]}, \hat{r}_{[d]}(+8)$

Summary so far

- Two classes of 2HDMs shaped by symmetry and L/R conditions
- SFCNC controlled by quark masses and unit vectors
- Moderate number of additional parameters
- Interesting phenomenology

Spontaneous CP & Complex CKM

- Original motivation for 2HDMs
  - CP *invariant* Lagrangian
  - CP *violation* arising from the *vacuum*

T.D. Lee, PRD8 (1973) 1226

- Here: realistic 2HDM of spontaneous CP violation
  - scalar potential with spontaneous CP breaking (vacuum phase θ)
  - θ generates a complex CKM matrix
  - SFCNC effects under control (in agreement with experiment)

Setup

- Yukawa couplings

$$\mathscr{L}_{\mathrm{Y}} = -\bar{Q}_L^0(\Gamma_1\Phi_1 + \Gamma_2\Phi_2)d_R^0 - \bar{Q}_L^0(\Delta_1\tilde{\Phi}_1 + \Delta_2\tilde{\Phi}_2)u_R^0 + \mathrm{H.c.}$$

- Symmetry under $\mathbb{Z}_2$ transformations (gBGL)

$$\Phi_1 \mapsto \Phi_1, \quad \Phi_2 \mapsto -\Phi_2, \quad Q_{L3}^0 \mapsto -Q_{L3}^0, \quad Q_{Lj}^0 \mapsto Q_{Lj}^0, \quad j = 1,2$$
$$d_{Rk}^0 \mapsto d_{Rk}^0, \quad u_{Rk}^0 \mapsto u_{Rk}^0, \quad k = 1,2,3$$

$$\Gamma_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}, \Delta_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}$$

- CP invariance of $\mathscr{L}_{\mathrm{Y}}$

$$\Gamma_j^* = \Gamma_j, \qquad \Delta_j^* = \Delta_j$$

Setup

Once again...

- Higgs basis

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \mathcal{R}_\beta \begin{pmatrix} e^{-i\theta_1}\Phi_1 \\ e^{-i\theta_2}\Phi_2 \end{pmatrix}, \text{ with } \mathcal{R}_\beta = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}, \mathcal{R}_\beta^T = \mathcal{R}_\beta^{-1}$$

$$\langle H_1 \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \langle H_2 \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

- Yukawas

$$\mathscr{L}_{\mathrm{Y}} = -\frac{\sqrt{2}}{v}\bar{Q}_L^0(M_d^0 H_1 + N_d^0 H_2)d_R^0 - \frac{\sqrt{2}}{v}\bar{Q}_L^0(M_u^0 \tilde{H}_1 + N_u^0 \tilde{H}_2)u_R^0 + \text{H.c.}$$

M_q^0 and N_q^0

$$M_d^0=\frac{ve^{i\theta_1}}{\sqrt{2}}(c_\beta\Gamma_1+e^{i\theta}s_\beta\Gamma_2)\,,\qquad N_d^0=\frac{ve^{i\theta_1}}{\sqrt{2}}(-s_\beta\Gamma_1+e^{i\theta}c_\beta\Gamma_2)$$

$$M_u^0=\frac{ve^{-i\theta_1}}{\sqrt{2}}(c_\beta\Delta_1+e^{-i\theta}s_\beta\Delta_2)\,,\quad N_u^0=\frac{ve^{-i\theta_1}}{\sqrt{2}}(-s_\beta\Delta_1+e^{-i\theta}c_\beta\Delta_2)$$

- Matrix relations

$$N_d^0=t_\beta M_d^0-(t_\beta+t_\beta^{-1})\mathrm{P}_3M_d^0,\quad N_u^0=t_\beta M_u^0-(t_\beta+t_\beta^{-1})\mathrm{P}_3M_u^0$$

$$\mathrm{P}_3=\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}$$

- Notice

$$M_d^0=\begin{pmatrix}1&0&0\\0&1&0\\0&0&e^{i\theta}\end{pmatrix}\hat{M}_d^0,\qquad M_u^0=\begin{pmatrix}1&0&0\\0&1&0\\0&0&e^{-i\theta}\end{pmatrix}\hat{M}_u^0$$

with $\hat{M}_d^0$ and $\hat{M}_u^0$ *real*

- Bidiagonalisation of M_d^0, M_u^0

$$\mathcal{U}_{d_L}^\dagger M_d^0 \mathcal{U}_{d_R} = \mathrm{diag}(m_{d_i}), \quad \mathcal{U}_{u_L}^\dagger M_u^0 \mathcal{U}_{u_R} = \mathrm{diag}(m_{u_i})$$

- $M_d^0 M_d^{0\dagger}$

$$M_d^0 M_d^{0\dagger} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\theta} \end{pmatrix} \hat{M}_d^0 \hat{M}_d^{0\,T} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\theta} \end{pmatrix}$$

$\hat{M}_d^0 \hat{M}_d^{0\,T}$ *real* and *symmetric*

$$\mathcal{O}_L^{d\,T}\, \hat{M}_d^0 \hat{M}_d^{0\,T}\, \mathcal{O}_L^d = \mathrm{diag}(m_{d_i}^2) \qquad \text{with real orthogonal } \mathcal{O}_L^d$$

$$\mathcal{U}_{d_L}^\dagger M_d^0 M_d^{0\dagger} \mathcal{U}_{d_L} = \mathrm{diag}(m_{d_i}^2)\,, \text{ with } \quad \mathcal{U}_{d_L} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\theta} \end{pmatrix} \mathcal{O}_L^d$$

- Similarly

$$\mathcal{U}_{u_L}^\dagger M_u^0 M_u^{0\dagger} \mathcal{U}_{u_L} = \mathrm{diag}(m_{u_i}^2)\,, \text{ with } \quad \mathcal{U}_{u_L} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\theta} \end{pmatrix} \mathcal{O}_L^u$$

- Right-handed transformations

$$M_d^{0\dagger}M_d^0=\hat{M}_d^{0\,T}\hat{M}_d^0,\qquad M_u^{0\dagger}M_u^0=\hat{M}_u^{0\,T}\hat{M}_u^0$$

$$\mathcal{O}_R^{d\,T}M_d^{0\dagger}M_d^0\mathcal{O}_R^d=\text{diag}(m_{d_i}^2),\quad \mathcal{O}_R^{u\,T}M_u^{0\dagger}M_u^0\mathcal{O}_R^u=\text{diag}(m_{u_i}^2)$$

with real orthogonal $\mathcal{O}_R^d$ and $\mathcal{O}_R^u$

- Finally

$$M_d=\text{diag}(m_{d_i})=\mathcal{U}_{d_L}^{\dagger}M_d^0\mathcal{O}_R^d,\quad M_u=\text{diag}(m_{u_i})=\mathcal{U}_{u_L}^{\dagger}M_u^0\mathcal{O}_R^u$$

- The CKM matrix $V\equiv\mathcal{U}_{u_L}^{\dagger}\mathcal{U}_{d_L}$ is

$$V=\mathcal{O}_L^{u\,T}\begin{pmatrix}1&0&0\\0&1&0\\0&0&e^{i2\theta}\end{pmatrix}\mathcal{O}_L^d$$

requires $e^{i2\theta}\neq\pm1$

N_d and N_u

- $N_d \equiv \mathcal{U}_{d_L}^\dagger N_d^0 \mathcal{O}_R^d$ with $N_d^0 = t_\beta M_d^0 - (t_\beta + t_\beta^{-1}) \mathrm{P}_3 M_d^0$

$$N_d = t_\beta \mathcal{U}_{d_L}^\dagger M_d^0 \mathcal{O}_R^d - (t_\beta + t_\beta^{-1}) \mathcal{U}_{d_L}^\dagger \mathrm{P}_3 M_d^0 \mathcal{O}_R^d = t_\beta M_d - (t_\beta + t_\beta^{-1}) \mathcal{U}_{d_L}^\dagger \mathrm{P}_3 \mathcal{U}_{d_L} M_d$$

- Unit vectors $\hat{r}_{[\mathrm{d}]}$ (real) and $\hat{n}_{[\mathrm{d}]}$ (complex)

$$\hat{r}_{[\mathrm{d}]j} \equiv [\mathcal{O}_L^d]_{3j}\,, \quad \hat{n}_{[\mathrm{d}]j} \equiv [\mathcal{U}_{d_L}]_{3j} = e^{i\theta} \hat{r}_{[\mathrm{d}]j}$$

$$\mathcal{U}_{d_L}^\dagger \mathrm{P}_3 \mathcal{U}_{d_L} = \mathcal{O}_L^{d\,T} \mathrm{P}_3 \mathcal{O}_L^d, \quad [\mathcal{U}_{d_L}^\dagger \mathrm{P}_3 \mathcal{U}_{d_L}]_{ij} = \hat{n}_{[\mathrm{d}]i}^* \hat{n}_{[\mathrm{d}]j} = \hat{r}_{[\mathrm{d}]i} \hat{r}_{[\mathrm{d}]j}$$

N_d and N_u

- Similarly for N_u with

$$\hat{r}_{[\mathrm{u}]j} \equiv [\mathcal{O}_L^u]_{3j} \quad \hat{n}_{[\mathrm{u}]j} \equiv [\mathcal{U}_{u_L}]_{3j} = e^{-i\theta}\hat{r}_{[\mathrm{u}]j}$$

$$\mathcal{U}_{u_L}^\dagger \mathrm{P}_3 \mathcal{U}_{u_L} = \mathcal{O}_L^{u\,T} \mathrm{P}_3 \mathcal{O}_L^u, \quad [\mathcal{U}_{u_L}^\dagger \mathrm{P}_3 \mathcal{U}_{u_L}]_{ij} = \hat{n}_{[\mathrm{u}]i}^* \hat{n}_{[\mathrm{u}]j} = \hat{r}_{[\mathrm{u}]i} \hat{r}_{[\mathrm{u}]j}$$

- N_d and N_u are *real* [N.B. phase convention dependent statement]

$$[N_d]_{ij} = t_\beta \delta_{ij} m_{d_i} - (t_\beta + t_\beta^{-1}) \hat{n}_{[\mathrm{d}]i}^* \hat{n}_{[\mathrm{d}]j} m_{d_j}$$
$$[N_u]_{ij} = t_\beta \delta_{ij} m_{u_i} - (t_\beta + t_\beta^{-1}) \hat{n}_{[\mathrm{u}]i}^* \hat{n}_{[\mathrm{u}]j} m_{u_j}$$

- $\hat{n}_{[\mathrm{d}]}$ and $\hat{n}_{[\mathrm{u}]}$ are not independent

$$\hat{n}_{[\mathrm{d}]i} = \hat{n}_{[\mathrm{u}]j} V_{ji}\,, \quad \hat{n}_{[\mathrm{u}]i} = V_{ij}^* \hat{n}_{[\mathrm{d}]j}$$

- In terms of $R \equiv \mathcal{O}_L^{u\,T} \mathcal{O}_L^d$

$$\hat{r}_{[\mathrm{d}]i} = \hat{r}_{[\mathrm{u}]j} R_{ji}\,, \quad \hat{r}_{[\mathrm{u}]i} = R_{ik} \hat{r}_{[\mathrm{d}]k}$$

CKM

- With

$$V = \mathcal{O}_L^{u\,T}[\mathbf{1} + (e^{i2\theta} - 1)\mathrm{P}_3]\mathcal{O}_L^d \Rightarrow V_{ij} = R_{ij} + (e^{i2\theta} - 1)S_{ij}$$

and

$$S_{ij} \equiv [\mathcal{O}_L^{u\,T}\mathrm{P}_3\mathcal{O}_L^d]_{ij} = \hat{r}_{[\mathrm{u}]i}\hat{r}_{[\mathrm{d}]j} = R_{ik}\hat{r}_{[\mathrm{d}]k}\hat{r}_{[\mathrm{d}]j} = \hat{r}_{[\mathrm{u}]i}\hat{r}_{[\mathrm{u}]k}R_{kj}$$

real and imaginary parts of V_{ij} read

$$\mathrm{Re}\big(V_{ij}\big) = R_{ij} - 2s_\theta^2 S_{ij}, \quad \mathrm{Im}\big(V_{ij}\big) = s_{2\theta}S_{ij}$$

- Compute the imaginary part of rephasing invariant quartet $V_{i_1j_1}V_{i_1j_2}^*V_{i_2j_2}V_{i_2j_1}^*$

CKM

$$\mathrm{Im}\left(V_{i_1j_1}V^*_{i_1j_2}V_{i_2j_2}V^*_{i_2j_1}\right)=\sin 2\theta\left\{\begin{matrix} 4s_\theta^2\begin{pmatrix} R_{i_1j_1}S_{i_2j_1}R_{i_2j_2}S_{i_1j_2}\\ -S_{i_1j_1}R_{i_2j_1}S_{i_2j_2}R_{i_1j_2}\end{pmatrix}\\ +4s_\theta^2\begin{pmatrix} S_{i_1j_1}S_{i_2j_1}S_{i_2j_2}R_{i_1j_2}\\ -S_{i_1j_1}S_{i_2j_1}R_{i_2j_2}S_{i_1j_2}\\ +S_{i_1j_1}R_{i_2j_1}S_{i_2j_2}S_{i_1j_2}\\ -R_{i_1j_1}S_{i_2j_1}S_{i_2j_2}S_{i_1j_2}\end{pmatrix}\\ +\begin{pmatrix} S_{i_1j_1}R_{i_2j_1}R_{i_2j_2}R_{i_1j_2}\\ -R_{i_1j_1}S_{i_2j_1}R_{i_2j_2}R_{i_1j_2}\\ +R_{i_1j_1}R_{i_2j_1}S_{i_2j_2}R_{i_1j_2}\\ -R_{i_1j_1}R_{i_2j_1}R_{i_2j_2}S_{i_1j_2}\end{pmatrix}\end{matrix}\right\}$$

. . . not very illuminating

- SFCNC and CP violating CKM
  - If $\hat{r}_{[\mathrm{q}]}$ had two vanishing components, *no SFCNC in that sector* but then *no CP violation in CKM*
  - That is having no tree level SFCNC in one quark sector is *incompatible* with a CP violating CKM matrix

CKM and SFCNC

$$\text{Im}\left(V_{i_1j_1}V^*_{i_1j_2}V_{i_2j_2}V^*_{i_2j_1}\right)=\sin 2\theta\left\{\begin{array}{c} 4s_\theta^2\begin{pmatrix}R_{i_1j_1}S_{i_2j_1}R_{i_2j_2}S_{i_1j_2}\\ -S_{i_1j_1}R_{i_2j_1}S_{i_2j_2}R_{i_1j_2}\end{pmatrix}\\ +4s_\theta^2\begin{pmatrix}S_{i_1j_1}S_{i_2j_1}S_{i_2j_2}R_{i_1j_2}\\ -S_{i_1j_1}S_{i_2j_1}R_{i_2j_2}S_{i_1j_2}\\ +S_{i_1j_1}R_{i_2j_1}S_{i_2j_2}S_{i_1j_2}\\ -R_{i_1j_1}S_{i_2j_1}S_{i_2j_2}S_{i_1j_2}\end{pmatrix}\\ +\begin{pmatrix}S_{i_1j_1}R_{i_2j_1}R_{i_2j_2}R_{i_1j_2}\\ -R_{i_1j_1}S_{i_2j_1}R_{i_2j_2}R_{i_1j_2}\\ +R_{i_1j_1}R_{i_2j_1}S_{i_2j_2}R_{i_1j_2}\\ -R_{i_1j_1}R_{i_2j_1}R_{i_2j_2}S_{i_1j_2}\end{pmatrix}\end{array}\right\}$$

- If $\hat{r}_{[\mathrm{q}]}$ has two vanishing components $\quad [S_{ij}=\hat{r}_{[\mathrm{u}]i}\hat{r}_{[\mathrm{d}]j}]$
  - S_{ij} has only a non vanishing row (column), for which $S_{ij}=R_{ij}$
  - with $i_1\neq i_2$ and $j_1\neq j_2$, only two terms $\neq 0$
  - they have opposite sign!
- At the end of the day,

  6 parameters + t_β for CKM and all SFCNC

Scalar sector

- 2HDM potential
  - CP invariant (all couplings are real)
  - $\mathbb{Z}_2$ symmetry, softly broken by $\mu_{12}^2 \neq 0$

$$\mathscr{V}(\Phi_1,\Phi_2) = \mu_{11}^2 \Phi_1^\dagger\Phi_1 + \mu_{22}^2 \Phi_2^\dagger\Phi_2 + \mu_{12}^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1)$$
$$+\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + 2\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + 2\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$
$$+ \lambda_5[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2]$$

- Vacuum expectation values for EWSB

$$\langle\Phi_1\rangle = \begin{pmatrix} 0 \\ e^{i\theta_1} v_1/\sqrt{2} \end{pmatrix}, \quad \langle\Phi_2\rangle = \begin{pmatrix} 0 \\ e^{i\theta_2} v_2/\sqrt{2} \end{pmatrix},$$

$\theta = \theta_2 - \theta_1$, $v^2 = v_1^2 + v_2^2$, $c_\beta \equiv v_1/v$, $s_\beta \equiv v_2/v$, $t_\beta \equiv \tan\beta$

- Minimization of $V(v_1, v_2, \theta) \equiv \mathscr{V}(\langle\Phi_1\rangle, \langle\Phi_2\rangle)$

$$\frac{\partial V}{\partial \theta} = -v_1 v_2 \sin\theta(\mu_{12}^2 + 2\lambda_5 v_1 v_2 \cos\theta) = 0$$
$$\frac{\partial V}{\partial v_1} = \mu_{11}^2 v_1 + \lambda_1 v_1^3 + (\lambda_3 + \lambda_4) v_1 v_2^2 + v_2(\mu_{12}^2 \cos\theta + \lambda_5 v_1 v_2 \cos 2\theta) = 0$$
$$\frac{\partial V}{\partial v_2} = \mu_{22}^2 v_2 + \lambda_2 v_2^3 + (\lambda_3 + \lambda_4) v_1^2 v_2 + v_1(\mu_{12}^2 \cos\theta + \lambda_5 v_1 v_2 \cos 2\theta) = 0$$

- For spontaneous CP violation, consider a solution $\{v_1, v_2, \theta\}$ with $\theta \neq 0, \pm\pi/2, \pm\pi$
- Trade

$$\mu_{12}^2 = -2\lambda_5 v_1 v_2 \cos\theta$$
$$\mu_{11}^2 = -(\lambda_1 v_1^2 + (\lambda_3 + \lambda_4 - \lambda_5) v_2^2)$$
$$\mu_{22}^2 = -(\lambda_2 v_2^2 + (\lambda_3 + \lambda_4 - \lambda_5) v_1^2)$$

- Expand fields

$$\Phi_j = e^{i\theta_j} \begin{pmatrix} \varphi_j^+ \\ \frac{1}{\sqrt{2}}(v_j + \rho_j + i\eta_j) \end{pmatrix}$$

- Higgs basis

$$H_1 = \begin{pmatrix} G^+ \\ (v + \mathrm{h}^0 + iG^0)/\sqrt{2} \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ (\mathrm{R}^0 + i\mathrm{I}^0)/\sqrt{2} \end{pmatrix}$$

$$\begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = \mathcal{R}_\beta \begin{pmatrix} \varphi_1^+ \\ \varphi_2^+ \end{pmatrix}, \begin{pmatrix} G^0 \\ \mathrm{I}^0 \end{pmatrix} = \mathcal{R}_\beta \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}, \begin{pmatrix} \mathrm{h}^0 \\ \mathrm{R}^0 \end{pmatrix} = \mathcal{R}_\beta \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}$$

- Identify would-be Goldstone bosons $G^\pm$ and G^0

$$G^\pm = c_\beta \varphi_1^\pm - s_\beta \varphi_2^\pm, \quad G^0 = c_\beta \eta_1 - s_\beta \eta_2$$

- Charged scalar $H^\pm = s_\beta \varphi_1^\pm - c_\beta \varphi_2^\pm$

$$\mathscr{V}(\Phi_1,\Phi_2) \supset v^2(\lambda_5-\lambda_4)H^+H^- \Rightarrow m^2_{H^\pm} = v^2(\lambda_5-\lambda_4)\,.$$

- Neutral scalars

$$\mathscr{V}(\Phi_1,\Phi_2) \supset \frac{1}{2}\begin{pmatrix}\mathrm{h}^0 & \mathrm{R}^0 & \mathrm{I}^0\end{pmatrix} \mathcal{M}_0^2 \begin{pmatrix}\mathrm{h}^0\\ \mathrm{R}^0\\ \mathrm{I}^0\end{pmatrix} \qquad \mathcal{M}_0^2 = \mathcal{M}_0^{2\,T}$$

with

$$[\mathcal{M}_0^2]_{11} = 2v^2\left\{\lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + 2c_\beta^2 s_\beta^2[\lambda_{345}+2\lambda_5 c_\theta^2]\right\}$$
$$[\mathcal{M}_0^2]_{22} = 2v^2\left\{c_\beta^2 s_\beta^2(\lambda_1+\lambda_2-2\lambda_{345}) + \lambda_5(c_\beta^2-s_\beta^2)^2 c_\theta^2\right\}$$
$$[\mathcal{M}_0^2]_{12} = 2v^2 s_\beta c_\beta\left\{-\lambda_1 c_\beta^2 + \lambda_2 s_\beta^2 + (c_\beta^2-s_\beta^2)[\lambda_{345}+2\lambda_5 c_\theta^2]\right)\right\}$$
$$[\mathcal{M}_0^2]_{13} = -v^2\lambda_5 s_{2\beta} s_{2\theta}$$
$$[\mathcal{M}_0^2]_{23} = -v^2\lambda_5 c_{2\beta} s_{2\theta}$$
$$[\mathcal{M}_0^2]_{33} = 2v^2\lambda_5 s_\theta^2$$

$$\lambda_{345} \equiv \lambda_3+\lambda_4-\lambda_5$$

- Physical neutral scalars

$$\begin{pmatrix} h \\ H \\ A \end{pmatrix} = \mathcal{R}^T \begin{pmatrix} \mathrm{h}^0 \\ \mathrm{R}^0 \\ \mathrm{I}^0 \end{pmatrix}$$

where

$$\mathcal{R}^T \mathcal{M}_0^2 \mathcal{R} = \mathrm{diag}(m_h^2, m_H^2, m_A^2)\,, \quad \mathcal{R}^{-1} = \mathcal{R}^T$$

- $\mathcal{R}$ "mixes", a priori, all three neutral scalars
- assume h is the lightest one, Higgs-like, $m_h = 125$ GeV
- Notice

$$\mathrm{Tr}[\mathcal{M}_0^2] = m_h^2 + m_H^2 + m_A^2 = 2v^2[\lambda_1 c_\beta^2 + \lambda_2 s_\beta^2 + \lambda_5]$$
$$\det[\mathcal{M}_0^2] = m_h^2\, m_H^2\, m_A^2 = 2v^6 \lambda_5 (\lambda_1\lambda_2 - \lambda_{345}^2) \sin^2 2\beta \sin^2\theta$$

No decoupling regime

Yukawa couplings

- $\mathscr{L}_{\mathrm{S\bar{q}q}}$, $S = h, H, A, H^\pm$ with $H_q \equiv \frac{N_q + N_q^\dagger}{2}$, $A_q \equiv \frac{N_q - N_q^\dagger}{2}$:

$$[H_q]_{ij} = t_\beta \delta_{ij} m_{q_i} - (t_\beta + t_\beta^{-1}) \hat{n}^*_{[\mathrm{q}]i} \hat{n}_{[\mathrm{q}]j} \frac{m_{d_i} + m_{d_j}}{2}$$

$$[A_q]_{ij} = (t_\beta + t_\beta^{-1}) \hat{n}^*_{[\mathrm{q}]i} \hat{n}_{[\mathrm{q}]j} \frac{m_{d_i} - m_{d_j}}{2}$$

- Neutral [$s = 1, 2, 3$ for $S = h, H, A$, respectively]

$$\mathscr{L}_{S\bar{\mathrm{q}}\mathrm{q}} = -\frac{S}{v} \left\{ \bar{d} \left[\mathcal{R}_{1s} M_d + \mathcal{R}_{2s} H_d + i\mathcal{R}_{3s} A_d \right] d + \bar{d} \left[\mathcal{R}_{2s} A_d + i\mathcal{R}_{3s} H_d \right] \gamma_5 d \right\}$$
$$- \frac{S}{v} \left\{ \bar{u} \left[\mathcal{R}_{1s} M_u + \mathcal{R}_{2s} H_u - i\mathcal{R}_{3s} A_u \right] u + \bar{u} \left[\mathcal{R}_{2s} A_u - i\mathcal{R}_{3s} H_u \right] \gamma_5 u \right\}$$

- Charged

$$\mathscr{L}_{H^\pm \bar{\mathrm{q}}\mathrm{q}} = -\frac{\sqrt{2} H^+}{v} \left[\bar{u}_L V N_d d_R - \bar{u}_R N_u^\dagger V d_L \right]$$
$$- \frac{\sqrt{2} H^-}{v} \left[\bar{d}_R N_d^\dagger V^\dagger u_L - \bar{d}_L V^\dagger N_u u_R \right]$$

Constraints

- Good CKM matrix:
  - $|V_{ij}|$ in first two rows
  - CP violating phase $\gamma \equiv \arg(-V_{ud}V_{ub}^{*}V_{cb}V_{cd}^{*})$ (only tree level one)
- Scalar sector
  - Oblique parameters S and T
  - Boundedness of the scalar potential and perturbative unitarity of scattering processes
  - $m_{H^{\pm}}$, m_H, $m_A \geq 150$ GeV

Constraints

- Scalars & Yukawas
  - Production $\times$ decay signal strengths of Higgs-like h [ATLAS+CMS from Run I + Run II]
  - Neutral meson mixings
    - B_d^0–$\bar{B}_d^0$ and B_s^0–$\bar{B}_s^0$: mass differences ΔM_{B_d}, ΔM_{B_s} and mixing $\times$ decay CP asymmetries in $B_d \to J/\Psi K_S$ and $B_s \to J/\Psi\Phi$
    - K^0–$\bar{K}^0$: scalar mediated contribution to M_{12}^K does not yield sizable contributions to ϵ_K and ΔM_K
    - D^0–$\bar{D}^0$: short distance contribution to M_{12}^D verifies $|M_{12}^D| < 3 \times 10^{-2}\ \text{ps}^{-1}$
  - $\text{Br}(B \to X_s\gamma)$ (i.e. $b \to s\gamma$)
  - Bounds on rare top decays $t \to hq$

Goals of the analysis

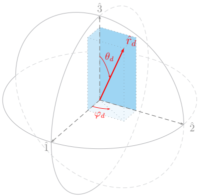
- establish that the model is viable
  (after imposing reasonable constraints)
- explore prospects for the observation
  of some definite non-SM signal
  - flavour changing decays $t \to hc, hu$ (LHC) and $h \to bs, bd$ (ILC)
  - representative low energy observable:
    time dependent CP violating asymmetry in $B_s \to J/\Psi\Phi$
    [SM prediction $A^{CP}_{J/\Psi\Phi} \simeq -0.04$, current exp. -0.030 ± 0.033]
  - not here: direct observation of new scalars

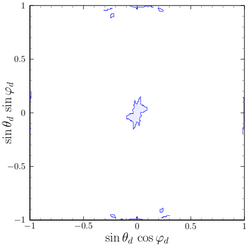
Parameters

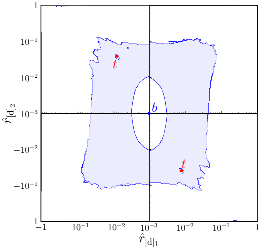
$$\{\hat{r}_{[\mathrm{d}]}, R, \theta, v^2, m_h, t_\beta, m^2_{H^\pm}, \mathcal{R}\}$$

$$\{2+3+1+0+0+1+1+3\} = 11$$

Results

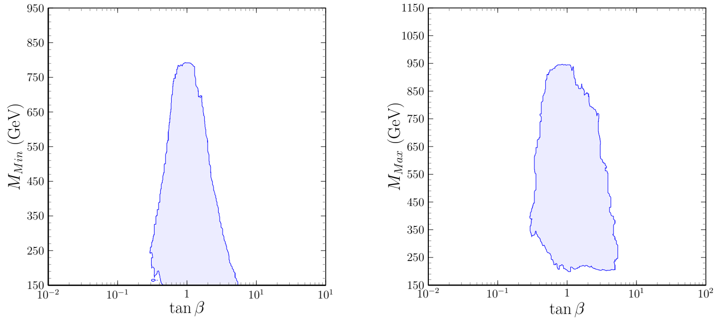






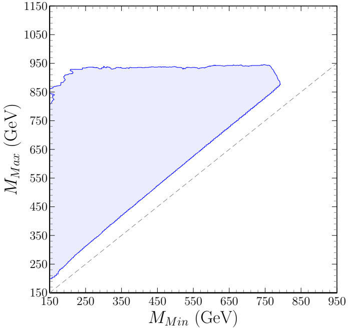
N.B. Allowed regions at 3σ in 2D-$\Delta\chi^2$

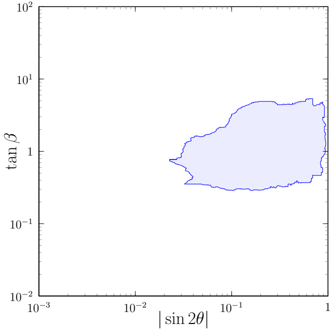
Results



$$M_{Min} \equiv \min(m_H, m_A, m_{H^\pm}), \quad M_{Max} \equiv \max(m_H, m_A, m_{H^\pm})$$

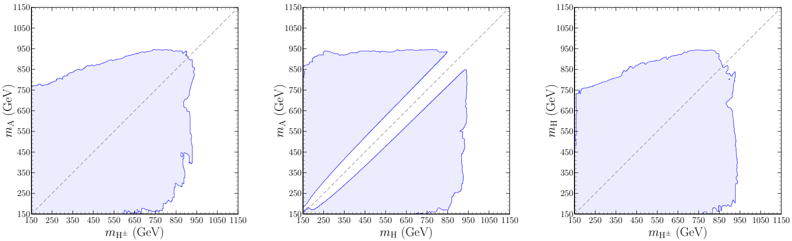
Results





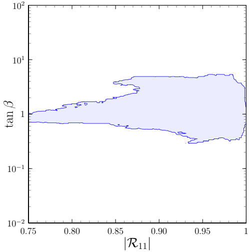
$$M_{Min} \equiv \min(m_H, m_A, m_{H^\pm}), \quad M_{Max} \equiv \max(m_H, m_A, m_{H^\pm})$$

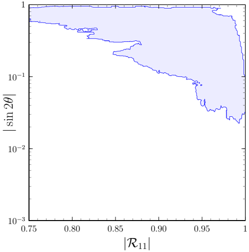
Results

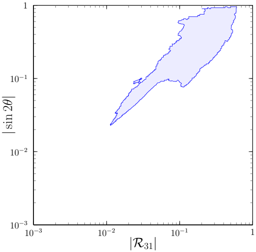


m_A (GeV)
$m_{H^\pm}$ (GeV)
m_A (GeV)
m_H (GeV)
m_H (GeV)
$m_{H^\pm}$ (GeV)
150 250 350 450 550 650 750 850 950 1050 1150

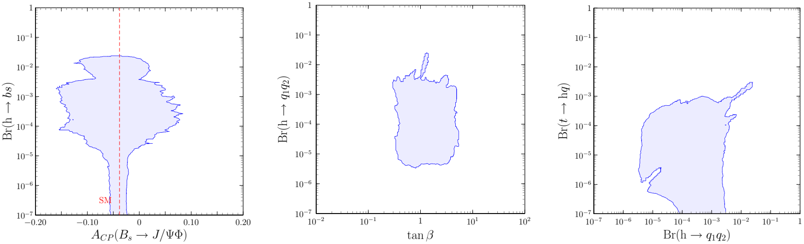
Results


10^2
10^1
1
10^{-1}
10^{-2}
$\tan\beta$
0.75
0.80
0.85
0.90
0.95
1
$|\mathcal{R}_{11}|$


1
10^{-1}
10^{-2}
10^{-3}
$|\sin 2\theta|$
0.75
0.80
0.85
0.90
0.95
1
$|\mathcal{R}_{11}|$


1
10^{-1}
10^{-2}
10^{-3}
$|\sin 2\theta|$
10^{-3}
10^{-2}
10^{-1}
1
$|\mathcal{R}_{31}|$

Results



$$\mathrm{Br}(t \to hq) \equiv \mathrm{Br}(t \to hc) + \mathrm{Br}(t \to hu)$$

$$\mathrm{Br}(h \to bq) \equiv \mathrm{Br}(h \to bs) + \mathrm{Br}(h \to bd)$$

$$\mathrm{Br}(h \to q_1 q_2) \equiv \mathrm{Br}(h \to bq) + \mathrm{Br}(h \to sd) + \mathrm{Br}(h \to cu)$$

Results – Summary

- Viable 2HDM with SCPV and complex CKM
- All new scalar masses below 950 GeV
- SFCNC mediated rare decays $t \to hq$ and $h \to q_1 q_2$ can be within experimental reach (LHC & ILC)
- Additional potential effects, e.g. $A^{CP}_{J/\Psi\Phi}$
- + correlations

Conclusions

- Part I: Two classes of 2HDM, shaped by symmetry and additional requirement (Left or Right conditions)
  controlled SFCNC (masses & unit vectors)
- Part II: Previous slide

Thank you!

Backup

Invariant conditions – Summary

Model	Invariant Conditions
dBGL	$\Gamma_1^\dagger \Gamma_2 = 0,\ \Delta_1^\dagger \Delta_2 = 0,\ \Gamma_1^\dagger \Delta_2 = 0,\ \Gamma_2^\dagger \Delta_1 = 0,\ \Gamma_1 \Gamma_2^\dagger = 0$
uBGL	$\Gamma_1^\dagger \Gamma_2 = 0,\ \Delta_1^\dagger \Delta_2 = 0,\ \Gamma_1^\dagger \Delta_2 = 0,\ \Gamma_2^\dagger \Delta_1 = 0,\ \Delta_1 \Delta_2^\dagger = 0$
gBGL	$\Gamma_1^\dagger \Gamma_2 = 0,\ \Delta_1^\dagger \Delta_2 = 0,\ \Gamma_1^\dagger \Delta_2 = 0,\ \Gamma_2^\dagger \Delta_1 = 0$
jBGL	$\Gamma_1^\dagger \Gamma_2 = 0,\ \Delta_1^\dagger \Delta_2 = 0,\ \Gamma_1^\dagger \Delta_1 = 0,\ \Gamma_2^\dagger \Delta_2 = 0$
Type A	$\Gamma_1 \Gamma_2^\dagger = 0,\ \Delta_1 \Delta_2^\dagger = 0,\ \text{rank}(\Gamma_1) = 3,\ \text{rank}(\Delta_1) = 2$
Type B	$\Gamma_1 \Gamma_2^\dagger = 0,\ \Delta_1 \Delta_2^\dagger = 0,\ \text{rank}(\Gamma_1) = 3,\ \text{rank}(\Delta_1) = 1$
Type C	$\Gamma_1 \Gamma_2^\dagger = 0,\ \Delta_1 \Delta_2^\dagger = 0,\ \text{rank}(\Gamma_1) = 2,\ \text{rank}(\Delta_1) = 3$
Type D	$\Gamma_1 \Gamma_2^\dagger = 0,\ \Delta_1 \Delta_2^\dagger = 0,\ \text{rank}(\Gamma_1) = 2,\ \text{rank}(\Delta_1) = 0$
Type E	$\Gamma_1 \Gamma_2^\dagger = 0,\ \Delta_1 \Delta_2^\dagger = 0,\ \text{rank}(\Gamma_1) = 2,\ \text{rank}(\Delta_1) = 2$
Type F	$\Gamma_1 \Gamma_2^\dagger = 0,\ \Delta_1 \Delta_2^\dagger = 0,\ \text{rank}(\Gamma_1) = 2,\ \text{rank}(\Delta_1) = 1$

Right models – A

- Transformation

$$\Phi_2 \mapsto e^{i\theta}\Phi_2, \quad u^0_{R3} \mapsto e^{i\theta}u^0_{R3}$$

- Yukawa matrices

$$\Gamma_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \Delta_1 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix}, \Delta_2 = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}$$

- *Right conditions*

$$N^0_d = M^0_d\, t_\beta \mathbf{1}, \quad N^0_u = M^0_u(t_\beta \mathrm{P}_1 + t_\beta \mathrm{P}_2 - t_\beta^{-1}\mathrm{P}_3)$$

- N_d and N_u parametrisation

$$(N_d)_{ij} = m_{d_i} t_\beta \delta_{ij}$$
$$(N_u)_{ij} = m_{u_i}(t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1})\hat{r}^*_{[\mathrm{u}]i}\hat{r}_{[\mathrm{u}]j})$$

Right models – B

- Transformation

$$\Phi_2 \mapsto e^{i\theta}\Phi_2, \quad u^0_{R1} \mapsto e^{i\theta}u^0_{R1}, \quad u^0_{R2} \mapsto e^{i\theta}u^0_{R2}$$

- Yukawa matrices

$$\Gamma_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \Delta_1 = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}, \Delta_2 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix}$$

- *Right conditions*

$$N^0_d = M^0_d\, t_\beta \mathbf{1}, \quad N^0_u = M^0_u(-t_\beta^{-1}\mathrm{P}_1 - t_\beta^{-1}\mathrm{P}_2 + t_\beta \mathrm{P}_3)$$

- N_d and N_u parametrisation

$$(N_d)_{ij} = m_{d_i} t_\beta \delta_{ij}$$
$$(N_u)_{ij} = m_{u_i}(-t_\beta^{-1}\delta_{ij} + (t_\beta + t_\beta^{-1})\hat{r}^*_{[\mathrm{u}]i}\hat{r}_{[\mathrm{u}]j})$$

Right models – C

- Transformation

$$\Phi_2 \mapsto e^{i\theta}\Phi_2, \quad d^0_{R3} \mapsto e^{-i\theta} d^0_{R3}$$

- Yukawa matrices

$$\Gamma_1 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}, \Delta_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- *Right conditions*

$$N^0_d = M^0_d (t_\beta \mathrm{P}_1 + t_\beta \mathrm{P}_2 - t_\beta^{-1} \mathrm{P}_3), \quad N^0_u = M^0_u \, t_\beta \mathbf{1}$$

- N_d and N_u parametrisation

$$(N_d)_{ij} = m_{d_i}(t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) \hat{r}^*_{[\mathrm{d}]i} \hat{r}_{[\mathrm{d}]j})$$
$$(N_u)_{ij} = t_\beta m_{u_i} \delta_{ij}$$

Right models – D

- Transformation

$$\Phi_2 \mapsto e^{i\theta}\Phi_2, \quad d^0_{R1} \mapsto e^{-i\theta} d^0_{R1}, \quad d^0_{R2} \mapsto e^{-i\theta} d^0_{R2}$$

- Yukawa matrices

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}, \Gamma_2 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix}, \Delta_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- *Right conditions*

$$N^0_d = M^0_d(-t_\beta^{-1}\mathrm{P}_1 - t_\beta^{-1}\mathrm{P}_2 + t_\beta \mathrm{P}_3), \quad N^0_u = M^0_u\, t_\beta \mathbf{1}\,.$$

- N_d and N_u parametrisation

$$(N_d)_{ij} = m_{d_i}(-t_\beta\delta_{ij} + (t_\beta + t_\beta^{-1})\hat{r}^*_{[\mathrm{d}]i}\hat{r}_{[\mathrm{d}]j})$$
$$(N_u)_{ij} = t_\beta m_{u_i}\delta_{ij}$$

Right models – E

- Transformation

$$\Phi_2 \mapsto e^{i\theta}\Phi_2, \quad d^0_{R3} \mapsto e^{-i\theta}d^0_{R3}, \quad u^0_{R3} \mapsto e^{i\theta}u^0_{R3}$$

- Yukawa matrices

$$\Gamma_1 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}, \Delta_1 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix}, \Delta_2 = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}$$

- *Right conditions*

$$N^0_d = M^0_d(t_\beta \mathrm{P}_1 + t_\beta \mathrm{P}_2 - t_\beta^{-1}\mathrm{P}_3), \quad N^0_u = M^0_u(t_\beta \mathrm{P}_1 + t_\beta \mathrm{P}_2 - t_\beta^{-1}\mathrm{P}_3)$$

- N_d and N_u parametrisation

$$(N_d)_{ij} = m_{d_i}(t_\beta\delta_{ij} - (t_\beta + t_\beta^{-1})\hat{r}^*_{[\mathrm{d}]i}\hat{r}_{[\mathrm{d}]j})$$
$$(N_u)_{ij} = m_{u_i}(t_\beta\delta_{ij} - (t_\beta + t_\beta^{-1})\hat{r}^*_{[\mathrm{u}]i}\hat{r}_{[\mathrm{u}]j})$$

Right models – F

- Transformation

$$\Phi_2 \mapsto e^{i\theta}\Phi_2, \quad d^0_{R3} \mapsto e^{-i\theta}d^0_{R3}, \quad u^0_{R1} \mapsto e^{i\theta}u^0_{R1}, \quad u^0_{R2} \mapsto e^{i\theta}u^0_{R2}$$

- Yukawa matrices

$$\Gamma_1 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}, \Delta_1 = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}, \Delta_2 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix}$$

- *Right conditions*

$$N^0_d = M^0_d(t_\beta \mathrm{P}_1 + t_\beta \mathrm{P}_2 - t_\beta^{-1}\mathrm{P}_3), \quad N^0_u = M^0_u(-t_\beta^{-1}\mathrm{P}_1 - t_\beta^{-1}\mathrm{P}_2 + t_\beta \mathrm{P}_3)$$

- N_d and N_u parametrisation

$$(N_d)_{ij} = m_{d_i}(t_\beta \delta_{ij} - (t_\beta + t$$

$$(N_u)_{ij} = m_{u_i}(-t_\beta^{-1}\delta_{ij} + (t_\beta + t_\beta^{-1})\hat{r}^*_{[\mathrm{u}]i}\hat{r}_{[\mathrm{u}]j})$$

Parameters for scalar sector

Parameters: $\{v^2,\, \beta,\, \theta,\, m_h^2,\, m_{H^\pm}^2,\, \alpha_j\}$

[N.B. $\mathcal{R}_{ij} = f(\alpha_1, \alpha_2, \alpha_3)$]

λ_5, m_H^2, m_A^2:

$$\lambda_5 = \frac{m_h^2}{2v^2}\frac{\mathcal{R}_{31}}{s_\theta}\frac{1}{s_\theta\mathcal{R}_{31} - c_\theta c_{2\beta}\mathcal{R}_{21} - c_\theta s_{2\beta}\mathcal{R}_{11}}$$

$$m_H^2 = m_h^2\frac{\mathcal{R}_{31}}{\mathcal{R}_{32}}\left[\frac{-c_\theta s_{2\beta}\mathcal{R}_{12} - c_\theta c_{2\beta}\mathcal{R}_{22} + s_\theta\mathcal{R}_{32}}{-c_\theta s_{2\beta}\mathcal{R}_{11} - c_\theta c_{2\beta}\mathcal{R}_{21} + s_\theta\mathcal{R}_{31}}\right]$$

$$m_A^2 = m_h^2\frac{\mathcal{R}_{31}}{\mathcal{R}_{33}}\left[\frac{-c_\theta s_{2\beta}\mathcal{R}_{13} - c_\theta c_{2\beta}\mathcal{R}_{23} + s_\theta\mathcal{R}_{33}}{-c_\theta s_{2\beta}\mathcal{R}_{11} - c_\theta c_{2\beta}\mathcal{R}_{21} + s_\theta\mathcal{R}_{31}}\right]$$

Parameters for scalar sector

With

$$[\mathcal{M}_0^2]_{11} = m_h^2\,\mathcal{R}_{11}^2 + m_H^2\,\mathcal{R}_{12}^2 + m_A^2\,\mathcal{R}_{13}^2$$
$$[\mathcal{M}_0^2]_{22} = m_h^2\,\mathcal{R}_{21}^2 + m_H^2\,\mathcal{R}_{22}^2 + m_A^2\,\mathcal{R}_{23}^2$$
$$[\mathcal{M}_0^2]_{12} = m_h^2\,\mathcal{R}_{11}\mathcal{R}_{21} + m_H^2\,\mathcal{R}_{12}\mathcal{R}_{22} + m_A^2\,\mathcal{R}_{13}\mathcal{R}_{23}$$

λ_1, λ_2 and λ_{345}:

$$\lambda_1 = \frac{1}{2v^2}\left[[\mathcal{M}_0^2]_{11} + t_\beta^2[\mathcal{M}_0^2]_{22} + 2t_\beta[\mathcal{M}_0^2]_{12}\right] - \lambda_5 c_\theta^2 t_\beta^2$$
$$\lambda_2 = \frac{1}{2v^2}\left[[\mathcal{M}_0^2]_{11} + t_\beta^{-2}[\mathcal{M}_0^2]_{22} - 2t_\beta^{-1}[\mathcal{M}_0^2]_{12}\right] - \lambda_5 c_\theta^2 t_\beta^{-2}$$
$$\lambda_{345} = \frac{1}{2v^2}\left[[\mathcal{M}_0^2]_{11} - [\mathcal{M}_0^2]_{22} + (t_\beta^{-1} - t_\beta)[\mathcal{M}_0^2]_{12}\right] - \lambda_5 c_\theta^2$$

$$\lambda_4 = \lambda_5 - m_{H^\pm}^2/v^2, \qquad \lambda_3 = \lambda_{345} - \lambda_4 + \lambda_5\,.$$