An SU(2)$_{\mu\tau}$ Model for Muon g-2 Anomaly

Koji Tsumura (Kyoto U.)

Workshop on Multi-Higgs Models,

Lisbon, Portugal, Sep 4-7, 2018

Based JHEP 05 (2018) 069 with C.-W. Chiang (NTU)

"Model with a gauged lepton flavor SU(2) symmetry"

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- Introduction
  - Muon g-2 anomaly
  - $U(1)_{\mu\text{-}\tau}$ Model
- $SU(2)_{\mu\tau}$ Model
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  - Higgs Sector (Symmetry breaking)
  - Michel Decay
  - Collider Signature
  - Neutrino Mass
- Conclusion

Magnetic Moment of Muon

- g-factor : Int. btw Spin-B (Magnetic field)
  - ✓ Classical (Dirac Eq.) → g=2
  - ✓ Quantum → g=2(1+a_μ) [a_μ=(g-2)/2]

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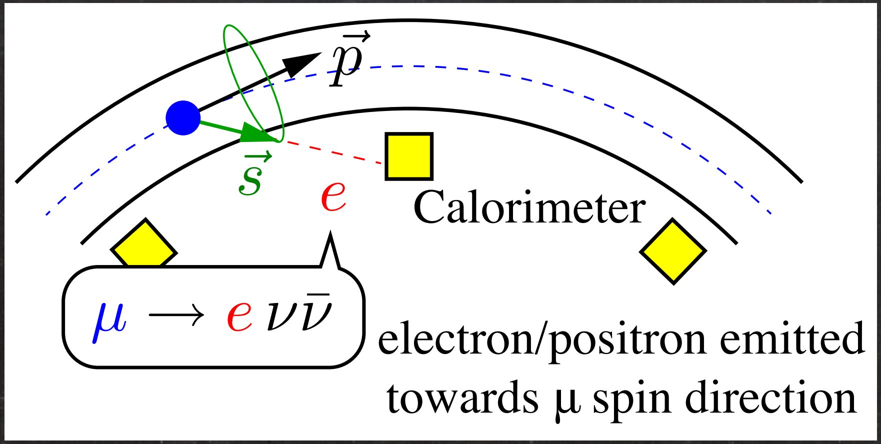
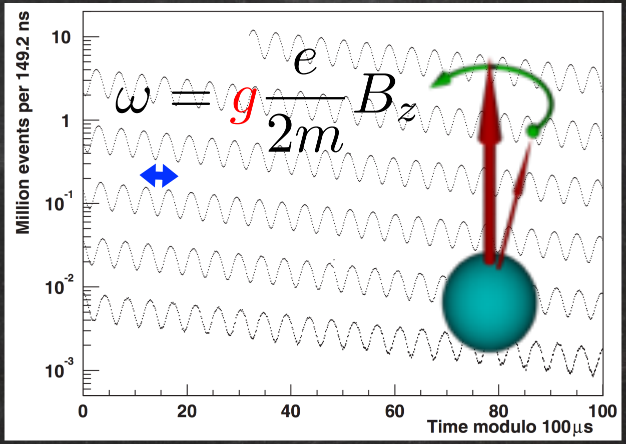
- Experiment : BNL E821



Larmor Precession



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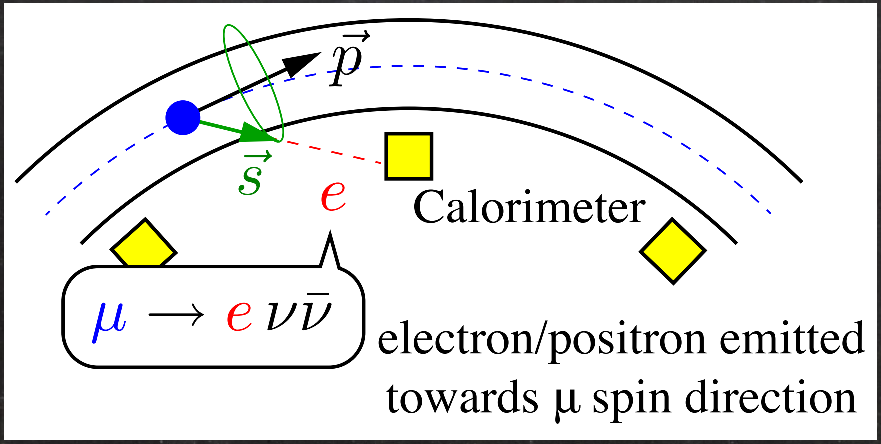

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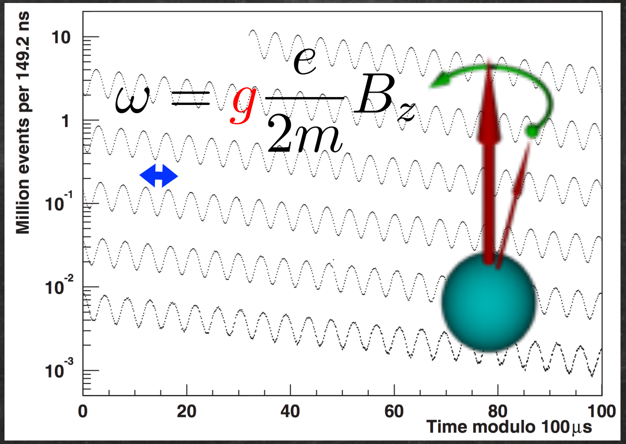

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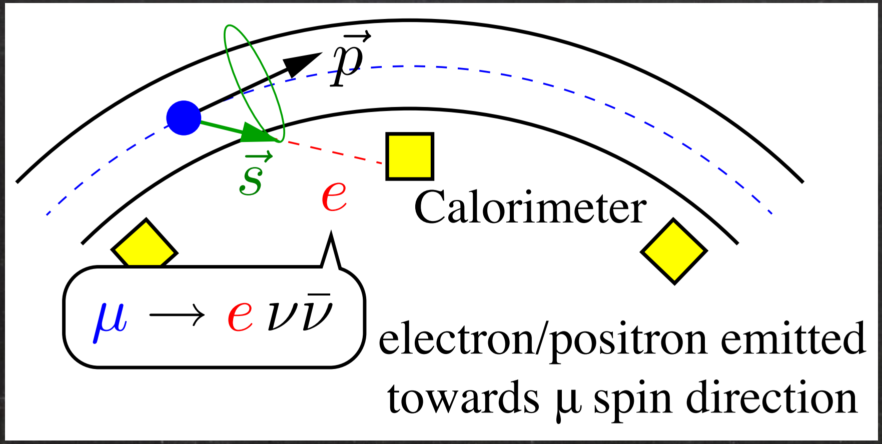

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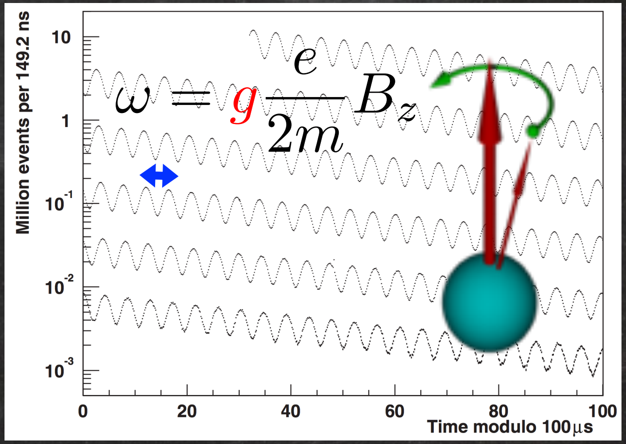

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- New Expt : FNAL E989 has been started

KNT18 a_μ^{SM} update [KNT18: arXiv:1802.02995, PRD (in press)]

	2011		2017	
QED	11658471.81 (0.02)	$\longrightarrow$	11658471.90 (0.01)	[arXiv:1712.06060]
EW	15.40 (0.20)	$\longrightarrow$	15.36 (0.10)	[Phys. Rev. D 88 (2013) 053005]
LO HLbL	10.50 (2.60)	$\longrightarrow$	9.80 **(2.60)**	[EPJ Web Conf. 118 (2016) 01016]
NLO HLbL			0.30 (0.20)	[Phys. Lett. B 735 (2014) 90]
	HLMNT11		**KNT18**	
LO HVP	694.91 (4.27)	$\longrightarrow$	693.27 **(2.46)**	this work
NLO HVP	-9.84 (0.07)	$\longrightarrow$	-9.82 (0.04)	this work
NNLO HVP			1.24 (0.01)	[Phys. Lett. B 734 (2014) 144]
Theory total	11659182.80 (4.94)	$\longrightarrow$	11659182.05 (3.56)	this work
Experiment			11659209.10 (6.33)	world avg
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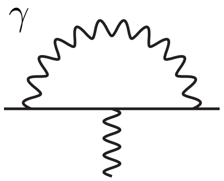

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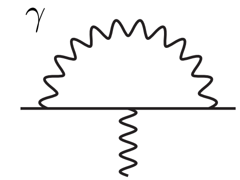

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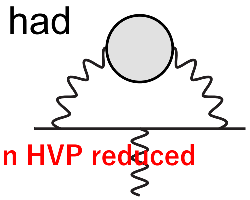

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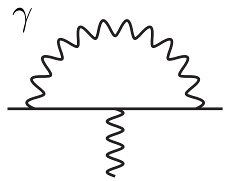

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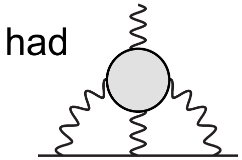

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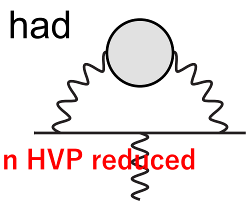

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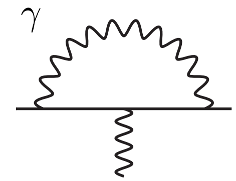

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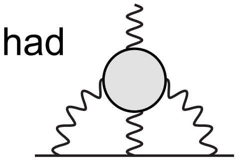

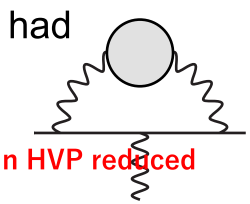

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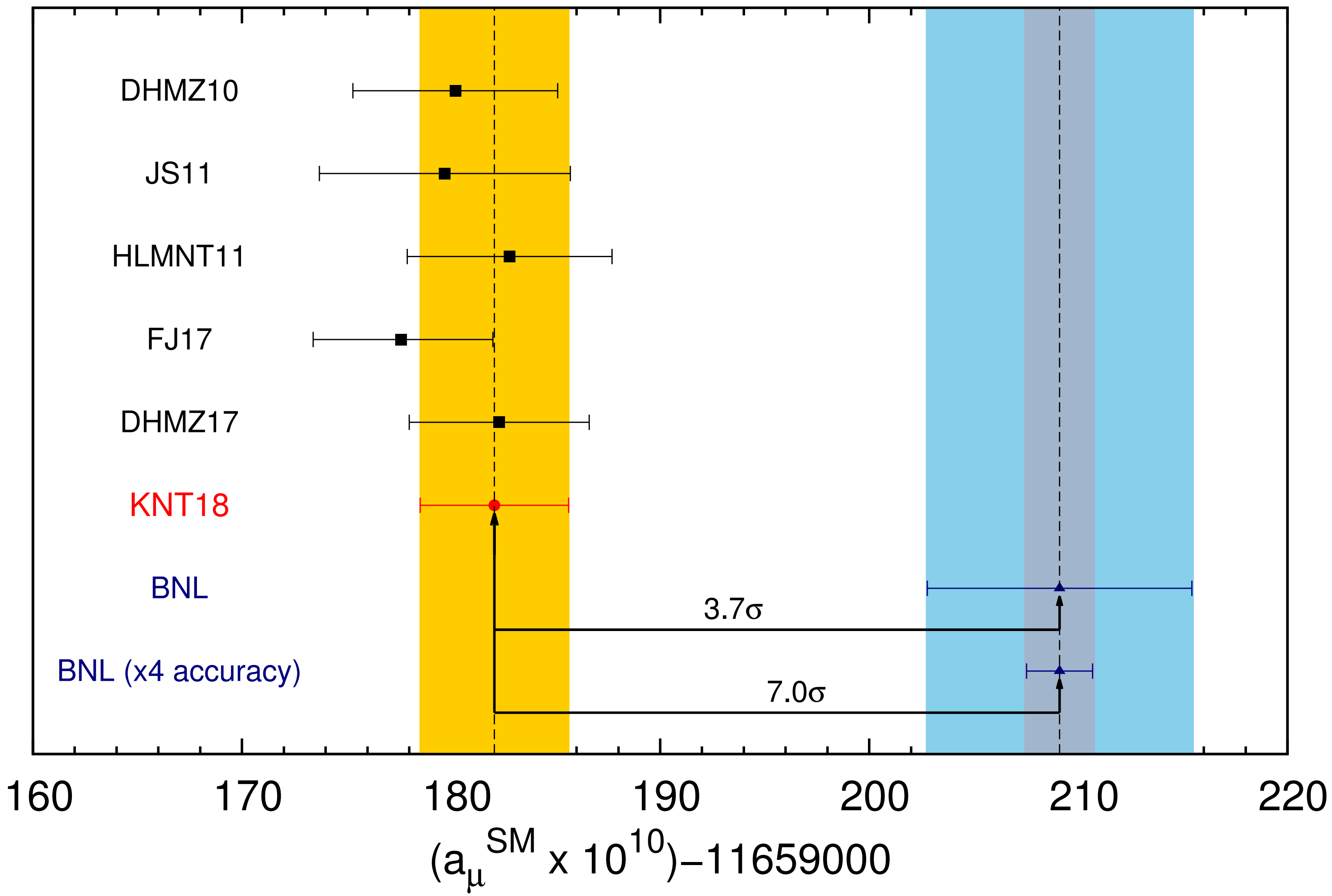

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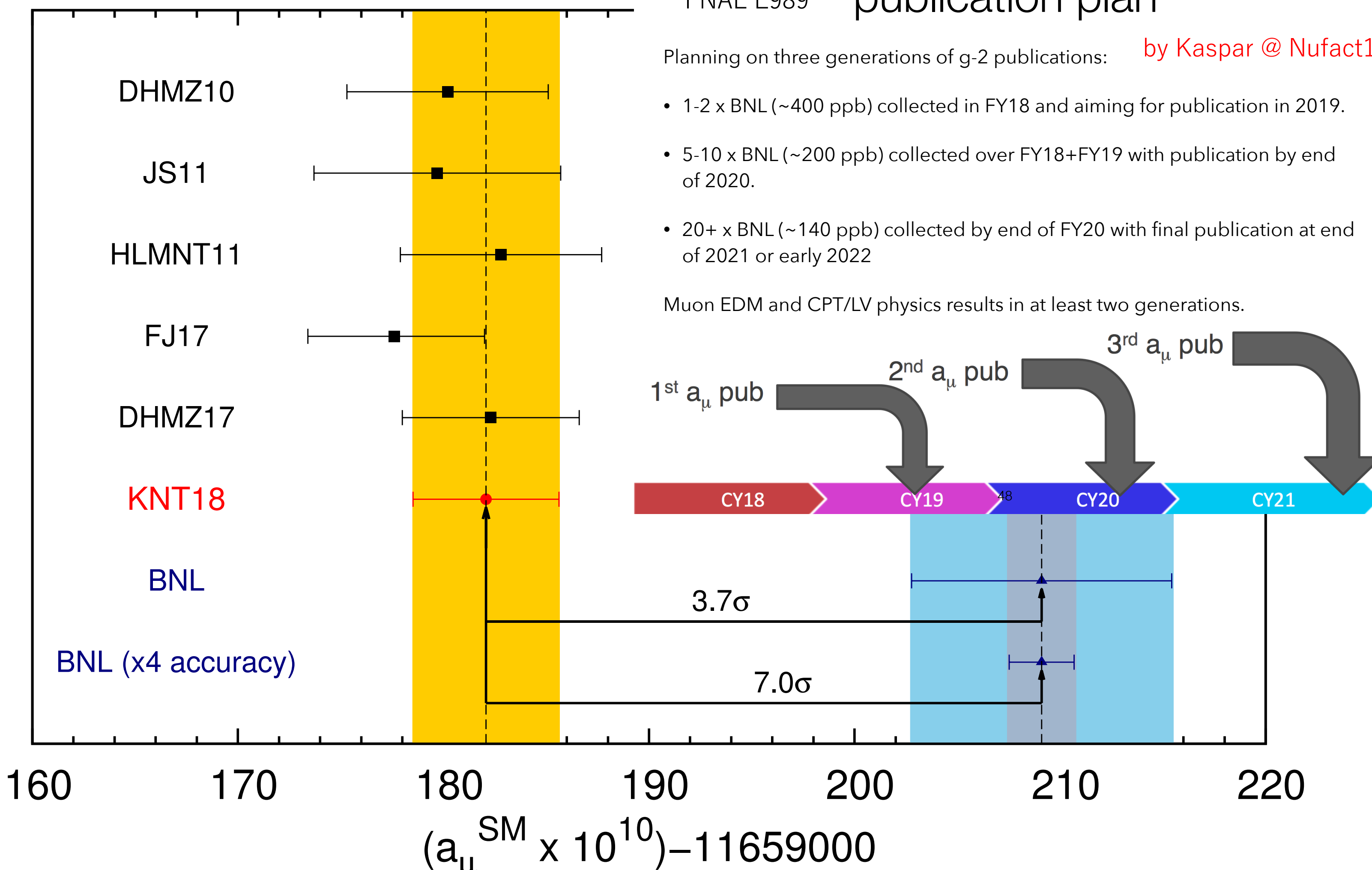


FNAL E989 publication plan

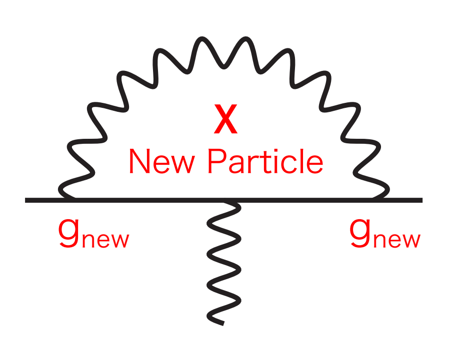
by Kaspar @ Nufact18

Planning on three generations of g-2 publications:

- 1-2 x BNL (~400 ppb) collected in FY18 and aiming for publication in 2019.
- 5-10 x BNL (~200 ppb) collected over FY18+FY19 with publication by end of 2020.
- 20+ x BNL (~140 ppb) collected by end of FY20 with final publication at end of 2021 or early 2022

Muon EDM and CPT/LV physics results in at least two generations.

New Physics



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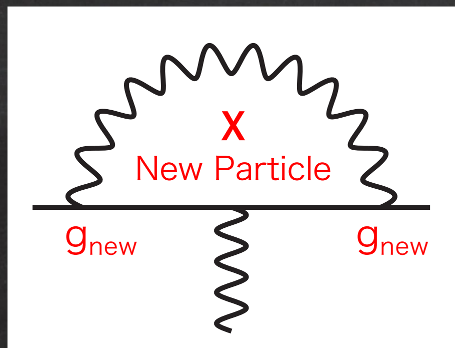
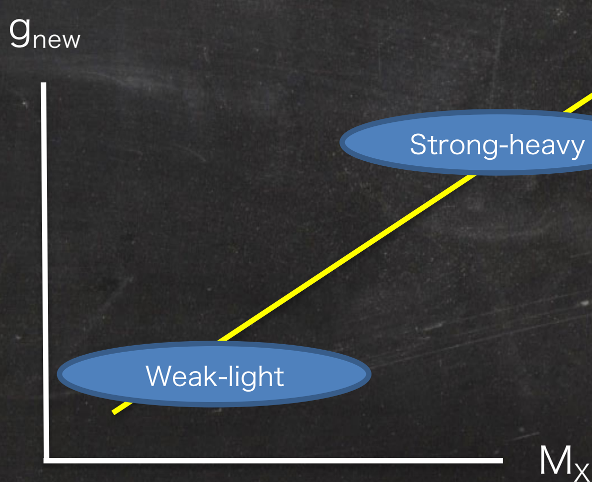


$$\Delta a_\mu^{\text{new}} \sim \frac{g_{\text{new}}^2}{(4\pi)^2} \frac{M_\mu^2}{M_X^2}$$

To fit the observed discrepancy

$g_{new} \approx g_{weak}$ and $M_X \approx M_W$

($g_{new} \approx 10^{-3}$ and $M_X \approx M_\mu$)



U(1)$_{\mu\text{-}\tau}$ Model

From Shimomura's slide @ NHWG20

Gauged U(1)$_{L\mu\text{-}L\tau}$ model

He, Joshi, Lew, Volkas, PRD (1991) ,
R. Foot, Mod.Phys.Lett. (1991)

- A minimal extension of the SM
- Anomaly free
- Large neutrino mixing (approx.)

  Choubey, Rodejohann, Eur.Phys.J, (2005)
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	l_e	e_R	l_μ	μ_R	l_τ	τ_R
L_μ	0	0	1	1	0	0
L_τ	0	0	0	0	1	1

The Lagrangian

Kinetic mixing with SM gauge boson

$$\mathcal{L}_{\text{new}} = -\frac{1}{4}Z'_{\mu\nu}Z'^{\mu\nu} + \frac{m^2_{Z'}}{2}Z'_\mu Z'^\mu + \underline{\frac{\epsilon}{2}B_{\mu\nu}Z'^{\mu\nu}}$$

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new interactions for *μ, τ* and *ν*

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SU(2)$_{\mu\tau}$ Model

- Gauged Lepton Flavor SU(2)
  e ~ singlet, (μ,τ) ~ doublet
- Flavor Higgs is also needed
  (discuss later)
- Flavor Gauge bosons X

$$(\mathcal{D}^\lambda)^\alpha{}_\beta = \delta^\alpha{}_\beta\, D^\lambda + i\, g_X (X^\lambda)^\alpha{}_\beta$$

	SM leptons			
	$L = L_e$	$L^\alpha = \begin{bmatrix} L_\mu \\ L_\tau \end{bmatrix}$	e_R	${e_R}^\alpha = \begin{bmatrix} \mu_R \\ \tau_R \end{bmatrix}$
$SU(2)_{\mu\tau}$	**1**	**2**	**1**	**2**
$SU(2)_L$	**2**	**2**	**1**	**1**
$U(1)_Y$	$-1/2$	$-1/2$	-1	-1

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$SU(2)_L$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)_Y$	$-1/2$	$-1/2$	-1	-1

$$(\mathcal{D}^\lambda)^\alpha{}_\beta = \delta^\alpha{}_\beta \, D^\lambda + i\, g_X (X^\lambda)^\alpha{}_\beta$$

$$(X^\lambda)^\alpha{}_\beta = \frac{(\sigma_a)^\alpha{}_\beta}{2} X^\lambda_a = \frac{1}{2}\begin{bmatrix} X^\lambda_3 & \sqrt{2}X^\lambda_+ \\ \sqrt{2}X^\lambda_- & -X^\lambda_3 \end{bmatrix}, \quad X^\lambda_\pm = \frac{1}{\sqrt{2}}(X^\lambda_1 \mp i\, X^\lambda_2)$$

X_3 is nothing but Z' in $U(1)_{L_\mu - L_\tau}$

The Lagrangian

Kinetic mixing with SM gauge boson

$$\mathcal{L}_{\rm new} = -\frac{1}{4} Z'_{\mu\nu} Z'^{\mu\nu} + \frac{m^2_{Z'}}{2} Z'_\mu Z'^\mu + \cancel{\frac{\epsilon}{2} B_{\mu\nu} Z'^{\mu\nu}}$$
$$+ g' Z'_\mu (\bar{\mu}\gamma^\mu \mu + \bar{\nu}_\mu \gamma^\mu \nu_\mu - \bar{\tau}\gamma^\mu \tau - \bar{\nu}_\tau \gamma^\mu \nu_\tau)$$

new interactions for μ, τ and ν

$SU(2)_{\mu\tau}$ Model

- Gauged Lepton Flavor SU(2)

  e ~ singlet, (μ,τ) ~ doublet

- Flavor Higgs is also needed

  (discuss later)

- Flavor Gauge bosons X

	SM leptons			
	$L = L_e$	$L^\alpha = \begin{bmatrix} L_\mu \\ L_\tau \end{bmatrix}$	e_R	${e_R}^\alpha = \begin{bmatrix} \mu_R \\ \tau_R \end{bmatrix}$
$SU(2)_{\mu\tau}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$
$SU(2)_L$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)_Y$	$-1/2$	$-1/2$	-1	-1

$$(\mathcal{D}^\lambda)^\alpha{}_\beta = \delta^\alpha{}_\beta\, D^\lambda + i\, g_X (X^\lambda)^\alpha{}_\beta$$

$$(X^\lambda)^\alpha{}_\beta = \frac{(\sigma_a)^\alpha{}_\beta}{2} X_a^\lambda = \frac{1}{2}\begin{bmatrix} X_3^\lambda & \sqrt{2}X_+^\lambda \\ \sqrt{2}X_-^\lambda & -X_3^\lambda \end{bmatrix}, \quad X_\pm^\lambda = \frac{1}{\sqrt{2}}(X_1^\lambda \mp i\, X_2^\lambda)$$

X_3 is nothing but Z' in $U(1)_{L_\mu - L_\tau}$

3

Flavor-charged X boson (New)

The Lagrangian

Kinetic mixing with SM gauge boson

$$\mathcal{L}_{\rm new} = -\frac{1}{4}Z'_{\mu\nu}Z'^{\mu\nu} + \frac{m_{Z'}^2}{2}Z'_\mu Z'^\mu + \cancel{\frac{\epsilon}{2}B_{\mu\nu}Z'^{\mu\nu}}$$
$$+g' Z'_\mu(\bar{\mu}\gamma^\mu\mu + \bar{\nu}_\mu\gamma^\mu\nu_\mu - \bar{\tau}\gamma^\mu\tau - \bar{\nu}_\tau\gamma^\mu\nu_\tau)$$

new interactions for μ, τ and ν

Flavor Changing X

- Flavor Changing Interactions

$$-\frac{g_X}{\sqrt{2}}[\,X_+^\lambda(\overline{\mu}\gamma_\lambda\tau+\overline{\nu_{\mu L}}\gamma_\lambda\nu_{\tau L})+X_-^\lambda(\overline{\tau}\gamma_\lambda\mu+\overline{\nu_{\tau L}}\gamma_\lambda\nu_{\mu L})]$$

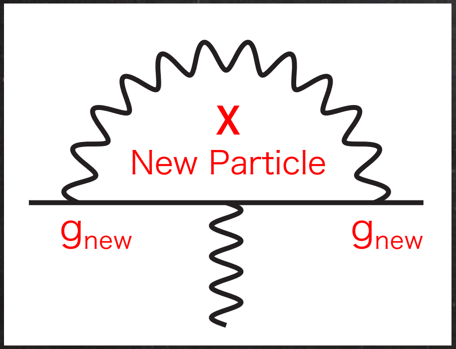
Flavor Changing X

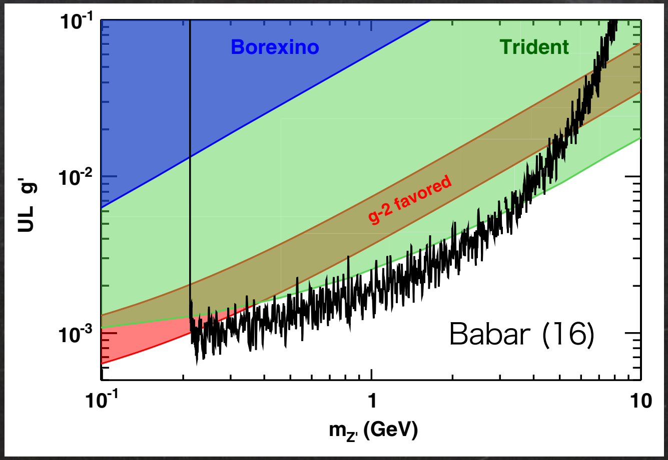
- Flavor Changing Interactions

$$-\frac{g_X}{\sqrt{2}}[X_+^\lambda(\overline{\mu}\gamma_\lambda\tau+\overline{\nu_{\mu L}}\gamma_\lambda\nu_{\tau L})+X_-^\lambda(\overline{\tau}\gamma_\lambda\mu+\overline{\nu_{\tau L}}\gamma_\lambda\nu_{\mu L})]$$

- Muon g-2

$$\Delta a_\mu(X_3)\approx\frac{g_X^2}{48\pi^2}\frac{M_\mu^2}{M_X^2}$$





Flavor Changing X

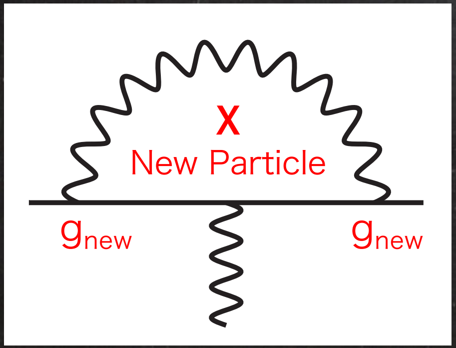
- Flavor Changing Interactions

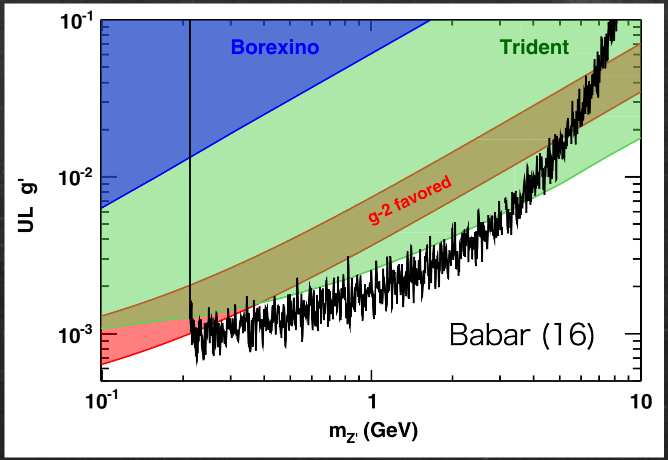
$$-\frac{g_X}{\sqrt{2}}[X_+^\lambda(\overline{\mu}\gamma_\lambda\tau+\overline{\nu_{\mu L}}\gamma_\lambda\nu_{\tau L})+X_-^\lambda(\overline{\tau}\gamma_\lambda\mu+\overline{\nu_{\tau L}}\gamma_\lambda\nu_{\mu L})]$$

- Muon g-2

Chirality Enhancement!!

$$\Delta a_\mu(X_3)\approx\frac{g_X^2}{48\pi^2}\frac{M_\mu^2}{M_X^2}\qquad\Delta a_\mu(X_\pm)\approx\frac{g_X^2}{48\pi^2}\frac{M_\mu^2}{M_X^2}\left(6\frac{M_\tau}{M_\mu}-4\right)\sim 96.9\times\Delta a_\mu(X_3)$$





Flavor Changing X

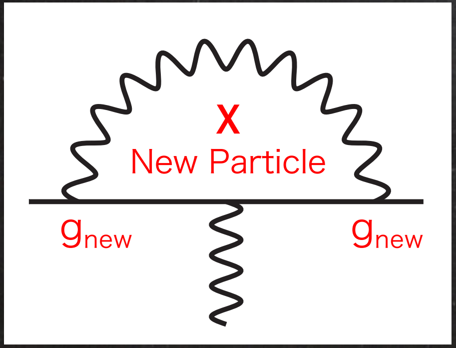
- Flavor Changing Interactions

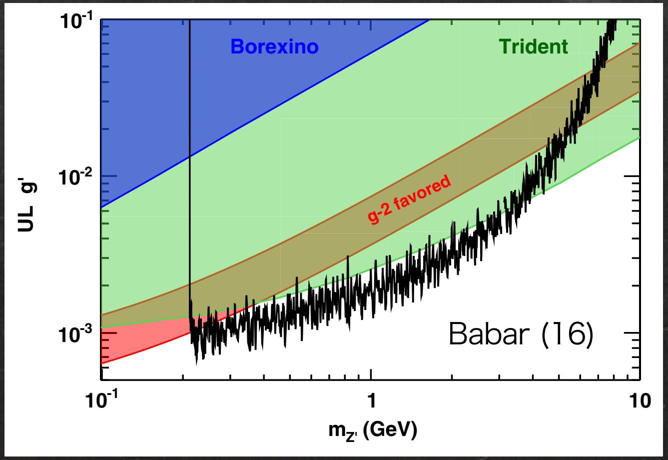
$$-\frac{g_X}{\sqrt{2}}[X_+^\lambda(\overline{\mu}\gamma_\lambda\tau+\overline{\nu_{\mu L}}\gamma_\lambda\nu_{\tau L})+X_-^\lambda(\overline{\tau}\gamma_\lambda\mu+\overline{\nu_{\tau L}}\gamma_\lambda\nu_{\mu L})]$$

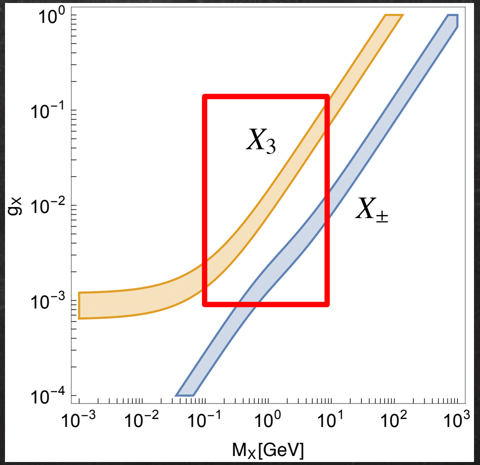
- Muon g-2

Chirality Enhancement!!

$$\Delta a_\mu(X_3)\approx\frac{g_X^2}{48\pi^2}\frac{M_\mu^2}{M_X^2}\qquad \Delta a_\mu(X_\pm)\approx\frac{g_X^2}{48\pi^2}\frac{M_\mu^2}{M_X^2}\left(6\frac{M_\tau}{M_\mu}-4\right)\sim 96.9\times\Delta a_\mu(X_3)$$







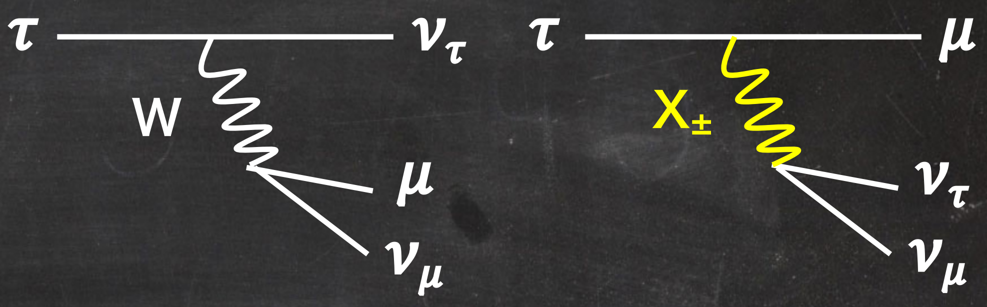
τ Michel Decay

- Flavor Changing Interactions

$$-\frac{g_X}{\sqrt{2}}[\,X_+^\lambda(\overline{\mu}\gamma_\lambda\tau+\overline{\nu_{\mu L}}\gamma_\lambda\nu_{\tau L})+X_-^\lambda(\overline{\tau}\gamma_\lambda\mu+\overline{\nu_{\tau L}}\gamma_\lambda\nu_{\mu L})]$$

- Lepton Universality

$$R_{\tau\to\mu/\tau\to e}=\frac{\Gamma(\tau\to\mu\overline{\nu_\mu}\nu_\tau)}{\Gamma(\tau\to e\overline{\nu_e}\nu_\tau)}$$

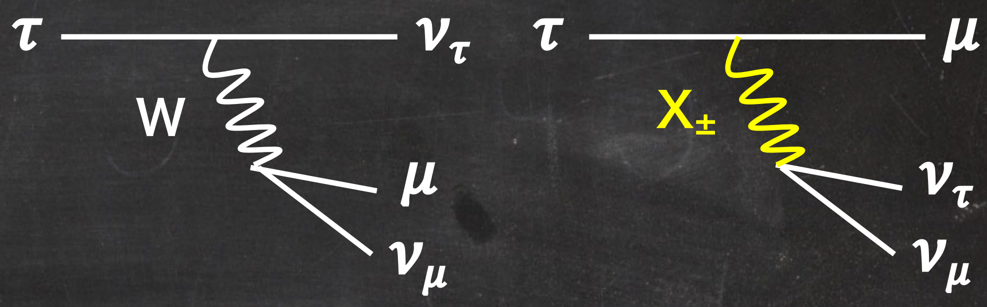


τ Michel Decay

- Flavor Changing Interactions

$$-\frac{g_X}{\sqrt{2}}[X_+^\lambda(\overline{\mu}\gamma_\lambda\tau+\overline{\nu_{\mu L}}\gamma_\lambda\nu_{\tau L})+X_-^\lambda(\overline{\tau}\gamma_\lambda\mu+\overline{\nu_{\tau L}}\gamma_\lambda\nu_{\mu L})]$$

- Lepton Universality



$$R_{\tau\to\mu/\tau\to e}=\frac{\Gamma(\tau\to\mu\overline{\nu_\mu}\nu_\tau)}{\Gamma(\tau\to e\overline{\nu_e}\nu_\tau)}$$

$$\eta^X\equiv\frac{R^{\mathrm{X}}_{\tau\to\mu/\tau\to e}}{R^{\mathrm{SM}}_{\tau\to\mu/\tau\to e}}=\left(1+\underbrace{\alpha_X}_{\text{Interference}}+\frac{1}{2}\alpha_X^2\right)-2\,\alpha_X\frac{M_\mu}{M_\tau}\frac{g(\rho^{-2})}{f(\rho^{-2})}$$

$$\frac{\alpha_X}{2}\equiv\frac{g_X^2}{4\sqrt{2}G_F M_X^2}$$

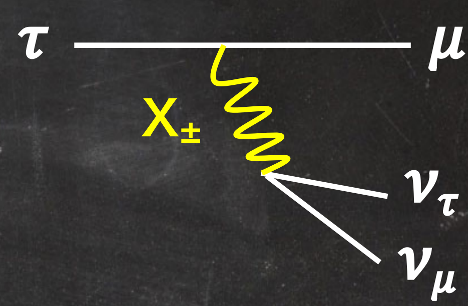
$$f(z)\equiv 1-8z+8z^3-z^4-12z^2\log z\approx 1$$
$$g(z)\equiv 1+9z-9z^2-z^3+6z\log z+6z^2\log z\approx 1$$

τ Michel Decay

- Flavor Changing Interactions

$$-\frac{g_X}{\sqrt{2}}[X_+^\lambda(\overline{\mu}\gamma_\lambda\tau+\overline{\nu_{\mu L}}\gamma_\lambda\nu_{\tau L})+X_-^\lambda(\overline{\tau}\gamma_\lambda\mu+\overline{\nu_{\tau L}}\gamma_\lambda\nu_{\mu L})]$$

- Lepton Universality

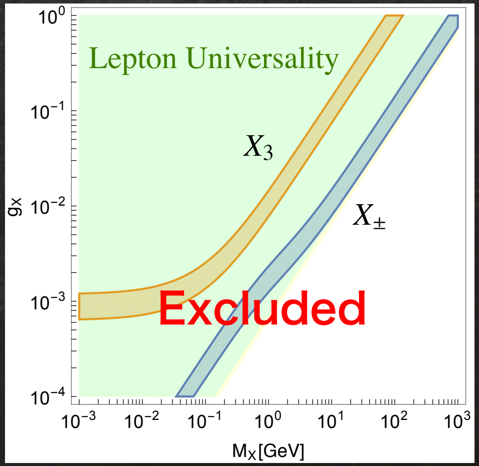


$$R_{\tau\to\mu/\tau\to e}=\frac{\Gamma(\tau\to\mu\overline{\nu_\mu}\nu_\tau)}{\Gamma(\tau\to e\overline{\nu_e}\nu_\tau)}$$

$$\eta^X\equiv\frac{R^{\mathrm{X}}_{\tau\to\mu/\tau\to e}}{R^{\mathrm{SM}}_{\tau\to\mu/\tau\to e}}=\left(1+\underline{\alpha_X}+\frac{1}{2}\alpha_X^2\right)-2\,\alpha_X\frac{M_\mu}{M_\tau}\frac{g(\rho^{-2})}{f(\rho^{-2})}$$

Interference

$$\frac{\alpha_X}{2}\equiv\frac{g_X^2}{4\sqrt{2}G_FM_X^2}$$



$$f(z)\equiv1-8z+8z^3-z^4-12z^2\log z\approx1$$

$$g(z)\equiv1+9z-9z^2-z^3+6z\log z+6z^2\log z\approx1$$

So far, No Higgs …

μ-τ Mass Degeneracy

- Yukawa sector

$$y_0 \overline{L}_\alpha \Phi_0 e_R^\alpha \to \frac{y_0 v_0}{\sqrt{2}} (\overline{\mu_L} \mu_R + \overline{\tau_L} \tau_R)$$

	SM leptons			
	$L = L_e$	$L^\alpha = \begin{bmatrix} L_\mu \\ L_\tau \end{bmatrix}$	e_R	${e_R}^\alpha = \begin{bmatrix} \mu_R \\ \tau_R \end{bmatrix}$
$SU(2)_{\mu\tau}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$
$SU(2)_L$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)_Y$	$-1/2$	$-1/2$	-1	-1

μ-τ Mass Degeneracy

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$$y_0 \overline{L}_\alpha \Phi_0 e_R^\alpha \to \frac{y_0 v_0}{\sqrt{2}} (\overline{\mu_L} \mu_R + \overline{\tau_L} \tau_R)$$

	SM leptons			
	$L = L_e$	$L^\alpha = \begin{bmatrix} L_\mu \\ L_\tau \end{bmatrix}$	e_R	${e_R}^\alpha = \begin{bmatrix} \mu_R \\ \tau_R \end{bmatrix}$
$SU(2)_{\mu\tau}$	**1**	**2**	**1**	**2**
$SU(2)_L$	**2**	**2**	**1**	**1**
$U(1)_Y$	$-1/2$	$-1/2$	-1	-1

- Flavor Adjoint Higgs

Scalars		
Φ_0	${\Phi^\alpha}_\beta$	S^α
1	**3**	**2**
2	**2**	**1**
$+1/2$	$+1/2$	0

$$\overline{L}_\alpha (y_0 \, \Phi_0 \, {\delta^\alpha}_\beta + 2 \, y \, {\Phi^\alpha}_\beta) e_R^\beta$$

$${\Phi^\alpha}_\beta = \frac{{(\sigma_a)^\alpha}_\beta}{2} \Phi_a = \frac{1}{2} \begin{bmatrix} \Phi_3 & \sqrt{2}\Phi_+ \\ \sqrt{2}\Phi_- & -\Phi_3 \end{bmatrix}, \quad \Phi_\pm = (\Phi_1 \mp i \, \Phi_2)/\sqrt{2}$$

μ-τ Mass Degeneracy

- Yukawa sector

$$y_0 \overline{L}_\alpha \Phi_0 e_R^\alpha \to \frac{y_0 v_0}{\sqrt{2}} (\overline{\mu_L} \mu_R + \overline{\tau_L} \tau_R)$$

	SM leptons			
	$L = L_e$	$L^\alpha = \begin{bmatrix} L_\mu \\ L_\tau \end{bmatrix}$	e_R	$e_R{}^\alpha = \begin{bmatrix} \mu_R \\ \tau_R \end{bmatrix}$
$SU(2)_{\mu\tau}$	**1**	**2**	**1**	**2**
$SU(2)_L$	**2**	**2**	**1**	**1**
$U(1)_Y$	$-1/2$	$-1/2$	-1	-1

- Flavor Adjoint Higgs

Scalars		
Φ_0	$\Phi^\alpha{}_\beta$	S^α
1	**3**	**2**
2	**2**	**1**
$+1/2$	$+1/2$	0

$$\overline{L}_\alpha (y_0 \, \Phi_0 \, \delta^\alpha{}_\beta + 2 \, y \, \Phi^\alpha{}_\beta) e_R^\beta$$

$$\Phi^\alpha{}_\beta = \frac{(\sigma_a)^\alpha{}_\beta}{2} \Phi_a = \frac{1}{2} \begin{bmatrix} \Phi_3 & \sqrt{2}\Phi_+ \\ \sqrt{2}\Phi_- & -\Phi_3 \end{bmatrix}, \quad \Phi_\pm = (\Phi_1 \mp i \, \Phi_2)/\sqrt{2}$$

$$y_0 = \frac{\sqrt{2}}{v_0} \frac{M_\mu + M_\tau}{2}$$

$$y = \frac{\sqrt{2}}{v_3} \frac{M_\mu - M_\tau}{2}$$

μ-τ Mass Degeneracy

- Yukawa sector

$$y_0 \overline{L}_\alpha \Phi_0 e_R^\alpha \to \frac{y_0 v_0}{\sqrt{2}} (\overline{\mu_L}\mu_R + \overline{\tau_L}\tau_R)$$

	SM leptons			
	$L = L_e$	$L^\alpha = \begin{bmatrix} L_\mu \\ L_\tau \end{bmatrix}$	e_R	${e_R}^\alpha = \begin{bmatrix} \mu_R \\ \tau_R \end{bmatrix}$
$SU(2)_{\mu\tau}$	**1**	**2**	**1**	**2**
$SU(2)_L$	**2**	**2**	**1**	**1**
$U(1)_Y$	$-1/2$	$-1/2$	-1	-1

- Flavor Adjoint Higgs

Scalars

Φ_0	${\Phi^\alpha}_\beta$	S^α
1	**3**	**2**
2	**2**	**1**
$+1/2$	$+1/2$	0

$$\overline{L}_\alpha (y_0 \, \Phi_0 \, {\delta^\alpha}_\beta + 2\, y\, {\Phi^\alpha}_\beta) e_R^\beta$$

$${\Phi^\alpha}_\beta = \frac{{(\sigma_a)^\alpha}_\beta}{2} \Phi_a = \frac{1}{2} \begin{bmatrix} \Phi_3 & \sqrt{2}\Phi_+ \\ \sqrt{2}\Phi_- & -\Phi_3 \end{bmatrix}, \quad \Phi_\pm = (\Phi_1 \mp i\, \Phi_2)/\sqrt{2}$$

- Flavor Fundamental SM singlet Higgs S^α

$$y_0 = \frac{\sqrt{2}}{v_0} \frac{M_\mu + M_\tau}{2}$$

$$y = \frac{\sqrt{2}}{v_3} \frac{M_\mu - M_\tau}{2}$$

$$\langle \Phi \rangle \propto \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \; : \; SU(2)_{\mu\tau} \to U(1)_{L_\mu - L_\tau}$$

$$\langle S \rangle \propto \begin{bmatrix} 0 \\ 1 \end{bmatrix} \; : \; SU(2)_{\mu\tau} \to \text{None}$$

Symmetry Breaking Sector

- Scalar Potential

$$
\begin{aligned}
V(\Phi_0, \Phi, S) = & -\mu_S^2|S|^2 - \mu_0^2 \Phi_{0i}^\dagger \Phi_0^i + 2m_\Phi^2 \,\mathrm{Tr}(\Phi_i^\dagger \Phi^i) + \lambda_S |S|^4 + \lambda_{0S}|S|^2\, \Phi_{0i}^\dagger \Phi_0^i \\
& + 2\lambda_{\Phi S1}|S|^2\, \mathrm{Tr}(\Phi_i^\dagger \Phi^i) + 2\lambda_{\Phi S2} S^\dagger [\Phi_i^\dagger, \Phi^i] S \\
& + 2\lambda_1' \big(\mathrm{Tr}(\Phi_i^\dagger \Phi^i)\big)^2 + 2\lambda_1'' \mathrm{Tr}(\Phi_i^\dagger \Phi^j)\mathrm{Tr}(\Phi_j^\dagger \Phi^i) + 2\lambda_1''' \mathrm{Tr}(\Phi_i^\dagger \Phi_j^\dagger)\mathrm{Tr}(\Phi^i \Phi^j) \\
& + \frac{\lambda_2}{2}(\Phi_{0i}^\dagger \Phi_0^i)^2 + 2\lambda_3 \mathrm{Tr}(\Phi_j^\dagger \Phi^j)\, \Phi_{0i}^\dagger \Phi_0^i + 2\lambda_4 \mathrm{Tr}(\Phi^i \Phi_j^\dagger)\, \Phi_{0i}^\dagger \Phi_0^j \\
& + \Big\{ \lambda_{\Phi S}' S\, i\,\sigma_2 [\Phi_i^\dagger, \Phi^i] S + \lambda_5 \mathrm{Tr}(\Phi_i^\dagger \Phi_j^\dagger)\, \Phi_0^i \Phi_0^j + 4\,\kappa\, \Phi_{0i}^\dagger \mathrm{Tr}(\Phi_j^\dagger \Phi^j \Phi^i) \\
& \quad + \lambda_\mu \Phi_{0i}^\dagger S^\dagger \Phi^i S + \lambda_{\mu 1}' \Phi_{0i}^\dagger S\, i\sigma_2 \Phi^i S + \lambda_{\mu 2}' S\, i\sigma_2 \Phi_i^\dagger S\, \Phi_0^i + \mathrm{H.c.} \Big\}
\end{aligned}
$$

Symmetry Breaking Sector

- Scalar Potential Assume Global Sym. for S → a simpler vacuum structure → Φ gets Diagonal VEV

$$
\begin{aligned}
V(\Phi_0, \Phi, S) = & -\mu_S^2 |S|^2 - \mu_0^2 \Phi_{0i}^\dagger \Phi_0^i + 2m_\Phi^2 \,\mathrm{Tr}(\Phi_i^\dagger \Phi^i) + \lambda_S |S|^4 + \lambda_{0S} |S|^2\, \Phi_{0i}^\dagger \Phi_0^i \\
& + 2\lambda_{\Phi S1} |S|^2 \,\mathrm{Tr}(\Phi_i^\dagger \Phi^i) + 2\lambda_{\Phi S2} S^\dagger [\Phi_i^\dagger, \Phi^i] S \\
& + 2\lambda_1' \left(\mathrm{Tr}(\Phi_i^\dagger \Phi^i)\right)^2 + 2\lambda_1'' \mathrm{Tr}(\Phi_i^\dagger \Phi^j) \mathrm{Tr}(\Phi_j^\dagger \Phi^i) + 2\lambda_1''' \mathrm{Tr}(\Phi_i^\dagger \Phi_j^\dagger) \mathrm{Tr}(\Phi^i \Phi^j) \\
& + \frac{\lambda_2}{2} (\Phi_{0i}^\dagger \Phi_0^i)^2 + 2\lambda_3 \mathrm{Tr}(\Phi_j^\dagger \Phi^j)\, \Phi_{0i}^\dagger \Phi_0^i + 2\lambda_4 \mathrm{Tr}(\Phi^i \Phi_j^\dagger)\, \Phi_{0i}^\dagger \Phi_0^j \\
& + \Big\{ \cancel{\lambda_{\Phi S}' S\, i\sigma_2 [\Phi_i^\dagger, \Phi^i]} S + \lambda_5 \mathrm{Tr}(\Phi_i^\dagger \Phi_j^\dagger)\, \Phi_0^i \Phi_0^j + 4\,\kappa\, \Phi_{0i}^\dagger \mathrm{Tr}(\Phi_j^\dagger \Phi^j \Phi^i) \\
& \quad + \lambda_\mu \Phi_{0i}^\dagger S^\dagger \Phi^i S + \cancel{\lambda_{\mu 1}' \Phi_{0i}^\dagger S\, i\sigma_2 \Phi^i S} + \cancel{\lambda_{\mu 2}' S\, i\sigma_2 \Phi_i^\dagger S\, \Phi_0^i} + \mathrm{H.c.} \Big\}
\end{aligned}
$$

Symmetry Breaking Sector

- Scalar Potential   Assume Global Sym. for S → a simpler vacuum structure → Φ gets Diagonal VEV

$$
\begin{aligned}
V(\Phi_0,\Phi,S) =& -\mu_S^2|S|^2 - \mu_0^2\Phi_{0i}^\dagger\Phi_0^i + 2m_\Phi^2\,\mathrm{Tr}(\Phi_i^\dagger\Phi^i) + \lambda_S|S|^4 + \lambda_{0S}|S|^2\,\Phi_{0i}^\dagger\Phi_0^i \\
&+ 2\lambda_{\Phi S1}|S|^2\,\mathrm{Tr}(\Phi_i^\dagger\Phi^i) + 2\lambda_{\Phi S2}S^\dagger[\Phi_i^\dagger,\Phi^i]S \\
&+ 2\lambda_1'\left(\mathrm{Tr}(\Phi_i^\dagger\Phi^i)\right)^2 + 2\lambda_1''\mathrm{Tr}(\Phi_i^\dagger\Phi^j)\mathrm{Tr}(\Phi_j^\dagger\Phi^i) + 2\lambda_1'''\mathrm{Tr}(\Phi_i^\dagger\Phi_j^\dagger)\mathrm{Tr}(\Phi^i\Phi^j) \\
&+ \frac{\lambda_2}{2}(\Phi_{0i}^\dagger\Phi_0^i)^2 + 2\lambda_3\mathrm{Tr}(\Phi_j^\dagger\Phi^j)\,\Phi_{0i}^\dagger\Phi_0^i + 2\lambda_4\mathrm{Tr}(\Phi^i\Phi_j^\dagger)\,\Phi_{0i}^\dagger\Phi_0^j \\
&+ \Big\{\cancel{\lambda_{\Phi S}'S\,i\sigma_2[\Phi_i^\dagger,\Phi^i]}S + \lambda_5\mathrm{Tr}(\Phi_i^\dagger\Phi_j^\dagger)\,\Phi_0^i\Phi_0^j + 4\,\kappa\,\Phi_{0i}^\dagger\mathrm{Tr}(\Phi_j^\dagger\Phi^j\Phi^i) \\
&\quad + \lambda_\mu\Phi_{0i}^\dagger S^\dagger\Phi^i S + \cancel{\lambda_{\mu1}'\Phi_{0i}^\dagger S\,i\sigma_2\Phi^i S} + \cancel{\lambda_{\mu2}'S\,i\sigma_2\Phi_i^\dagger S\,\Phi_0^i} + \mathrm{H.c.}\Big\}
\end{aligned}
$$

$$\langle\Phi\rangle = \begin{pmatrix} v_3 & 0 \\ 0 & v_3 \end{pmatrix} \;:\; SU(2)_{\mu\tau} \xrightarrow{\langle\Phi\rangle} U(1)_{\mu-\tau}$$

$$\begin{cases} (\Phi_0, \Phi_3) & \text{2HDM} \\ (\Phi_+, \Phi_-) & \text{2SDM} \end{cases}$$

... "flavor charged"

Scalar (w/o VEV)

Symmetry Breaking Sector

- Scalar Potential   Assume Global Sym. for S → a simpler vacuum structure → Φ gets Diagonal VEV

$$
\begin{aligned}
V(\Phi_0, \Phi, S) = & -\mu_S^2 |S|^2 - \mu_0^2 \Phi_{0i}^\dagger \Phi_0^i + 2 m_\Phi^2 \,\mathrm{Tr}(\Phi_i^\dagger \Phi^i) + \lambda_S |S|^4 + \lambda_{0S} |S|^2\, \Phi_{0i}^\dagger \Phi_0^i \\
& + 2\lambda_{\Phi S1} |S|^2\, \mathrm{Tr}(\Phi_i^\dagger \Phi^i) + 2\lambda_{\Phi S2} S^\dagger [\Phi_i^\dagger, \Phi^i] S \\
& + 2\lambda_1' \left(\mathrm{Tr}(\Phi_i^\dagger \Phi^i)\right)^2 + 2\lambda_1'' \mathrm{Tr}(\Phi_i^\dagger \Phi^j)\mathrm{Tr}(\Phi_j^\dagger \Phi^i) + 2\lambda_1''' \mathrm{Tr}(\Phi_i^\dagger \Phi_j^\dagger)\mathrm{Tr}(\Phi^i \Phi^j) \\
& + \frac{\lambda_2}{2} (\Phi_{0i}^\dagger \Phi_0^i)^2 + 2\lambda_3 \mathrm{Tr}(\Phi_j^\dagger \Phi^j)\, \Phi_{0i}^\dagger \Phi_0^i + 2\lambda_4 \mathrm{Tr}(\Phi^i \Phi_j^\dagger)\, \Phi_{0i}^\dagger \Phi_0^j \\
& + \Big\{ \cancel{\lambda_{\Phi S}' S\, i\sigma_2 [\Phi_i^\dagger, \Phi^i] S} + \lambda_5 \mathrm{Tr}(\Phi_i^\dagger \Phi_j^\dagger)\, \Phi_0^i \Phi_0^j + 4\,\kappa\, \Phi_{0i}^\dagger \mathrm{Tr}(\Phi_j^\dagger \Phi^j \Phi^i) \\
& \quad + \lambda_\mu \Phi_{0i}^\dagger S^\dagger \Phi^i S + \cancel{\lambda_{\mu 1}' \Phi_{0i}^\dagger S\, i\sigma_2 \Phi^i S} + \cancel{\lambda_{\mu 2}' S\, i\sigma_2 \Phi_i^\dagger S\, \Phi_0^i} + \mathrm{H.c.} \Big\}
\end{aligned}
$$

- Classification of Scalar fields (Assume CP inv. Scalar sector for simplicity)

Original field	Mass eigenstates	Electric charge	CP	Lepton Flavor
(S_0^h, h_3, h_0)	(s, H, h)	0	even	conserving
(S_0^z, z_3, z_0)	$((X_3)_L, Z_L, A)$	0	odd	conserving
(ω_3^+, ω_0^+)	$((W^+)_L, H^+)$	+1	—	conserving
$(S_+, \phi_+^0, \phi_-^{0\star})$	$((X_+)_L, H_+, h_+)$	0	—	changing
(ϕ_+^+, ϕ_-^+)	(ϕ_+^+, ϕ_-^+)	+1	—	changing

Flavor Changing Scalar

- New interactions

$$\tan\beta \equiv \frac{v_0}{v_3} \qquad \tan\beta' \equiv \frac{2v_3}{v_S} \ll 1$$

$$\frac{M_\mu - M_\tau}{\sqrt{2}\, v c_\beta}\left\{ s_{\beta'}\, \overline{\mu}\, \tau\, \underbrace{S_+}_{\text{NG boson}} + \overline{\mu}(c_{\alpha'} c_{\beta'} - s_{\alpha'}\gamma_5)\tau H_+ - \overline{\mu}(s_{\alpha'} c_{\beta'} + c_{\alpha'}\gamma_5)\tau h_+ \right\}$$

Original field	Mass eigenstates	Electric charge	CP	Lepton Flavor
(S_0^h, h_3, h_0)	(s, H, h)	0	even	conserving
(S_0^z, z_3, z_0)	$((X_3)_L, Z_L, A)$	0	odd	conserving
(ω_3^+, ω_0^+) α'	$((W^+)_L, H^+)$	$+1$	—	conserving
$(S_+, \phi_+^0, \phi_-^{0\star})$	NG $((X_+)_L, H_+, h_+)$	0	—	changing
(ϕ_+^+, ϕ_-^+)	(ϕ_+^+, ϕ_-^+)	$+1$	—	changing

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- Muon g-2

$$R = M_{h_\pm}^2 / M_{H_\pm}^2$$

Chirality enhancement

$$\Delta a_{\mu\pm} \simeq \frac{1}{48\pi^2}\frac{M_\mu^2}{M_{h_\pm}^2}\left(\frac{M_\mu - M_\tau}{v}\sqrt{1+t_\beta^2}\right)^2 \left\{1 + R + 3\frac{M_\tau}{M_\mu}\, \underline{c_{2\alpha'}} \left[(1-R)\left(\ln\frac{M_\mu^2}{M_{h_\pm}^2} + \frac{3}{2}\right) - R\ln R\right]\right\}$$

Yukawa w/ tanβ enhancement

Mixing angle dependence

Original field	Mass eigenstates	Electric charge	CP	Lepton Flavor
(S_0^h, h_3, h_0)	(s, H, h)	0	even	conserving
(S_0^z, z_3, z_0)	$((X_3)_L, Z_L, A)$	0	odd	conserving
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$(S_+, \phi_+^0, \phi_-^{0\star})$	$((X_+)_L, H_+, h_+)$ NG	0	—	changing
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Flavor Changing Scalar

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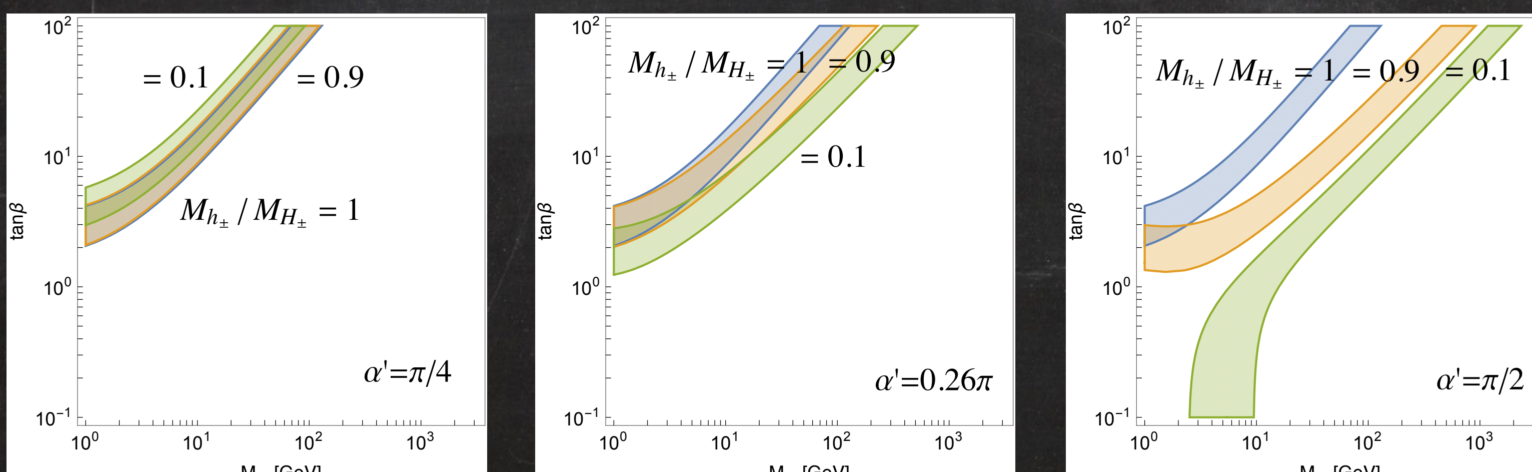
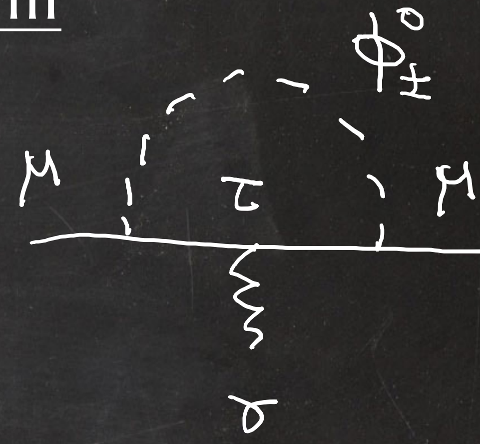


Comparison w/ 2HDM-III

2SDM ... Flavor charged



$$\Delta a_\mu \simeq \frac{1}{(4\pi)^2}\frac{m_\mu^2}{m_\phi^2}\times\underbrace{\left(\frac{m_\tau}{v}t_\beta\right)^2}_{\text{coupling}}\times\underbrace{\left(\frac{m_\tau}{m_\mu}\right)}_{\text{Chirality Enhancement}}$$

2HDM-III (Omura, Senaha, Tobe)

$$\Delta a_\mu \sim \frac{1}{(4\pi)^2}\frac{m_\mu^2}{m_\phi^2}\times\underbrace{(\rho_{\mu\tau}\rho^*_{\tau\mu})}\times\left(\frac{m_\tau}{m_\mu}\right)$$

$\rho_{\mu\tau}\neq\rho_{\tau\mu}$ (2 independent complex parameters)

$\Updownarrow$

EDM constraint

$\propto\left(\frac{s^2_{\beta-\alpha}}{m_H^2}-\frac{1}{m_A^2}\right)$ relate to h125 phys.

$$\tan\beta \equiv \frac{v_0}{v_3}$$

Flavor Conserving Scalar

- New interactions

$$-\left(\overline{\mu_L}\left\{\frac{M_\mu}{v}\frac{1}{t_{2\beta}}+\frac{M_\tau}{v}\frac{1}{s_{2\beta}}\right\}\mu_R+\overline{\tau_L}\left\{\frac{M_\tau}{v}\frac{1}{t_{2\beta}}+\frac{M_\mu}{v}\frac{1}{s_{2\beta}}\right\}\tau_R\right)(H+i\,A)$$

Set to 0 (simplicity)

SM-like limit sin(β-α)=1 (simplicity)

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(ϕ_+^+, ϕ_-^+)	(ϕ_+^+, ϕ_-^+)	+1	—	changing

$$\tan\beta \equiv \frac{v_0}{v_3}$$

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- New interactions

$$-\left(\overline{\mu_L}\left\{\frac{M_\mu}{v}\frac{1}{t_{2\beta}} + \frac{M_\tau}{v}\frac{1}{s_{2\beta}}\right\}\mu_R + \overline{\tau_L}\left\{\frac{M_\tau}{v}\frac{1}{t_{2\beta}} + \frac{M_\mu}{v}\frac{1}{s_{2\beta}}\right\}\tau_R\right)(H + i\,A)$$

- Muon g-2

No Chirality enhancement

$$\Delta a_{\mu 0} \simeq \frac{1}{12\pi^2}\frac{M_\mu^2}{M_\varphi^2}\left(\frac{M_\mu}{v}\frac{1}{t_{2\beta}} + \frac{M_\tau}{v}\frac{1}{s_{2\beta}}\right)^2, \qquad (M_\varphi \equiv M_H = M_A)$$

$$1/t_{2\beta} = (1 - t_\beta^2)/2t_\beta$$
$$1/s_{2\beta} = (1 + t_\beta^2)/2t_\beta$$

Yukawa w/ tanβ enhancement

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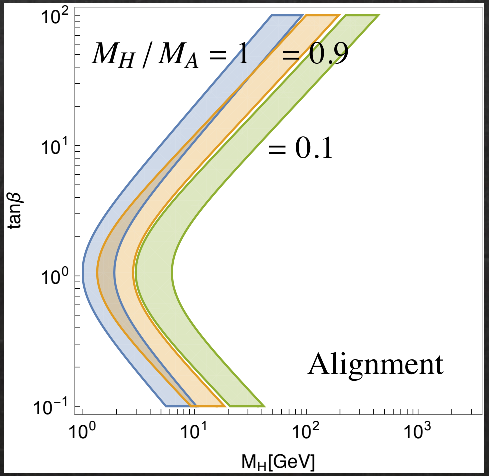
Yukawa w/ tanβ enhancement



$$\tan\beta \equiv \frac{v_0}{v_3}$$

Flavor Conserving Scalar

- New interactions

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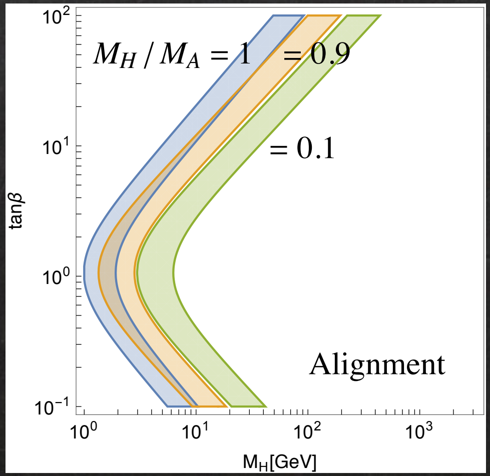
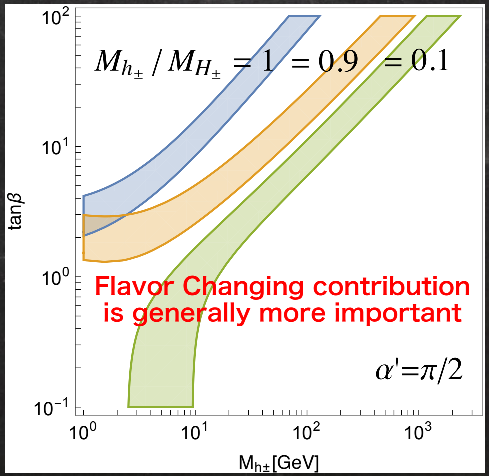
Yukawa w/ tanβ enhancement

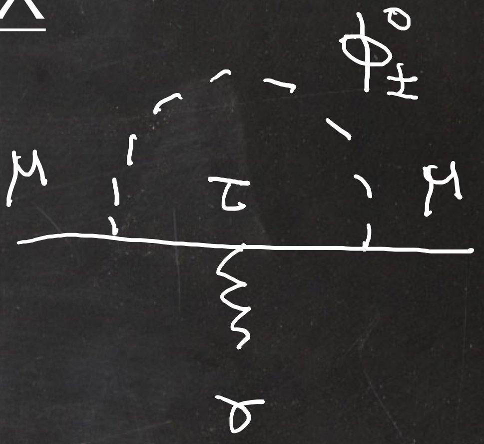
$$M \sim M_\varphi \sim M_{h_\pm}$$





Comparison w/ 2HDM-X

2SDM … Flavor charged



$$\Delta a_\mu \simeq \frac{1}{(4\pi)^2}\frac{m_\mu^2}{m_\phi^2}\times\underbrace{\left(\frac{m_\tau}{v}t_\beta\right)^2}_{\text{coupling}}\times\underbrace{\left(\frac{m_\tau}{m_\mu}\right)}_{\text{Chirality Enhancement}}$$

2HDM-X (Abe, Sato, Yagyu)

$$\Delta a_\mu \sim \frac{1}{(4\pi)^2}\frac{m_\mu^2}{m_\phi^2}\times\left(\frac{m_\mu}{v}t_\beta\right)^2\times 1$$

light ϕ (= A) $\sim$ 50 GeV

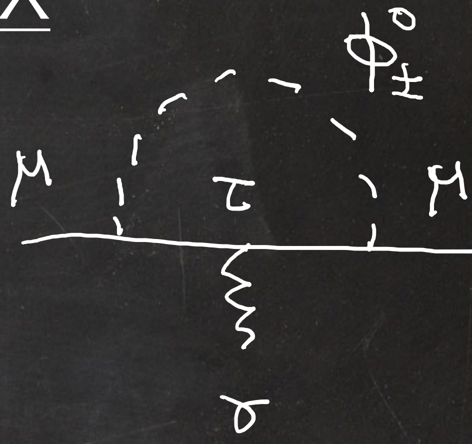
moderate t_β $\sim 10^2$

$$\left[\frac{m_\tau}{v}t_\beta \lesssim 4\pi\right]$$

Comparison w/ 2HDM-X

2SDM … Flavor charged

$$\Delta a_\mu \simeq \frac{1}{(4\pi)^2}\frac{m_\mu^2}{M_\phi^2} \times \underbrace{\left(\frac{m_\tau}{v} t_\beta\right)^2}_{\text{coupling}} \times \underbrace{\left(\frac{m_\tau}{m_\mu}\right)}_{\text{Chirality Enhancement}}$$



$$\times \left(\frac{m_\tau}{m_\mu}\right)^3 \sim 5{,}000$$

2HDM-X (Abe, Sato, Yagyu)

$$\Delta a_\mu \sim \frac{1}{(4\pi)^2}\frac{m_\mu^2}{M_\phi^2} \times \left(\frac{m_\mu}{v} t_\beta\right)^2 \times 1$$

light ϕ (= A) ~ 50 GeV

moderate t_β $\sim 10^2$

$$\left[\frac{m_\tau}{v} t_\beta \lesssim 4\pi\right]$$

Comparison w/ 2HDM-X

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μ-2HDM (Abe, Sato, Yagyu)

$$\Delta a_\mu \sim \frac{1}{(4\pi)^2}\frac{m_\mu^2}{m_\phi^2} \times \left(\frac{m_\mu}{v} t_\beta\right)^2 \times 1$$

heavy ϕ ~ 500 GeV

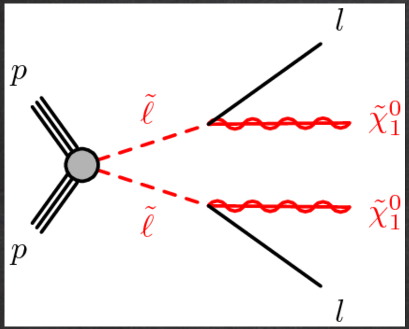
large t_β $\sim 10^3$

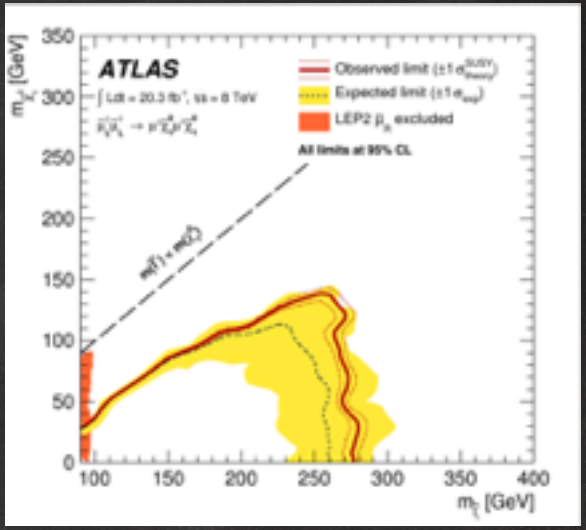
$$\left[\frac{m_\mu}{v} t_\beta \lesssim 4\pi\right]$$

Slepton Search

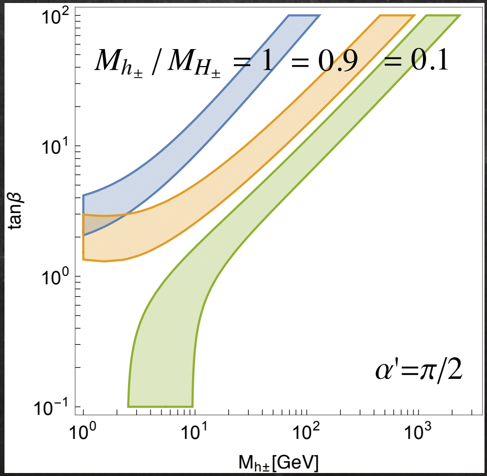
- Pair Production of Charged Higgs

H± has the same quantum charge with left-handed slepton





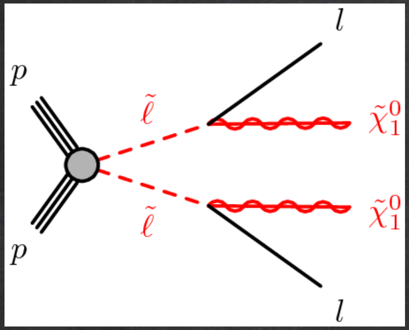
$$M \sim M_\varphi \sim M_{h_\pm}$$

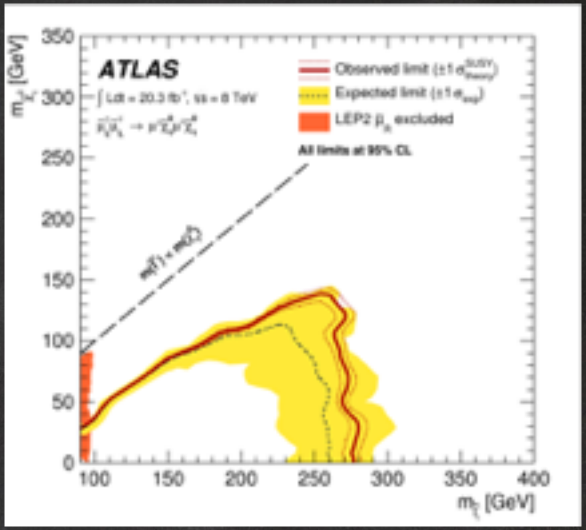


Slepton Search

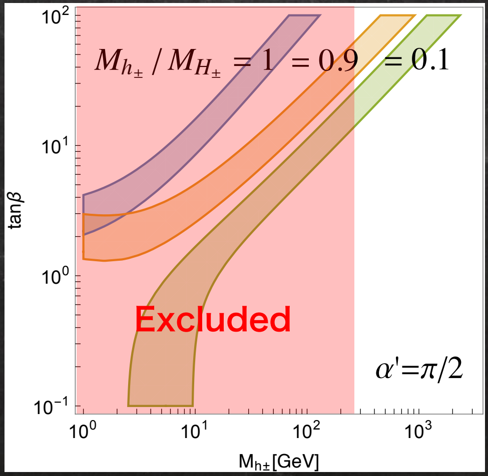
- Pair Production of Charged Higgs

$H^{\pm}$ has the same quantum charge with left-handed slepton





$$M \sim M_{\varphi} \sim M_{h_{\pm}}$$



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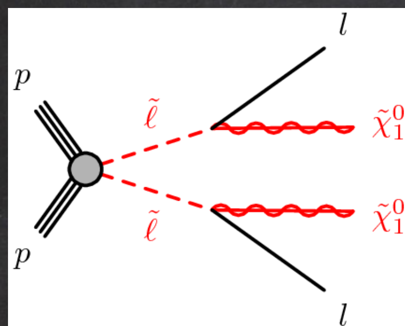

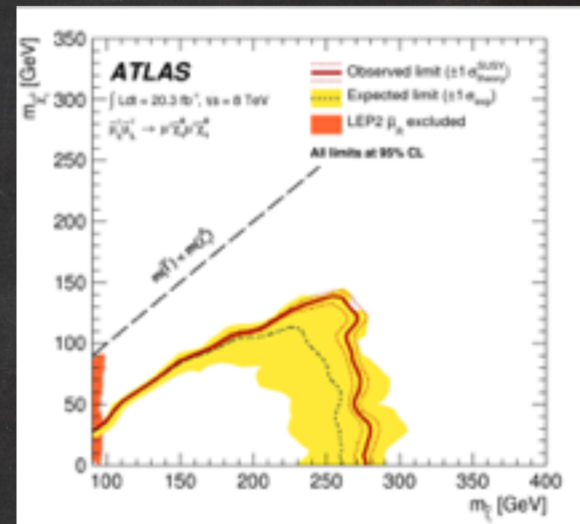

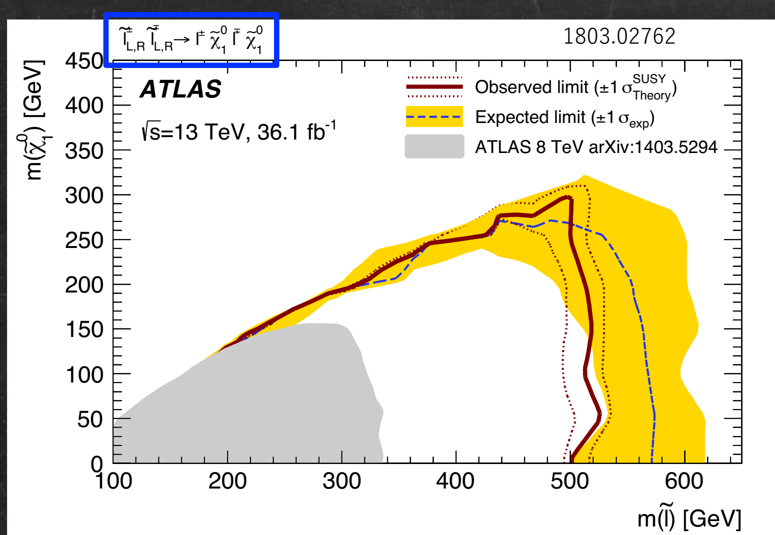

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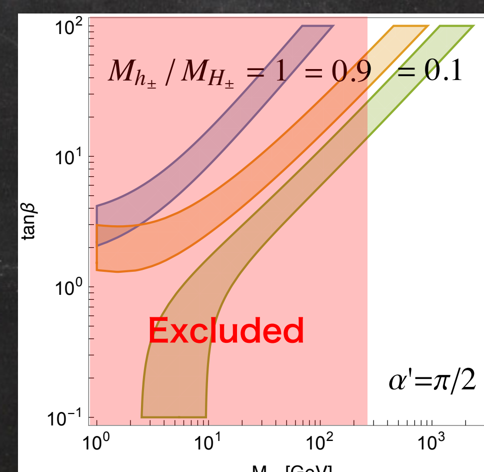
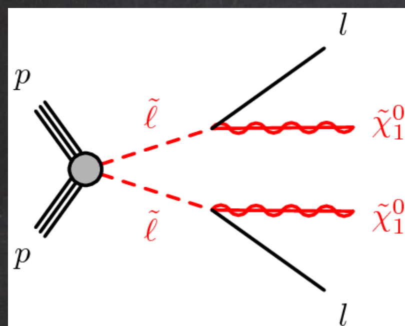
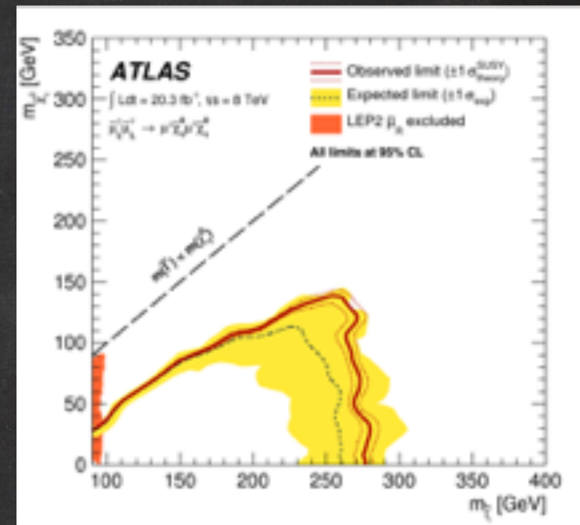


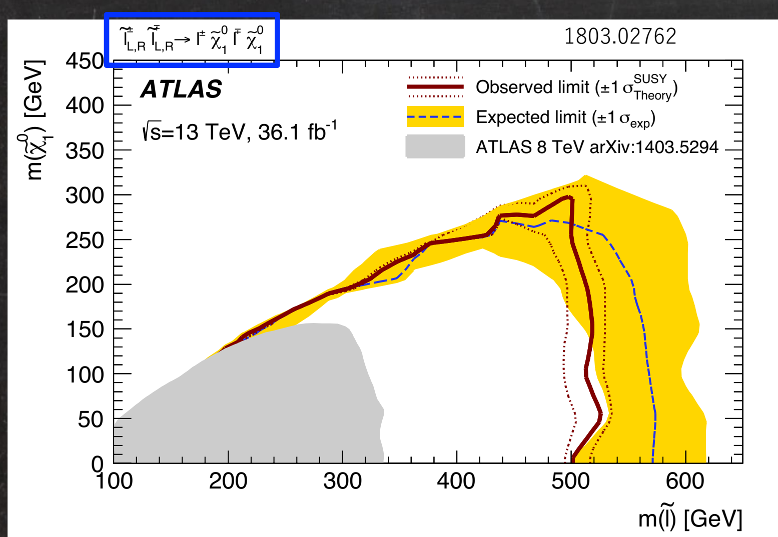
Slepton Search

- Pair Production of Charged Higgs

H± has the same quantum charge with left-handed slepton







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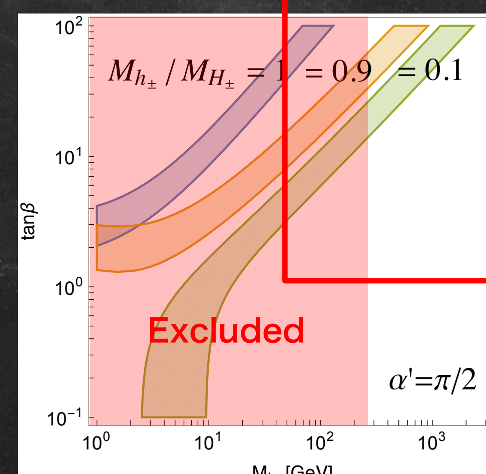


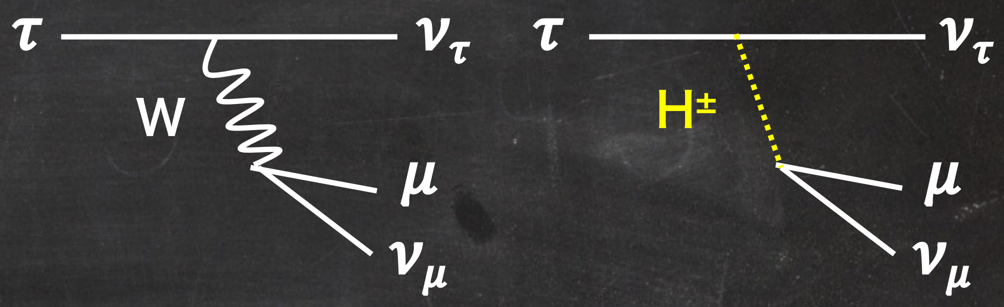
τ Michel Decay

- Charged Scalar interactions

$$+i\sqrt{2}\,\overline{\nu_{\mu L}}\Big\{\frac{M_\mu}{v}\frac{1}{t_{2\beta}}+\frac{M_\tau}{v}\frac{1}{s_{2\beta}}\Big\}\mu_R H^+ + i\sqrt{2}\,\overline{\nu_{\tau L}}\Big\{\frac{M_\tau}{v}\frac{1}{t_{2\beta}}+\frac{M_\mu}{v}\frac{1}{s_{2\beta}}\Big\}\tau_R H^+$$

- Lepton Universality

$$R_{\tau\to\mu/\tau\to e}=\frac{\Gamma(\tau\to\mu\overline{\nu_\mu}\nu_\tau)}{\Gamma(\tau\to e\overline{\nu_e}\nu_\tau)}$$



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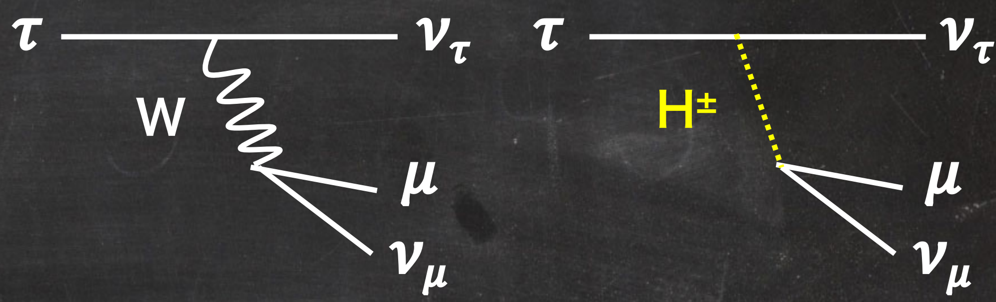
- Lepton Universality



$$R_{\tau\to\mu/\tau\to e} = \frac{\Gamma(\tau\to\mu\overline{\nu_\mu}\nu_\tau)}{\Gamma(\tau\to e\overline{\nu_e}\nu_\tau)}$$

$$\eta^{H^\pm} \equiv \frac{R^{H^\pm}_{\tau\to\mu/\tau\to e}}{R^{\rm SM}_{\tau\to\mu/\tau\to e}}$$

$$\approx \left\{1+\frac{1}{4}\left[-\frac{v^2}{M^2_{H^\pm}}\left(\frac{M_\mu}{v}\frac{1}{t_{2\beta}}+\frac{M_\tau}{v}\frac{1}{s_{2\beta}}\right)\left(\frac{M_\tau}{v}\frac{1}{t_{2\beta}}+\frac{M_\mu}{v}\frac{1}{s_{2\beta}}\right)\right]^2 \right.$$

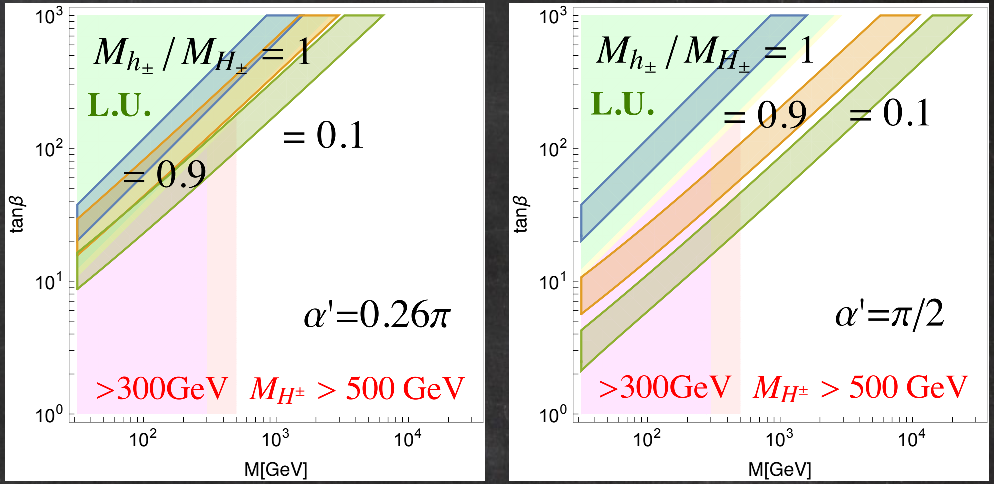
$$\left. +2\left[-\frac{v^2}{M^2_{H^\pm}}\left(\frac{M_\mu}{v}\frac{1}{t_{2\beta}}+\frac{M_\tau}{v}\frac{1}{s_{2\beta}}\right)\left(\frac{M_\tau}{v}\frac{1}{t_{2\beta}}+\frac{M_\mu}{v}\frac{1}{s_{2\beta}}\right)\right]\underline{\frac{M_\mu}{M_\tau}}\frac{g(\rho^{-2})}{f(\rho^{-2})}\right\}$$

Chirality Suppressed Interference

τ Michel Decay

- Charged Scalar interactions

$$M^2 \equiv M_H^2 = M_A^2 = M_{H^\pm}^2 = M_{h_\pm}^2$$



$$\eta^{H^\pm} \equiv \frac{R^{H^\pm}_{\tau\to\mu/\tau\to e}}{R^{\rm SM}_{\tau\to\mu/\tau\to e}}$$

$$\approx \left\{ 1 + \frac{1}{4}\left[-\frac{v^2}{M_{H^\pm}^2}\left(\frac{M_\mu}{v}\frac{1}{t_{2\beta}} + \frac{M_\tau}{v}\frac{1}{s_{2\beta}}\right)\left(\frac{M_\tau}{v}\frac{1}{t_{2\beta}} + \frac{M_\mu}{v}\frac{1}{s_{2\beta}}\right)\right]^2 \right.$$

$$\left. + 2\left[-\frac{v^2}{M_{H^\pm}^2}\left(\frac{M_\mu}{v}\frac{1}{t_{2\beta}} + \frac{M_\tau}{v}\frac{1}{s_{2\beta}}\right)\left(\frac{M_\tau}{v}\frac{1}{t_{2\beta}} + \frac{M_\mu}{v}\frac{1}{s_{2\beta}}\right)\right] \frac{M_\mu}{M_\tau}\frac{g(\rho^{-2})}{f(\rho^{-2})} \right\}$$

Chirality Suppressed Interference

Collider Signature

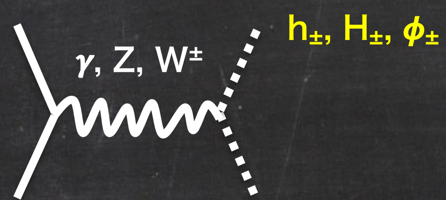
- Flavor conserving μ-τ Specific Higgs bosons

EW production

$$\mathcal{B}(H/A \to \tau\tau) = \mathcal{B}(H/A \to \mu\mu) = 50\%$$
$$\mathcal{B}(H^{\pm} \to \tau\nu) = \mathcal{B}(H^{\pm} \to \mu\nu) = 50\%$$

- Flavor changing Scalar bosons

EW production



$$\mathcal{B}(h_{\pm}/H_{\pm} \to \tau\mu) = 100\%$$
$$\mathcal{B}(\phi^{\pm}_{\pm} \to \tau\nu) = \mathcal{B}(\phi^{\pm}_{\pm} \to \mu\nu) = 50\%$$

Singlet SM Higgs

Original field	Mass eigenstates	Electric charge	CP	Lepton Flavor
(S_0^h, h_3, h_0)	(s, H, h)	0	even	conserving
(S_0^z, z_3, z_0)	$((\text{NG}\,X_3)_L, \text{NG}\,Z_L, A)$	0	odd	conserving
(ω_3^+, ω_0^+)	$((\text{NG}\,W^+)_L, H^+)$	+1	—	conserving
$(S_+, \phi_+^0, \phi_-^{0\star})$	$((\text{NG}\,X_+)_L, H_+, h_+)$	0	—	changing
(ϕ_+^+, ϕ_-^+)	(ϕ_+^+, ϕ_-^+)	+1	—	changing

Implication for Neutrinos

Witten Anomaly ?

- Odd Number of Weyl Fermions → Witten Anomaly

	SM leptons			
	$L = L_e$	$L^\alpha = \begin{bmatrix} L_\mu \\ L_\tau \end{bmatrix}$	e_R	${e_R}^\alpha = \begin{bmatrix} \mu_R \\ \tau_R \end{bmatrix}$
$SU(2)_{\mu\tau}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$
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$$Z = \int dA_\mu (\det i\not{D})^{1/2} \exp\left(-\frac{1}{2g^2} \int d^4x \mathrm{Tr} F_{\mu\nu}^2\right) \to 0$$

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➔ A simple solution : introduce a flavor doublet N_R

Neutrino Mass

- ν Dirac Yukawa $+\overline{L}\,\lambda_e\widetilde{\Phi_0}N_R+\overline{L}_\alpha(\lambda_+\widetilde{\Phi_0}\delta^\alpha{}_\beta+2\lambda_-\widetilde{\Phi}^\alpha{}_\beta)N_R^\beta$
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Scalars

Φ_0	$\Phi^\alpha{}_\beta$	S^α	$\Sigma^\alpha{}_\beta$
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ν mass and mixing are predicted in a specific parameter region

[The prediction is the same as in the $U(1)_{\mu-\tau}$ model]

Asai-Hamaguchi-Nagata (17)

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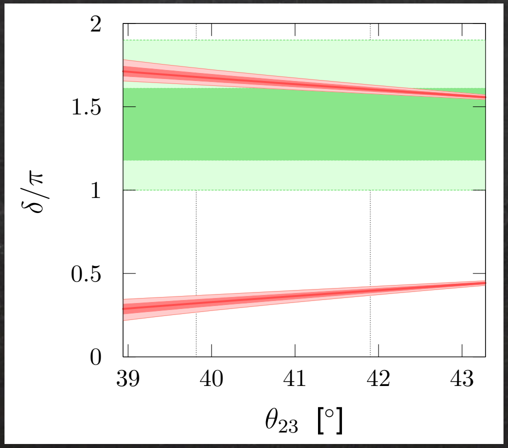
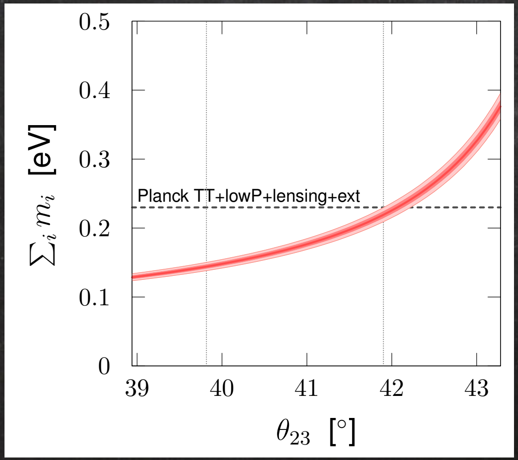
Strong correlation among ν params

Red : correlation

Green : favored by expt





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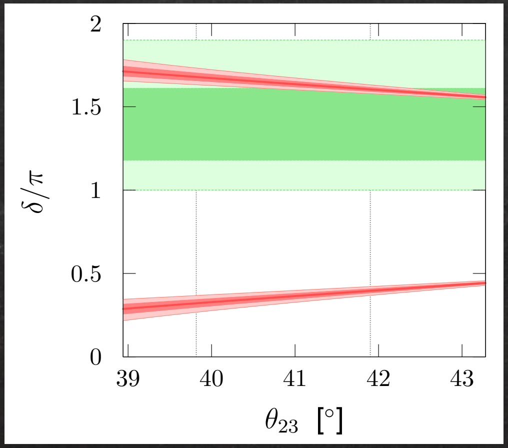

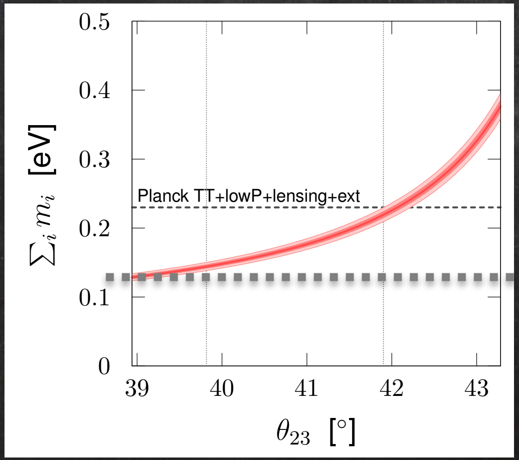

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A higher dimensional operator

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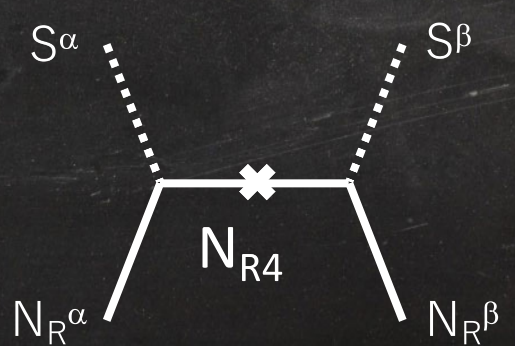

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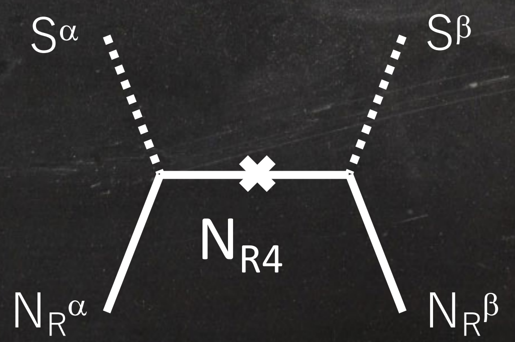

Summary

- Introduction
  - Muon g-2 anomaly
  - $U(1)_{\mu-\tau}$ Model
- $SU(2)_{\mu\tau}$ Model
  - X_3 and $X_\pm$
  - Higgs Sector
  - Michel Decay
  - Collider Signature
  - Neutrino Mass
- Conclusion